Ground states and excitations of spatially anisotropic quantum antiferromagnets

Oleg Starykh, University of Utah
Leon Balents, KITP
Masanori Kohno, NIMS, Tsukuba
Jason Alicea, Caltech/UC Irvine
Andrey Chubukov, U Wisconsin







Ground states and excitations of spatially anisotropic *triangular lattice* antiferromagnets

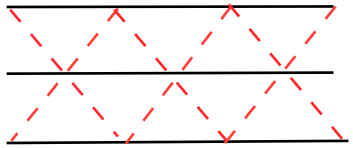
Oleg Starykh, University of Utah
Leon Balents, KITP
Masanori Kohno, NIMS, Tsukuba
Jason Alicea, Caltech/UC Irvine
Andrey Chubukov, U Wisconsin



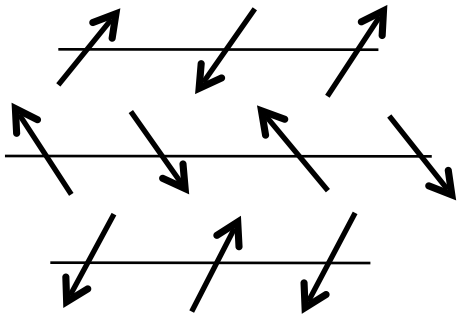




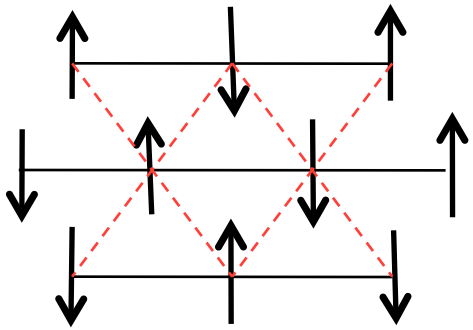
Main idea: exploit spatial anisotropy J'/J << 1

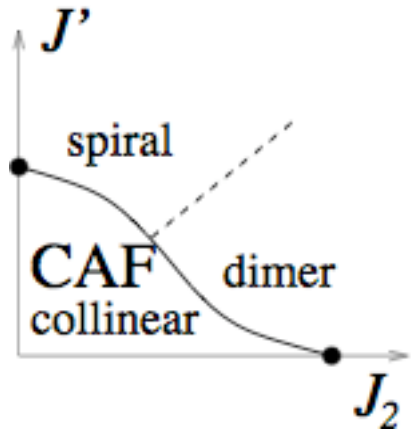


- Approach from the limit of decoupled chains
  (frustration helps -- it greatly enhances spatial anisotropy)
- Allow for ALL symmetry-allowed inter-chain interactions to develop
- Most relevant perturbations of decoupled chains drive spin order
- High energy excitations: not very sensitive to details of the ground state



J'/J





Decoupled chains J'=0

Coupled chains J'/J = 0.34

critical (infinitely degenerate) state

quantum collinear order

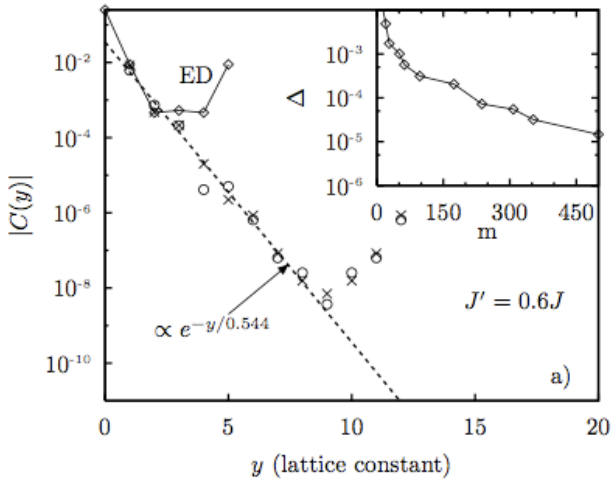
(OAS and Balents, PRL **98** 077205 (2007))

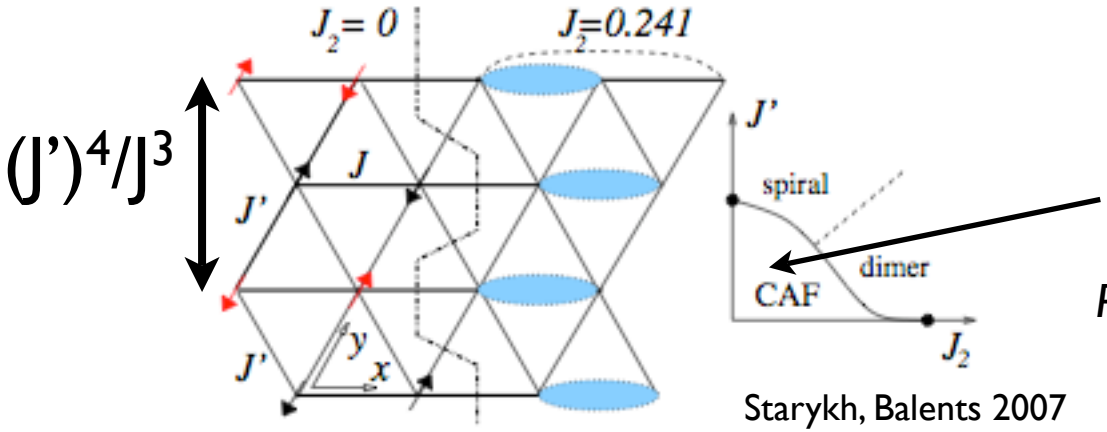
How small should J'/J be ?

- Frustration greatly enhances region of "small" interchain J'
- Example: *spatially anisotropic triangular* AFM
- Numerics: *no* interchain correlations for J'/J < 0.6 - 0.7

Weng et al 2006, Hayashi Ogata 2007, Heidarian Sorella Becca 2009

interchain spin correlations,
J'=0.6 J;
exponential decay
Weng et al 2006

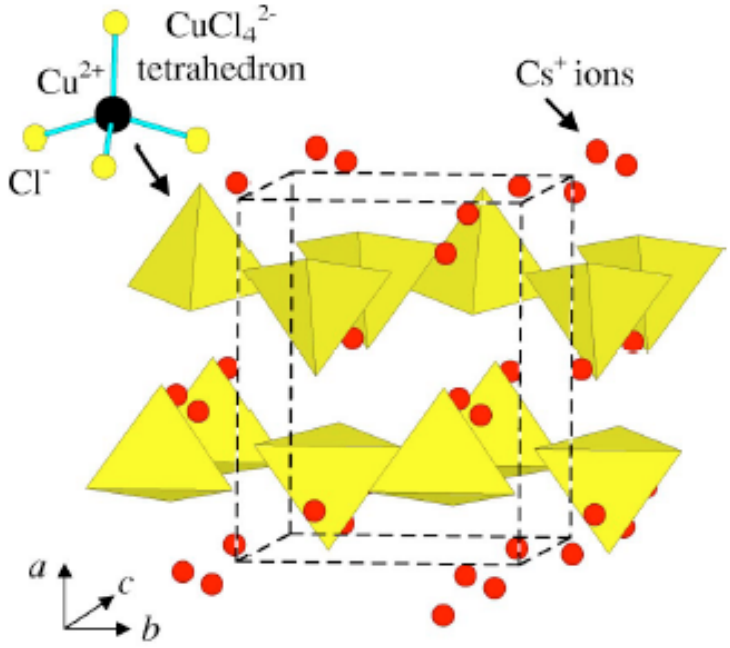


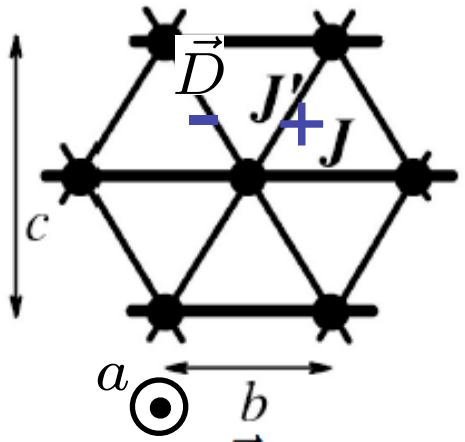


Starykh, Balents 2007

Collinear AFM state,
generated by quantum fluctuations,
coupling between NN chains $(J'/J)^4$
Pardini Singh 2008 - no; Bishop et al 2008 - yes

Cs_2CuCl_4: structure and parameters





J = 0.374 meV

J'/J = 0.34

D/J = 0.05

J''/J= 0.05

$$\vec{D} = (\pm D, 0, 0)$$

$$\mathcal{H} = \sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{(ij)} \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

moderate frustration f = 6

VOLUME 86, NUMBER 7 PHYSICAL REVIEW LETTERS 12 FEBRUARY 2001

Experimental Realization of a 2D Fractional Quantum Spin Liquid

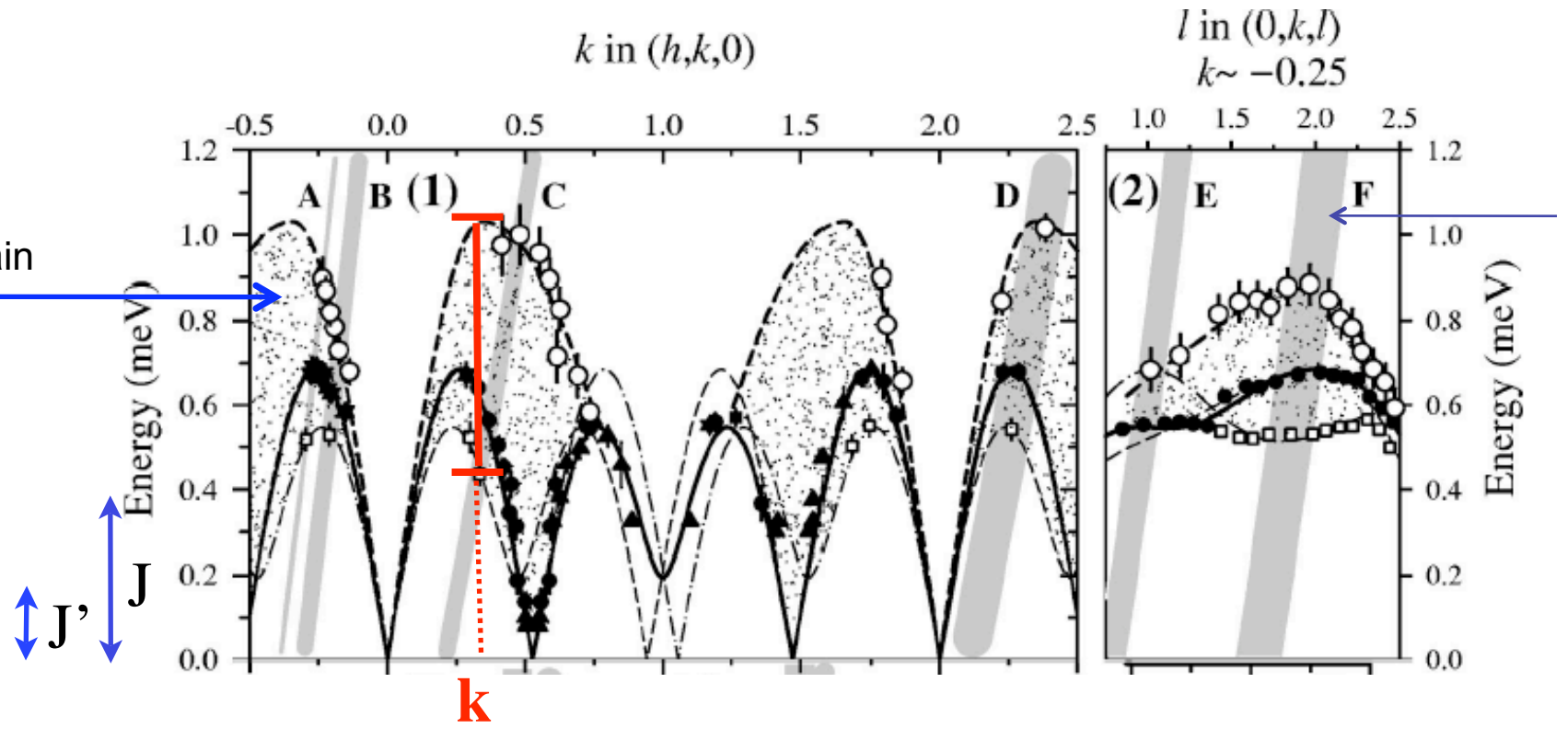
R. Coldea,[1,2] D. A. Tennant,[2,3] A. M. Tsvelik,[4] and Z. Tylczynski[5]

PHYSICAL REVIEW B 68, 134424 (2003)

Extended scattering continua characteristic of spin fractionalization in the two-dimensional frustrated quantum magnet Cs_2CuCl_4 observed by neutron scattering

R. Coldea,[1,2,3] D. A. Tennant,[1,3] and Z. Tylczynski[4]

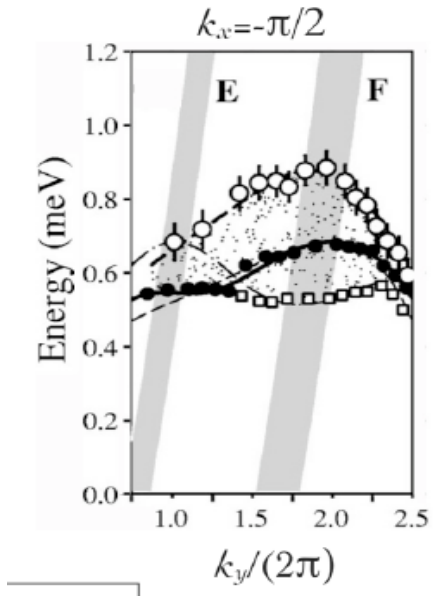
Along the chain



transverse to chain

Very unusual response: broad and strong continuum!

Numerous 2D theories



- Arguments for 2D:
  - J'/J = 0.3 not very small
  - Significant transverse dispersion

★ Exotic theories:

- J.Alicea, O.I.Motrunich & M.P.Fisher: Phys. Rev. Lett. **95**, 247203 (2005).
- S.V.Isakov, T.Senthil & Y.B.Kim: Phys. Rev. B **72**, 174417 (2005).
- Y.Zhou & X.-G.Wen: cond-mat/0210662.
- F.Wang & A.Vishwanath: Phys. Rev. B **74**, 174423 (2006).
- C.-H.Chung, K.Voelker & Y. B. Kim: Phys. Rev. B **68**, 094412 (2003).

★ **d=1:** Bocquet, Essler, Tsvelik, Gogolin, PRB 64, 094425 (2001)

★ Series expansion:

- W. Zheng, J. Fjaerestad, R. R. P. Singh, R. H. McKenzie, R. Coldea 2006.
- S. Yunoki and S. Sorella, 2006.
- M. Q. Weng, D. N. Sheng, Z. Y. Weng, R. J. Bursill, 2006

★ Spin waves:

- M.Y.Veillette, A.J.A.James & F.H.L.Essler: Phys. Rev. B **72**, 134429 (2005).
- D.Dalidovich, R.Sknepnek, A.J.Berlinsky, J.Zhang & C.Kallin: Phys. Rev. B **73**, 184403 (2006).
- R.Coldea, D.A.Tennant & Z.Tylczynski: Phys. Rev. B **68**, 134424 (2003).

