Phases of spin chains with uniform Dzyaloshinskii-Moriya interactions

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RIKEN condensed matter seminar, Nov.21, 2017

Outline

Spin 1/2 chains with uniform DM interaction

Electron spin resonance

Phase diagram of the quantum model with orthogonal magnetic field

Weakly coupled chains

Conclusions

Dzyaloshinskii-Moriya (DM) interaction

J. Phys. Chem. Solids Pergamon Press 1958. Vol. 4. pp. 241–255.

A THERMODYNAMIC THEORY OF "WEAK" FERROMAGNETISM OF ANTIFERROMAGNETICS

D

I. DZYALOSHINSKY

Institute for Physical Problems, Academy of Sciences of the U.S.S.R., Moscow

(*Received* 19 *February* 1957)

PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Anisotropic Superexchange Interaction and Weak Ferromagnetism

M

TÔRU MORIYA*

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received May 25, 1960)

$$\mathbf{D}_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j$$

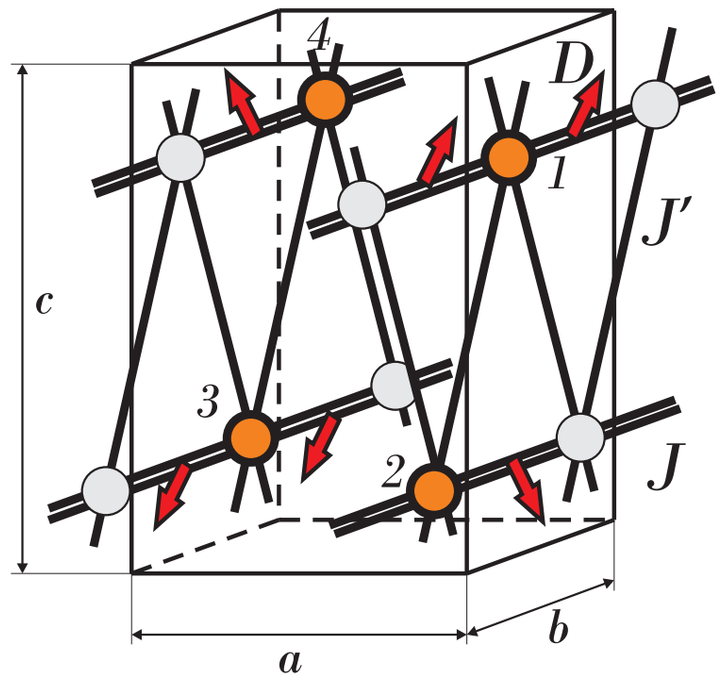
- Reduces symmetry to U(1) [rotations about D axis]
- Easy plane anisotropy (perp. to D)
- Promotes magnetic order
- Generically stabilizes incommensurate non-collinear (spiral) states

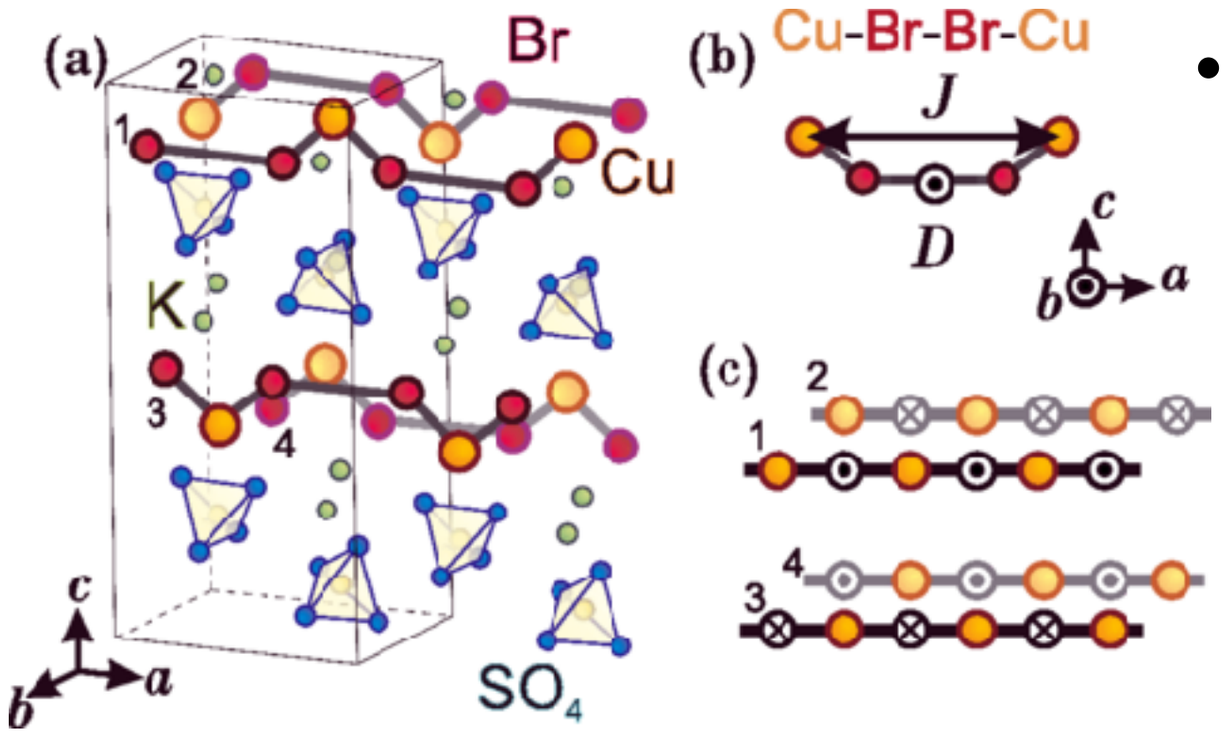
Cs_2CuCl_4: uniform Dzyaloshinskii-Moriya interaction

$$\vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j$$

Focus on the in-chain DM: for a given chain (y,z) vector D is constant

$$D_{y,z} = (-1)^y D_c \hat{c} + (-1)^z D_a \hat{a}$$





- Spin chain material $K_2CuSO_4Br_2$.

Spin chain along a axis.
Intra-chain exchange J=20.5K,
Inter-chain exchange J'=0.03K.

Smirnov et al., Phys. Rev. B, **92**,134417, 2015

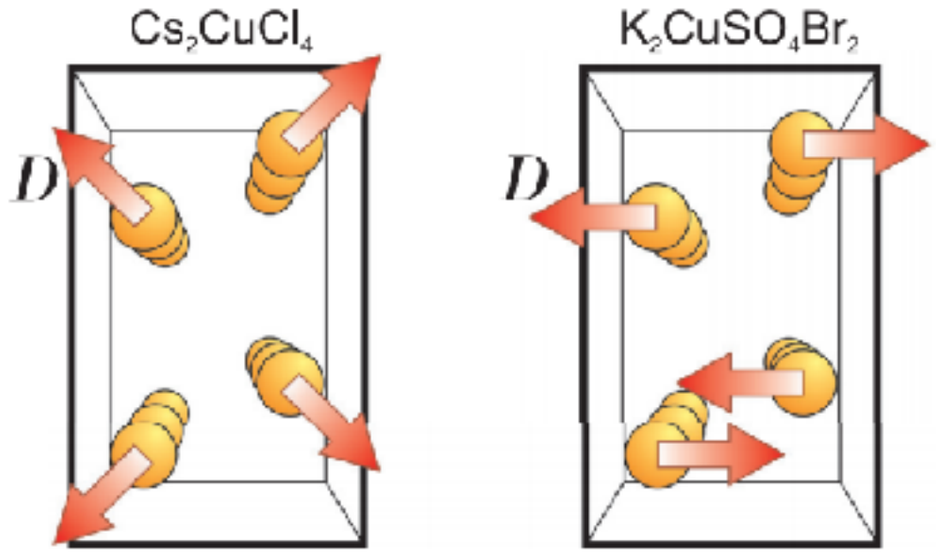


FIG. 2. (Color online) Comparison of the Dzyaloshinskii-Moriya vectors in Cs_2CuCl_4 and $K_2CuSO_4Br_2$. Perspective view along the spin chains.

ESR - electron spin resonance

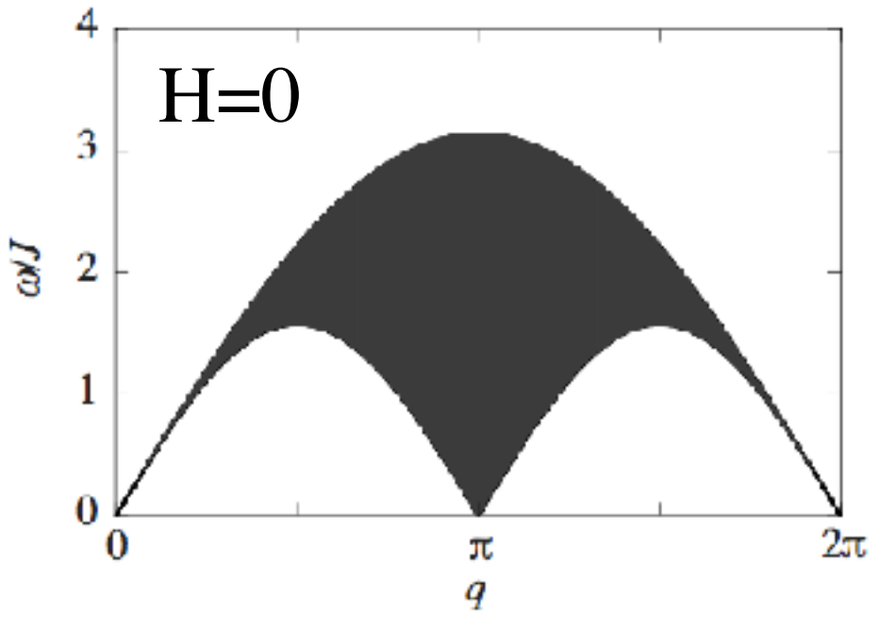
- Simple (in principle) and sensitive probe of magnetic *anisotropies* (and, also, **q=0** probe: $S = \Sigma_r S_r$)

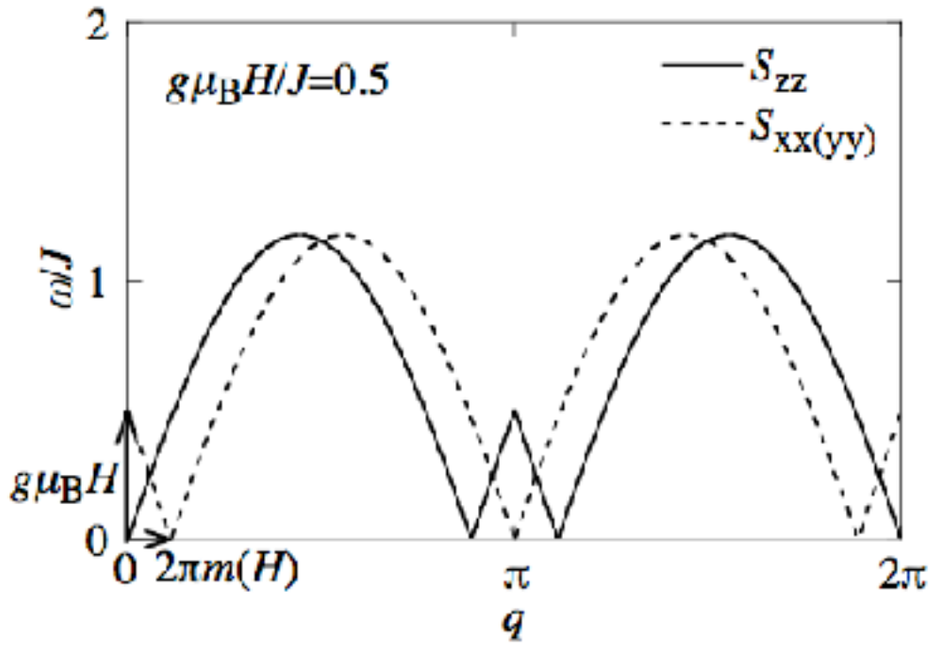
$$I(h,\omega) = \frac{\omega}{4L}\int dt e^{i\omega t}\langle [S^+(t), S^-]\rangle$$

Zeeman **H** along **z**, microwave radiation **h** polarized perpendicular to it.

- For SU(2) invariant chain in paramagnetic phase

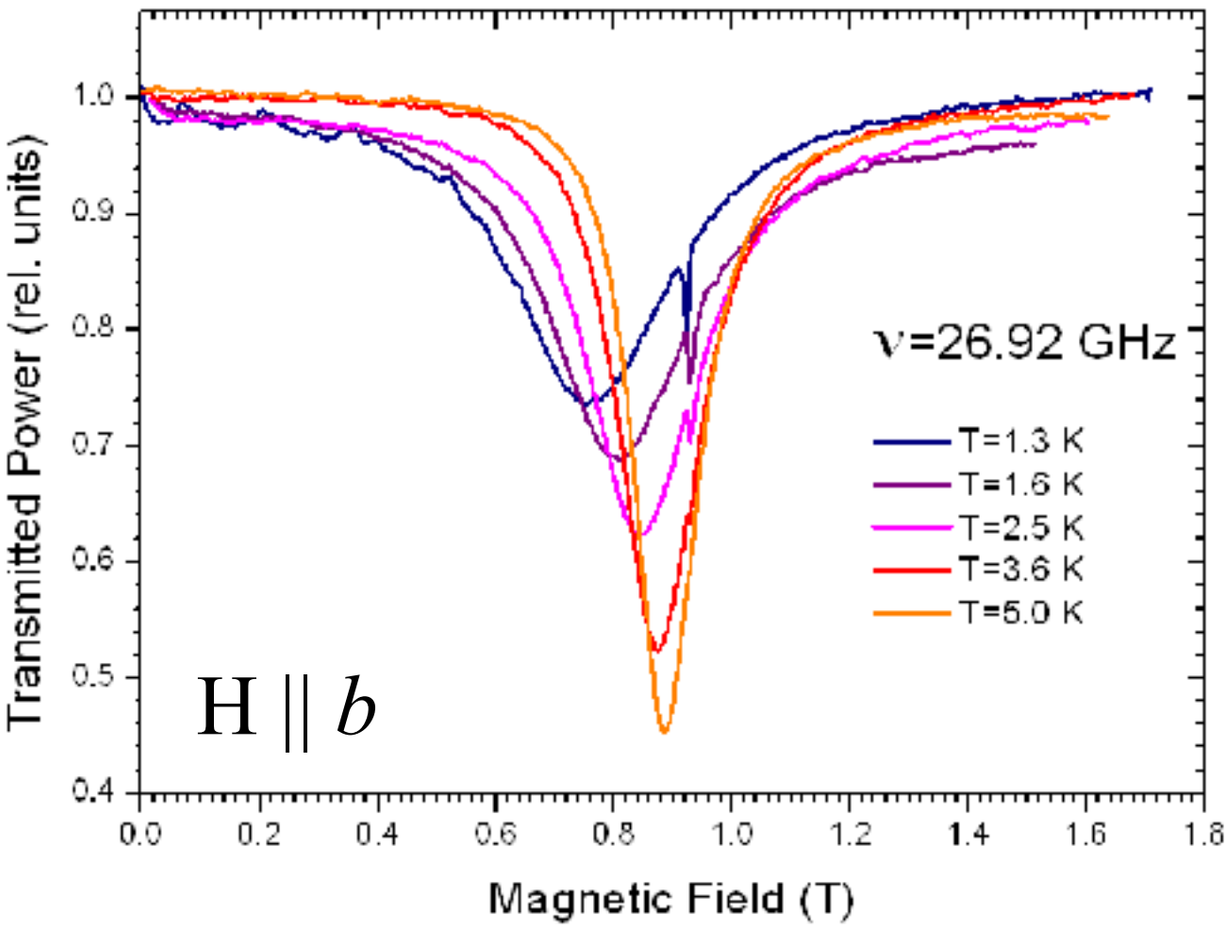
  $I(H,\omega) \sim \delta(\omega\text{-}H)\ m(H)$ [Kohn's Th] Oshikawa, Affleck PRB 2002



