Spin-current and other unusual phases in magnetized triangular lattice antiferromagnets



Oleg Starykh, University of Utah

collaborators:
Jason Alicea, Leon Balents, Andrey Chubukov, Zhihao Hao,
Ru Chen, Hyejin Ju, Hong-Chen Jiang

Instabilities of 1/3 magnetization plateau



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Outline

- Context: interesting experiments and theories
- Magnetization curve
- classical antiferromagnet in a field: entropic selection
- Quantum spins - plateau due to quantum fluctuations
- Instability: two-particle condensation at large S
- Instability: transition to half-metal on lowering U

★ Conclusions

Experiments S=1/2

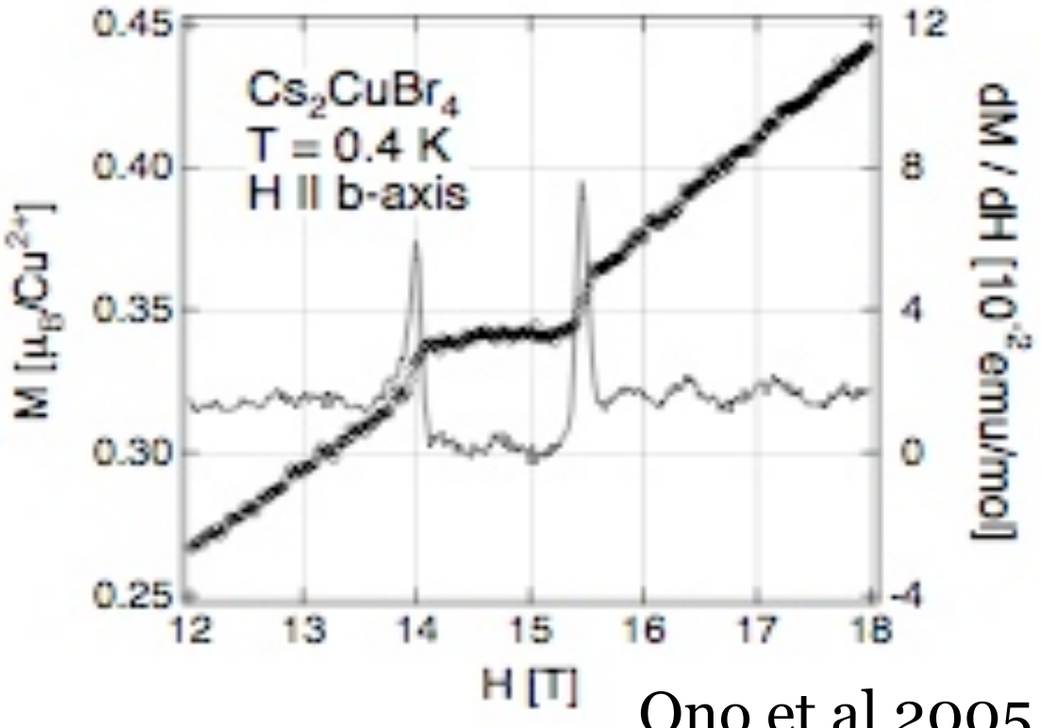


Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel$ measured at $T = 0.4$ K in magnetic fields up to 20 T.