Two-spinon continuum

Spinon energy $\omega(k_x) = \frac{\pi J}{2}|\sin(k_x)|$

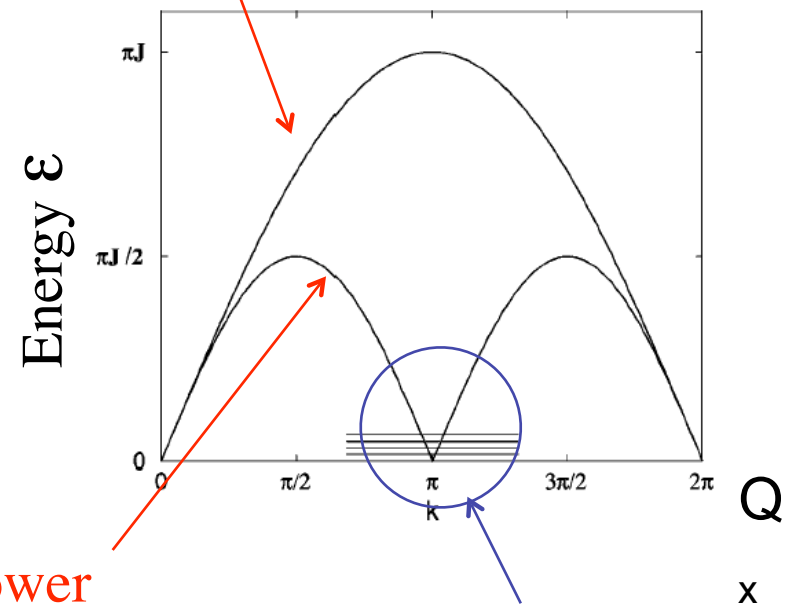
S=1 excitation $\begin{cases} \varepsilon = \omega(k_{x1}) + \omega(k_{x2}) \\ Q_x = k_{x1} + k_{x2} \end{cases}$

$$\varepsilon = \pi J \sin(\frac{Q_x}{2}) \cos(\frac{k_{x1} - k_{x2}}{2})$$

Variables:
k_{x1} and k_{x2}
or
ε and Q_x

$\omega_{2,\text{upper}} = \pi J \sin(Q_x/2)$
$k_{x1} = k_{x2} = Q_x/2$

Upper boundary



Lower boundary

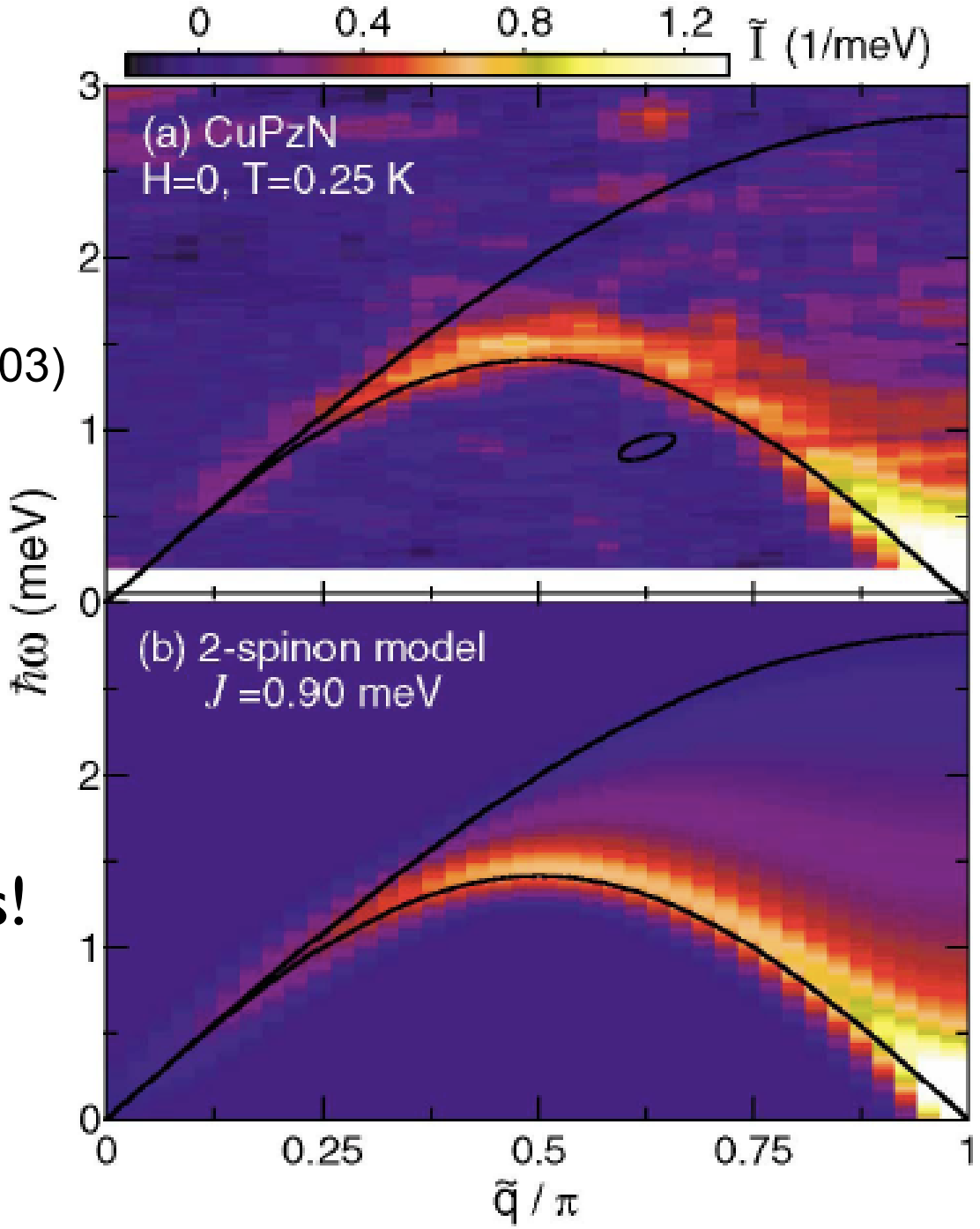
Low-energy sector $Q_{AFM}=\pi$

$\omega_{2,\text{lower}} = \frac{\pi J}{2}\sin(Q_x)$
$k_{x1} = 0, k_{x2} = Q_x$

Dynamic structure factor of copper pyrazine dinitirate (CuPzN)

Stone et al,
PRL 91, 037205 (2003)

No single particle peaks!



Effective Schrödinger equation in two-spinon basis

- Study two spinon subspace

$$|k_x, k_y; \epsilon\rangle = \sum_y e^{ik_y y} |k_x, \epsilon\rangle_y \otimes_{y' \neq y} |0\rangle_{y'}$$

(two spinons on chain **y** with S^z_{tot} =+1)

  - Momentum conservation: 1d Schrödinger equation in ε space (**k** = (k_x, k_y))

$$\epsilon \psi_{\mathbf{k}}(\epsilon) + \int d\tilde{\epsilon} D_{k_x}(\tilde{\epsilon})\, J'(\mathbf{k}) A^*_{k_x}(\epsilon) A_{k_x}(\tilde{\epsilon})\, \psi_{\mathbf{k}}(\tilde{\epsilon}) = E \psi_{\mathbf{k}}(\epsilon)$$

- Crucial matrix elements known exactly

$$A_{k_x}(\epsilon) \equiv \frac{1}{\sqrt{2}} \langle 0 | S^-_{-k_x, y} | k_x, \epsilon \rangle_y$$

Bougourzi *et al*, 1996

Structure Factor

- Spectral Representation

J.S. Caux *et al*, 2006

$$S(k,\omega) \propto \sum_n \left|\langle n|S_k^+|0\rangle\right|^2 \delta(\omega - E_n)$$

Weight in 1d:
73% in 2 spinon states
99% in 2+4 spinons

  - Can obtain closed-form "RPA-like" expression for 2d S(k,ω) in 2-spinon approximation

$$S(k,\omega) = \frac{S_{1d}(k,\omega)}{[1 + J'(k)\chi'_{1d}(k,\omega)]^2 + [\pi J'(k) S_{1d}(k,\omega)]^2}$$

$$S_{1d}(k,\omega) = |A_k(\omega)|^2 / \sqrt{\omega^2_{2,\text{upper}} - \omega^2} \qquad \chi'_{1d}(k,\omega) = P\int_0^\infty d\omega' \frac{S_{1d}(k,\omega')}{\omega' - \omega}$$

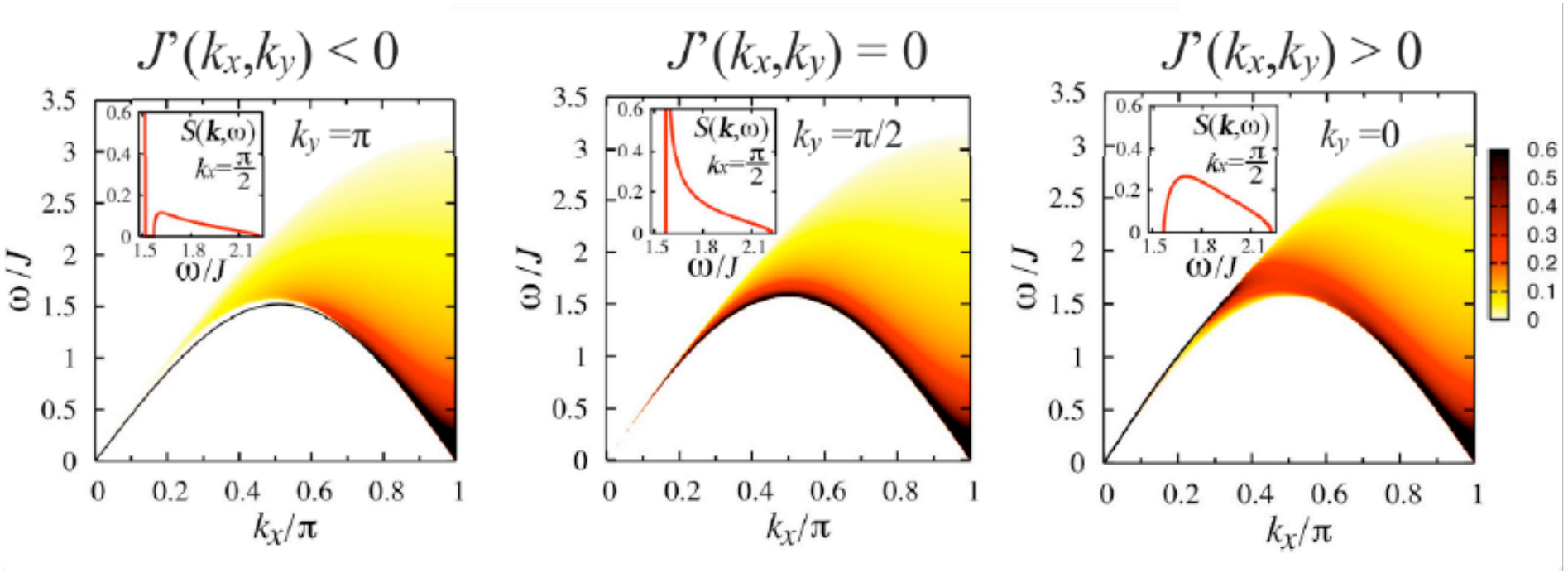
$$S_{1d}(k,\omega) = \frac{1}{2\pi} \frac{e^{-I(\rho(k,\omega))}}{\sqrt{\omega_U^2(k) - \omega^2}} \Theta[\omega_k^U - \omega]\Theta[\omega_k^L - \omega]$$

$$I(\rho) = \int_0^\infty dt \frac{e^t}{t} \frac{\cosh(2t)\cos(4\rho t) - 1}{\cosh t \sinh(2t)}$$

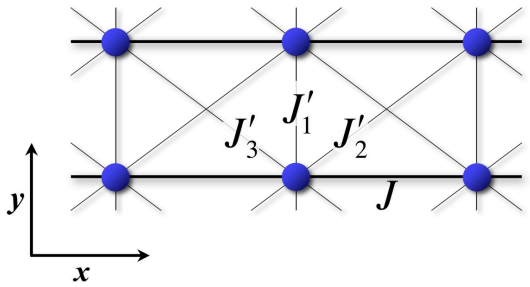
$$\rho(k,\omega) = \frac{4}{\pi} \text{acosh} \sqrt{\frac{\omega_U^2(k) - \omega_L^2(k)}{\omega^2 - \omega_L^2(k)}}$$

Types of behavior

- Behavior depends upon spinon interaction $J'(k_x,k_y)$



Bound "triplon"



Identical to 1D

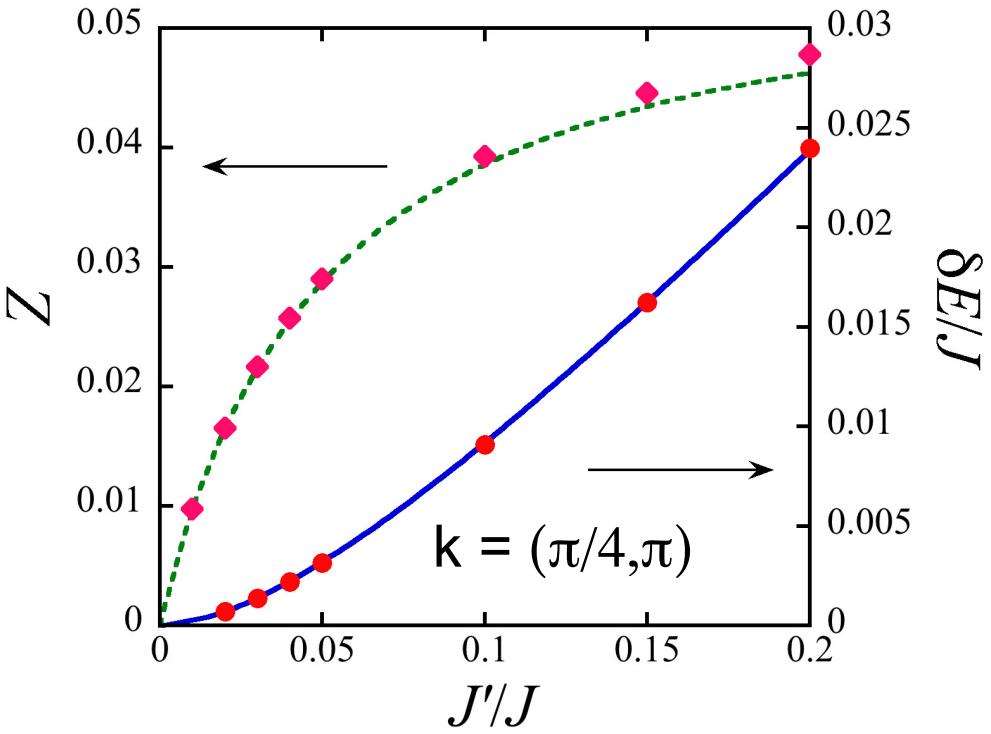
$J'(k) = 4J_1'\cos^2[k_x/2]\cos[k_y]$

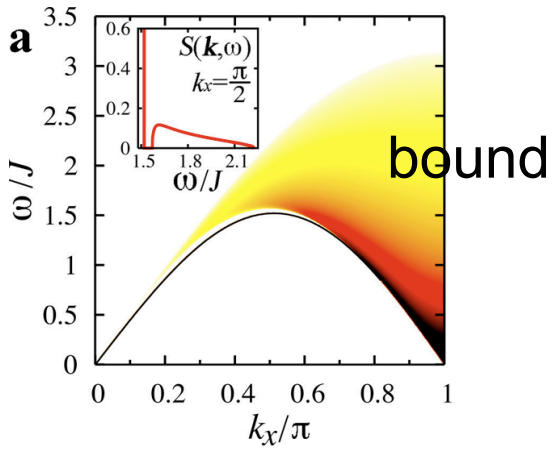
$J'_2 = J'_3 = J'_1/2$

Upward shift of spectral weight. Broad resonance in continuum or anti-bound state (small k)

Details of triplon dispersion

- Energy separation from the continuum $\delta E \sim [J'(k)]^2$
- Spectral weight in the triplon pole $Z \sim |J'(k)|$