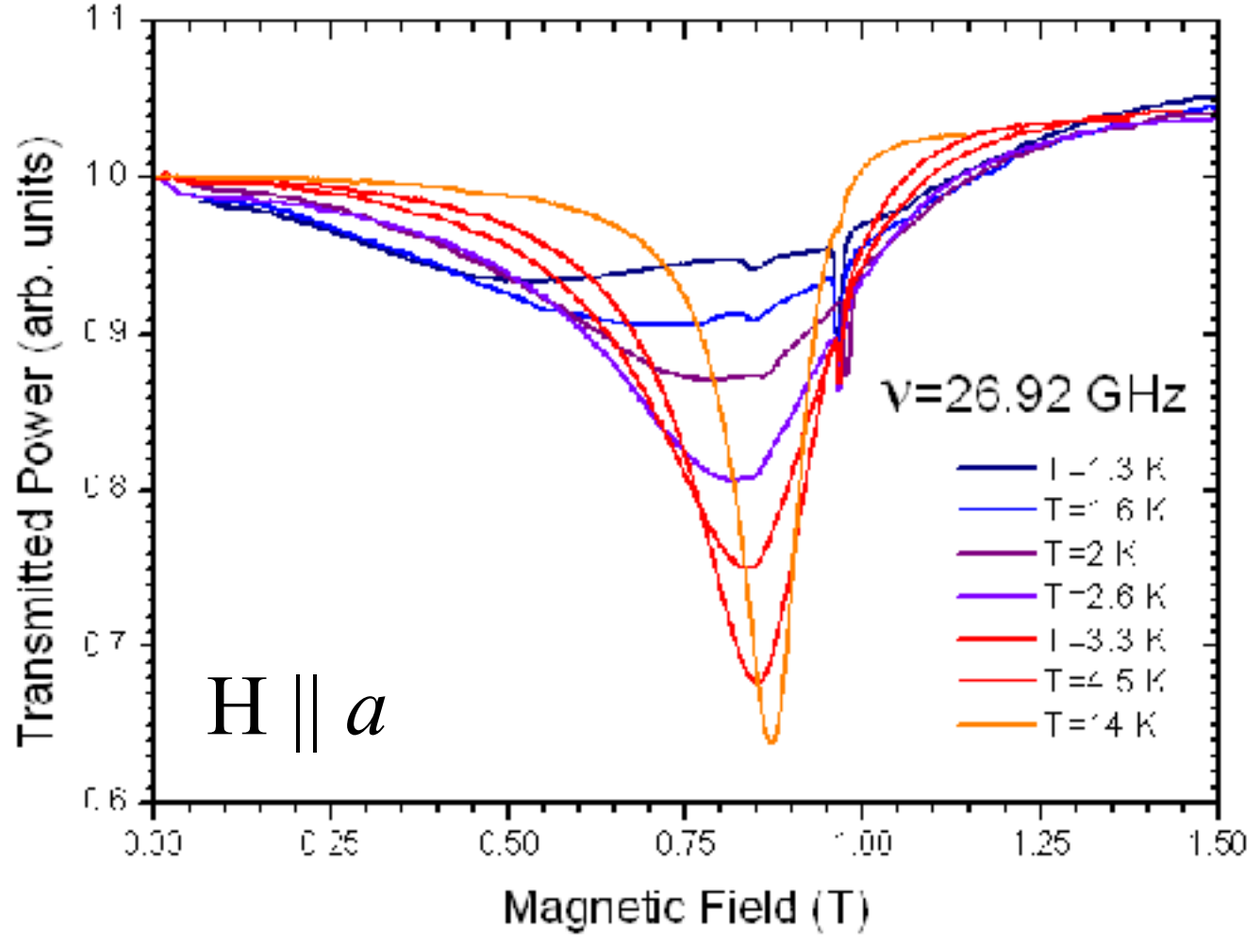


ESR data (Povarov and Smirnov, Kapitza Institute)

Shift for H || *b*



Shift and splitting for H || *a*, *c*



Resonance line is significantly modified with lowering the temperature; modification is strongly anisotropic with respect to field.

The lowest temperature T=1.3 K is still twice higher than ordering T_N

Cs_2CuCl_4 ESR data: T=1.3 K

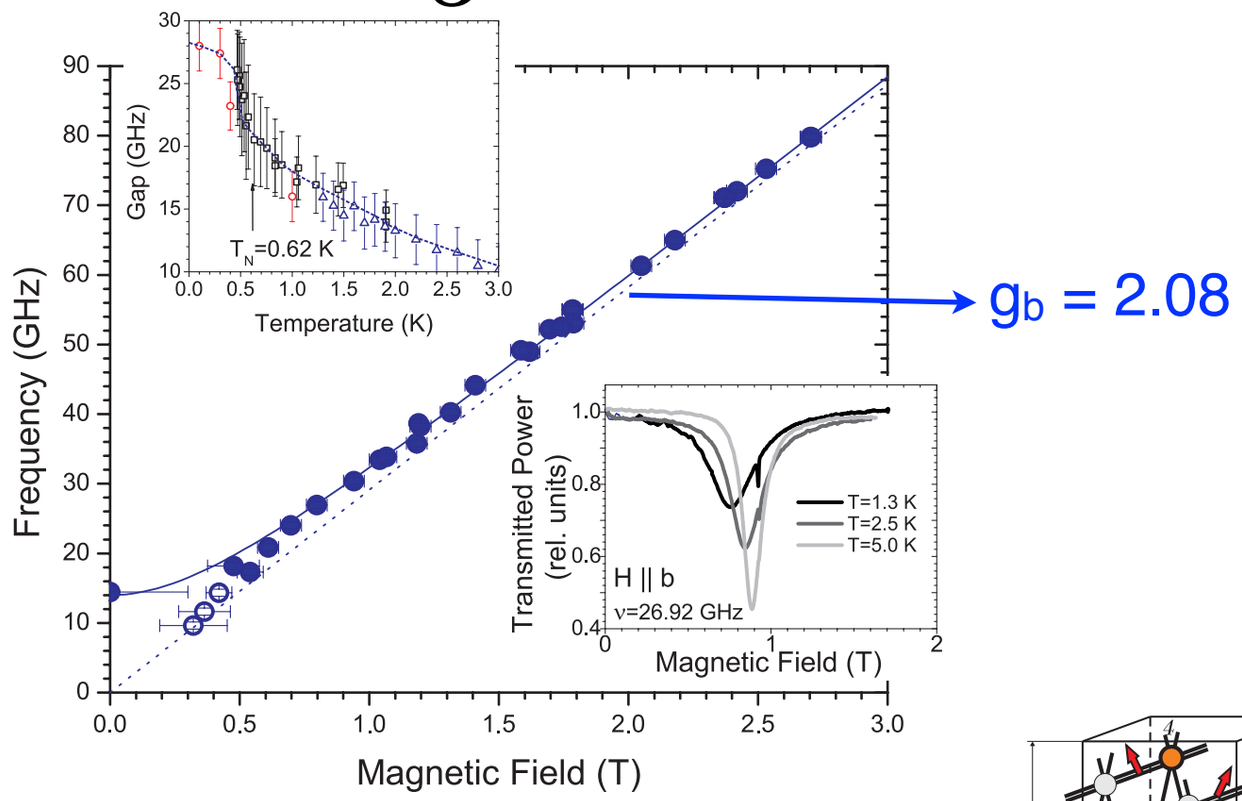
Povarov et al, 2011

- H along b-axis

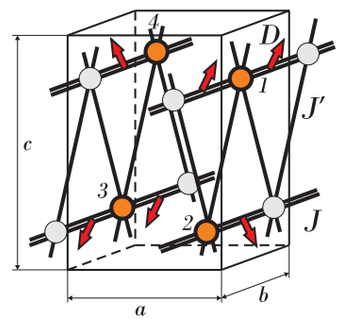
✓ **gap**-like behavior for ν > 17 GHz

$$2\pi\hbar\nu = \sqrt{(g_b\mu_B H)^2 + \Delta^2}$$

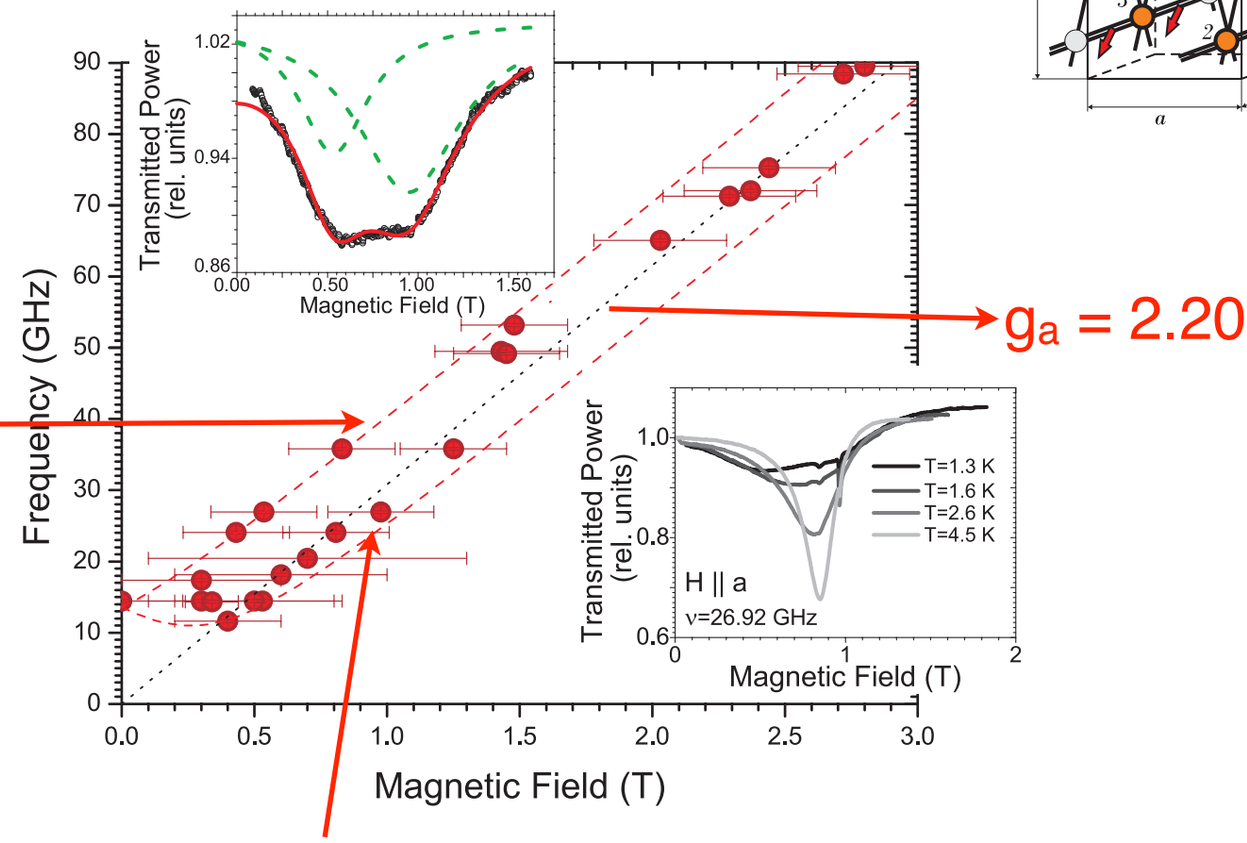
✓ loss of intensity for ν < 17 GHz



$g_b = 2.08$

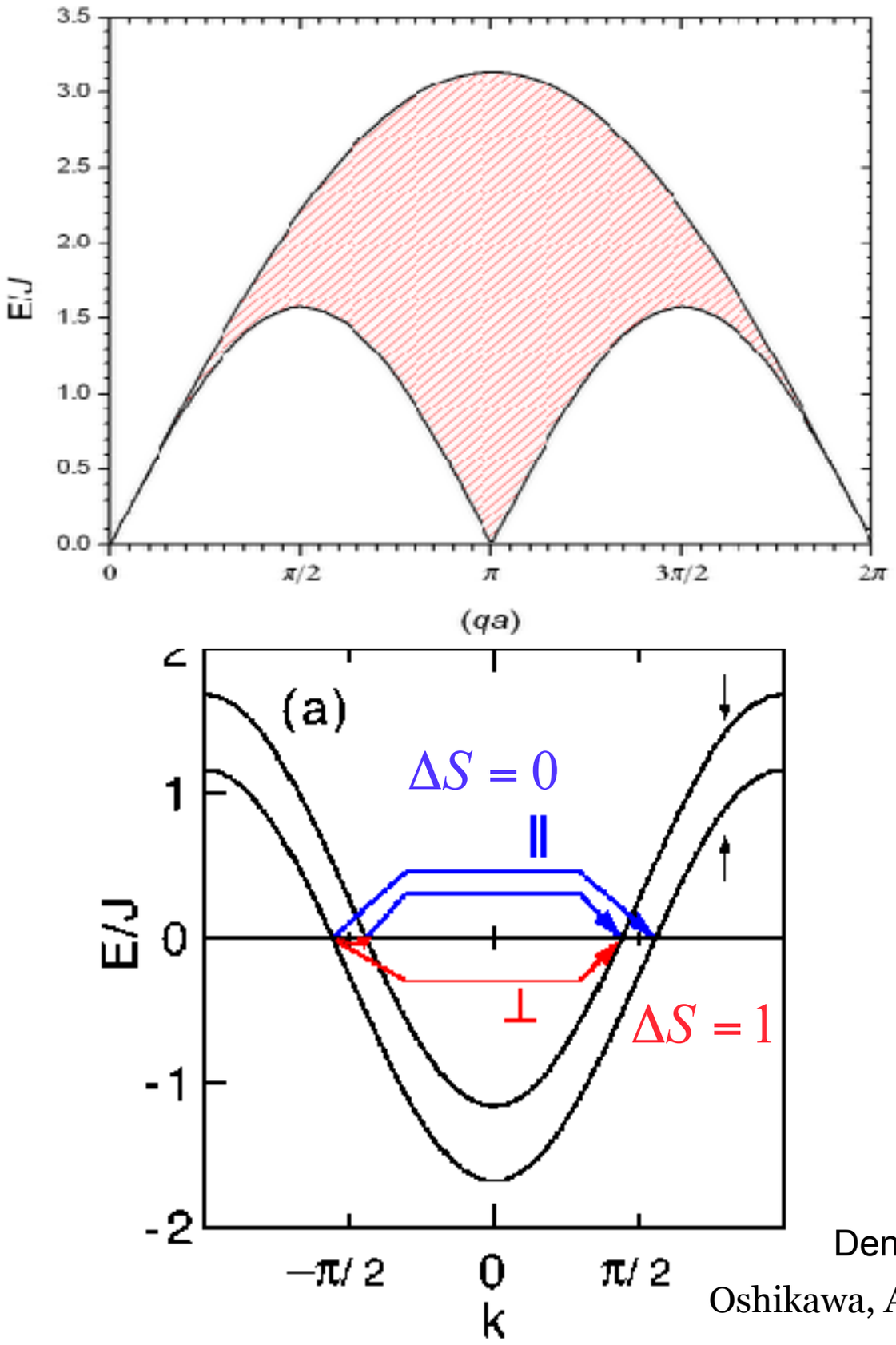


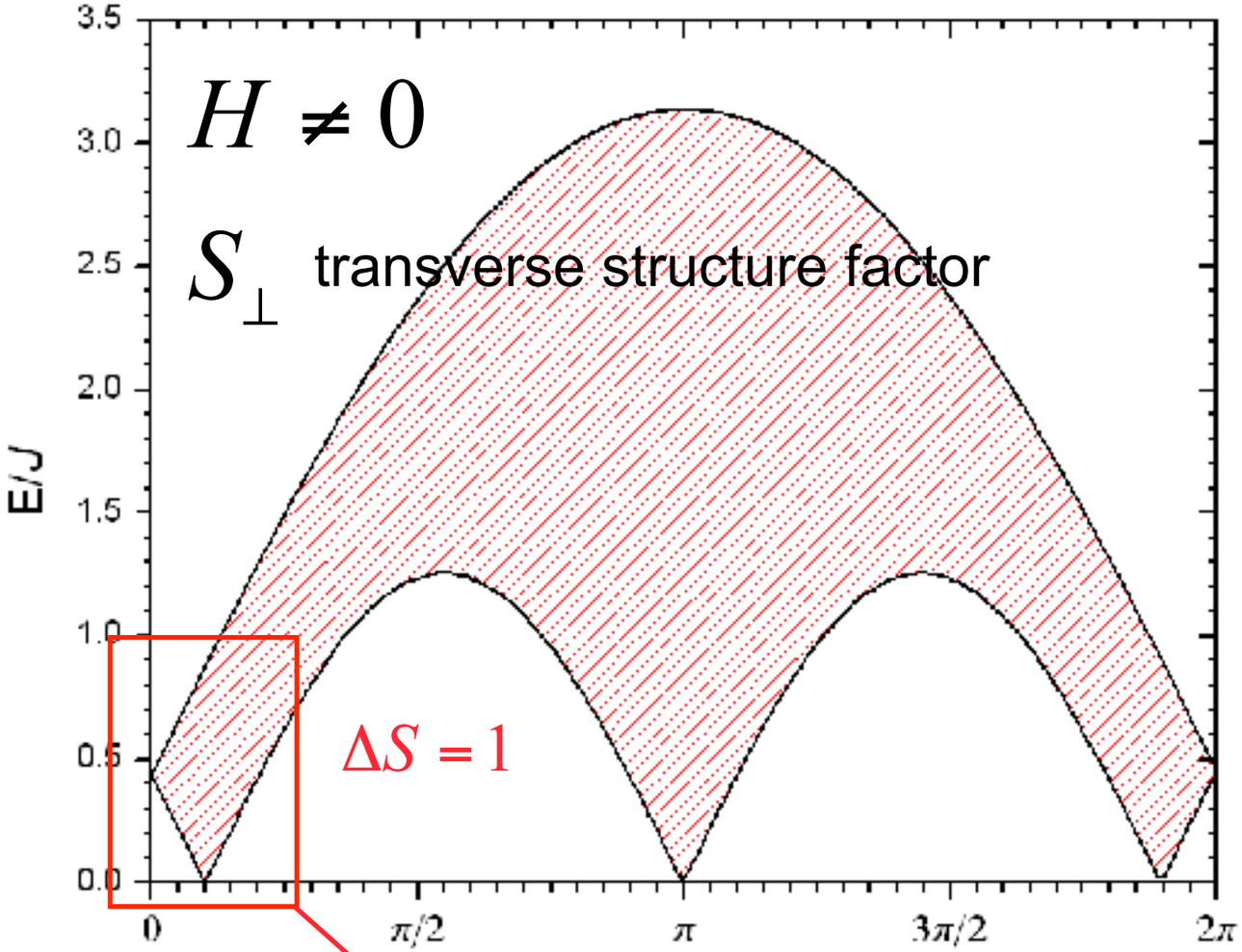
- H along a-axis: splitting of the ESR line