Ono et al 2005

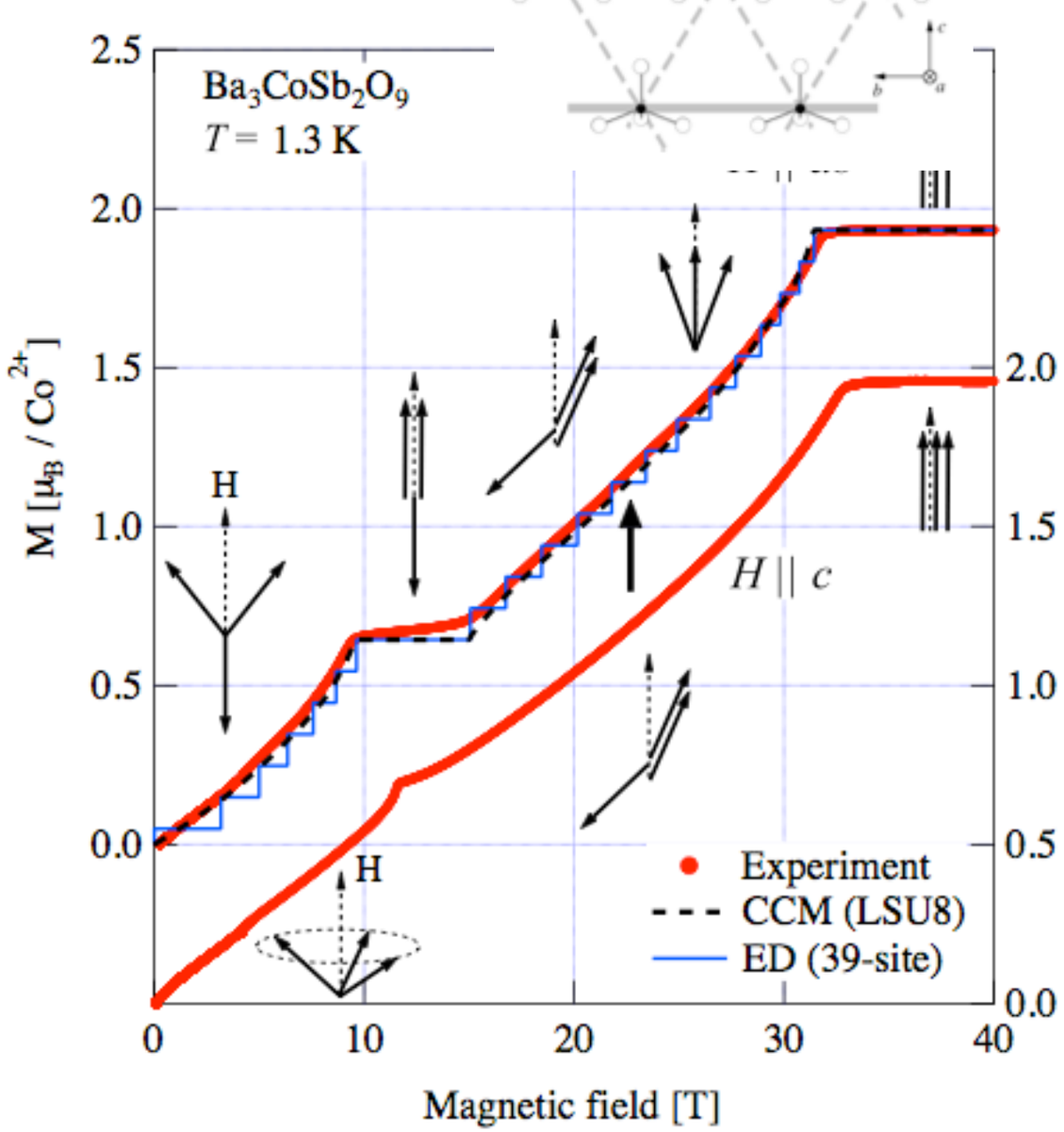
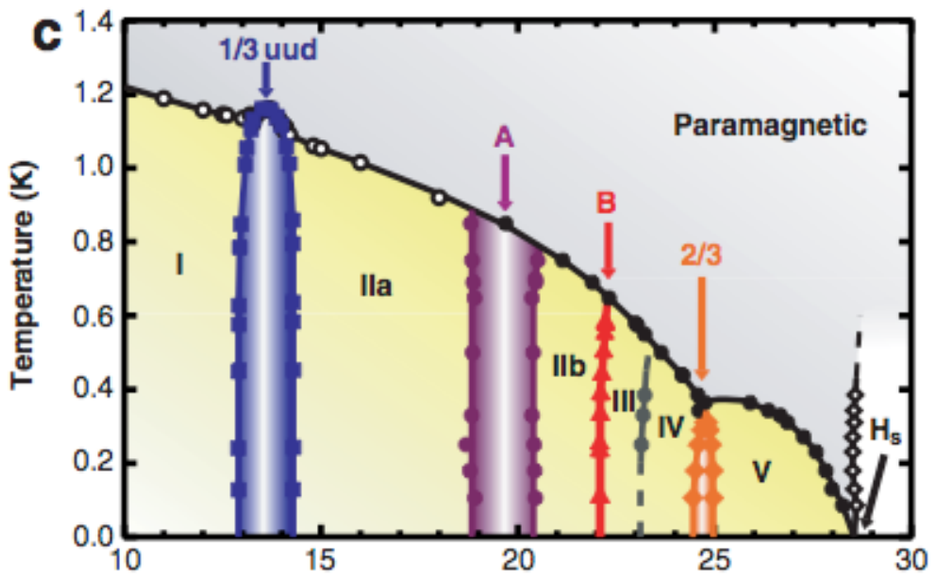