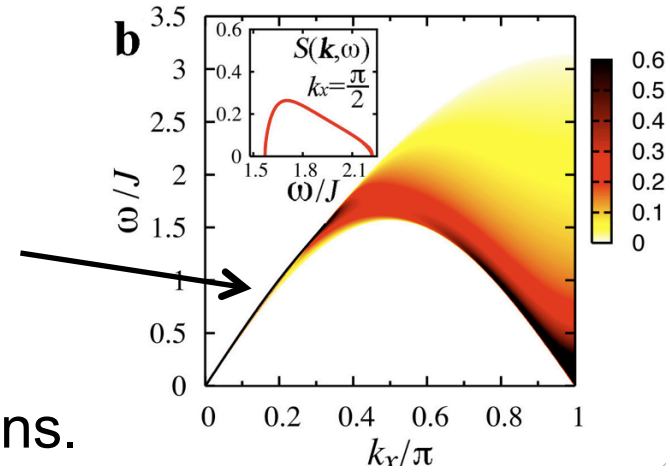


bound

- *Anti-bound* triplon when $J'(k) > J'_{\text{critical}}(k)$ and

$$1 + J'_{\text{critical}}(k)\chi'(k_x, \omega_{2,\text{upper}}) = 0$$

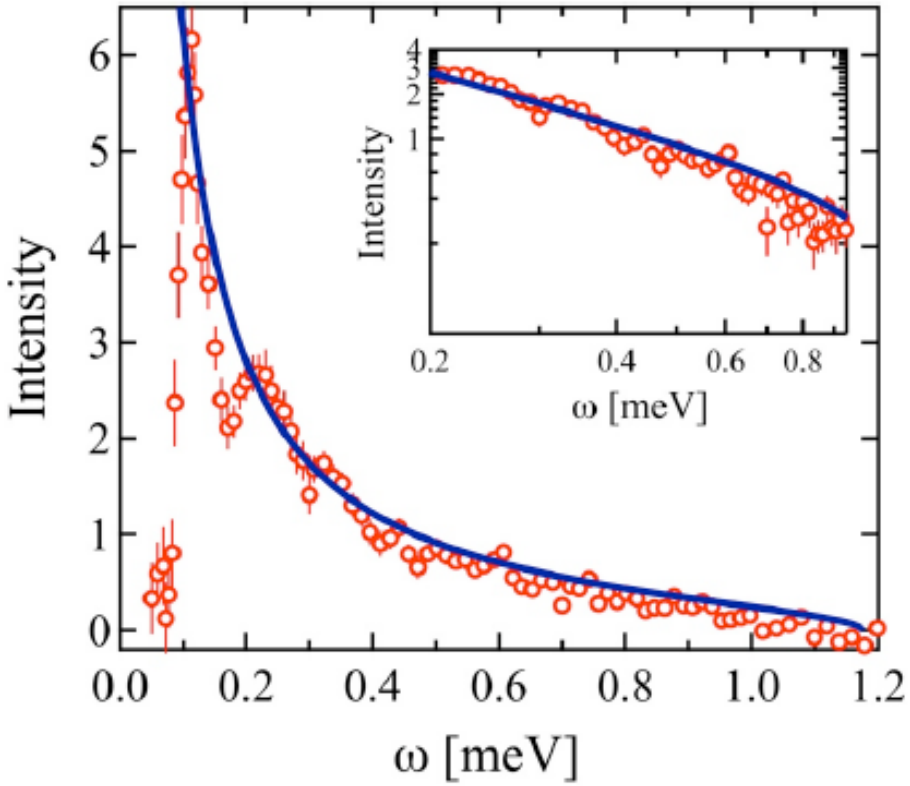
- Expect at small k~0 where continuum is narrow.
- Always of finite width due to 4-spinon contributions.



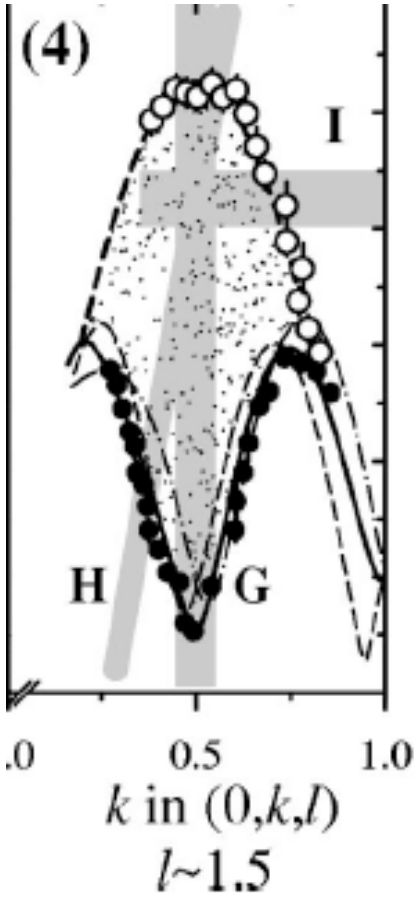
Cs_2CuCl_4 -- Broad lineshape: "free spinons"

- "Power law" fits well to free spinon result
  - Fit determines normalization

J'(k)=0 here !



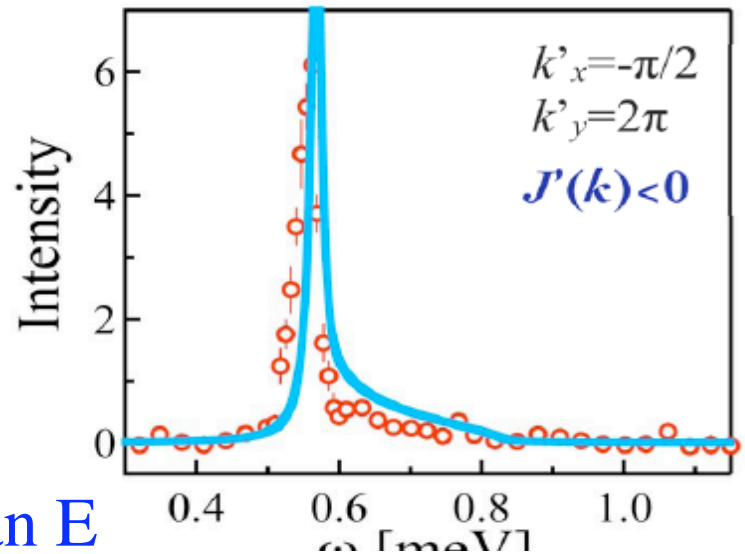
scan G



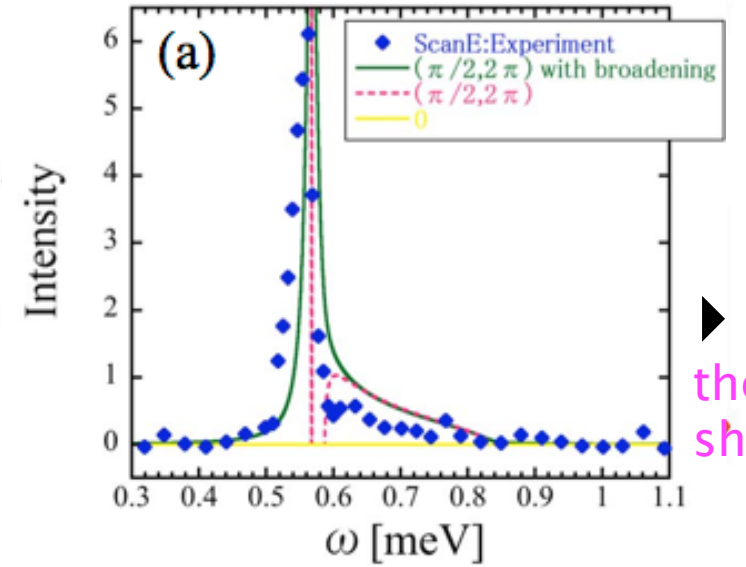
triangular geometry: $k_x' = k_x$, $k_y' = k_x + 2k_y$

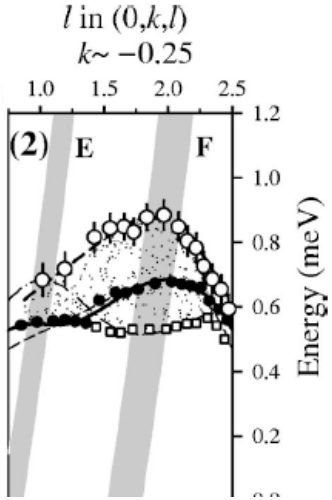
Triplon: S=1 bound state of two spinons

- Effect of finite experimental resolution:

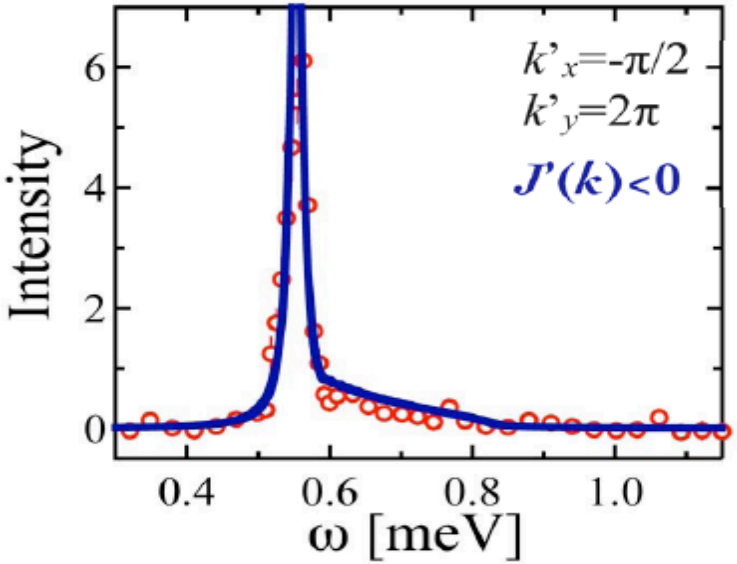


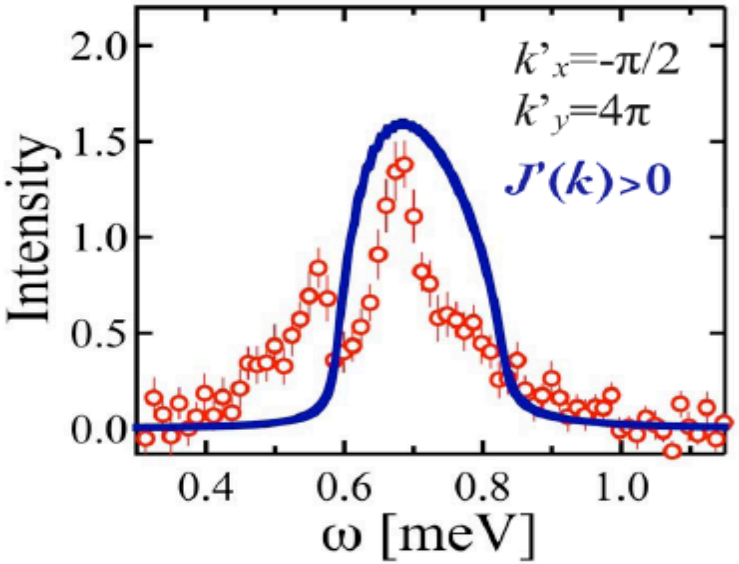
scan E





Dashed curve: theoretical line shape

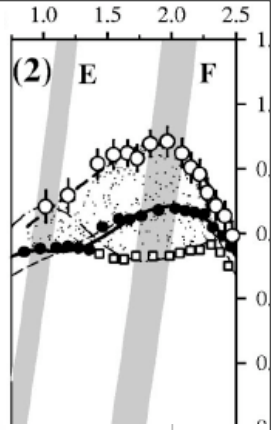


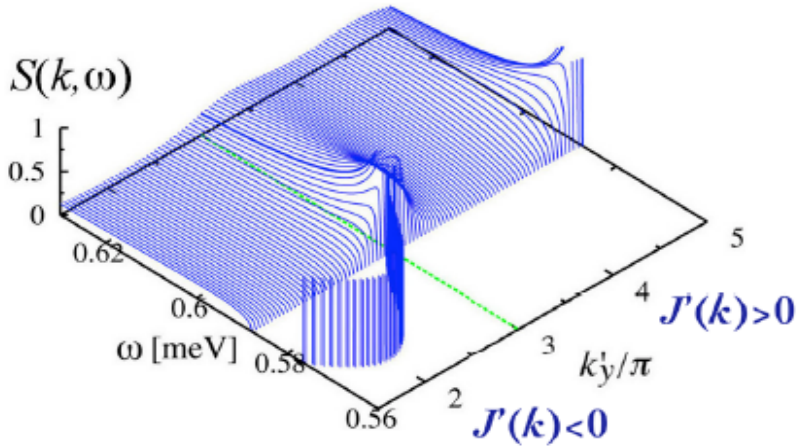


scan F

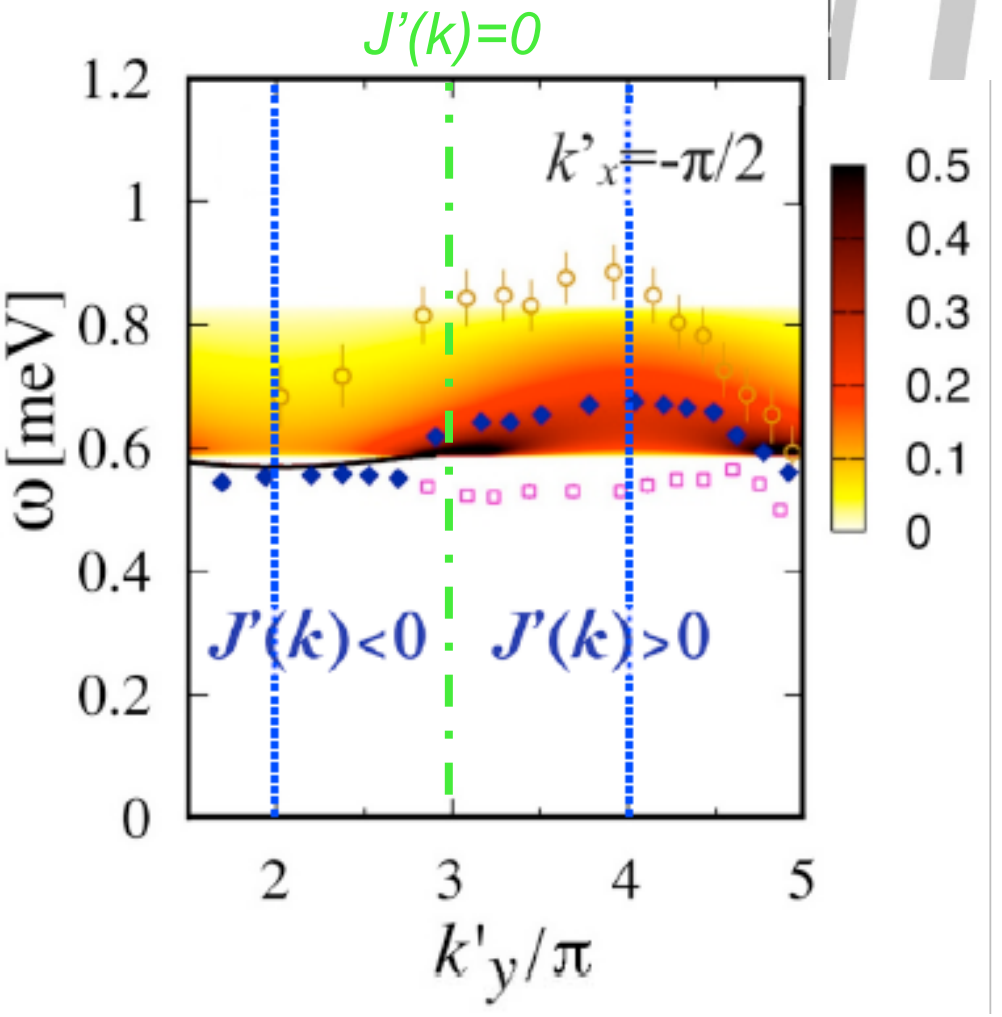
- Curves: 2-spinon RPA with experimental resolution
- Curves: 4-spinon RPA with experimental resolution