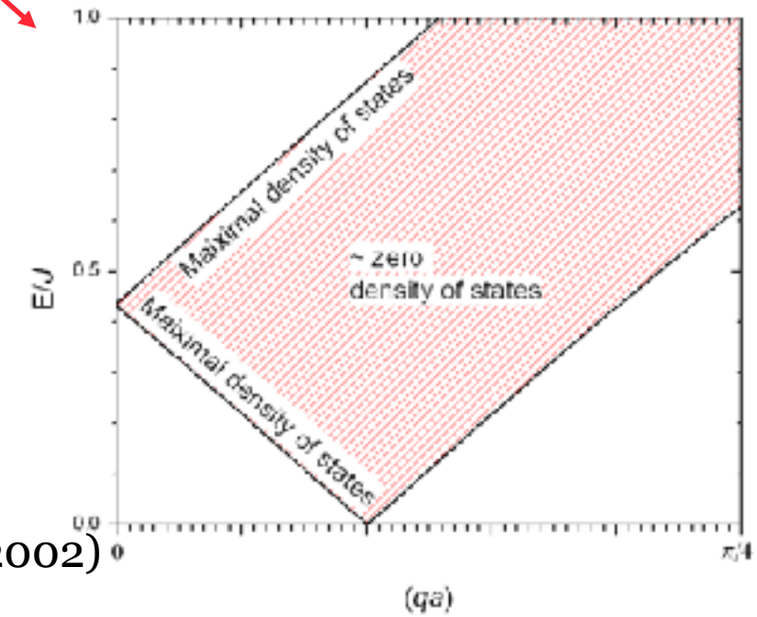


$g_a = 2.20$

Continuum in magnetic field







Dender et al, PRL 1997

Oshikawa, Affleck, PRB **65** 134410 (2002)

Explanation I: H along DM axis

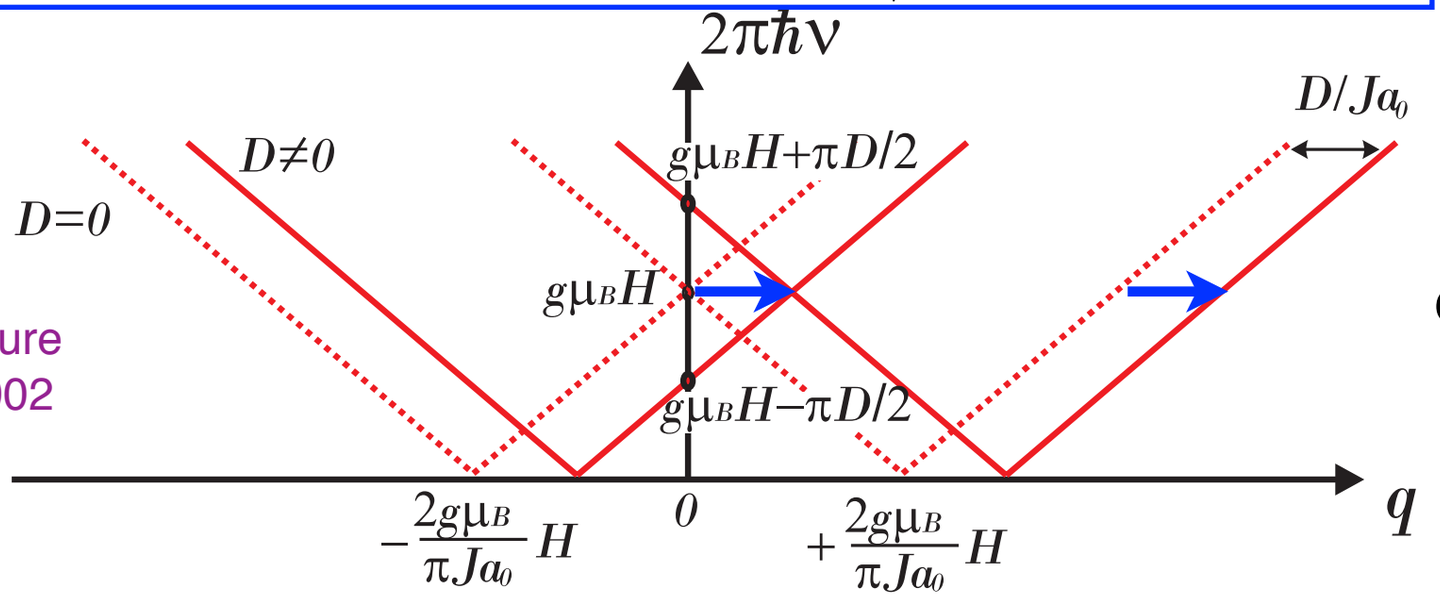
$$H = \sum_{x,y,z} \boxed{JS_{x,y,z} \cdot S_{x+1,y,z}} - \boxed{D_{y,z} \cdot S_{x,y,z} \times S_{x+1,y,z}} - g\mu_B H \cdot S_{x,y,z}$$

chain — uniform DM along the chain — magnetic field

Unitary rotation about **z**-axis $S^+(x) \rightarrow S^+(x)e^{i(D/J)x}, S^z(x) \rightarrow S^z(x)$

- removes DM term from the Hamiltonian (to D^2 accuracy)
- ***boosts*** momentum to $D/(J\,a_0)$

$$q = 0 \rightarrow q = D/(Ja_0) \Rightarrow 2\pi\hbar\nu_{R/L} = g\mu_B H \pm \pi D/2$$



dotted lines: D=0 picture
Oshikawa, Affleck 2002

rotated basis: q=0
original basis: q=D/J

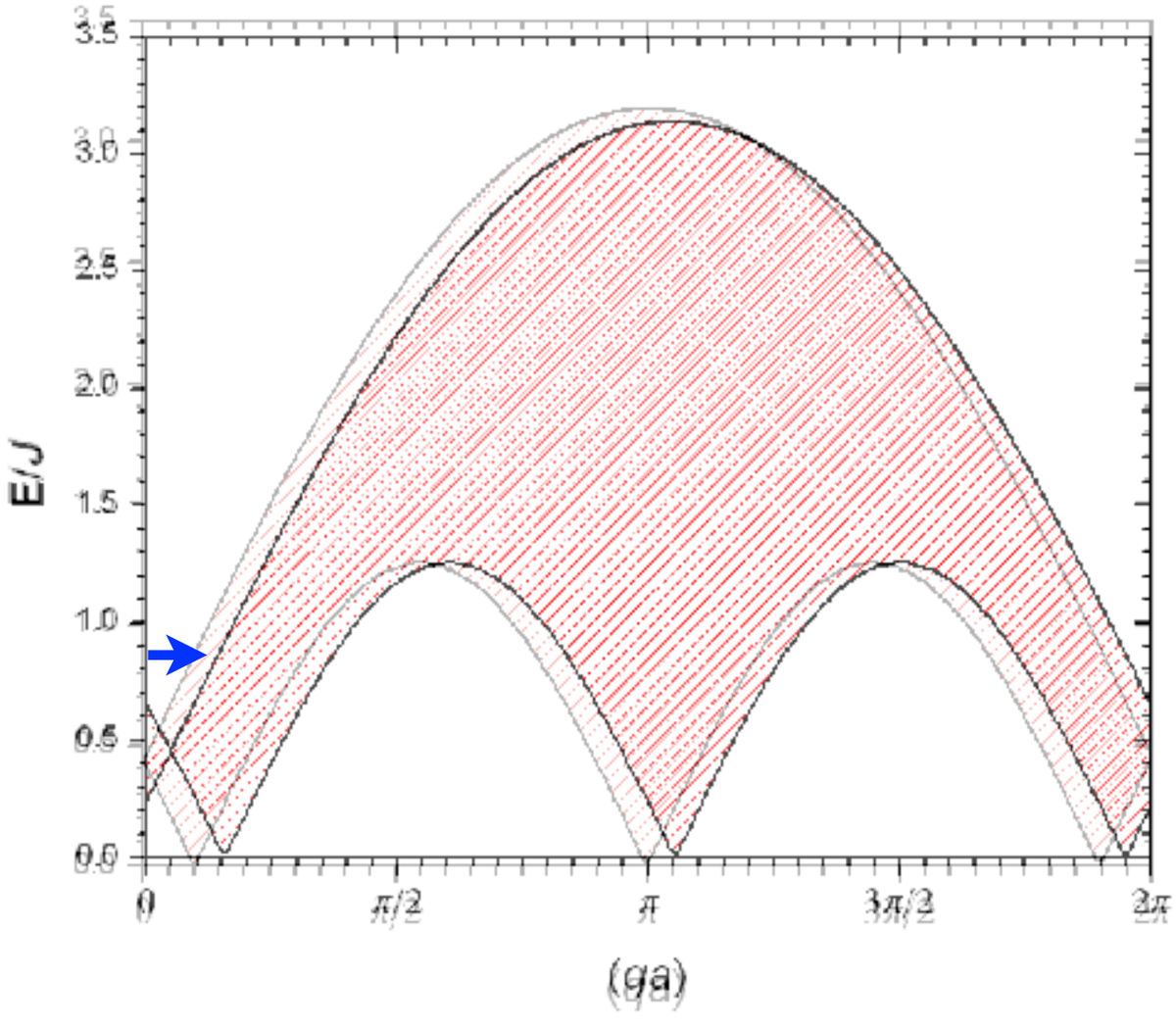
Chiral probe: **ESR probes *right*- and *left*-moving modes (spinons) independently**

Spectrum shift due to the *uniform* DM

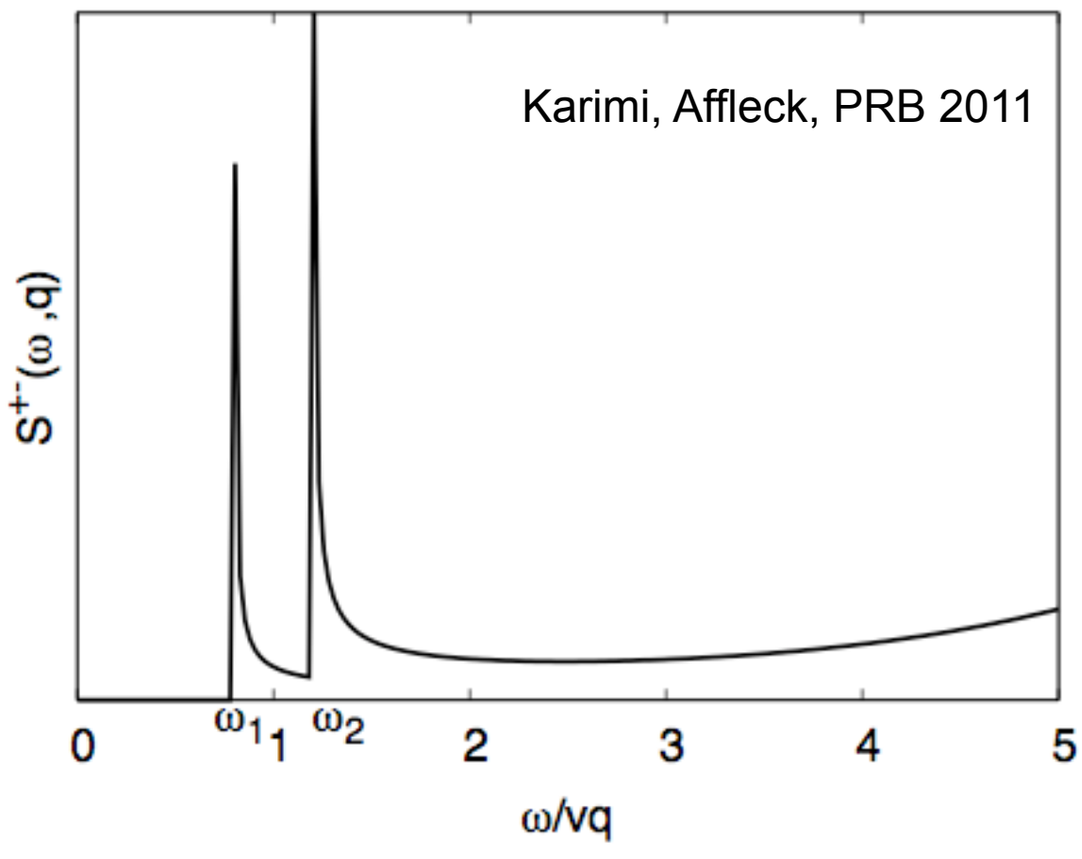
$$\mathcal{H} = \sum_n J(\mathbf{S}_n \cdot \mathbf{S}_{n+1}) + (\mathbf{D} \cdot [\mathbf{S}_n \times \mathbf{S}_{n+1}]) - \mu_B g(\mathbf{H} \cdot \mathbf{S}_n)$$

Gangadharaiah, Sun, Starykh, PRB **78** 054436 (2008)

Spin transformation $S_n^+ = \widetilde{S}_n^+ e^{i\alpha n}$ $\alpha = -D/J$ **excludes DM interaction**, but **results in the spectrum shift** by momentum *D/J*. This leads to **two** ESR peaks.



$$\mathbf{D} \parallel \mathbf{H} \parallel z$$

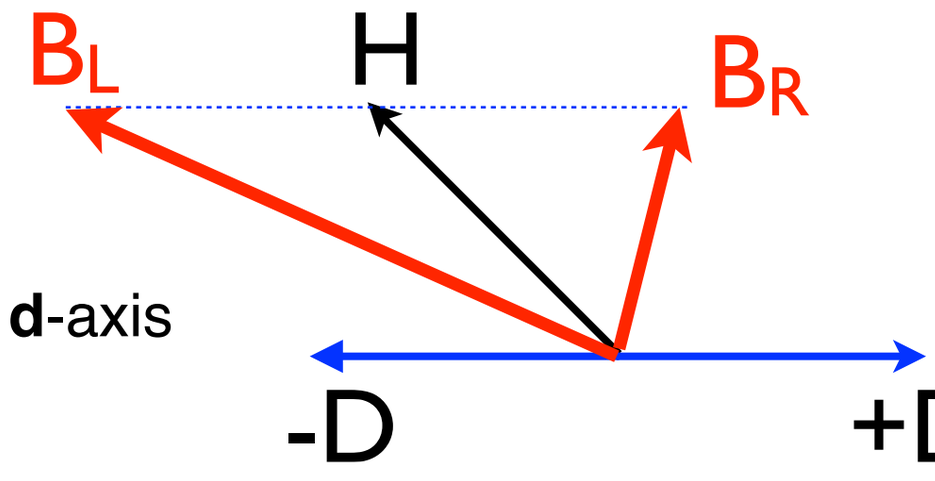


Explanation II: arbitrary orientation

Field theory in terms of spin currents **M**

$$H = \boxed{\frac{2\pi v}{3}[(\vec{M}_R)^2 + (\vec{M}_L)^2]} \quad \boxed{-\frac{vD}{J}[M_R^d - M_L^d]} \quad -g\mu_B H[M_R^z + M_L^z]$$

unperturbed chain — uniform DM along the chain — magnetic field



Uniform DM produces internal momentum-dependent magnetic field along **d**-axis

- Total field acting on right/left movers $g\mu_B\vec{H} \pm \hbar v\vec{D}/J$
- Hence ESR signals at $2\pi\hbar\nu_{R/L} = |g\mu_B\vec{H} \pm \hbar v\vec{D}/J|$
- Polarization: for H=0 maximal absorption when microwave field h_{mw} is perpendicular to the internal (DM) one. Hence **h_{mw} ll b** is most effective.