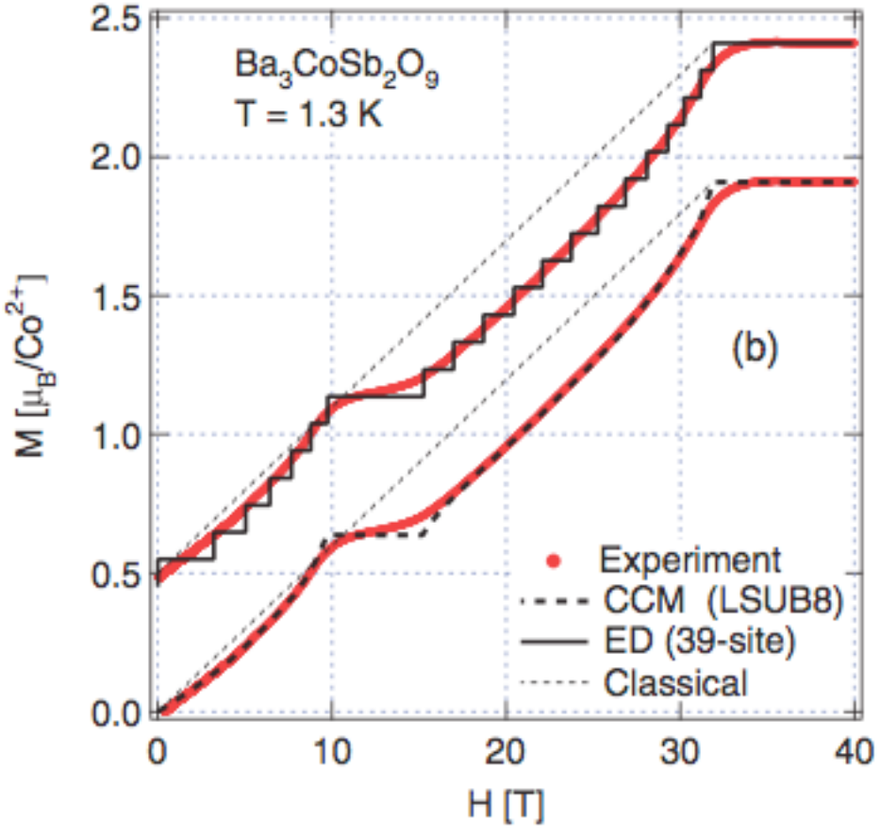
FIG. 2: (Color online) Magnetization curves in $Ba_3CoSb_2O_9$ at 1.3 K for $H \parallel ab$ (left) and $H \parallel c$ (right), corrected for the Van Vleck paramagnetism. The dashed curve and steplike line denote the results calculated by a higher-order coupled cluster method (CCM) [11] and exact diagonalization (ED) for a 39-site rhombic cluster [14], respectively. Spin structures at various magnetic fields are illustrated.