Transverse dispersion





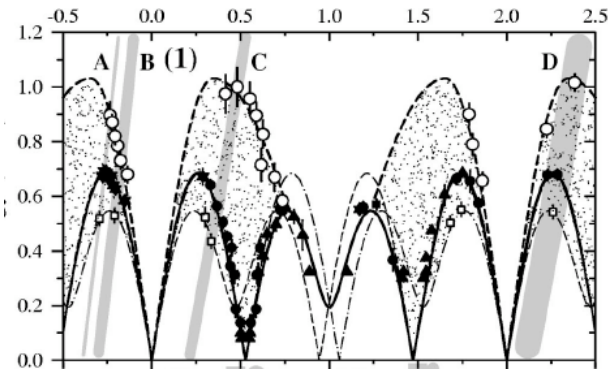
Bound state and resonance

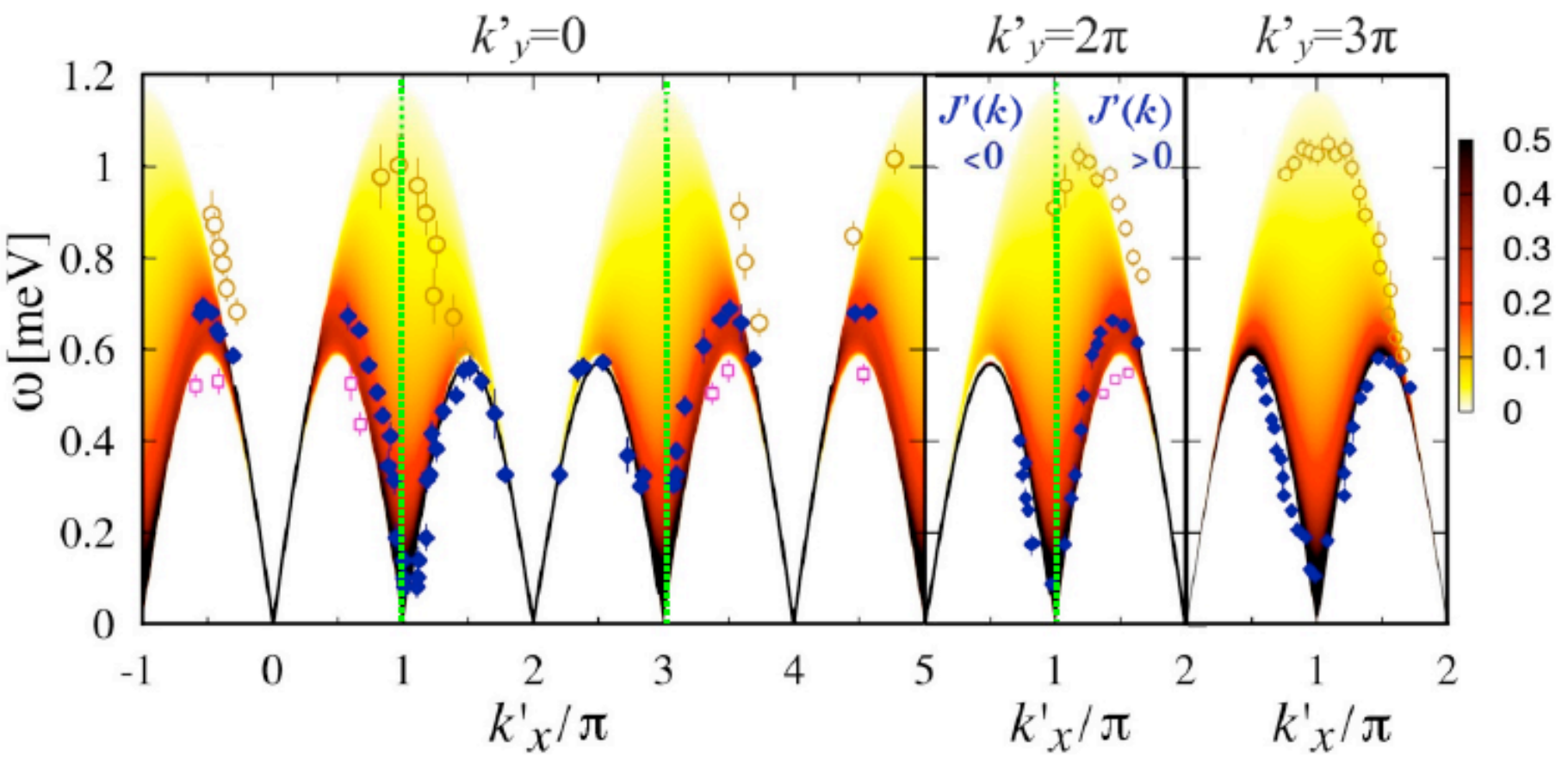


Solid symbols: experiment
Note that peak (blue diamonds) coincides with bottom edge only for J'(k)<0

Kohno, Starykh, Balents, *Nature Physics* 2007

Spectral asymmetry





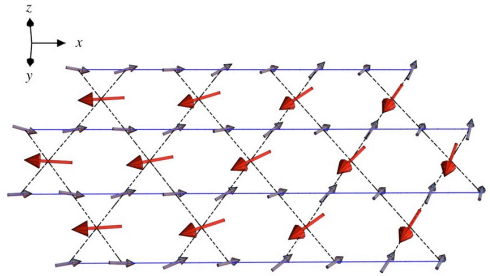
- Vertical green lines: $J'(k)=4J' \cos[k'_x/2] \cos[k'_y/2] = 0$.

Summary (I)

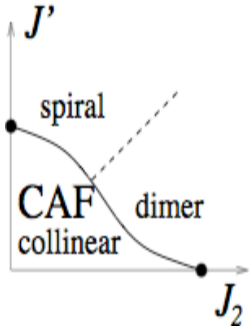
- ❖Mystery of Cs_2CuCl_4 is (almost) solved:
  - not a 2d spin liquid
  - Dynamic response: high-energy elementary excitations are descendants of 1d spinons
    - (kinematically) bind in S=1 pairs (triplons)
    - significant variation of spectral weight with two-dimensional momentum (k_x,k_y)
- ★Geometric frustration preserves 1d features, promotes multi-particle excitations

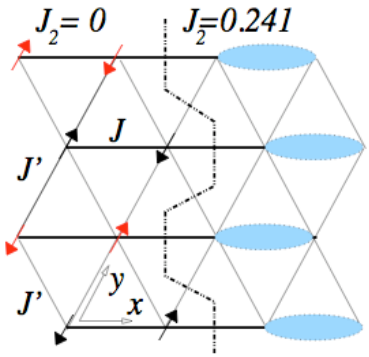
Low-energy properties

- Ordered ground state
  - understood as an instability of weakly coupled chains
  - need to follow renormalization of all (bare and dynamically generated) symmetry-permitted interactions



- Examples:
  - *dimerized* phases in quasi-1d J_1-J_2 and checkerboard models
  - *non-coplanar* in quasi-1d kagome
  - *collinear AF* (CoAF) in triangular antiferromagnet





- Here: ground states of Cs_2CuCl_4 and Cs_2CuBr_4 in magnetic field

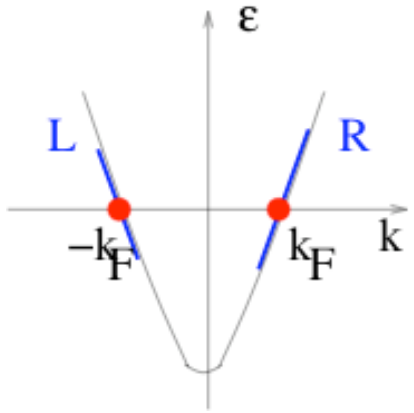
Heisenberg spin chain via free Dirac fermions

- Spin-1/2 AFM chain = half-filled (1 electron per site, $k_F=\pi/2a$) fermion chain



- Spin-charge separation

$$H_{\text{dirac}} = iv \int dx \sum_{s=\uparrow,\downarrow} (\Psi^+_{L,s}\partial_x \Psi_{L,s} - \Psi^+_{R,s}\partial_x \Psi_{R,s})$$



- *q=0 fluctuations: right- and left- spin currents*

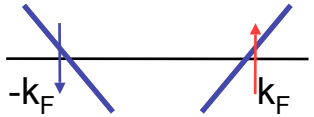
$$\vec{J}_R = \Psi^+_{Rs}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Rs'} \ , \ \vec{J}_L = \Psi^+_{Ls}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Ls'}$$

- *$2k_F$ (= π/a) fluctuations: charge density wave ε , spin density wave N*

Staggered Magnetization N

$$\begin{cases} N^+ \sim \Psi^+_{R\uparrow}\Psi_{L\downarrow} + \text{h.c.} \\ N^z \sim \Psi^+_{R\uparrow}\Psi_{L\uparrow} - \Psi^+_{R\downarrow}\Psi_{L\downarrow} + \text{h.c.} \end{cases}$$

Spin flip ΔS=1



Susceptibility

1/q $\quad \chi_{1d}(q)$

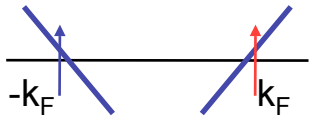
1/q

Staggered Dimerization

$\varepsilon = (-1)^x S_x S_{x+a}$

$$\varepsilon \sim i\left(\Psi^\dagger_{R\uparrow}\Psi_{L\uparrow} + \Psi^\dagger_{R\downarrow}\Psi_{L\downarrow} - \text{h.c.}\right)$$

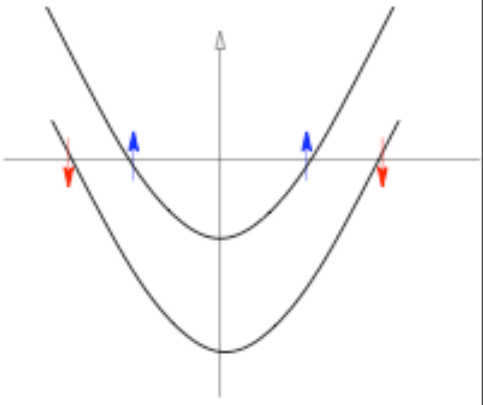
ΔS=0



1/q

- Must be careful: **often** spin-charge separation must be enforced by hand

S=1/2 AFM Chain in a Field



$$\mathcal{H} = J\sum_x \vec{S}(x)\cdot\vec{S}(x+1) - h\sum_x S^z(x)$$

- Field-split Fermi momenta: $k_{F\uparrow} = \frac{\pi}{2} - \delta\ ,\ k_{F\downarrow} = \frac{\pi}{2} + \delta$
  - ✓ Uniform magnetization $M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$
  - ✓ Half-filled condition $n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$
- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$ scaling dimension $1/4\pi R^2$ **increases**

$S^z_{\pi\pm 2\delta}$

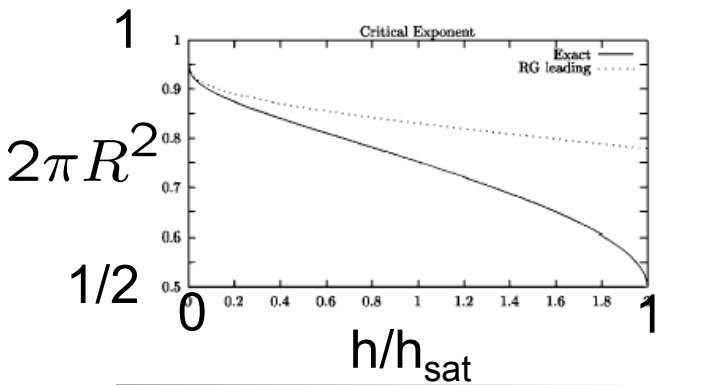
- $S^{x,y}$ components ($\Delta S=1$) remain at π scaling dimension πR^2 **decreases**

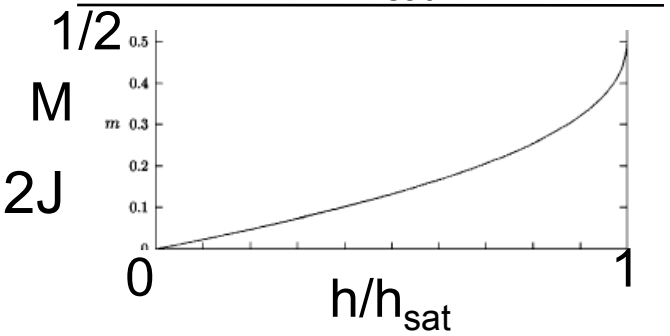
$S^{\pm}_{\pi}$

- Derived for free electrons but correct always - *Luttinger Theorem*

1

Affleck and Oshikawa, 1999





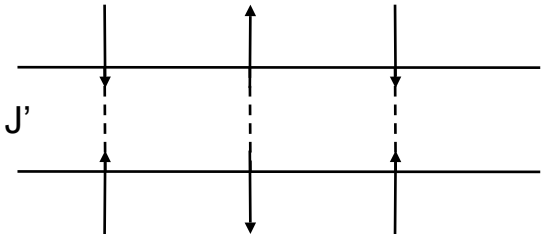
- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

Weakly coupled Heisenberg chains in magnetic field

- non - frustrated inter-chain coupling $\vec{S}_r \cdot \vec{S}_{r'} \rightarrow N_r^x N_{r'}^x + N_r^y N_{r'}^y + N_r^z N_{r'}^z$

most relevant

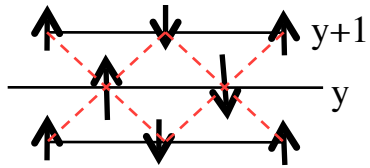
less relevant



$2\pi R^2 < 1/(2\pi R^2)$

spins order in the plane perpendicular to the direction of magnetic field (z): umbrella / cone / spin-flop states

- frustrated inter-chain coupling



$\vec{S}_{x,y} \cdot (\vec{S}_{x,y+1} + \vec{S}_{x+1,y+1}) \rightarrow \underbrace{N_y^x \partial_x N_{y+1}^x + N_y^y \partial_x N_{y+1}^y}_{\text{less relevant}} + \sin(\delta) S_{\pi-2\delta}^z(y) S_{\pi+2\delta}^z(y+1)$

most relevant (small to intermediate fields)

$1+2\pi R^2 > 1/(2\pi R^2)$

★ frustration promotes collinear SDW order

Real material: *relevant* inter-chain DM interaction, B||D

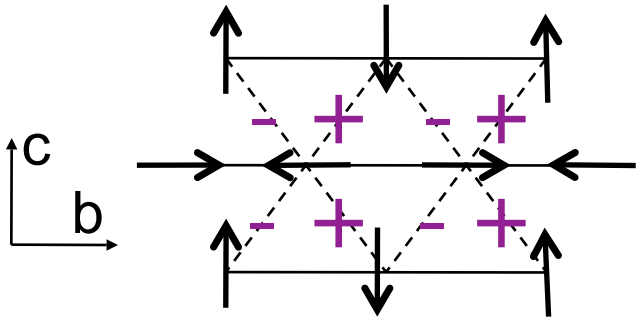
- Even though D=0.05 : DM beats CoAF and dimerization zero-field instabilities

$$\mathcal{H}_{DM} = D\sum_{y}(-1)^{y}\hat{z}\cdot\vec{N}(y)\times\vec{N}(y+1)$$

[y = chain index]

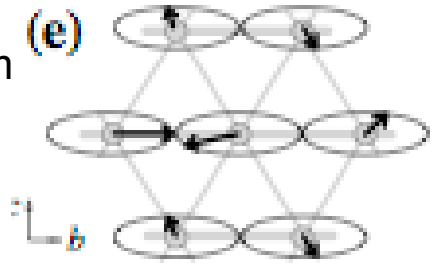
relevant: dim = $2\pi R^2$

- DM allows *relevant* coupling of N^x and N^y on neighboring chains
  - immediately stabilizes spiral state
    - *orthogonal spins* on neighboring chains



Finite D, but J'=0

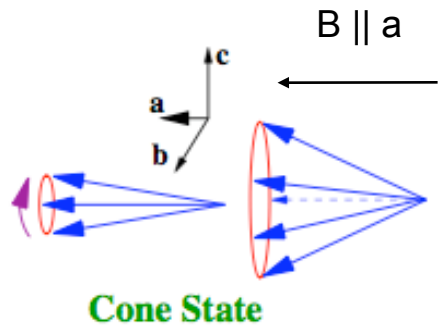
  - small J' *perturbatively* makes spiral weakly incomm



Finite D and J'

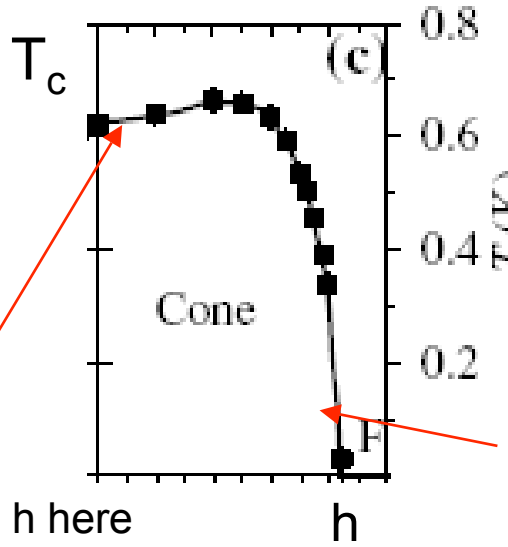
Dzyaloshinskii-Moriya interaction (DM) controls phase diagram of Cs_2CuCl_4 for B || D (B along crystal **a** axis)

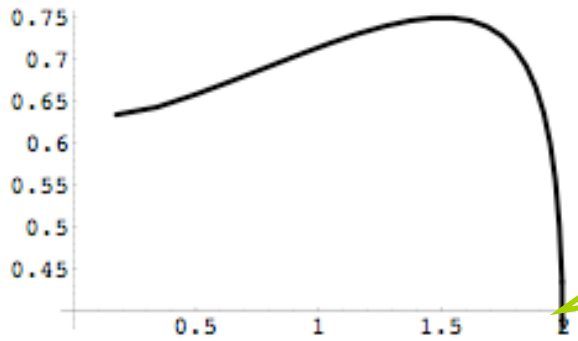
Transverse Field: B || D

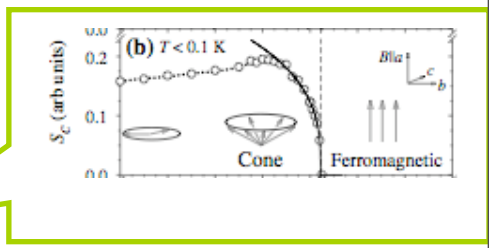


- DM term becomes *more relevant*
- *b-c* spin components (XY) remain commensurate: spin simply tilt in the direction of the field
- Spiral (cone) state just persists for all fields.