Povarov et al, PRL 2011

Gangadharaiah, Sun, OS, PRB 2008

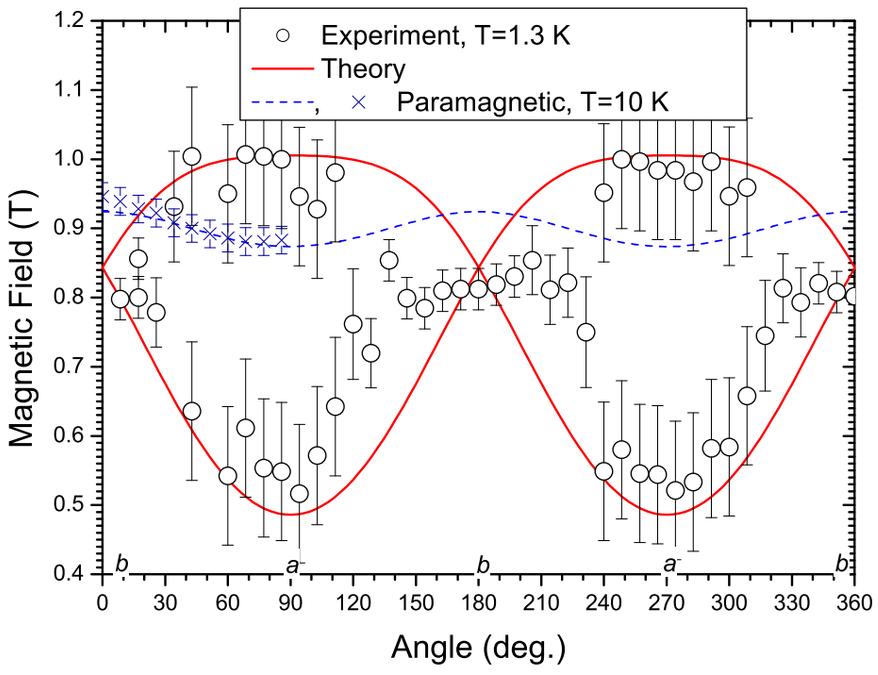
Explanation III: arbitrary orientation

- General orientation of **H** and **D**
- 4 sites/chains in unit cell

$$(2\pi\hbar\nu_R)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a + (-1)^z \pi D_a/2]^2 + [g_c\mu_B H_c + (-1)^y \pi D_c/2]^2,$$

$$(2\pi\hbar\nu_L)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a - (-1)^z \pi D_a/2]^2 + [g_c\mu_B H_c - (-1)^y \pi D_c/2]^2.$$

a-b plane



$$D_a/(4\hbar) = 8 \text{ GHz}$$
$$D_c/(4\hbar) = 11 \text{ GHz}$$

0.3 Tesla
0.4 Tesla

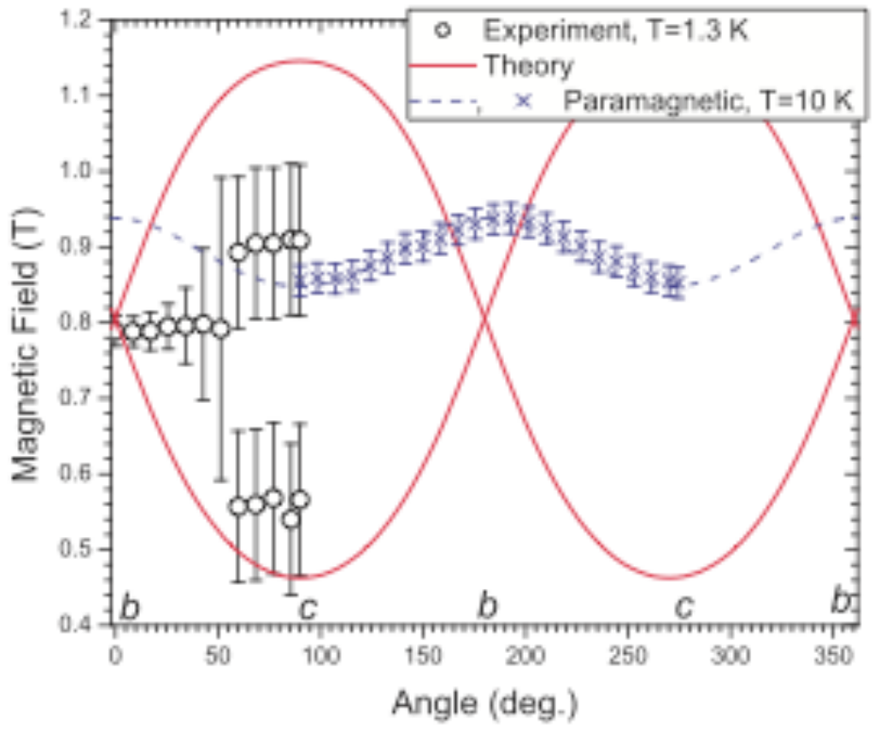
D ~ J/10

- for **H** along b-axis only: the "gap" is determined by the DM interaction strength

$$\Delta = \frac{\pi}{2}\sqrt{D_a^2 + D_c^2} \rightarrow (2\pi\hbar)13.6 \text{ GHz}$$

T-dependent line width
S. C. Furuya Phys. Rev. B 95, 014416 (2017)

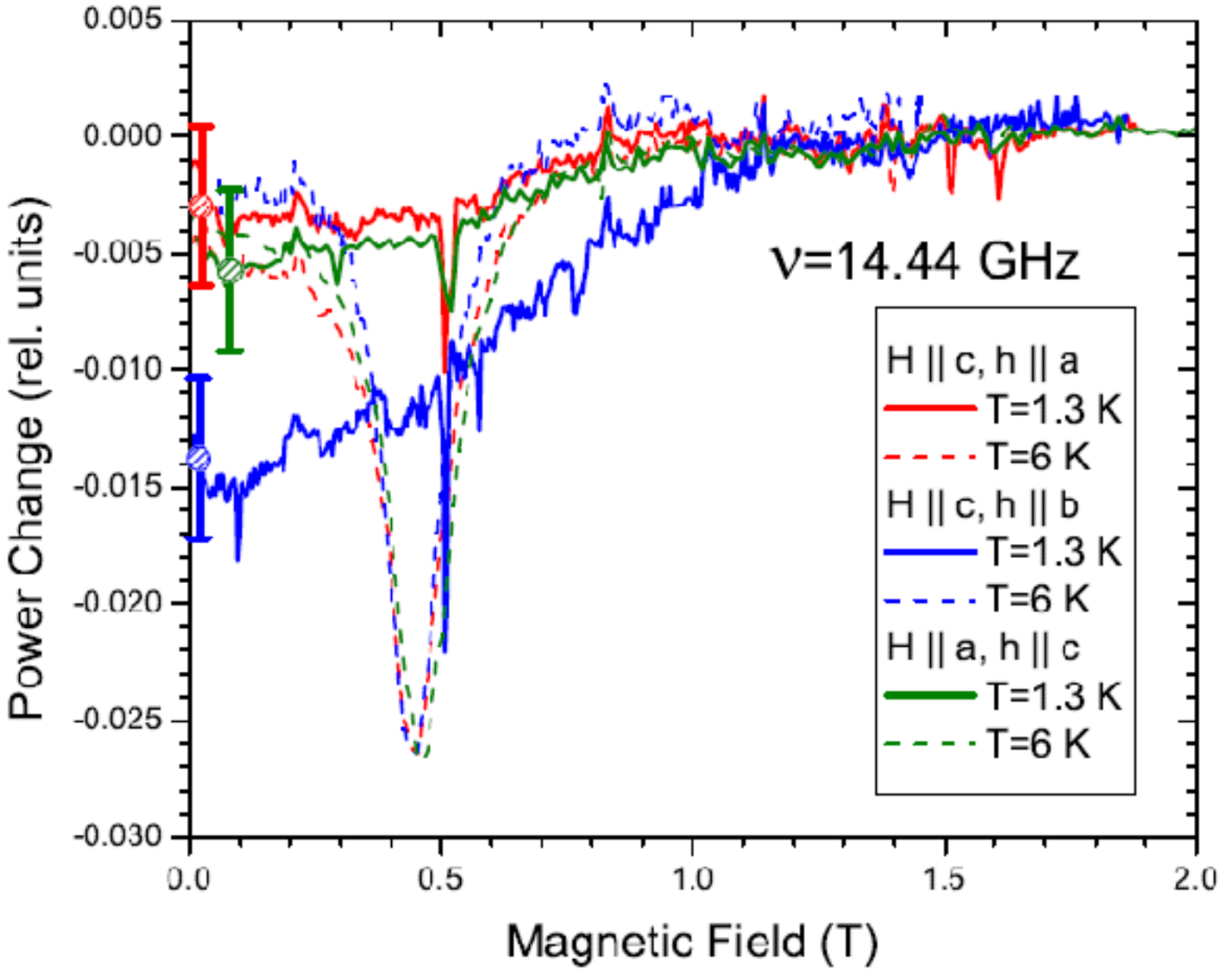
b-c plane

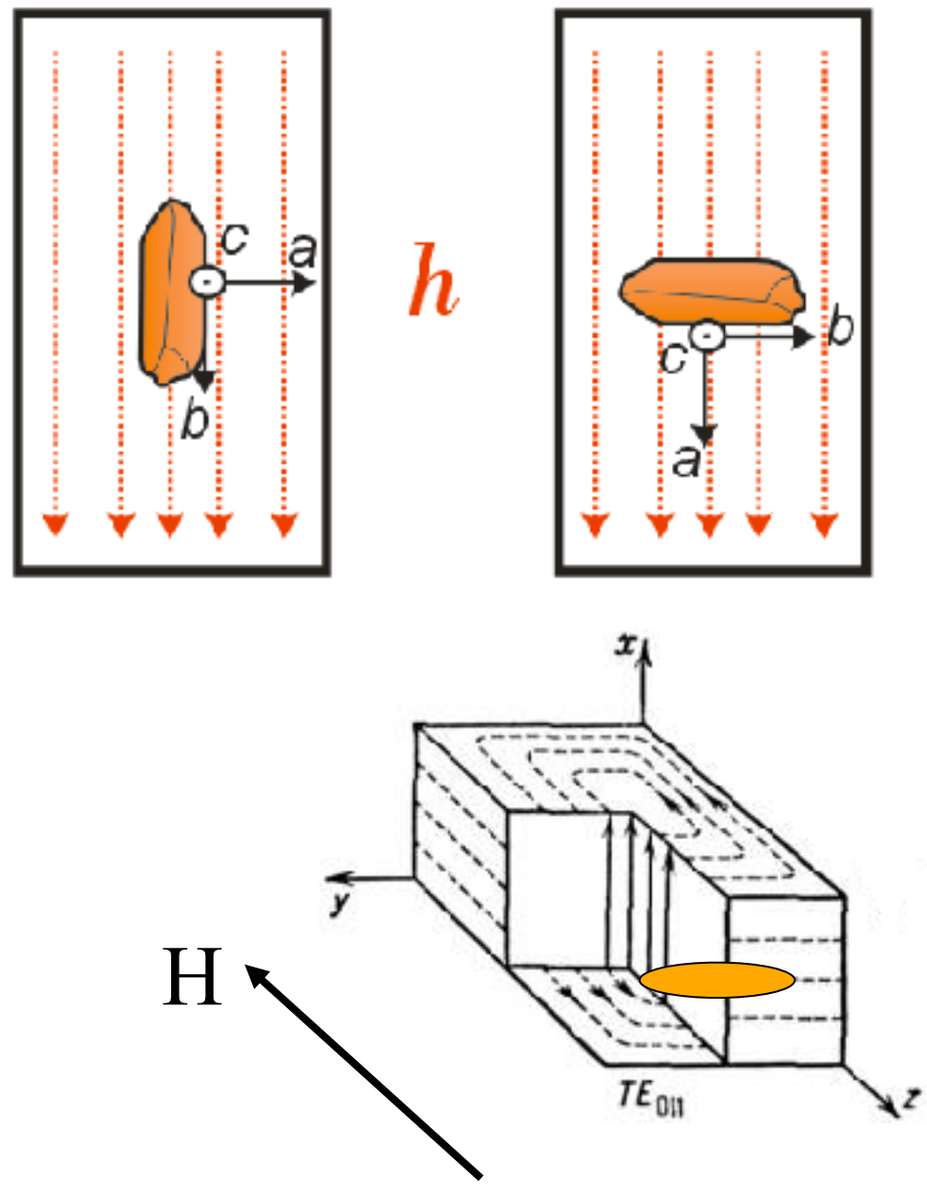


This explanation suggests:

✔ 1) ESR absorption in the absence of H $\nu \sim \sqrt{D_a^2 + D_c^2}$

✔ 2) strong polarization dependence in zero field





The largest absorption occurs when microwave field h(t) is lined along crystal b-axis, h || *b* [so that it is perpendicular to the **D** vector in *a*-*c* plane]

Extension to 2d spin liquids

Spinon magnetic resonance of quantum spin liquids

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[1] *Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA*
[2] *Department of Materials Science and Engineering, University of Utah, Salt Lake City, Utah 84112, USA*
(Dated: June 7, 2017)

We describe electron spin resonance in a quantum spin liquid with significant spin-orbit coupling. We find that the resonance directly probes spinon continuum which makes it an efficient and informative probe of exotic excitations of the spin liquid. Specifically, we consider spinon resonance of three different spinon mean-field Hamiltonians, obtained with the help of projective symmetry group analysis, which model a putative quantum spin liquid state of the triangular rare-earth antiferromagnet $YbMgGaO_4$. The band of absorption is found to be very broad and exhibit strong van Hove singularities of single spinon spectrum as well as pronounced polarization dependence.

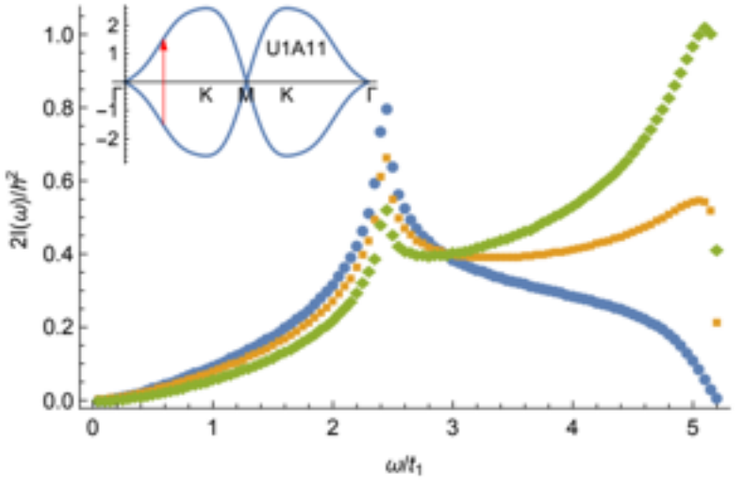


FIG. 1. (Color online) Plot of $2I(\omega)/|h|^2$ vs ω/t_1 for different polarizations $\theta = 0$ (blue dots), $\pi/4$ (orange squares), and $\pi/2$ (green rhombi) for U1A11 state. The insert shows spinon band structure along the high-symmetry path Γ-K-M-K-Γ in the Brillouin zone. Vertical red line illustrates optical transitions between spinon bands.

Outline

Spin 1/2 chains with uniform DM interaction

Electron spin resonance

Phase diagram of the quantum model with orthogonal magnetic field

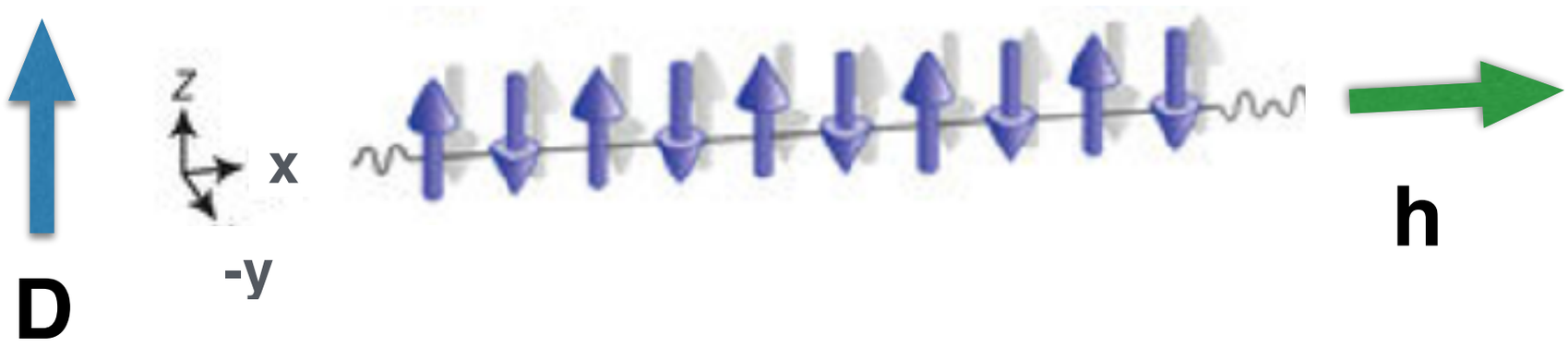
Weakly coupled chains

Conclusions

Model Hamiltonian

$$H = J\sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right] - \sum_i D\hat{z}\cdot(\mathbf{S}_i \times \mathbf{S}_{i+1}) - h\sum_i S_i^x,$$

- J > 0, the spins antiferromagnetically interact with nearest neighbor.
- The DM interaction is uniform along chain.
- The external magnetic field is perpendicular to the DM vector.
- Small anisotropy $\Delta \sim 1$.