Susuki 2013

Tsuji 2007

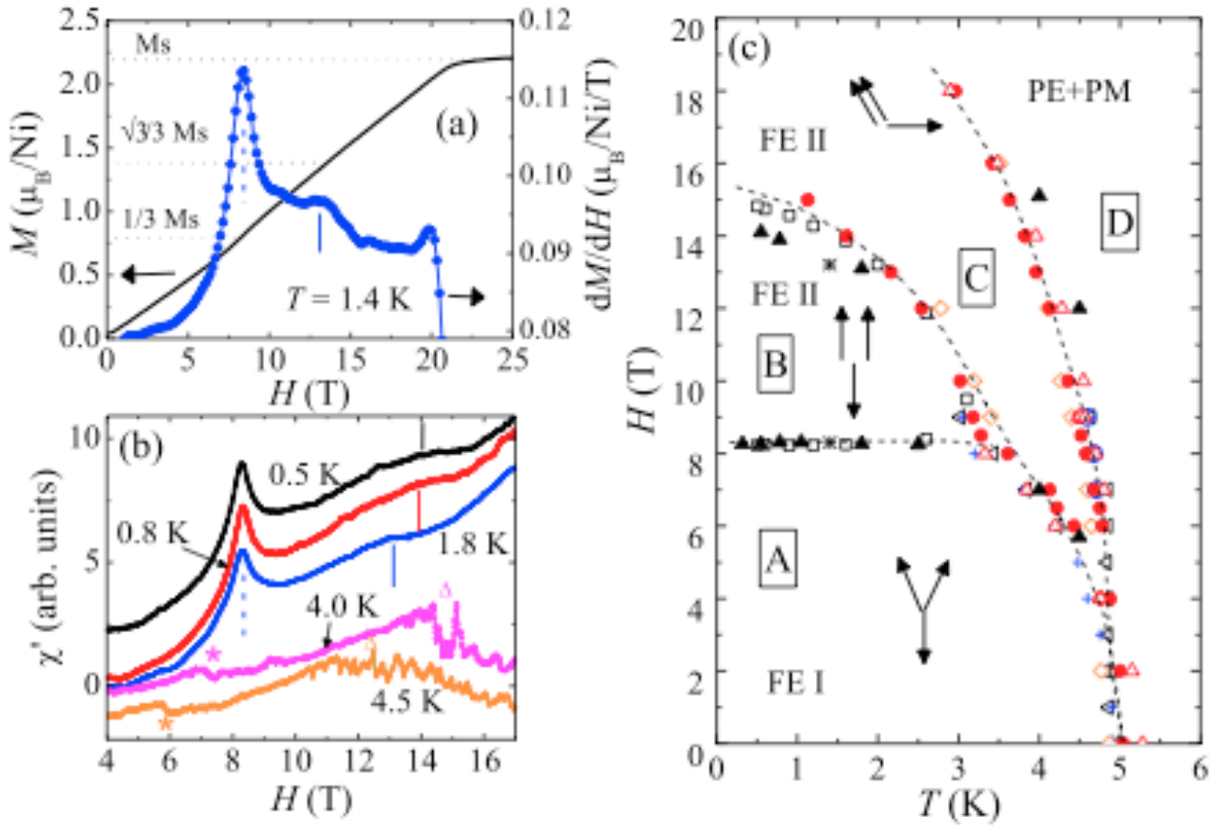


s=1/2, $Ba_3CoNb_2O_9$



Shirata et al, PRL 2012

s=1, $Ba_3NiNb_2O_9$

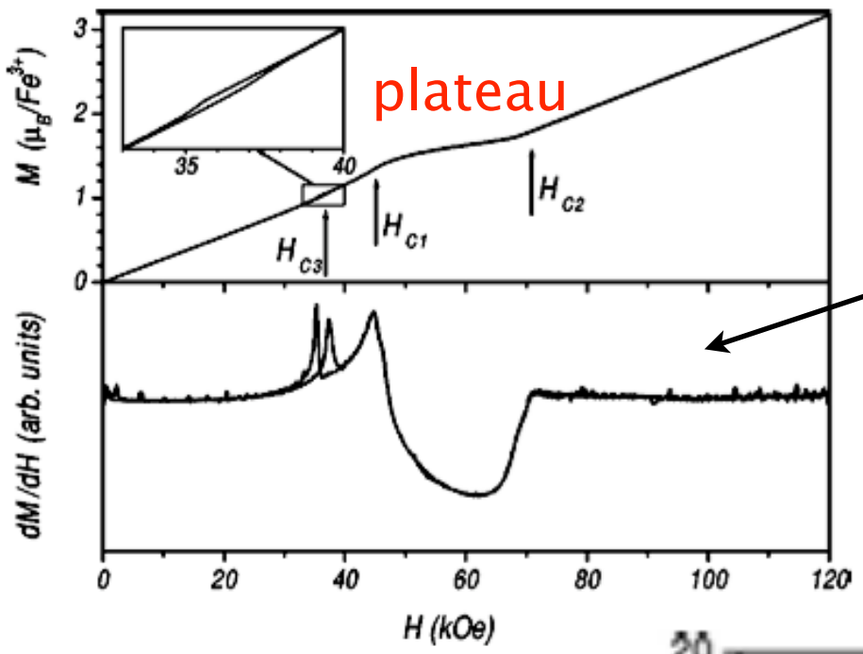