Experiment *vs* Theory







Order *increases* with h here due to increasing relevance of DM term

Order *decreases* with h here due to vanishing amplitude as h_{sat} is approached (density of magnons -> 0)

BEC

Coldea et al 2002;
Radu et al 2005;
Veillette et al 2006;
Kovrizhin et al 2006

physics is controlled by weak (~1/20 of J) Dzyaloshinskii-Moriya interaction

Longitudinal Field: B ⊥ D removes DM

- DM term involves S^z (at $\pi - 2\delta$) and S^x (at π):
  - ✓ Leads to momentum mis-match for h>0: DM "irrelevant" for **h > D**
- With DM killed, sub-dominant instabilities take hold
- Two important couplings for h>0: $k_{F\downarrow} - k_{F\uparrow} = 2\delta = 2\pi M$

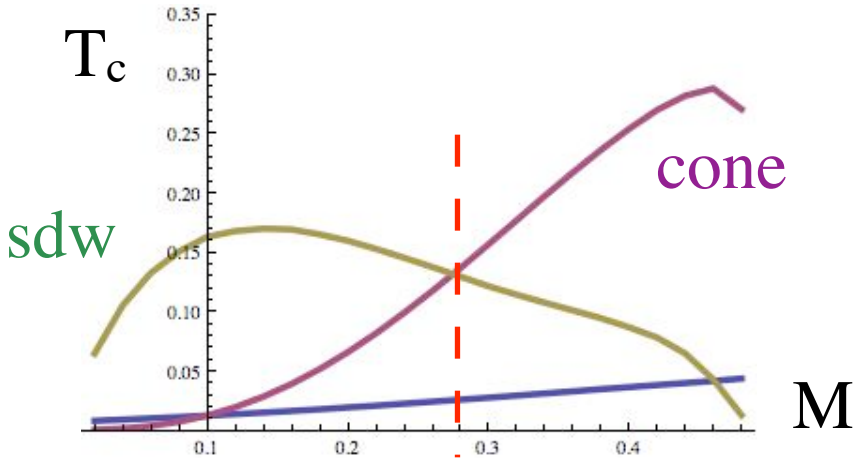
Magnetic field relieves frustration!

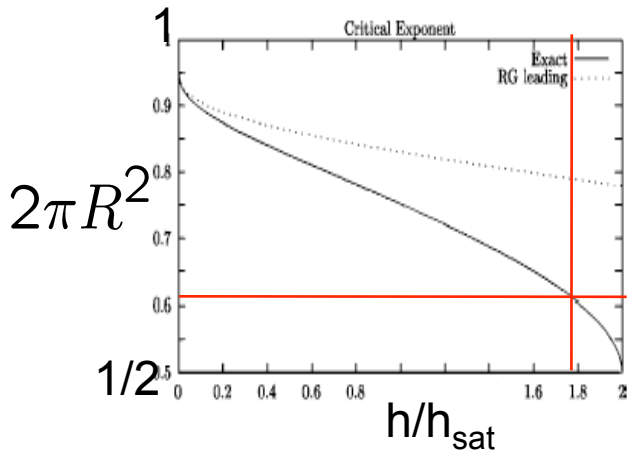
$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2 — "collinear" SDW

dim $1+2\pi R^2$: 2 -> 3/2 — spiral "cone" state

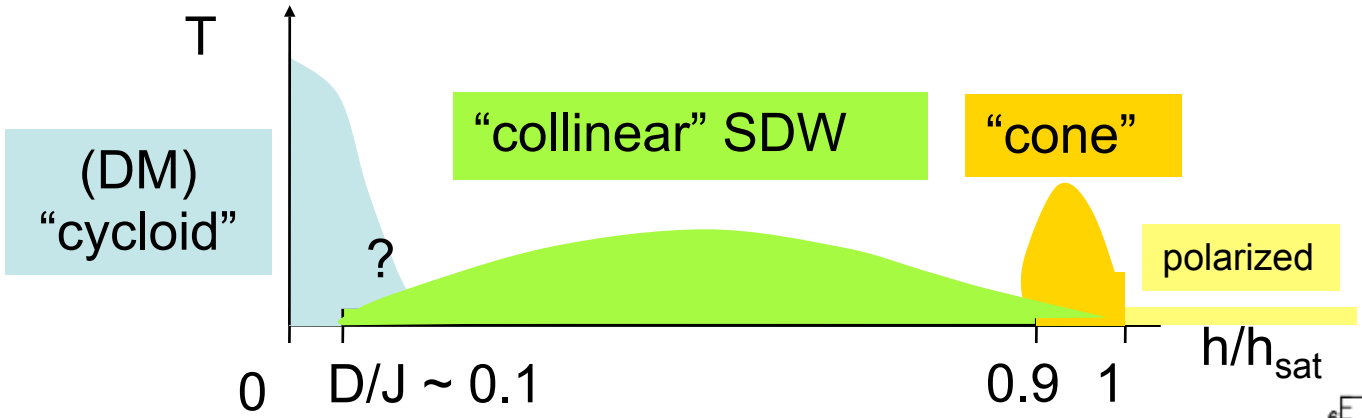
- "Critical point": $1+2\pi R^2 = 1/2\pi R^2$ gives $2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$ at M = 0.3



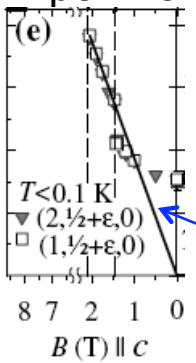


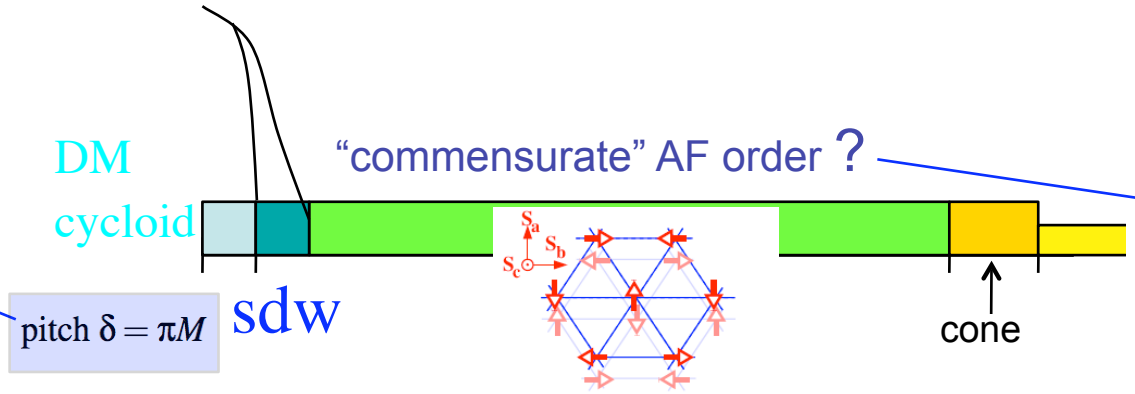
(somewhat naïve) Phase Diagram

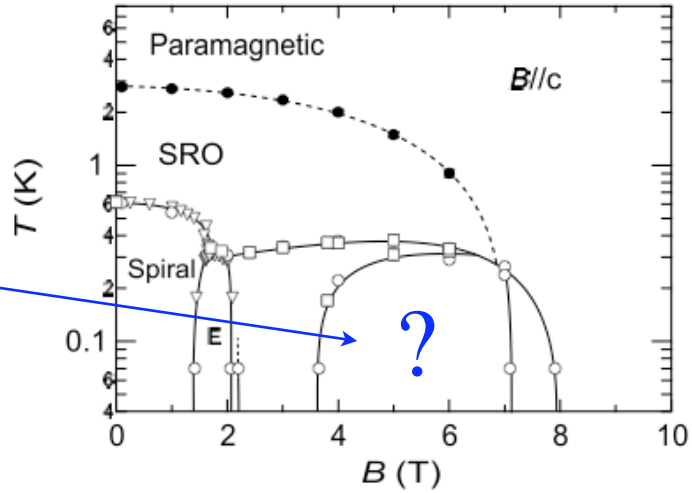
Theory



Experiment

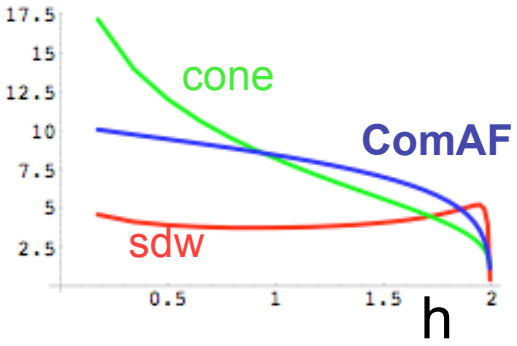






? "Commensurate Collinear" order of some sort has been observed recently:
- seems to have *orthogonal spins* on neighboring chains (Coldea // Veillette Chalker 2006)

RG length:
shortest wins
sdw / **cone**



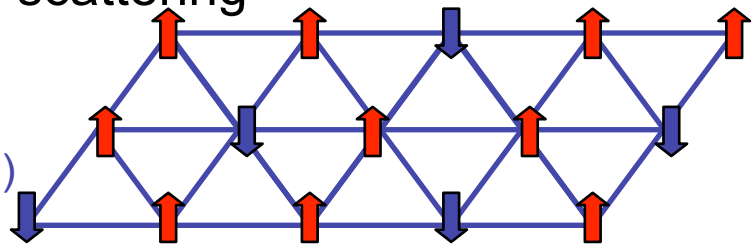
Our theory does predict fluctuations-induced term of right sign, but *too small an amplitude*

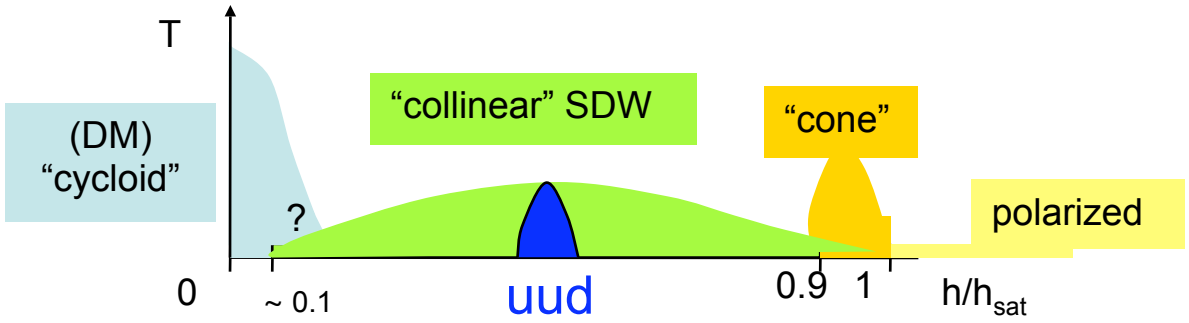
$$+\ c\frac{(J')^4}{J^3}\sum_y S^+_\pi(y)S^-_\pi(y+2)+\text{h.c.}$$

Beyond the naïve: commensurate locking

- "Collinear" SDW state *locks* to the lattice at low-T
  - -"irrelevant" (1d) umklapp terms become relevant once SDW order is present (when *commensurate*): multiparticle umklapp scattering

  - -strongest locking is at M=1/3 M_{sat}
  ✓Observed in **Cs_2CuBr_4** (Ono 2004, Tsuji 07, Fortune 09)
    - ***down***-spins at the centers of hexagons



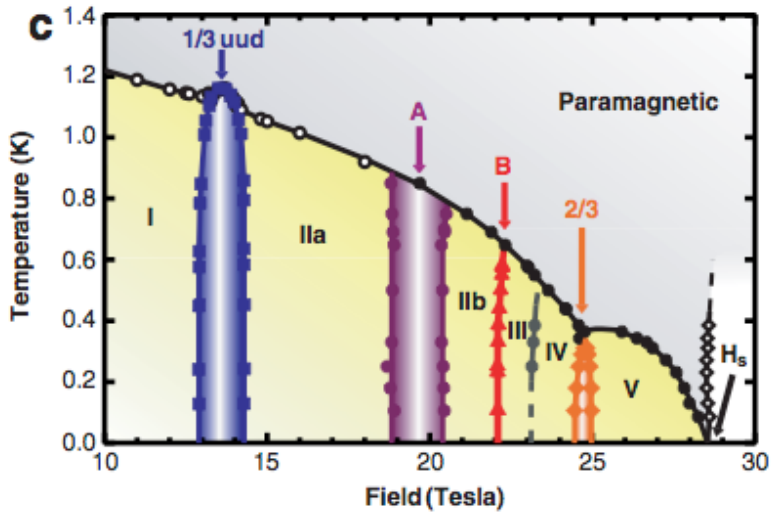


$$\left(\Psi_R^\dagger \Psi_L\right)^n \rightarrow (\pi - 2\delta)n = 2\pi m \rightarrow 2M = 1 - 2m/n$$

n	3	4	5	5	6
m	1	1	1	2	1
2M	1/3	1/2	3/5	1/5	2/3

Cs_2CuBr_4 Fortune et al PRL 2009

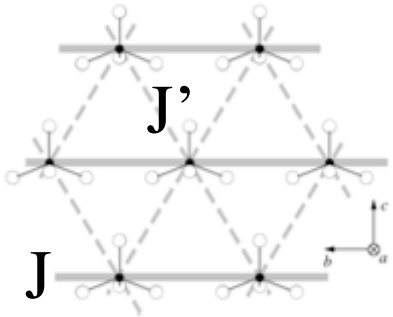
1/3 ~1/2 5/9 2/3



M=1/3 magnetization plateau

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.75**
but not Cs_2CuCl_4 [**J'/J = 0.34**]