Classical version: chiral soliton lattice

- In classical version, the spin can be treated as vectors with fixed length.

$$\mathbf{S}_i = S(\sin\theta_i \cos\phi_i, \sin\theta_i \sin\phi_i, \cos\theta_i)$$

$$H = J\sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right] - \sum_i D\hat{z}\cdot(\mathbf{S}_i \times \mathbf{S}_{i+1}) - h\sum_i S_i^x,$$

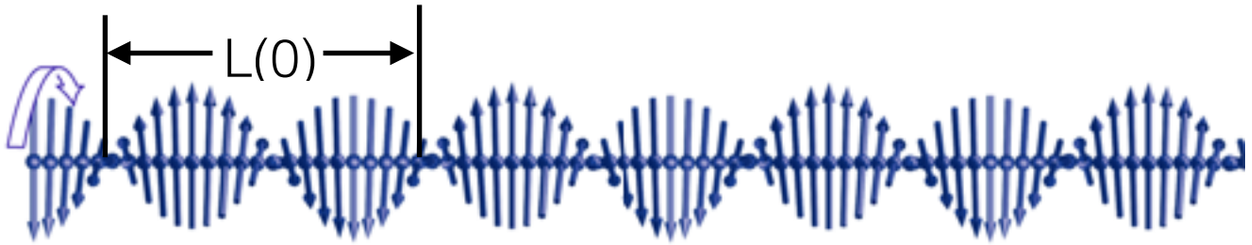
SOVIET PHYSICS JETP VOLUME 20, NUMBER 3 MARCH, 1965

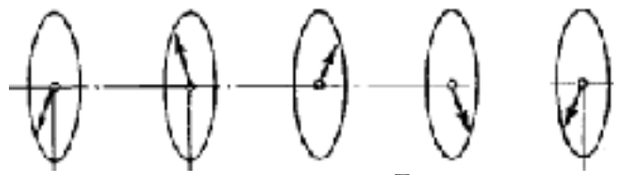
THEORY OF HELICOIDAL STRUCTURES IN ANTIFERROMAGNETS. III.

I. E. DZYALOSHINSKIĬ

h=0

helical structure

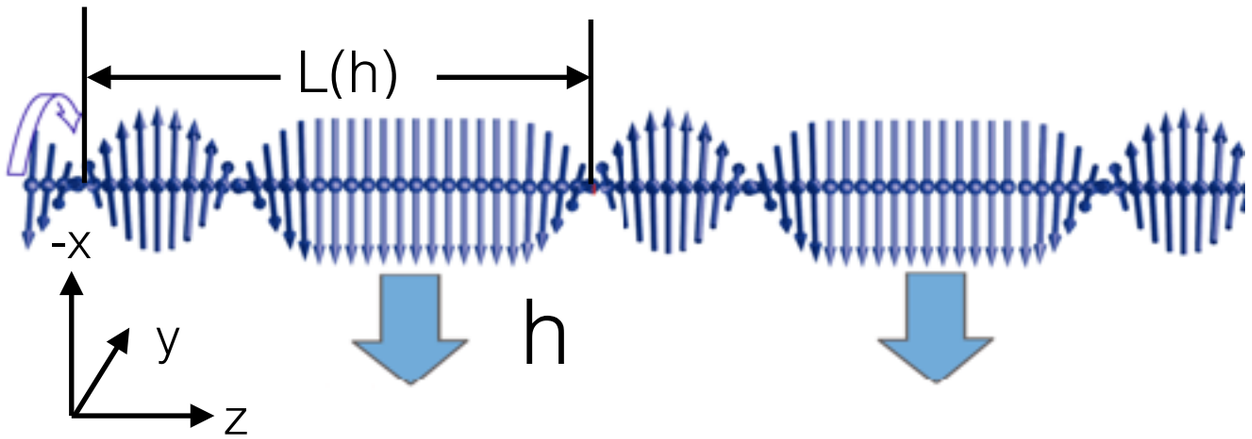


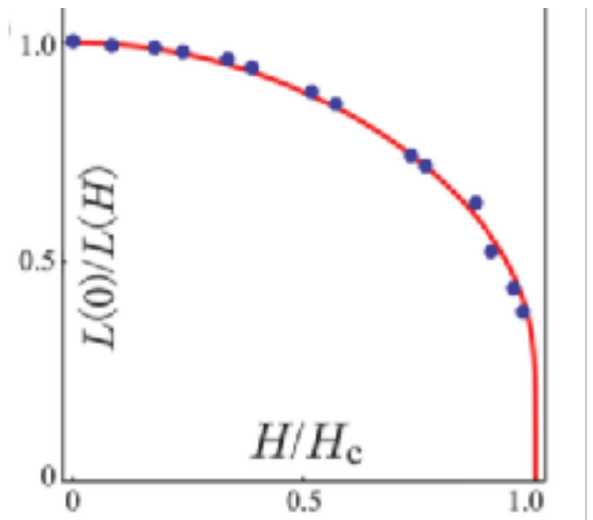


$$L(0) \sim \frac{J}{D}$$

h<hc

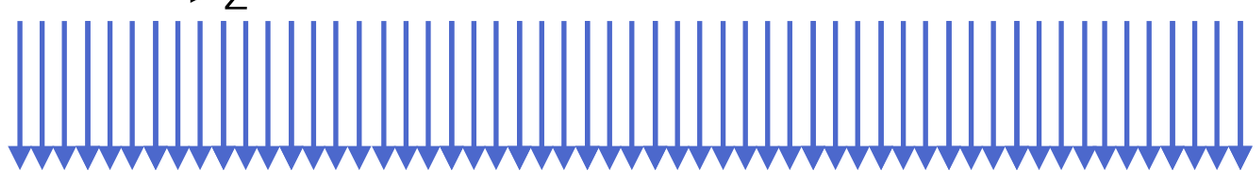
chiral soliton lattice





h≥hc

fully polarized



Togawa et al., Phys. Rev.Lett, 2012.

Quantum version

- In quantum mechanics, spins are operators satisfy commutation relation, $[S^\alpha, S^\beta] = i\epsilon_{\alpha\beta\gamma}S^\gamma$.

- Spin-1/2 operator can be mapped to fermionic one by Jordan-Wigner transformation:

$$S_j^+ = \psi_j^\dagger \exp(i\pi \sum_{k<j} \psi_k^\dagger \psi_k),$$

$$S_j^- = \exp(-i\pi \sum_{k<j} \psi_k^\dagger \psi_k)\psi_j,$$

$$S_j^z = \psi_j^\dagger \psi_j - 1/2,$$

- Spin Hamiltonian transforms to interaction fermions:

$$H = J\sum_{\langle i,j\rangle}\left[\frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + \Delta S_i^z S_j^z\right].$$

$$H_F = J\sum_{i}\left[\frac{1}{2}(\psi_i^+ \psi_{i+1} + \psi_i \psi_{i+1}^+) + \Delta(\psi_i^\dagger \psi_i - \frac{1}{2})(\psi_{i+1}^\dagger \psi_{i+1} - \frac{1}{2})\right].$$

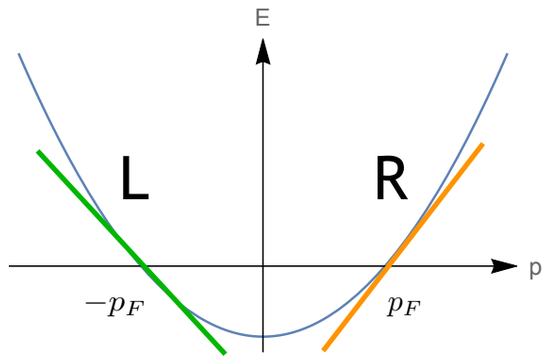
Bosonization

- Aa low-energy effective field theory, and non-perturbative approach.
  - Jordan-Wigner transformation, $S_j^z = \psi_j^+ \psi_j - 1/2.$
  - Spin operator coincides with density operator for fermions.
  - Fermions with linearized spectrum near two Fermi points,

$$\psi(x) = R(x)e^{ip_F x} + L(x)e^{-ip_F x}$$

  - Low-energy excitations:
    described by Fourier components of the density operator

$$\rho_q = \sum_p \psi_p^\dagger \psi_{p+q} \quad \text{with q close to 0 and } \pm 2p_F \qquad (2p_F = \pi).$$



$E(p + p_F) \simeq vp,$
$E(p - p_F) \simeq= -vp,$
$|p| \ll p_F.$

- Spin operator in uniform spin current $J_{R/L}$ and staggered magnetization N,

$$\boldsymbol{S}(x) \to \boldsymbol{J}_L(x) + \boldsymbol{J}_R(x) + (-1)^{x/a}\boldsymbol{N}(x),$$ *a* is lattice spacing

with $J_L(x) = L^+(x)L(x), \quad J_R(x) = R^+(x)R(x), \quad N(x) = R^+(x)L(x) + L^+(x)R(x).$

Here v is Fermi velocity.

Hamiltonian in terms of bosonization

- Low-energy theory $\boldsymbol{S}(x) \to \boldsymbol{J}_L(x) + \boldsymbol{J}_R(x) + (-1)^{x/a}\boldsymbol{N}(x),$

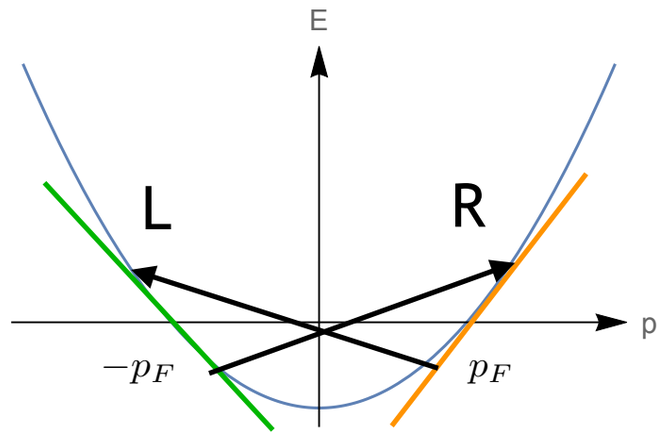
$$H = H_0 + V + H_{\rm bs}, \quad H = J\sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right] - \sum_i D\hat{z}\cdot(\mathbf{S}_i \times \mathbf{S}_{i+1}) - h\sum_i S_i^x,$$

- Isotropic Heisenberg chain

$$H_0 = \frac{2\pi v}{3}\int dx(\mathbf{J}_R\cdot\mathbf{J}_R + \mathbf{J}_L\cdot\mathbf{J}_L),$$

- V contains DM interaction and Zeeman term

$$V = -D\int dx(J_R^z - J_L^z) - h\int dx(J_R^x + J_L^x),$$



- Back-scattering interaction between right- and left- spin current.

$$H_{\rm bs} = -g_{\rm bs}\int dx[J_R^x J_L^x + J_R^y J_L^y + (1+\lambda)J_R^z J_L^z],$$

with coupling constant $g_{\rm bs} \approx 0.23\times(2\pi v),$ Garate and Affleck (2010)

Exchange and DM-induced anisotropy: $\lambda = c(1 - \Delta + \frac{D^2}{2J^2}).$

Chiral rotation

- Perform a chiral rotation $\mathbf{J}_{R/L} = \mathcal{R}(\theta_{R/L})\mathbf{M}_{R/L}$,

$$\mathcal{R}(\theta_{R/L}) = \begin{pmatrix} \cos\theta_{R/L} & 0 & \sin\theta_{R/L} \\ 0 & 1 & 0 \\ -\sin\theta_{R/L} & 0 & \cos\theta_{R/L}. \end{pmatrix}, \qquad \begin{aligned} \theta_R &= \theta_0 + \pi/2, \\ \theta_L &= -\theta_0 + \pi/2, \\ \theta_0 &\equiv \arctan\left(\frac{-D}{h}\right). \end{aligned}$$

- Simplified Hamiltonian,

$$H = H_0 + V + H_{\rm bs}, \rightarrow H = \tilde{H}_0 + \tilde{H}_{\rm bs},$$

$$\tilde{H}_{\rm bs} = H_A + H_B + H_C + H_\sigma,$$

Oscillating factors

$$H_A = \pi v y_A \int \mathrm{d}x (M_R^z M_L^+ e^{it_\varphi x} - M_R^+ M_L^z e^{-it_\varphi x} + \mathrm{h.c.}),$$

$$H_B = \pi v y_B \int \mathrm{d}x (M_R^+ M_L^- e^{-i2t_\varphi x} + \mathrm{h.c.}),$$

$$H_C = \pi v y_C \int \mathrm{d}x (M_R^+ M_L^+ + \mathrm{h.c.}),$$

$$H_\sigma = -2\pi v y_\sigma \int \mathrm{d}x M_R^z M_L^z.$$

$$t_\varphi \equiv \frac{\sqrt{D^2 + h^2}}{v}$$

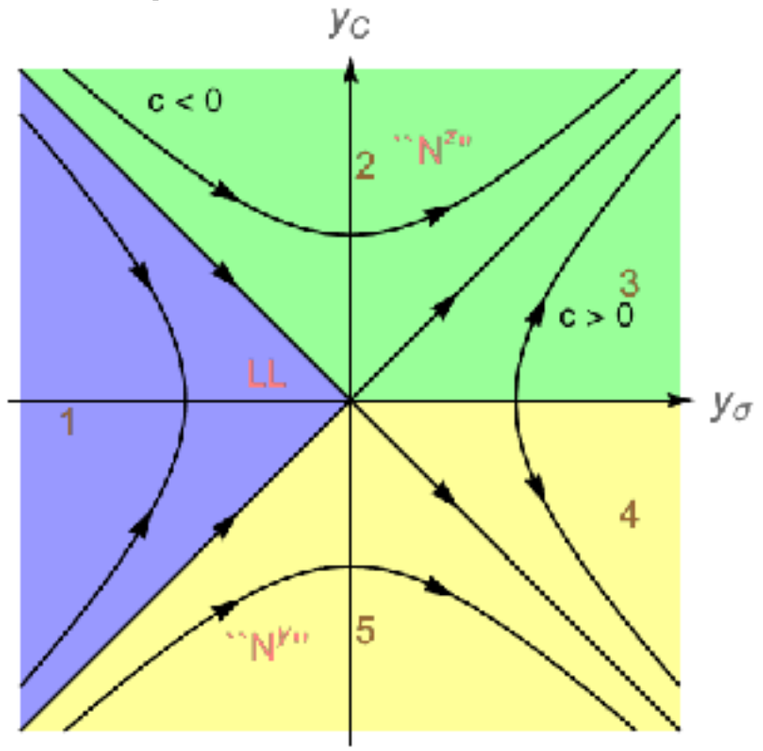
Renormalization group (RG) analysis

- RG equations are well-known Kosterlitz-Thouless equations,

$$\frac{dy_C}{d\ell} = y_C y_\sigma, \quad \frac{dy_\sigma}{d\ell} = y_C^2.$$

$$H_C = \pi v y_C \int \mathrm{d}x (M_R^+ M_L^+ + \mathrm{h.c.}),$$

$$H_\sigma = -2\pi v y_\sigma \int \mathrm{d}x M_R^z M_L^z.$$



- Ground state is determined by the initial values.

$$y_C(0) \propto [(1+\frac{\lambda}{2})\cos\theta^- - 1 + \frac{\lambda}{2}],$$

$$y_\sigma(0) \propto [(1+\frac{\lambda}{2})\cos\theta^- - \frac{\lambda}{2}],$$

$$C = y_\sigma^2(0) - y_C^2(0).$$

- The initial values are determined by ratio D/h, and also influenced by anisotropy.

$$\cos\theta^- = (h^2 - D^2)/(h^2 + D^2)$$

$$\lambda = c(1 - \Delta + \frac{D^2}{2J^2}).$$

Region	1	2	3	4	5
$y_C(0)$	$+/-$	$+$	$+$	$-$	$-$
$y_\sigma(0)$	$-$	$-/+$	$+$	$+$	$-/+$
C	$+$	$-$	$+$	$+$	$-$
$y_C(\ell^*)$	0	$+\infty$	$+\infty$	$-\infty$	$-\infty$
$y_\sigma(\ell^*)$	finite	$+\infty$	$+\infty$	$+\infty$	$+\infty$
$y_B(\ell^*)$	finite	finite	finite	finite	finite
State	LL	"N^z"	"N^z"	"N^y"	"N^y"

$\Delta - h/D$ phase diagram

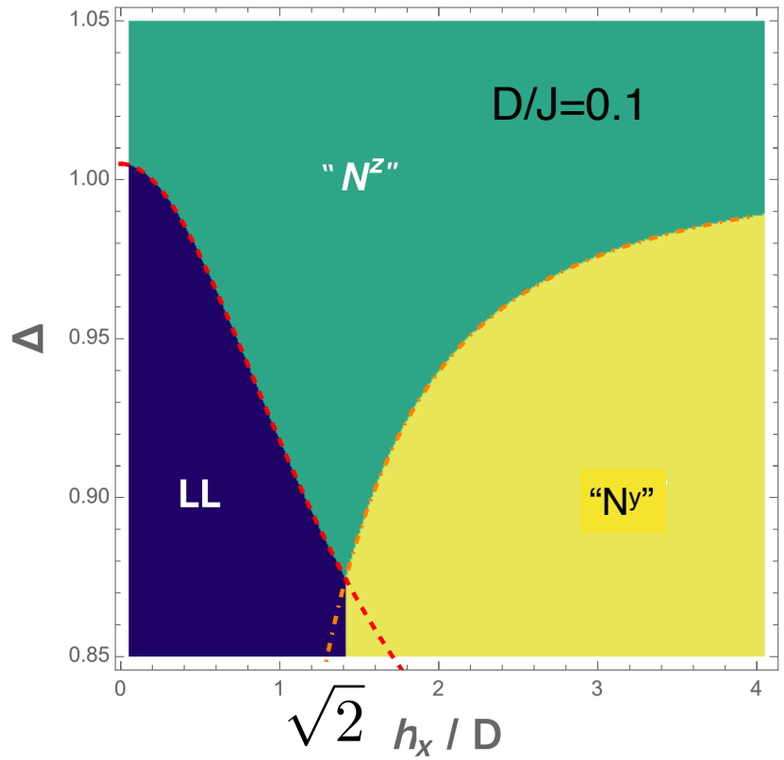
- There are three distinct phases.