Hwang et al, PRL 2012

Experiments S >> 1

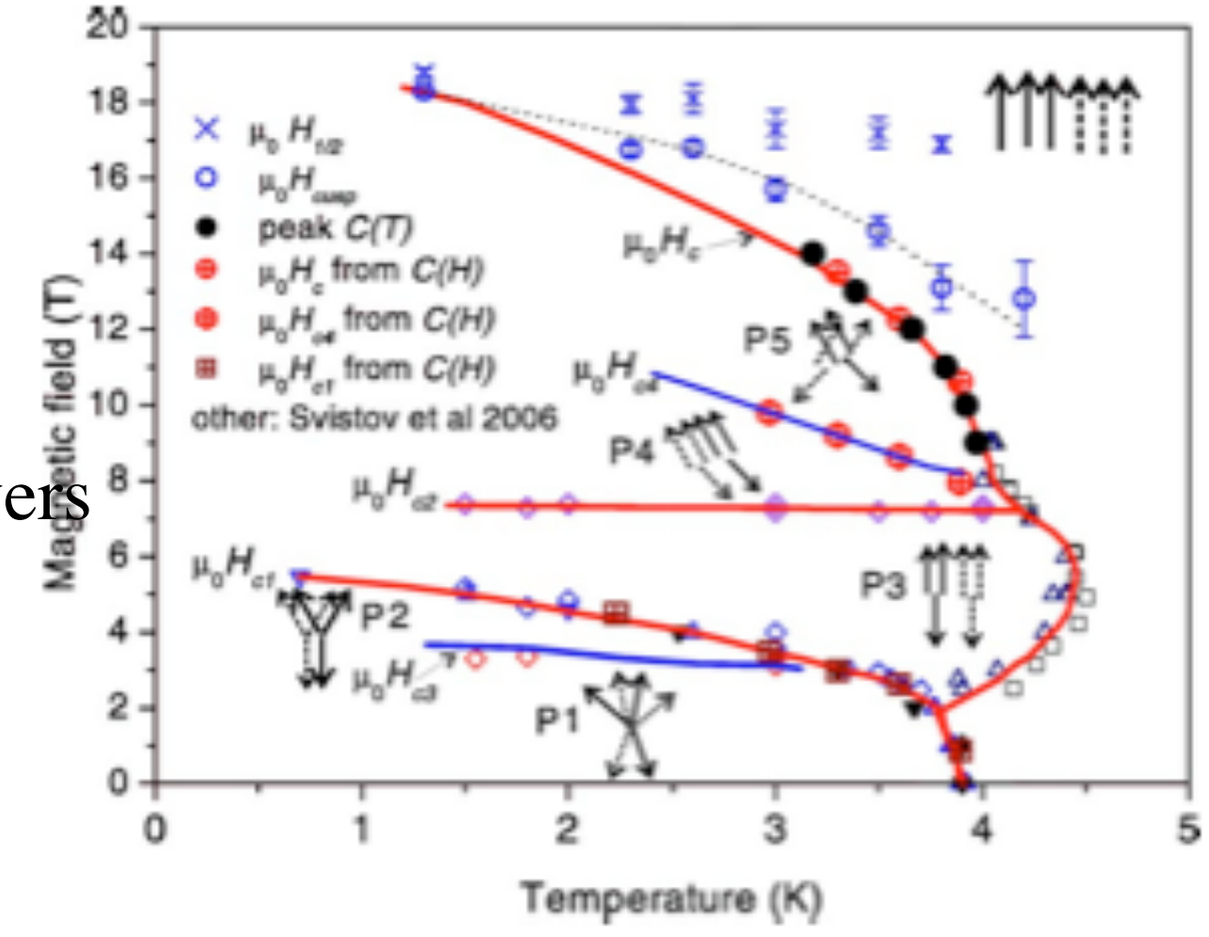
$RbFe(MoO_4)_2$:
$S=5/2$ Fe^{3+}

Svistov et al PRB (2003)
Smirnov et al PRB (2007)



plateau is signaled by depression in dM/dH