S=1/2



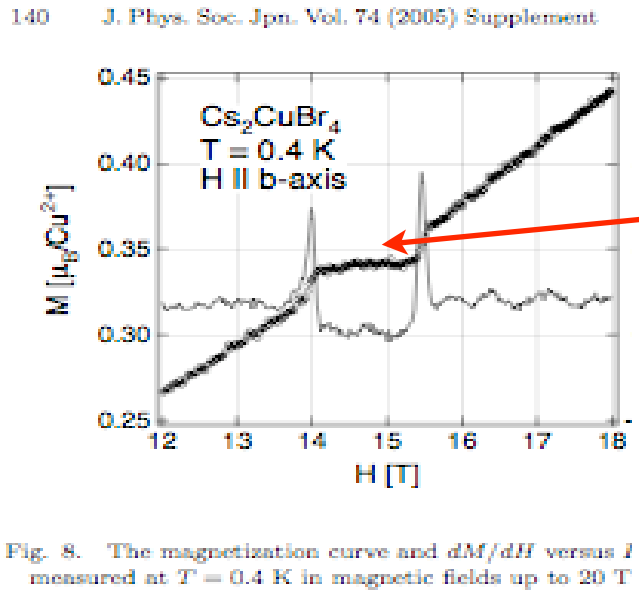
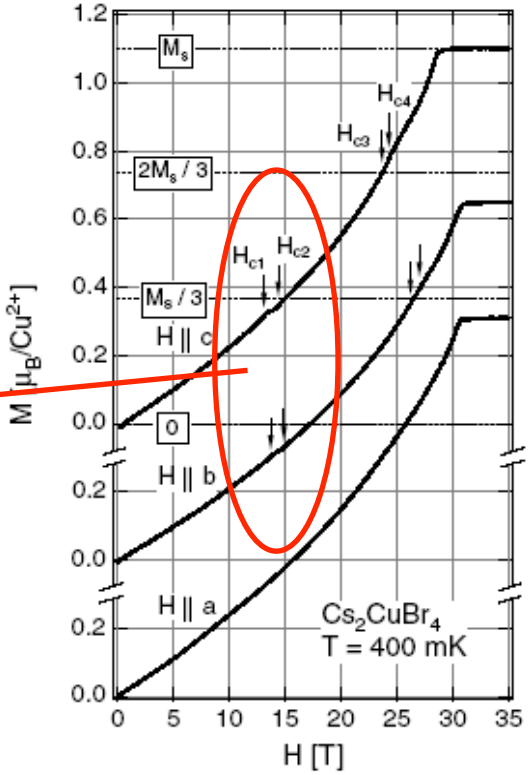
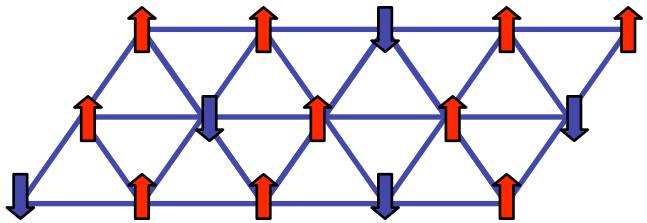
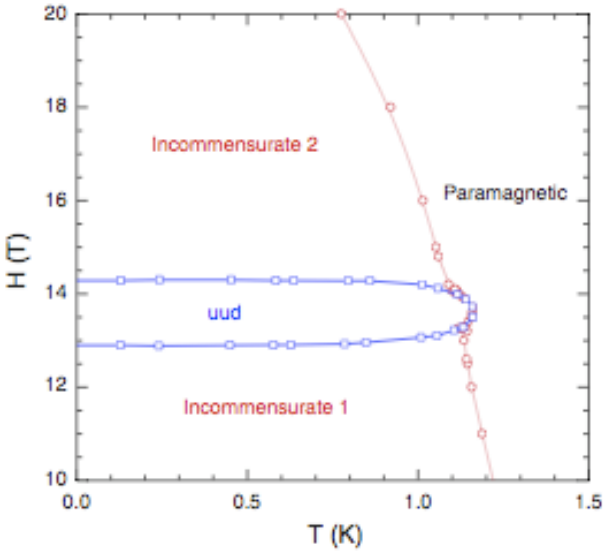


Fig. 8. The magnetization curve and dM/dH versus H measured at $T = 0.4$ K in magnetic fields up to 20 T



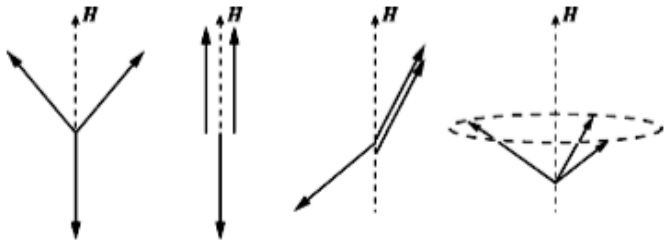
• UUD (up-up-down) structure -- down-spins at the centers of hexagons; commensurate structure -- one down spin per every triangle





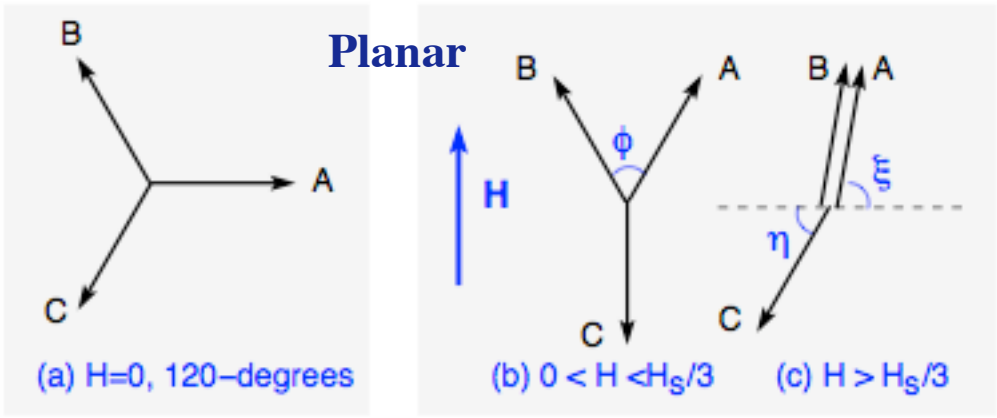
★ "**up-up-down**" state predicted by large-S expansion for spatially isotropic triangular lattice antiferromagnet (Chubukov, Golosov 1991): special to J=J' situation

Classical isotropic Δ AFM in magnetic field

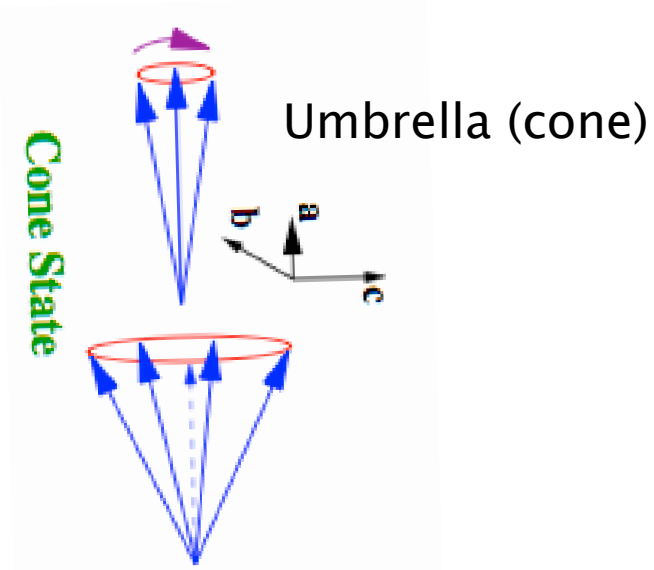


- Spiral state in the absence of the field
- Apply field H: two degenerate states
  - ✦ planar - magnetic field is in the plane of spin spiral
  - ✦ cone (umbrella) - field perpendicular to the spiral plane, spins tilt out of the plane towards the field direction
- Energies (and susceptibilities) of the two states are ***equal*** for J=J': $E_{class} = - c J S^2$

  (accidental degeneracy)
- 1/3 plateau is possible at collinear point (ϕ=0): develops into finite interval by **thermal and quantum fluctuations**



plateau is possible at "collinear" point

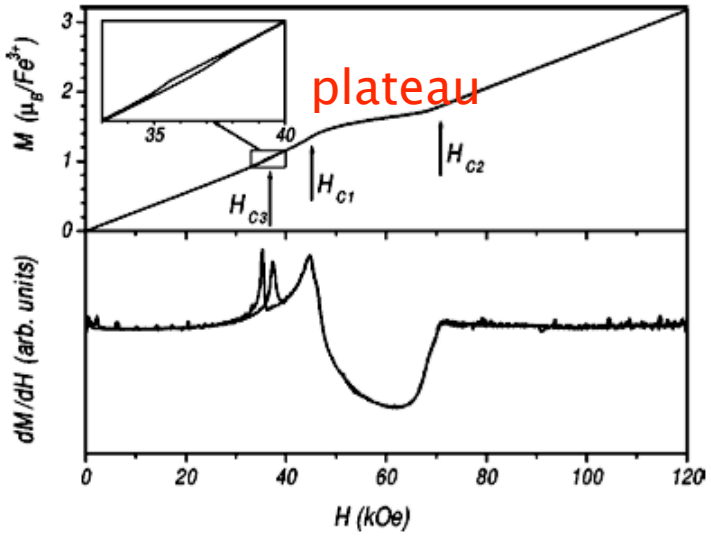


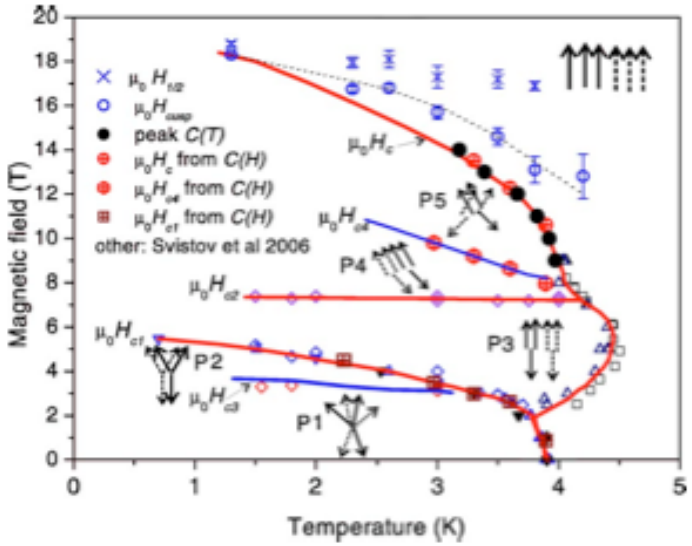
No plateau possible

Order-by-Disorder mechanism

$RbFe(MoO_4)_2$: $S=5/2$ Fe^{3+}

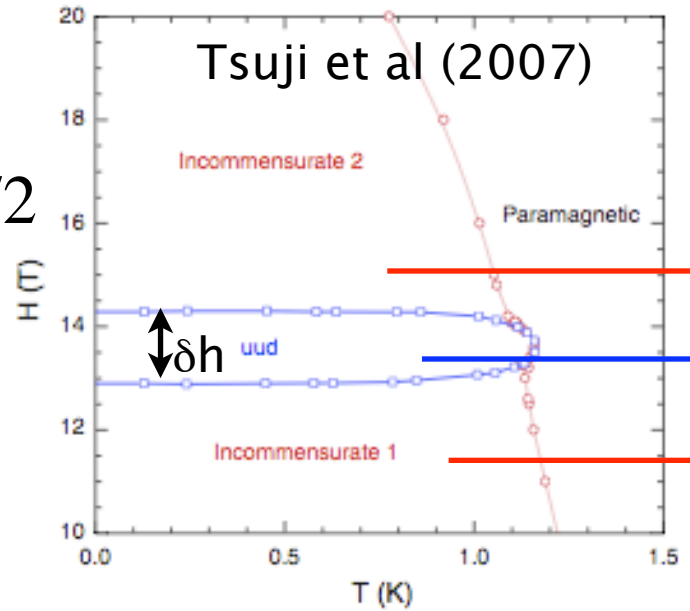
Svistov et al PRB (2003)
Smirnov et al PRB (2007)



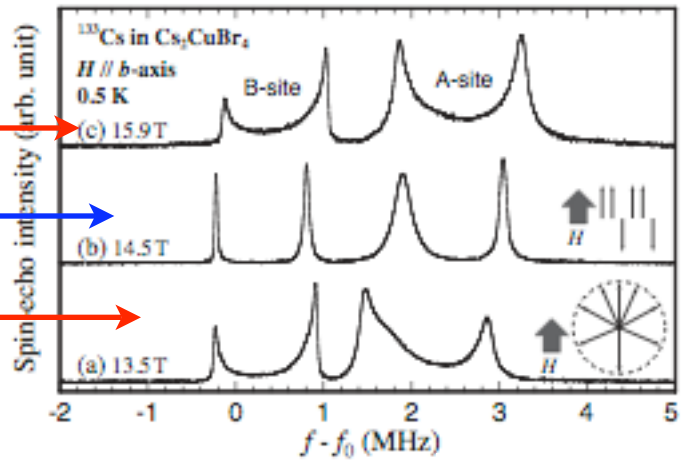


Cs_2CuBr_4: $S=1/2$

Ono et al 2003



NMR spectra: Fujii et al (2004)

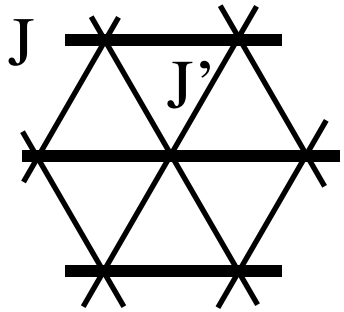


The problem: spatial anisotropy stabilizes umbrella !