$$\langle \mathbf{S}(x) \rangle = M\hat{\mathbf{x}} + (-1)^x \langle \mathbf{N}(x) \rangle.$$

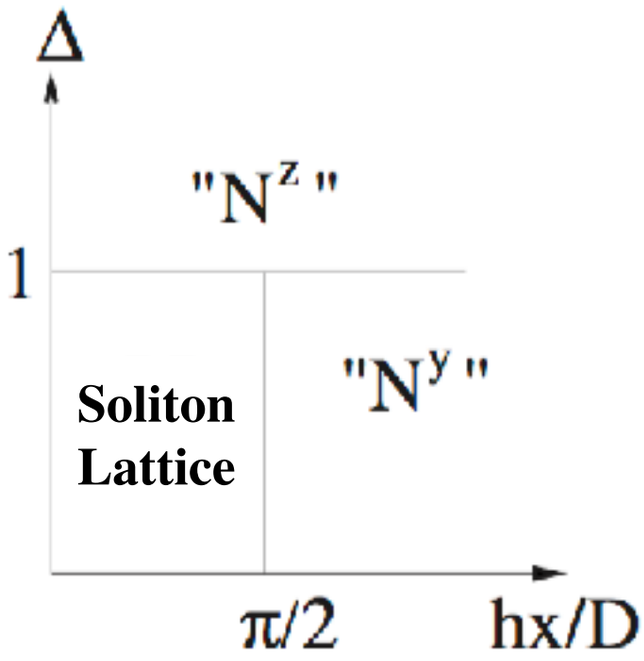
  - "N^z": $\langle \mathbf{N}(x) \rangle \propto (-1)^{k_1+1}\mathbf{z}.$
  - "N^y": $\langle \mathbf{N}(x) \rangle \propto \cos\theta_0 (-1)^{k_2}\mathbf{y},$
  - Luttinger-liquid (LL): $\langle \mathbf{N}(x) \rangle = 0,$ $\langle \mathbf{N}^a(x)\mathbf{N}^a(0) \rangle \sim \frac{C^a}{x^\beta}.$

- Phase boundaries.
  - "N^z"-"N^y": $\Delta_{c1} = 1 + \frac{1}{2}(\frac{D}{J})^2 - \frac{2}{c}(\frac{D}{h})^2.$
  - "N^z"-LL: $\Delta_{c2} = 1 + \frac{1}{2}(\frac{D}{J})^2 - \frac{2}{c}\frac{1}{1+2(D/h)^2}.$
  - "N^z"-LL: $\Delta_{c3} = \sqrt{2}.$

- Compared to classical phase diagram, LL replaces soliton lattice.



Quantum phase diagram

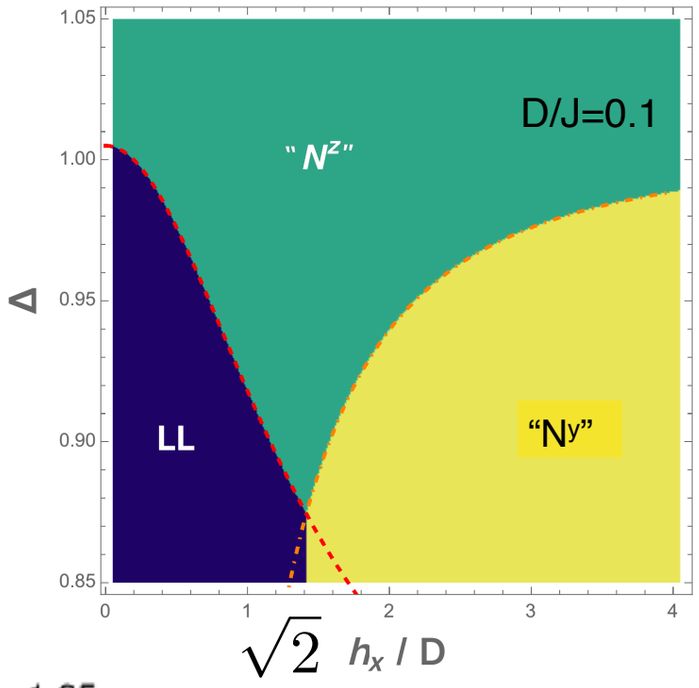


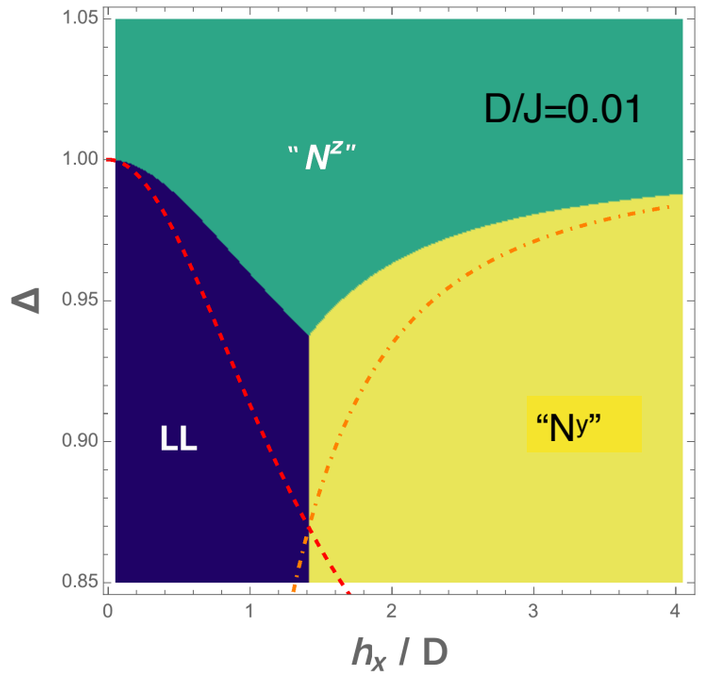
Classical phase diagram

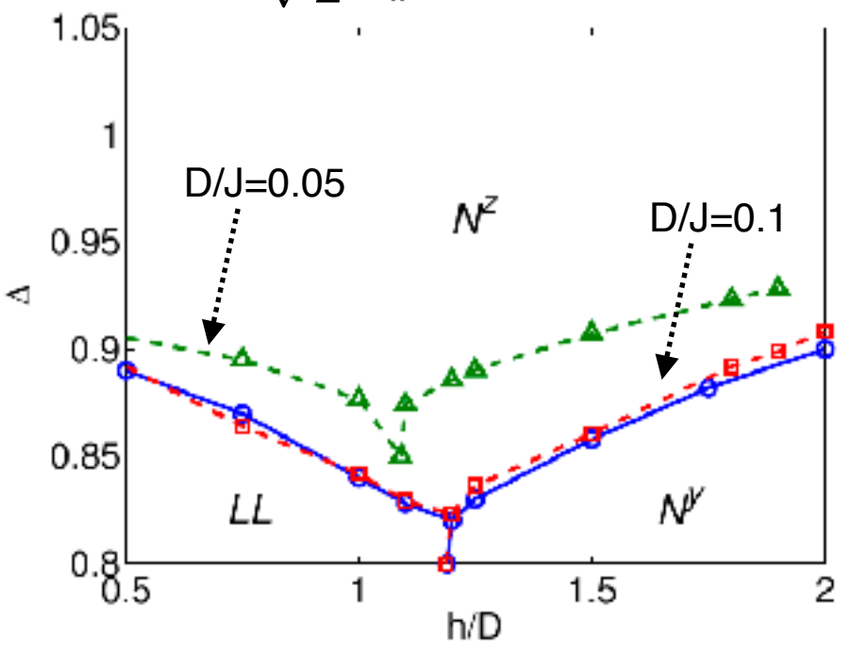
Garate and Affleck (2010)

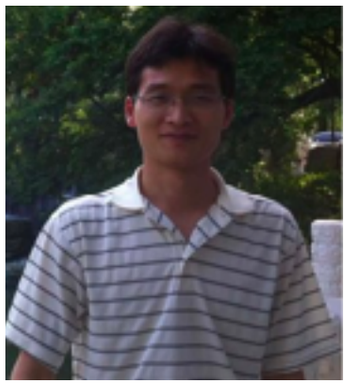
$\Delta - h/D$ phase diagram

- Large DM interaction promotes the "N^z" state.
- Phase boundary between LL and "N^y" is independent of DM interaction.
- Phase diagrams are verified by numerical study (DMRG and iTEBD).









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Outline

Spin 1/2 chains with uniform DM interaction

Electron spin resonance

Phase diagram of the quantum model with orthogonal magnetic field

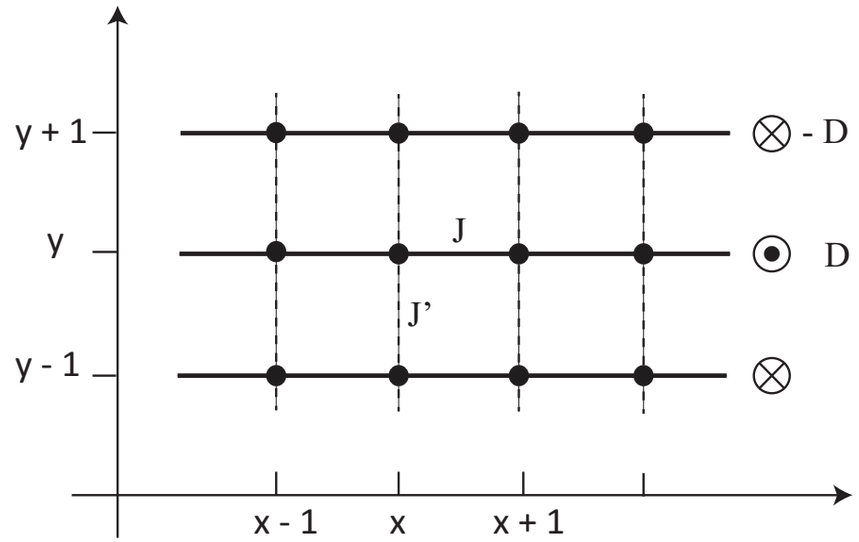
Weakly coupled chains

Conclusions

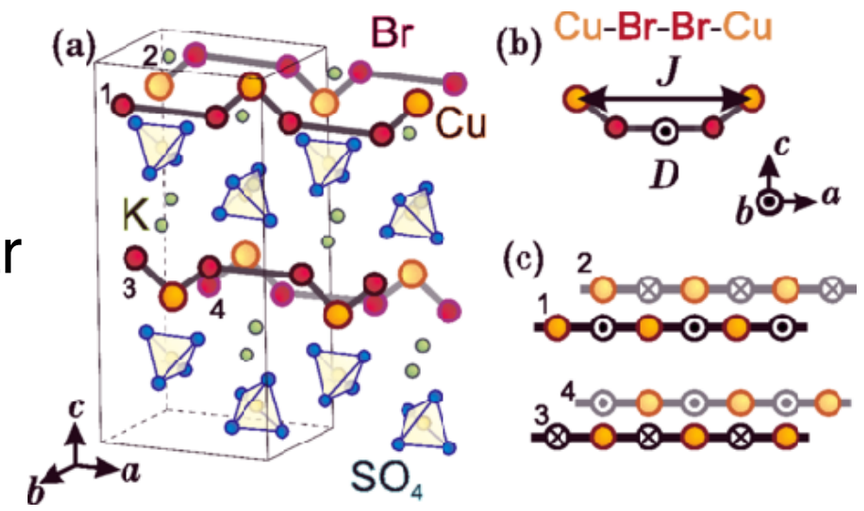
$h \perp D$

Model Hamiltonian

$$H = \sum_{x,y} [J\mathbf{S}_{x,y} \cdot \mathbf{S}_{x+1,y} + J'\mathbf{S}_{x,y} \cdot \mathbf{S}_{x,y+1}]$$
$$+ \mathbf{D} \cdot \sum_{x,y} (-1)^y \mathbf{S}_{x,y} \times \mathbf{S}_{x+1,y} - \mathbf{h} \cdot \sum_{x,y} \mathbf{S}_{x,y},$$



- J > 0. Also J' >0, the interchain exchange interaction, and J'<< J.
- The DM interaction is uniform along chain, but antiparallel in neighboring chains.
- Real magnetic insulator $K_2CuSO_4(Br/Cl)_2$.
- There is no exchange anisotropy, $\Delta = 1$.
- Here we only consider magnetic field perpendicular to DM interaction.



Intra-chain exchange J=20.5K,
Inter-chain exchange J'=0.03K,
DM interaction D=0.28K.

Smirnov et al., Phys. Rev. B, 92,134417, 2015

Hamiltonian in terms of bosonization

- Low-energy theory $\mathbf{S}_{x,y} \to \mathbf{J}_{yL}(x) + \mathbf{J}_{yR}(x) + (-1)^{x/a}\mathbf{N}_y(x),$

$$\mathcal{H} = \sum_y [\mathcal{H}_0 + \mathcal{V} + \mathcal{H}_{\rm bs} + \mathcal{H}_{\rm inter}], \qquad \mathcal{H} = \sum_{x,y} [J\mathbf{S}_{x,y}\cdot\mathbf{S}_{x+1,y} + J'\mathbf{S}_{x,y}\cdot\mathbf{S}_{x,y+1}] + \mathbf{D}\cdot\sum_{x,y}(-1)^y \mathbf{S}_{x,y}\times\mathbf{S}_{x+1,y} - \mathbf{h}\cdot\sum_{x,y}\mathbf{S}_{x,y},$$

$$\mathcal{H}_0 = \frac{2\pi v}{3}\int \mathrm{d}x(\mathbf{J}_{yR}\cdot\mathbf{J}_{yR} + \mathbf{J}_{yL}\cdot\mathbf{J}_{yL}),$$

$$\mathcal{V} = -h_z\int \mathrm{d}x(J^z_{yR} + J^z_{yL}) - h_x\int \mathrm{d}x(J^x_{yR} + J^x_{yL}) + (-1)^y\tilde{D}\int \mathrm{d}x(J^z_{yR} - J^z_{yL}),$$

Isotropic Heisenberg chain.

Contains DM interaction and Zeeman term.

$$\mathcal{H}_{\rm bs} = -g_{\rm bs}\int \mathrm{d}x[J^x_{yR}J^x_{yL} + J^y_{yR}J^y_{yL} + (1+\lambda)J^z_{yR}J^z_{yL}]$$

$$\mathcal{H}_{\rm inter} = J'\int \mathrm{d}x\mathbf{N}_y\cdot\mathbf{N}_{y+1},$$

$\mathbf{S}_{x,y}\cdot\mathbf{S}_{x,y+1} \to \mathbf{N}_y(x)\cdot\mathbf{N}_{y+1}(x)$

Back-scattering interaction between right- and left- spin current.

Most relevant part of interchain coupling.

DM-induced anisotropy $\lambda = c'\frac{D^2}{J^2}$, where $c' = (\frac{2\sqrt{2}v}{g_{\rm bs}})^2 \approx 3.83.$ Garate and Affleck (2010)

Ground state is determined by $\mathcal{H}_{\rm inter}$, which is strongly affected by $\mathcal{H}_{\rm bs}$.