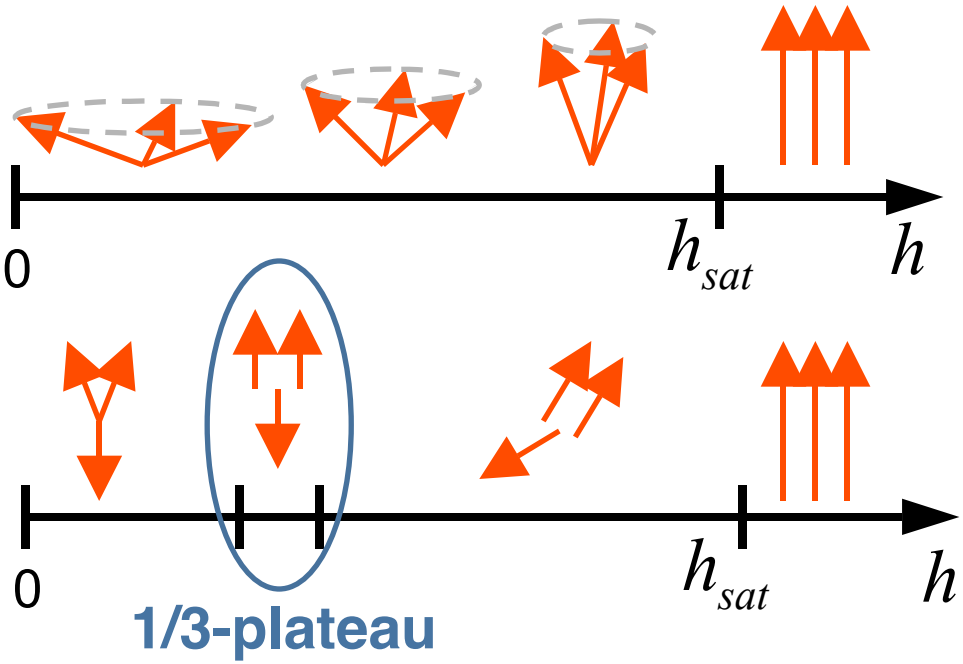


Spatially anisotropic model: classical analysis *fails*

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$J' \neq J$



Umbrella state: favored classically

Planar states: favored by quantum fluctuations

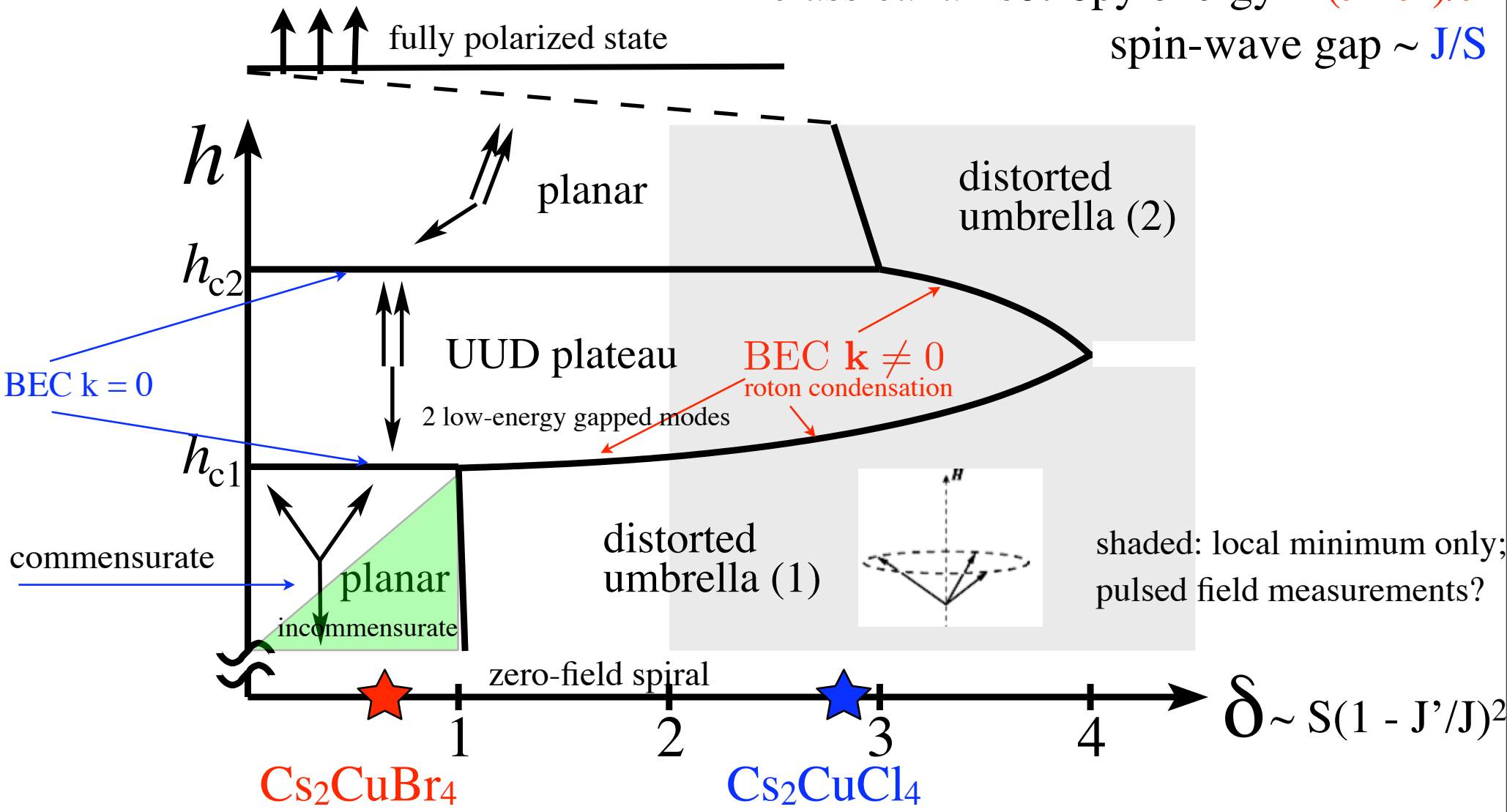
- J'/J = 0.75 not particularly small: OK to use semiclassical spin wave expansion
- Technical formulation: spatial anisotropy J-J' causes softening of interacting (including 1/S correction) spin waves

Our *semiclassical* approach: treat spatial anisotropy (J–J') as a perturbation to **interacting** spin waves

- single dimensionless parameter $\delta=(40/3)S(1 - J'/J)^2$:

classical anisotropy energy ~ (J - J')/J

spin-wave gap ~ J/S



Alicea, Chubukov, Starykh PRL 102, 137201 (2009)

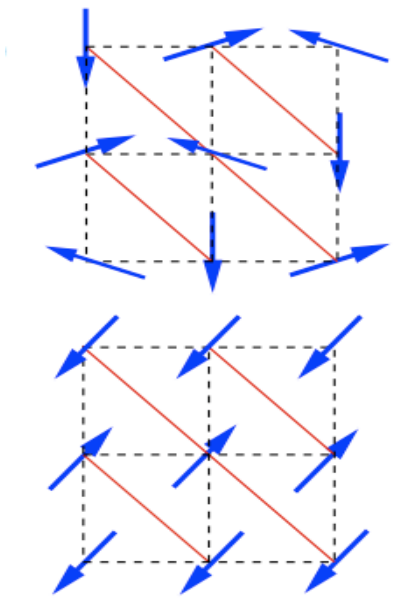
Summary (II)

- ✦ Rich, interesting physics: much can be understood by viewing the problem/material from 1d perspective
- ✦ Good for weakly ordered states: abundance of multi-particle excitations
- ✦ Delicate details of 3d ordering are determined by minute, and often anisotropic, sub-leading interactions
- • T-H phase diagram of Cs_2CuCl_4 and Cs_2CuBr_4 remain to be understood
  - – commensurate AF state in Cs_2CuCl_4
  - – 2/3 and other fractional states in Cs_2CuBr_4
- • Evolution of high-energy continuum into low-energy spin waves?

NMR data: persistence of **planar** structures

- RG for quantum spins:
  - collinear tendency persists among orthogonal to the field components ($S^{x,y}$)
  - generates collinear coupling between next-nearest chains

  $(J'/J)^4 [S^{+}_{x,y} S^{-}_{x,y+2} + h.c.]$
- Observed in magnetic field in Cs_2CuCl_4 (M. Takigawa, presentation at HFM-08): 2-sublattice collinear order between chains in the plane perpendicular to the applied magnetic field

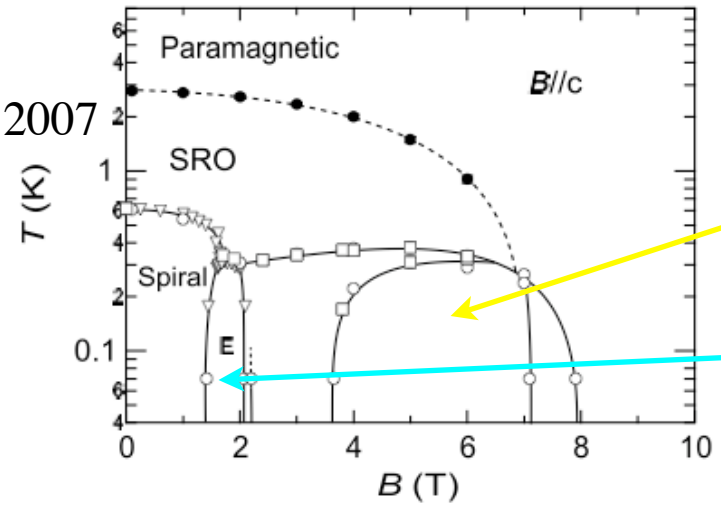


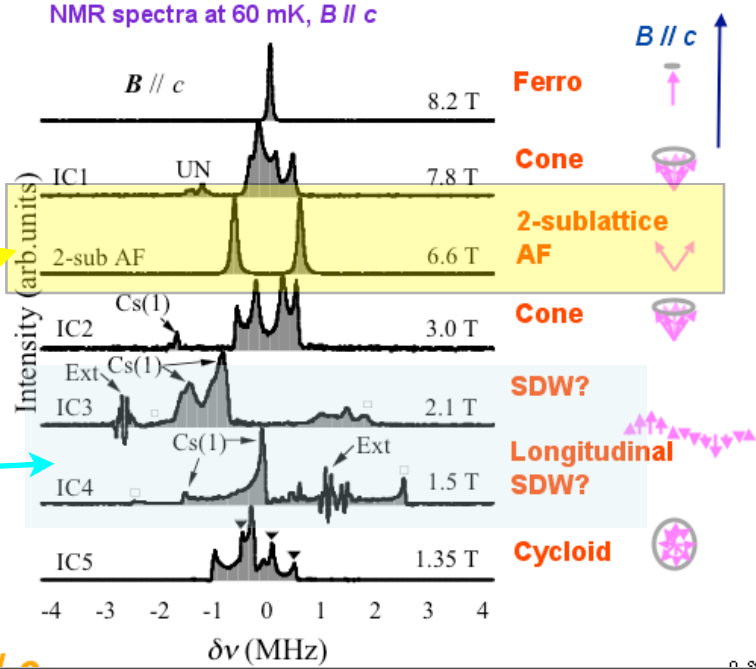
top view:

umbrella

2-sublattice collinear

Tokiwa et al, 2007

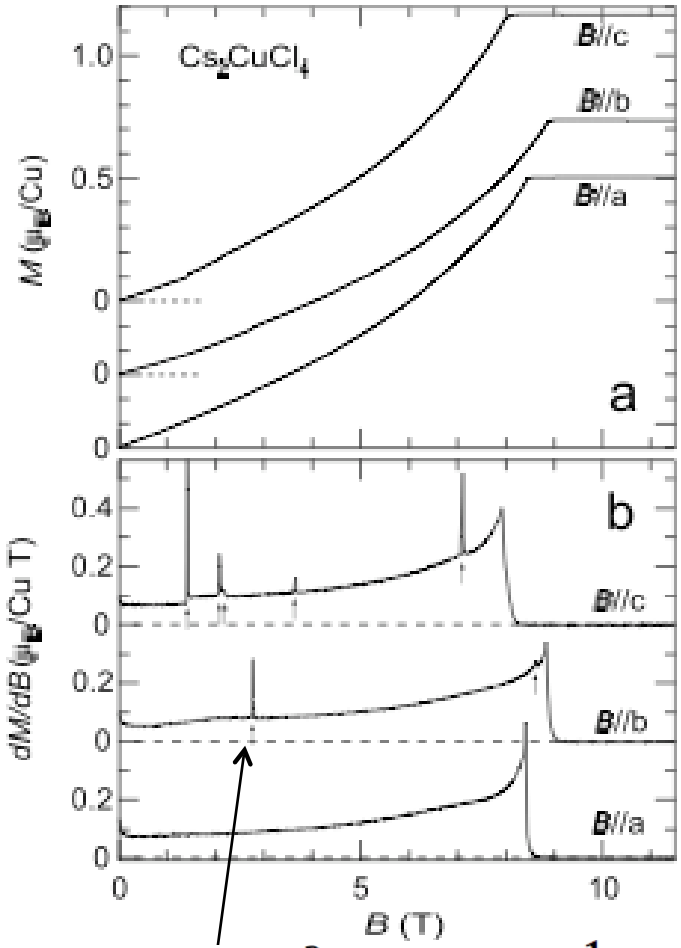


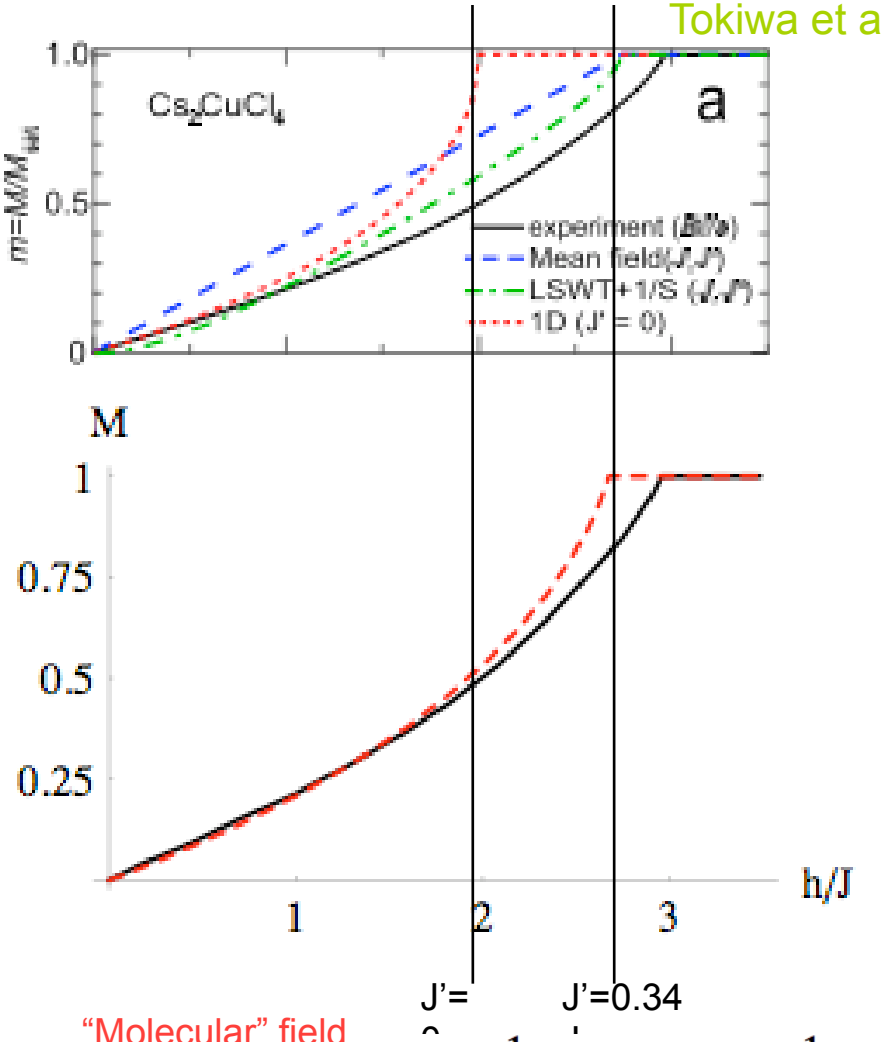


Magnetization measurements

- M(h) is smooth: not sensitive to low-energy (long-distance) fluctuations. Determined by *uncorrelated* (but magnetized) chains.

Tokiwa et al, 2006





$$\delta E(M) = 2J'M^2 \;,\; h = \frac{\partial E}{\partial M} \qquad M_{1d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h}}\right)$$

"Molecular" field →

$$M_{2d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h-4J'M_{2d}}}\right)$$

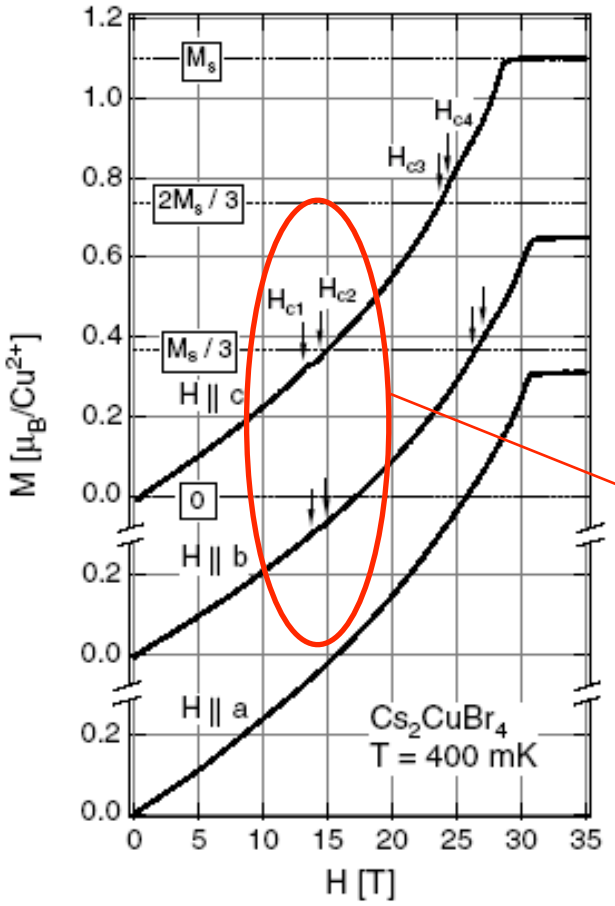
✓Error in saturation field is $(J')^2/2J \approx 2\%$

- dM/dh delineates phase boundaries: divergent derivative = phase transition

Cs_2CuBr_4

- Isostructural to Cs_2CuCl_4 but believed to be less quasi-1d: J'/J = 0.75

T. Ono *et al*, 2004



- Magnetization plateau at M=1/3 M_{sat} observed for longitudinal but not transverse fields