$h \perp D$ Hamiltonian in terms of bosonization

- Perform a chiral rotation $\mathbf{J}_{R/L} = \mathcal{R}(\theta_{R/L})\mathbf{M}_{R/L},$

$$\mathcal{R}(\theta_{R/L}) = \begin{pmatrix} \cos\theta_{R/L} & 0 & \sin\theta_{R/L} \\ 0 & 1 & 0 \\ -\sin\theta_{R/L} & 0 & \cos\theta_{R/L}. \end{pmatrix}, \quad \begin{aligned} \theta_R &= \frac{\pi}{2} + \theta_0^y, \\ \theta_L &= \frac{\pi}{2} - \theta_0^y, \\ \theta_0^y &\equiv (-1)^y \tan^{-1}[\frac{D}{h}]. \end{aligned}$$

- Simplified Hamiltonian,

$$\mathcal{H} = \sum_y [\mathcal{H}_0 + \mathcal{V} + \mathcal{H}_{\rm bs} + \mathcal{H}_{\rm inter}] \to \mathcal{H} = \sum_y \left[\tilde{\mathcal{H}}_0 + \tilde{\mathcal{H}}_{\rm bs} + \tilde{\mathcal{H}}_{\rm inter}\right]$$

$$\tilde{H}_{\rm bs} = H_A + H_B + H_C + H_\sigma,$$

$$H_A = \pi v y_A \int \mathrm{d}x (M_R^z M_L^+ e^{it_\varphi x} - M_R^+ M_L^z e^{-it_\varphi x} + \mathrm{h.c.}),$$

$$H_B = \pi v y_B \int \mathrm{d}x (M_R^+ M_L^- e^{-i2t_\varphi x} + \mathrm{h.c.}),$$

$$H_C = \pi v y_C \int \mathrm{d}x (M_R^+ M_L^+ + \mathrm{h.c.}),$$

$$H_\sigma = -2\pi v y_\sigma \int \mathrm{d}x M_R^z M_L^z. \qquad t_\varphi \equiv \frac{\sqrt{D^2+h^2}}{v}$$

$$\tilde{\mathcal{H}}_{\rm inter} = H_x + H_y + H_\varphi$$

$$H_x = 2\pi v g_x \int dx \mathcal{N}_y^x \mathcal{N}_{y+1}^x,$$

$$H_y = 2\pi v g_y \int dx \mathcal{N}_y^y \mathcal{N}_{y+1}^y,$$

$$H_{{\rm inter},\varphi} \propto \; g_{\varphi_1} \int dx \cos[\sqrt{2\pi}(\varphi_y - \varphi_{y+1})].$$

$h \perp D$

RG equations

- Backscattering RG equations,

$$\frac{dy_C}{dl} = y_C y_\sigma, \quad \frac{dy_\sigma}{dl} = y_C^2,$$

$$H_C = \pi v y_C \int \mathrm{d}x (M_R^+ M_L^+ + \mathrm{h.c.}),$$

$$H_\sigma = -2\pi v y_\sigma \int \mathrm{d}x M_R^z M_L^z.$$

- Interchain interaction RG equations,

$$\frac{dg_x}{dl} = g_x(1 + y_C + \frac{1}{2}y_\sigma),$$
$$\frac{dg_y}{dl} = g_y(1 - y_C + \frac{1}{2}y_\sigma),$$
$$\frac{dg_{\varphi_1}}{dl} = g_{\varphi_1}(1 - \frac{1}{2}y_\sigma).$$

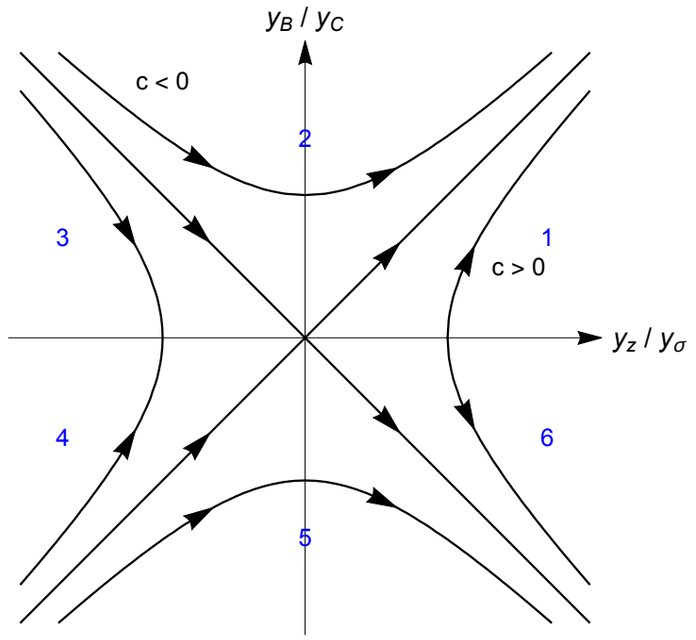
$$\rightarrow g(l) \propto g(0)e^l$$

Interaction term	Coupling operator	Coupling constant	Induced state
$\mathcal{H}_x$	$\mathcal{N}_y^x \mathcal{N}_{y+1}^x$	g_x	SDW(z)
$\mathcal{H}_y$	$\mathcal{N}_y^y \mathcal{N}_{y+1}^y$	g_y	SDW (y)
$\mathcal{H}_{\mathrm{inter},\varphi}$	$\cos[\sqrt{2\pi}(\varphi_y - \varphi_{y+1})]$	g_{φ_1}	Distorted-cone

- Ground state is determined by the fasted growing *g*,

$$y_C(0) \propto [(1 + \frac{\lambda}{2})\cos\theta^- - 1 + \frac{\lambda}{2}],$$
$$y_\sigma(0) \propto [(1 + \frac{\lambda}{2})\cos\theta^- - \frac{\lambda}{2}],$$
$$C = y_\sigma^2(0) - y_C^2(0).$$

θ^- is function of h and D.



$h \perp D$ $\lambda - (h/D)$ phase diagram

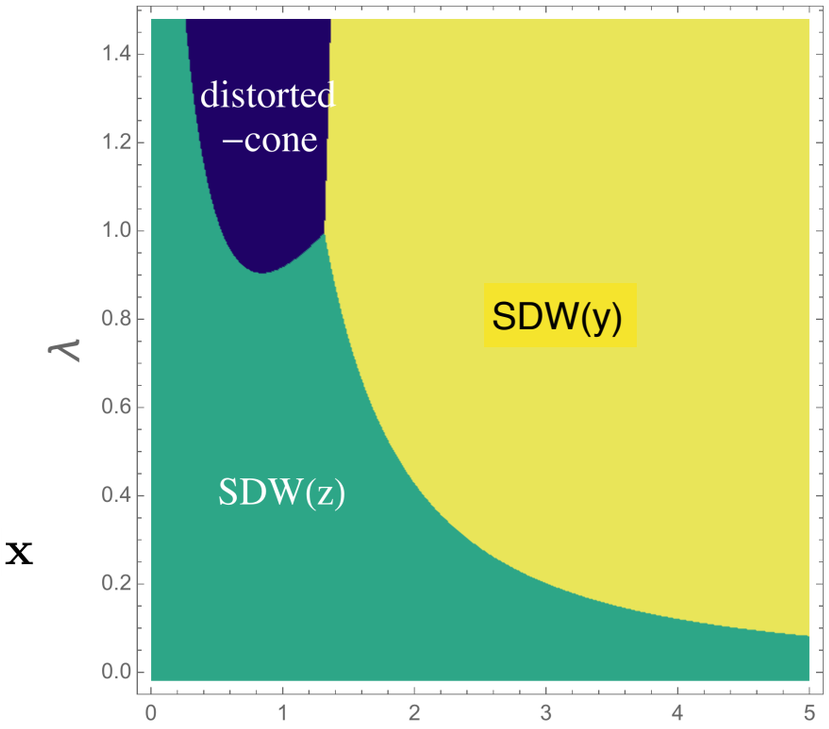
- There are three distinct phases.

- SDW(z): $\langle \mathbf{S}_{x,y} \rangle \sim M\mathbf{x} + (-1)^{x+y} \Psi_{\mathrm{sdw(z)}} \mathbf{z},$

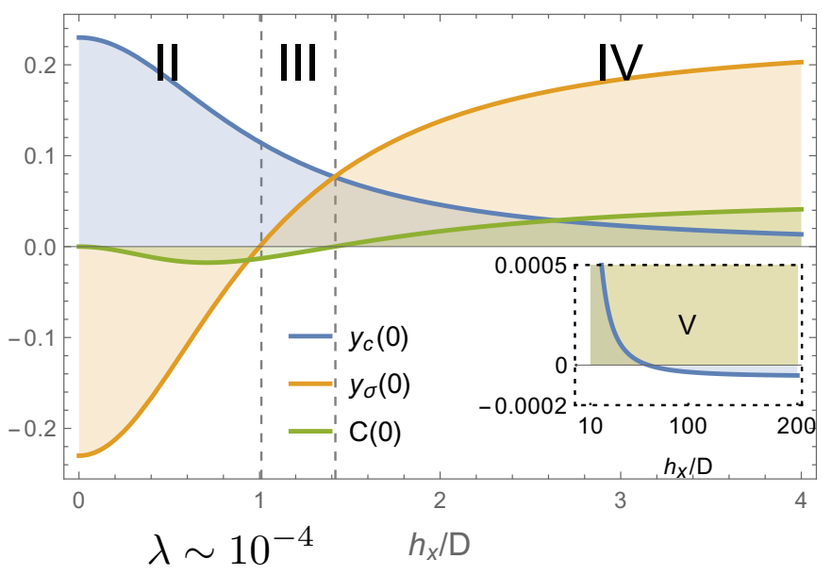
- SDW(y): $\langle \mathbf{S}_{x,y} \rangle \sim M\mathbf{x} + (-1)^{x+y} \dfrac{h}{\sqrt{h^2 + D^2}} \Psi_{\mathrm{sdw(y)}} \mathbf{y}.$

- distorted-cone:

$$\langle \mathbf{S}_{x,y} \rangle \sim M\mathbf{x} + (-1)^{x+y} \Psi_{\mathrm{dist-cone}} \Big(\sin[\sqrt{2\pi}\hat{\varphi} + t_\varphi x]\mathbf{x} - \frac{(-1)^y D}{\sqrt{h^2 + D^2}} \cos[\sqrt{2\pi}\hat{\varphi} + t_\varphi x]\mathbf{y} \Big).$$

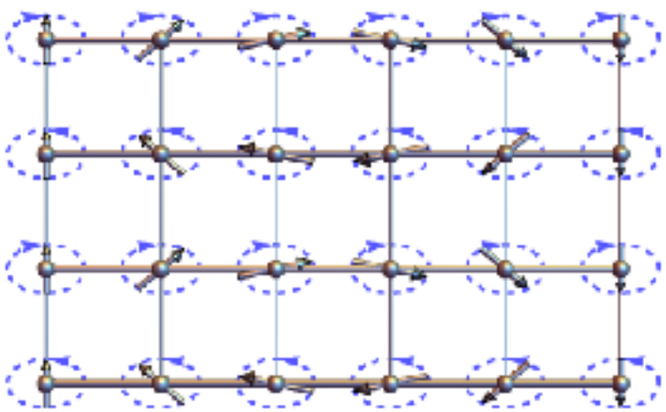


- Distinguish strong coupling by the initial values of y_C and y_σ.



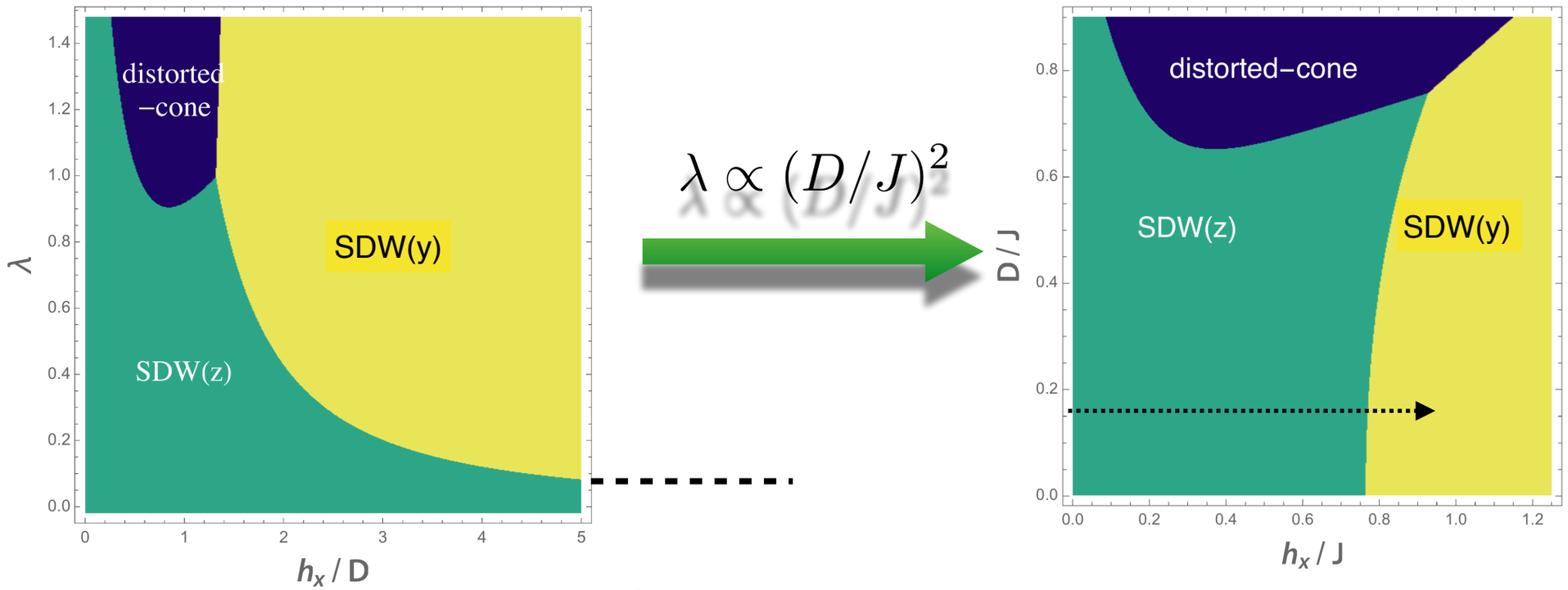
Region	I	II	III	IV	V
$y_C(0)$	+	+	+	+	−
$y_\sigma(0)$	−	−	+	+	+
C	+	−	−	+	+
Fastest growing	g_{φ_1}		g_x		g_y

distorted-cone



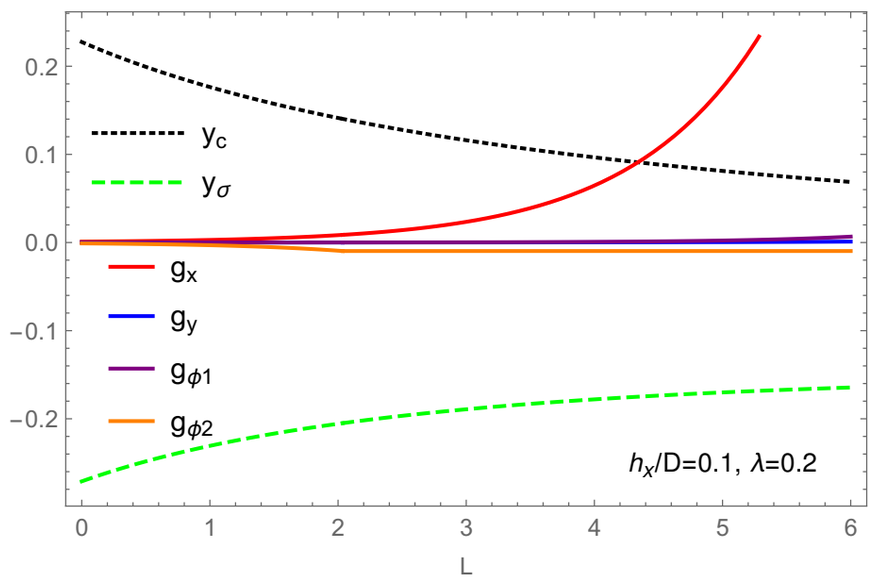
$\langle \boldsymbol{N} \rangle$ in $x - y$ plane

$h \perp D$ $D-h$ phase diagram



real material

- Phase boundary between two SDWs is independent of D, when D/J is small.
- Phase boundary between SDW(y) and distorted-cone is linear with h/D~1.5.
- Distorted-cone is unlikely to realize in real materials with small D/J.

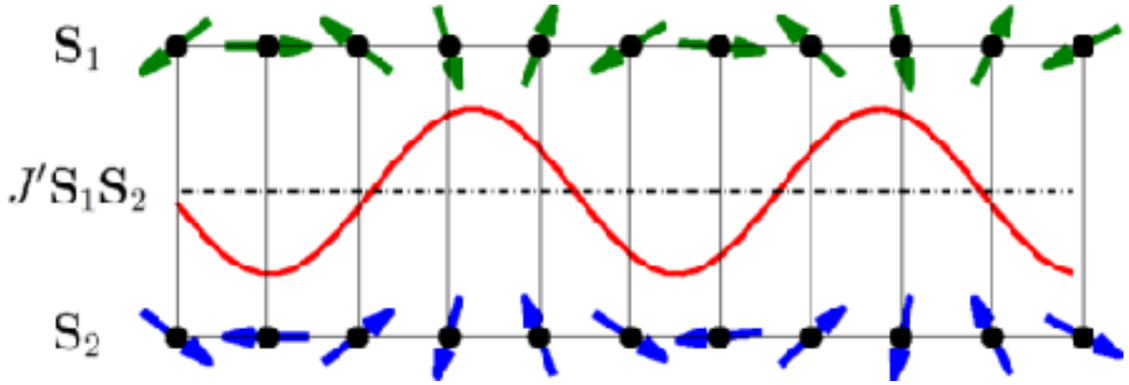


Interaction term	Coupling operator	Coupling constant	Induced state
$\mathcal{H}_x$	$\mathcal{N}_y^x \mathcal{N}_{y+1}^x$	g_x	SDW(z)
$\mathcal{H}_y$	$\mathcal{N}_y^y \mathcal{N}_{y+1}^y$	g_y	SDW (y)
$\mathcal{H}_{\text{inter},\varphi}$	$\cos[\sqrt{2\pi}(\varphi_y - \varphi_{y+1})]$	g_{φ_1}	Distorted-cone

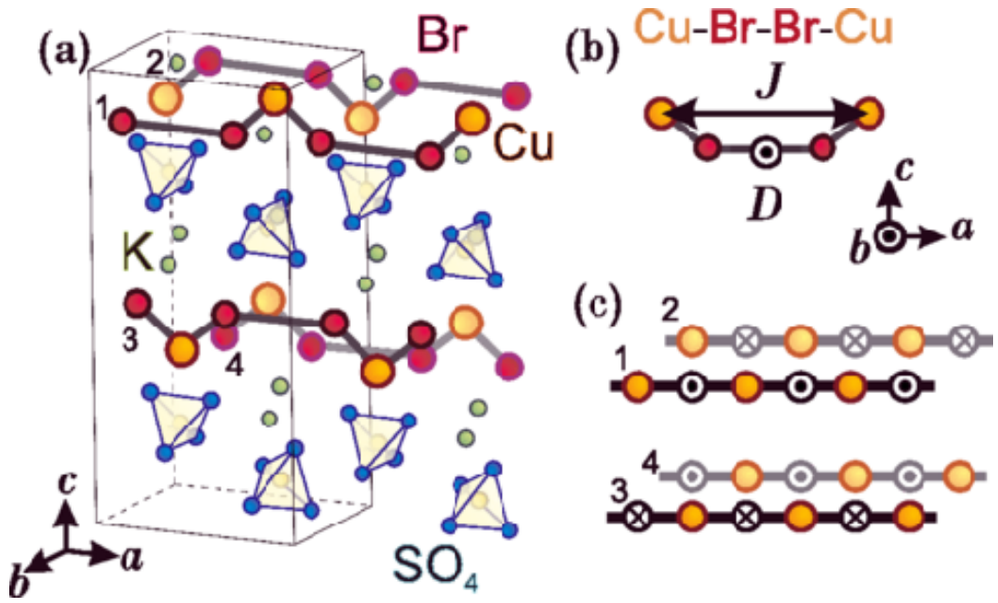
Typical RG flows for SDW(z).

h ∥ D ∥ z DM-induced frustration

- Identical-between-chains DM interactions stabilize 2D helical structure.
- Staggered-between-chains DM interactions effectively cancel the transverse interchain coupling.