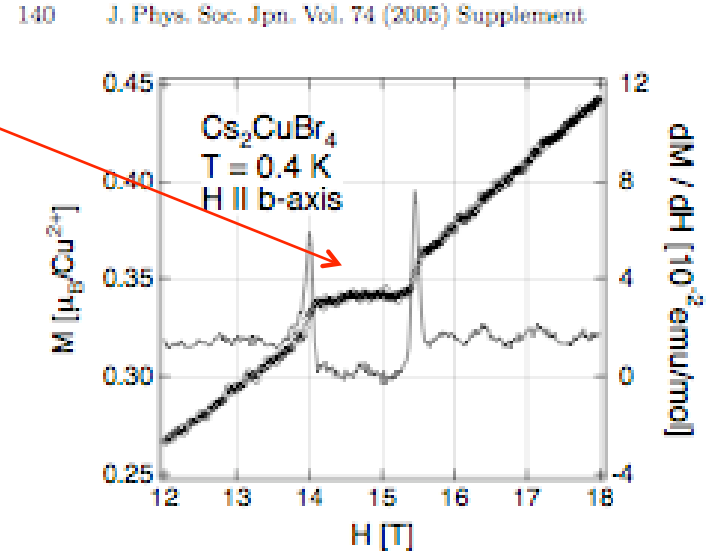
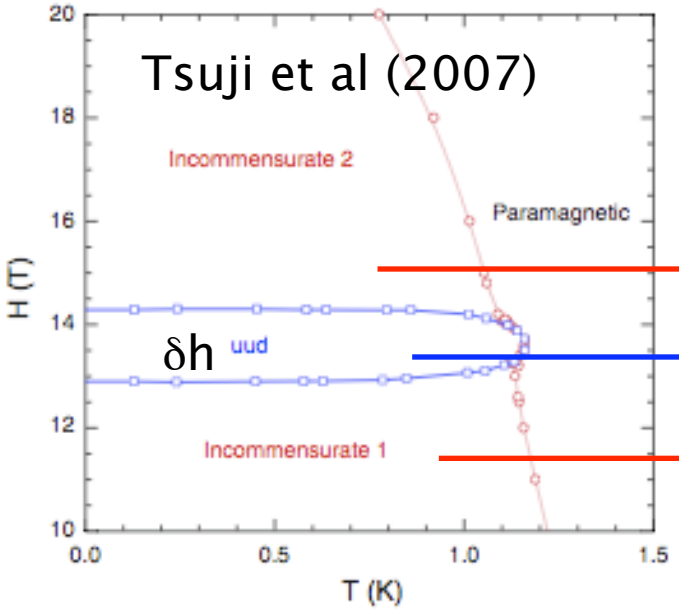


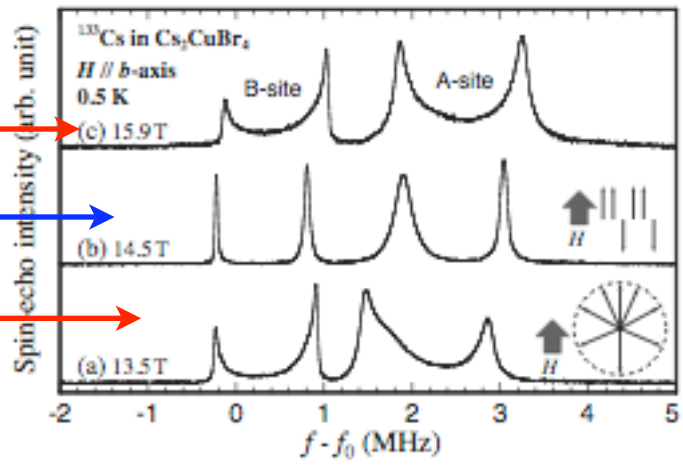
Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel b$ measured at $T = 0.4$ K in magnetic fields up to 20 T.

Alicea, Chubukov, Starykh 2008

Order-by-Disorder: Quantum fluctuations (1/S)

- fluctuation spectra of *different* spin structures are *different*: $E = E_{class} + \Delta E^{sw}$
- quantum fluctuations prefer planar arrangement $\Delta E^{sw}_{planar} = \frac{S}{2}\sum_k \omega_{planar}(k) < \Delta E^{sw}_{umbrella} = \frac{S}{2}\sum_k \omega_{umbrella}(k)$
- prefer collinear configuration even more, when possible: state with maximum number of soft modes wins.
- plateau is a quantum effect, width $\delta h = 1.8\ J/S$ ($h_{saturation} = 9\ J$) Chubukov, Golosov (1991)
- plateau is the effect of interactions (hence, width ~ 1/S) between spin waves

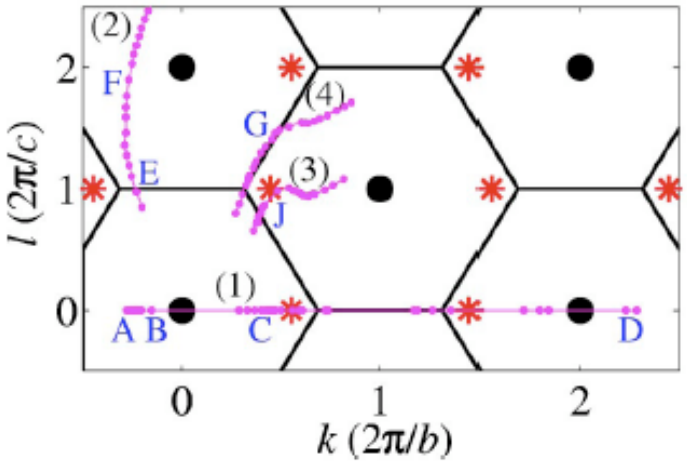


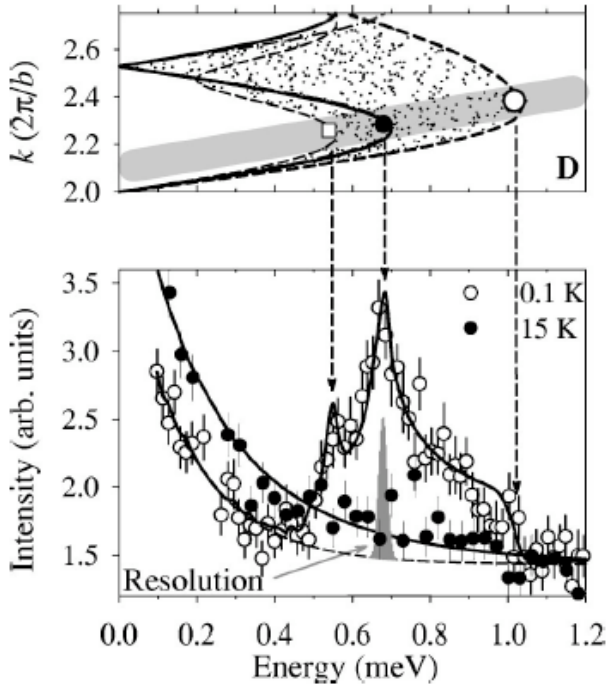


The problem: spatial anisotropy stabilizes umbrella

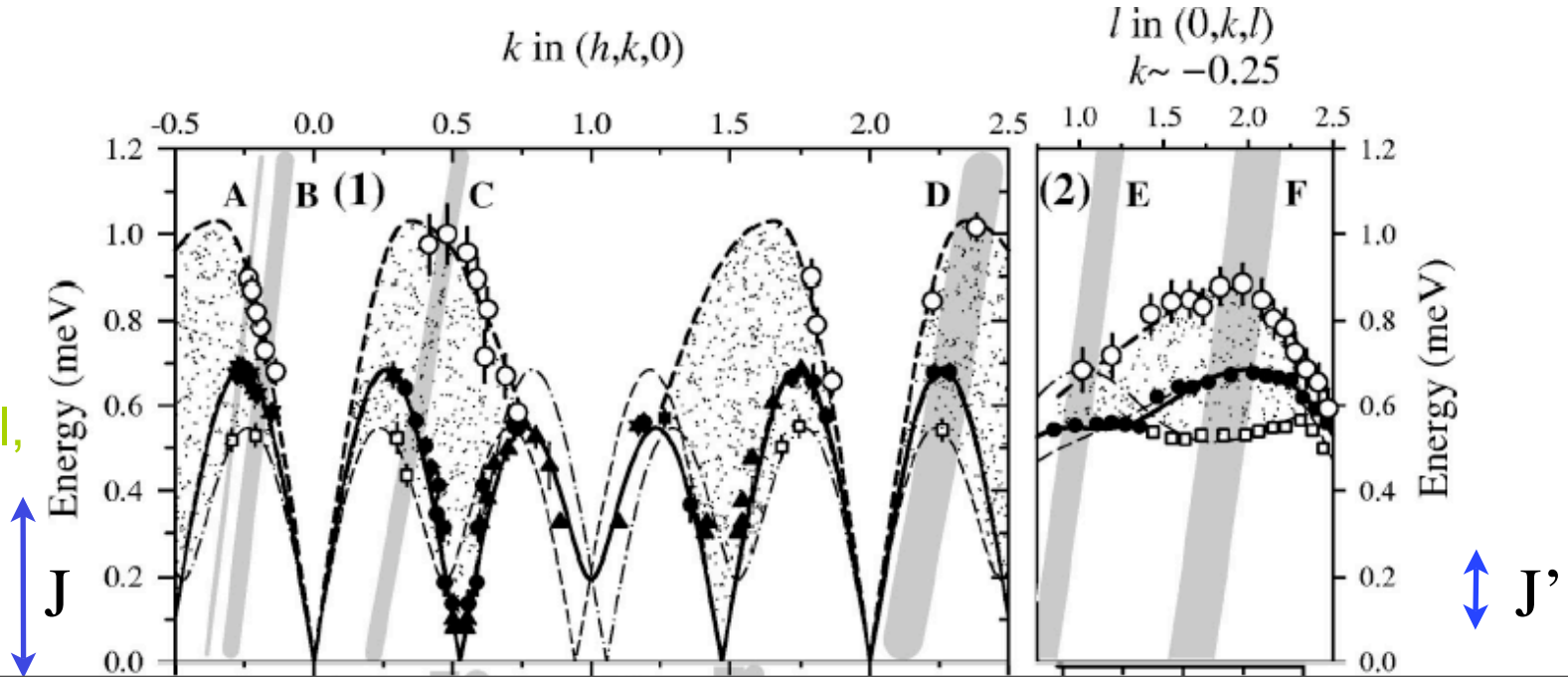


Continuum of excitations is a high-energy feature!





Coldea et al, PRB 2003



Low-energy degrees of freedom

- Quantum triad: uniform magnetization $\mathbf{M} = \mathbf{J_R} + \mathbf{J_L}$,

staggered magnetization **N** and staggered dimerization $\varepsilon = (-1)^x S_x S_{x+1}$

✓Components of Wess-Zumino-Witten-Novikov SU(2) matrix $G = \varepsilon + i\vec{N}\cdot\vec{\sigma}$

- Hamiltonian $H \sim \mathbf{J_R J_R} + \mathbf{J_L J_L} + \underbrace{\gamma_{bs}\, \mathbf{J_R J_L}}_{\text{marginal perturbation}}$

- Operator product expansion $z = v\,\tau - ix$ (similar to commutation relations)

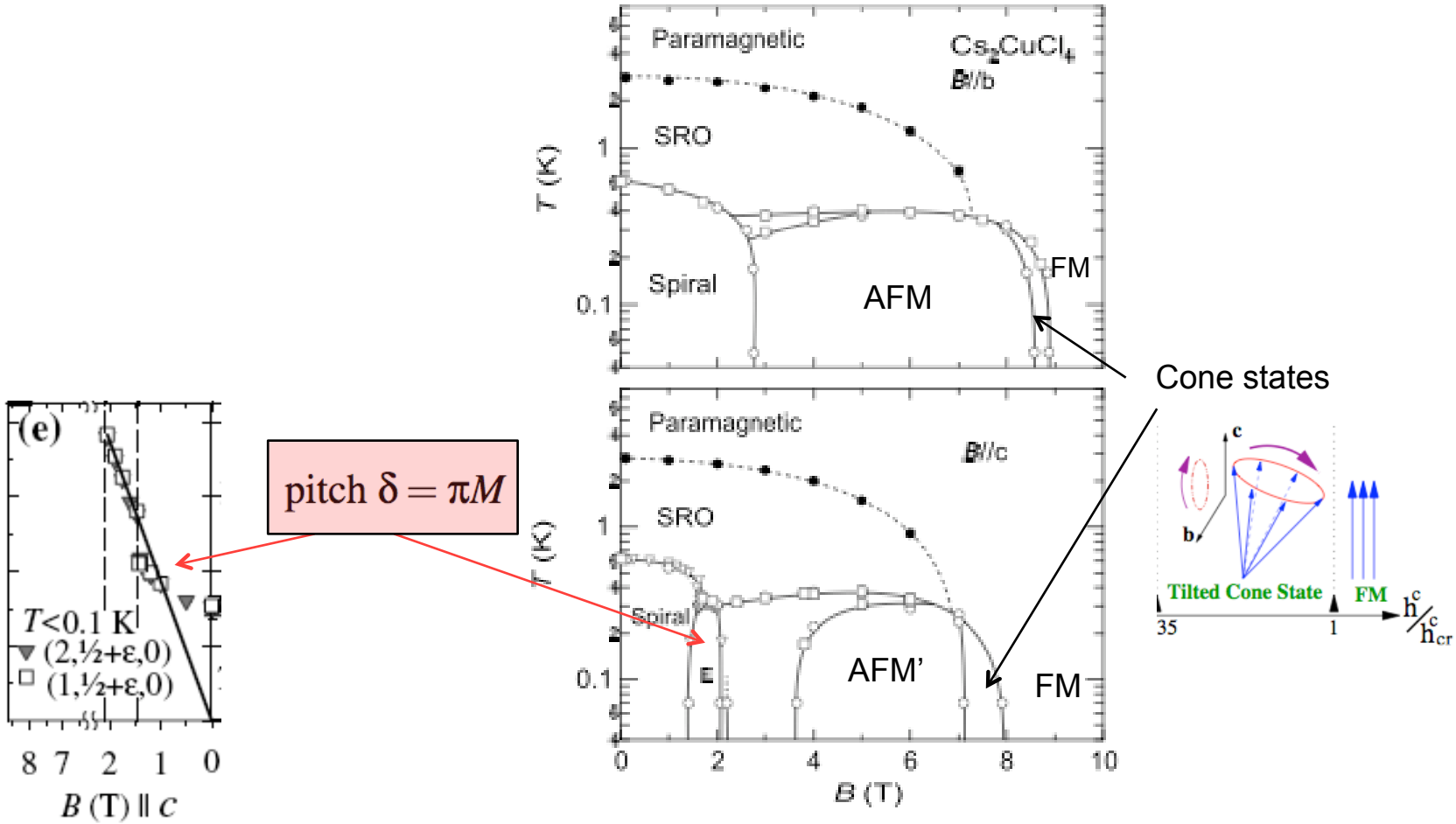
$$\begin{cases} J_R^a(x,\tau)J_R^b(0,0) = \dfrac{\delta^{ab}}{8\pi^2 z^2} + \dfrac{i\varepsilon^{abc}J_R^c(x,\tau)}{2\pi z} \\ J_R^a(x,\tau)N^b(0,0) = \dfrac{-i\delta^{ab}\varepsilon(x,\tau) + i\varepsilon^{abc}N^c(x,\tau)}{4\pi z}\,,\; J_R^a(x,\tau)\varepsilon(0,0) = \dfrac{iN^a(x,\tau)}{4\pi z} \end{cases}$$

- Scaling dimension 1/2 (relevant) $\langle N^a(x,\tau)N^a(0,0)\rangle \sim \dfrac{1}{\sqrt{v^2\tau^2+x^2}} \sim \langle \varepsilon(x,\tau)\varepsilon(0,0)\rangle$

- Scaling dimension 1 (marginal) $\langle M^a(x,\tau)M^a(0,0)\rangle \sim \dfrac{1}{z^2} + \dfrac{1}{\bar{z}^2}$

EXPERIMENT: Longitudinal Field ,T_c *vs* B; *SDW, …, cone*

- Very different behavior for B along *b, c* axes (both orthogonal to DM direction *a*)
  - *Additional anisotropies in the problem?*
  - *Nature of AFM and AFM' phases - the biggest puzzle?*



R. Coldea *et al*, 2001;
T. Radu *et al*, 2005;
Y. Tokiwa *et al*, 2006