Hälg et al., Phys. Rev. B, 2014.



Intra-chain exchange J=20.5K,
Inter-chain exchange J'=0.03K,
DM interaction D=0.28K.

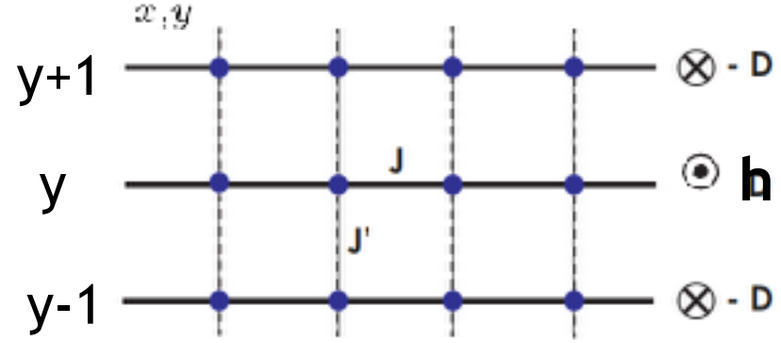
Smirnov et al., Phys. Rev. B, **92**,134417, 2015

Model: h ∥ D ∥ z

- Weakly coupled S = 1/2 AFM Heisenberg chains with a uniform DMI and magnetic field, $J' \ll J$.

$$H = \sum_{x,y} [J \boldsymbol{S}_{x,y} \cdot \boldsymbol{S}_{x+1,y} + J' \boldsymbol{S}_{x,y} \cdot \boldsymbol{S}_{x,y+1}] - \boldsymbol{D} \cdot \sum_{x,y} (-1)^y \boldsymbol{S}_{x,y} \times \boldsymbol{S}_{x+1,y} - \boldsymbol{h} \cdot \sum_{x,y} \boldsymbol{S}_{x,y},$$

J, J' : Intra-chain and inter-chain exchange, and
$(-1)^y D$: staggered between chains.



- Low-energy theory:
  - ➢ Spin operator in spin current $J_{R/L}$ and staggered magnetization **N**, x -1 x x+1

    $\boldsymbol{S}(x) \to \boldsymbol{J}_L(x) + \boldsymbol{J}_R(x) + (-1)^{x/a} \boldsymbol{N}(x)$, *a* is lattice spacing
  - ➢ DMI ($\sim (-1)^y D \partial_x \theta$) can be absorbed into H_0 by a shift of bosonic field,

$$H = H_0 + H_{\text{inter}}, \qquad H_{\text{inter}} = J' \int dx \boldsymbol{N}_y \cdot \boldsymbol{N}_{y+1} \to H_{\text{cone}} + H_{\text{sdw}}, \qquad N_y^+ \to N_y^+ e^{i(-1)^y Dx/v},$$

$$H_{\text{cone}} = J' \int dx (N_y^+ \cdot N_{y+1}^- e^{i2(-1)^y Dx/v} + h.c.), \qquad H_{\text{sdw}} = J' \int dx N_y^z \cdot N_{y+1}^z$$

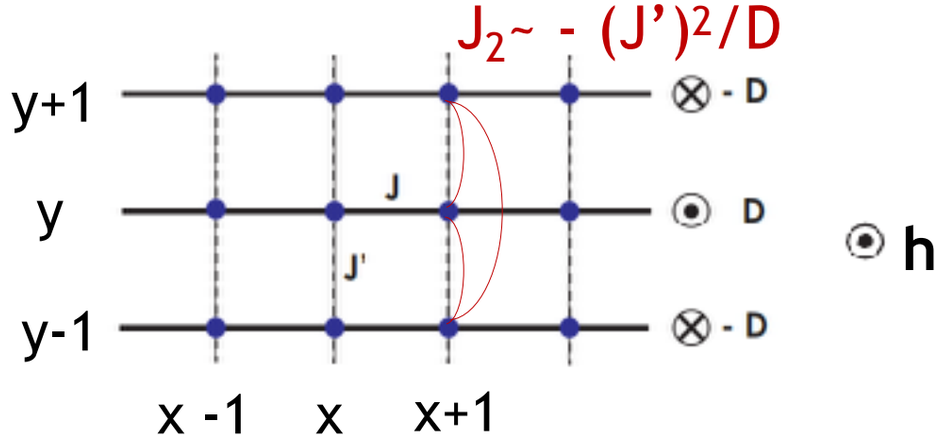
Cone: spiral in XY plane **Rapidly oscillates when D is large.** Collinear SDW

Strong DMI: quantum fluctuations generate interaction between next-nearest (NN) chains

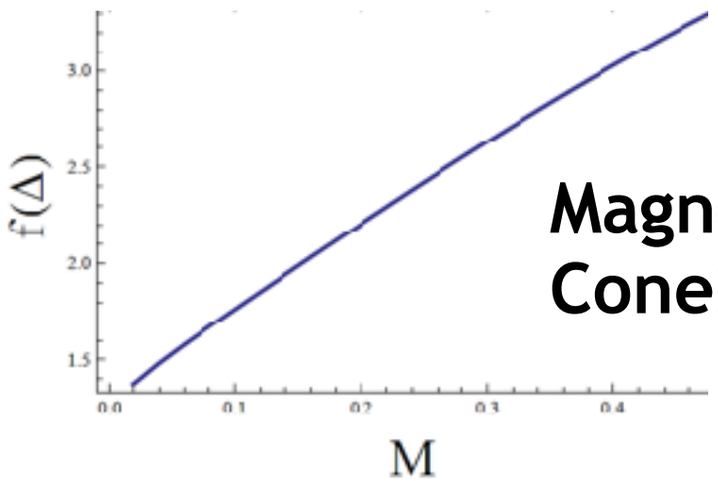
- Effective interaction between NN chain, **no frustration** due to DMI,

$$H_{\text{c_nn}} = J_2 \int dx (N_y^+ \cdot N_{y+2}^- + h.c.),$$

with $J_2 \sim -f(\Delta)\frac{J'^2}{D}$.

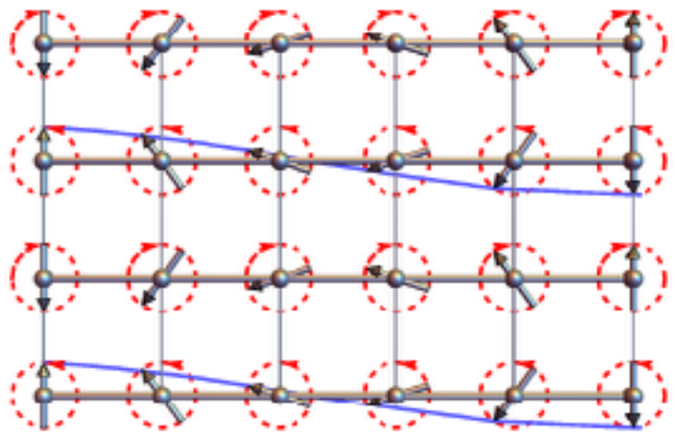


Δ : scaling dimension of N^+ , modulated by magnetic field



Magnetic field enhances Cone2N.

- Induced state: Cone2N

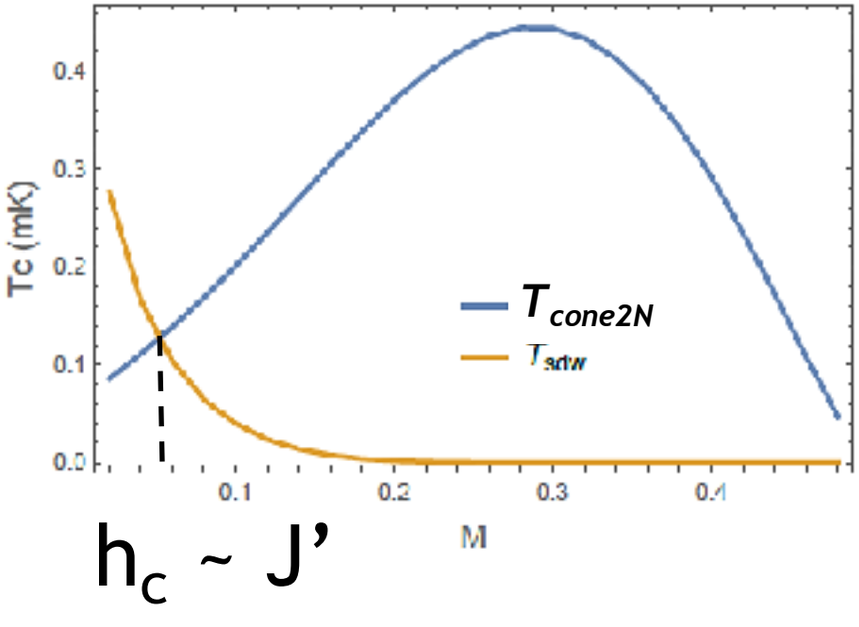


$J_2 < 0$

Strong DMI: competition between SDW and Cone2N

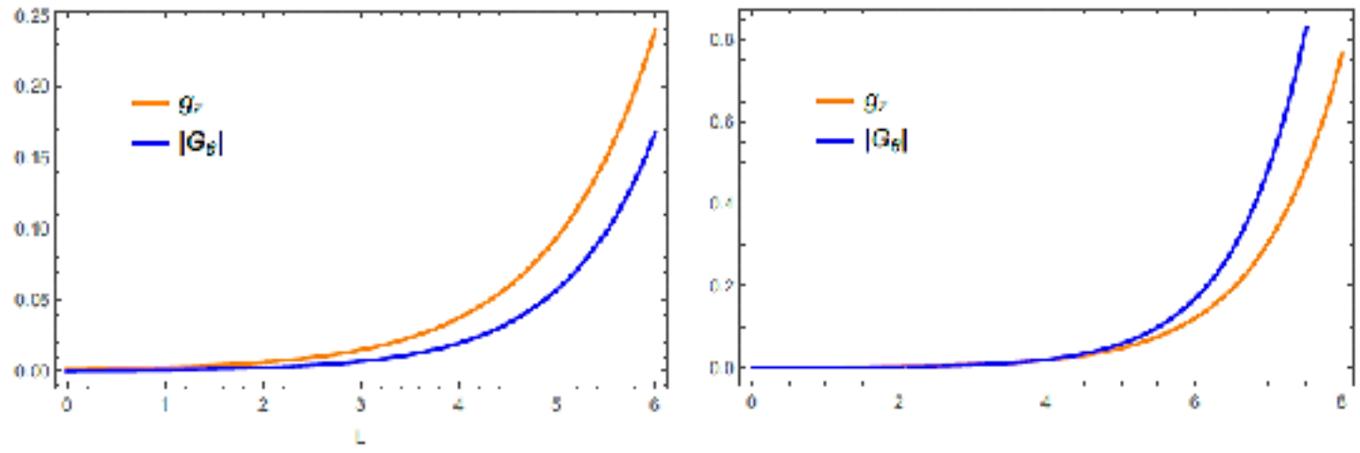
Interaction term	Coupling operator	Coupling constant	Ordering temperature	Induced state
H_{sdw}	$N_y^z \cdot N_{y+1}^z$	g_z	T_{sdw}	SDW
H_{cone2N}	$N_y^+ \cdot N_{y+1}^-$	G_6	T_{cone2N}	Cone2N

Ordering temperatures: for D/J'=10,



$h_c \sim J'$

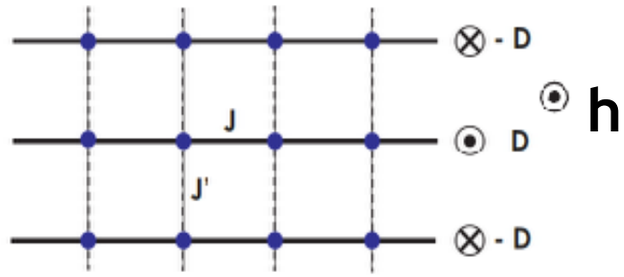
RG flows: for D/J'=10,



Small field, h/D=0.005. Large field, h/D=5.

SDW - cone transition

Summary h || D



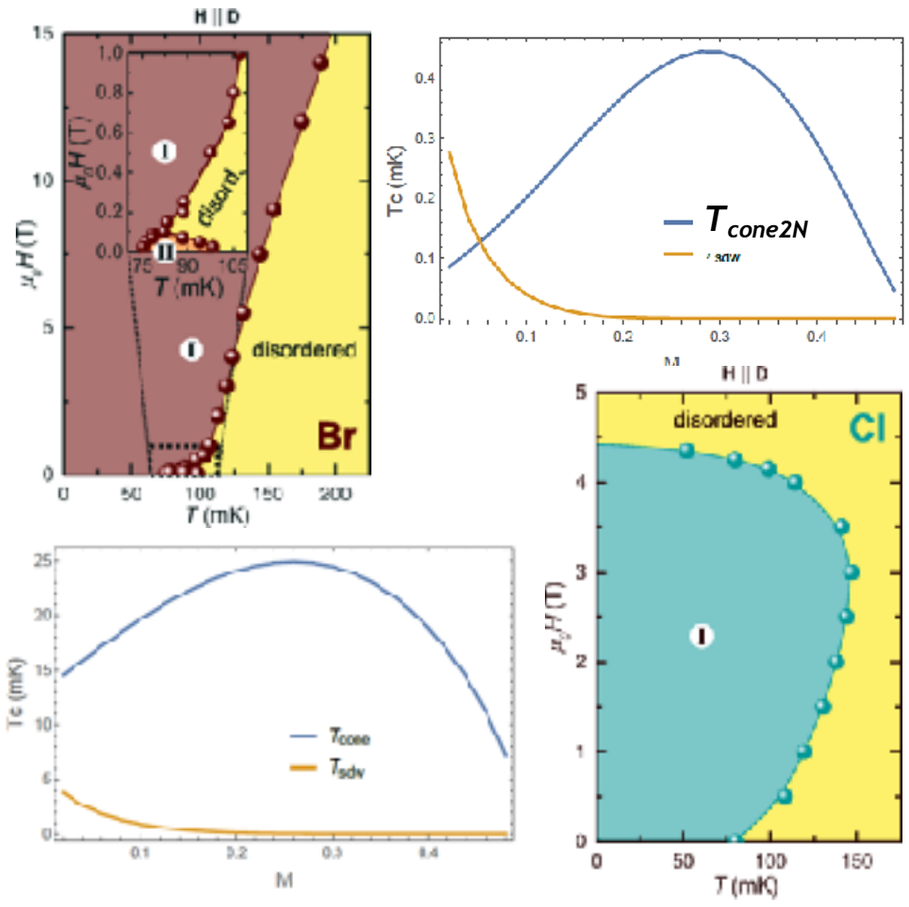
- Strong DMI promotes collinear SDW.
- Magnetic field enhances Cone2N, quantum fluctuation generate J_2 among NN chains.
- C-IC phase transition between SDW and Cone2N.

Strong DMI
$K_2CuSO_4Br_2$

- 1st chain $N_y^z \cdot N_{y+1}^z$
- NN chain $N_y^+ \cdot N_{y+2}^-$

Weak DMI
$K_2CuSO_4Cl_2$

- 1st chain $N_y^+ \cdot N_{y+1}^-$



Conclusions

Simple model but many interesting properties

Interesting dependence on the angle between h and D

Strong in-chain DM frustrates interchain exchange

Experimental checks are available