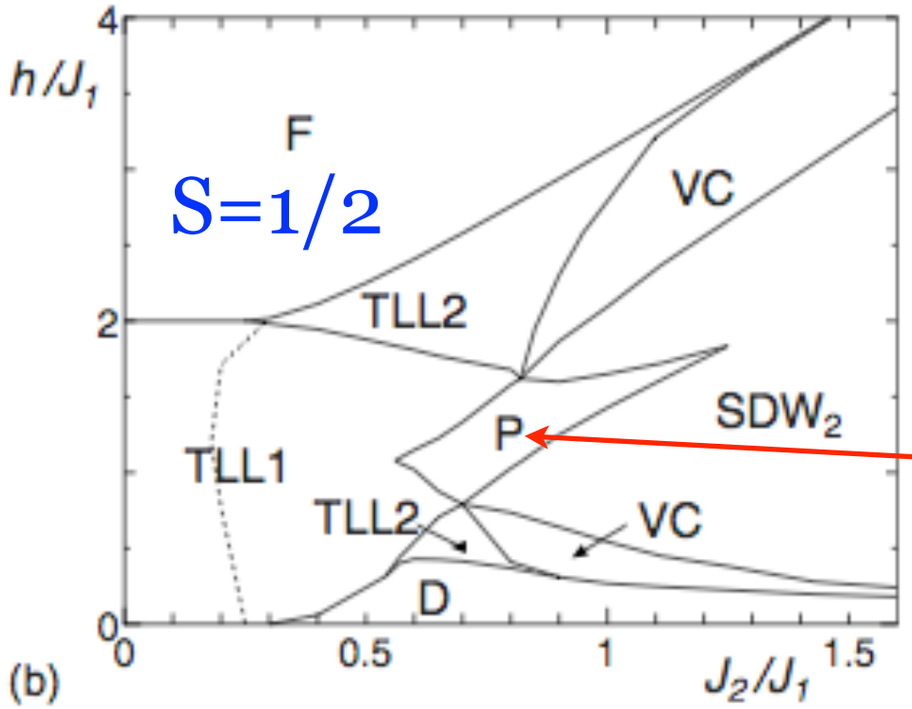
Phase diagram contains only co-planar states (while classically - only non-planar cone state!)



Two antiferromagnetically coupled layers

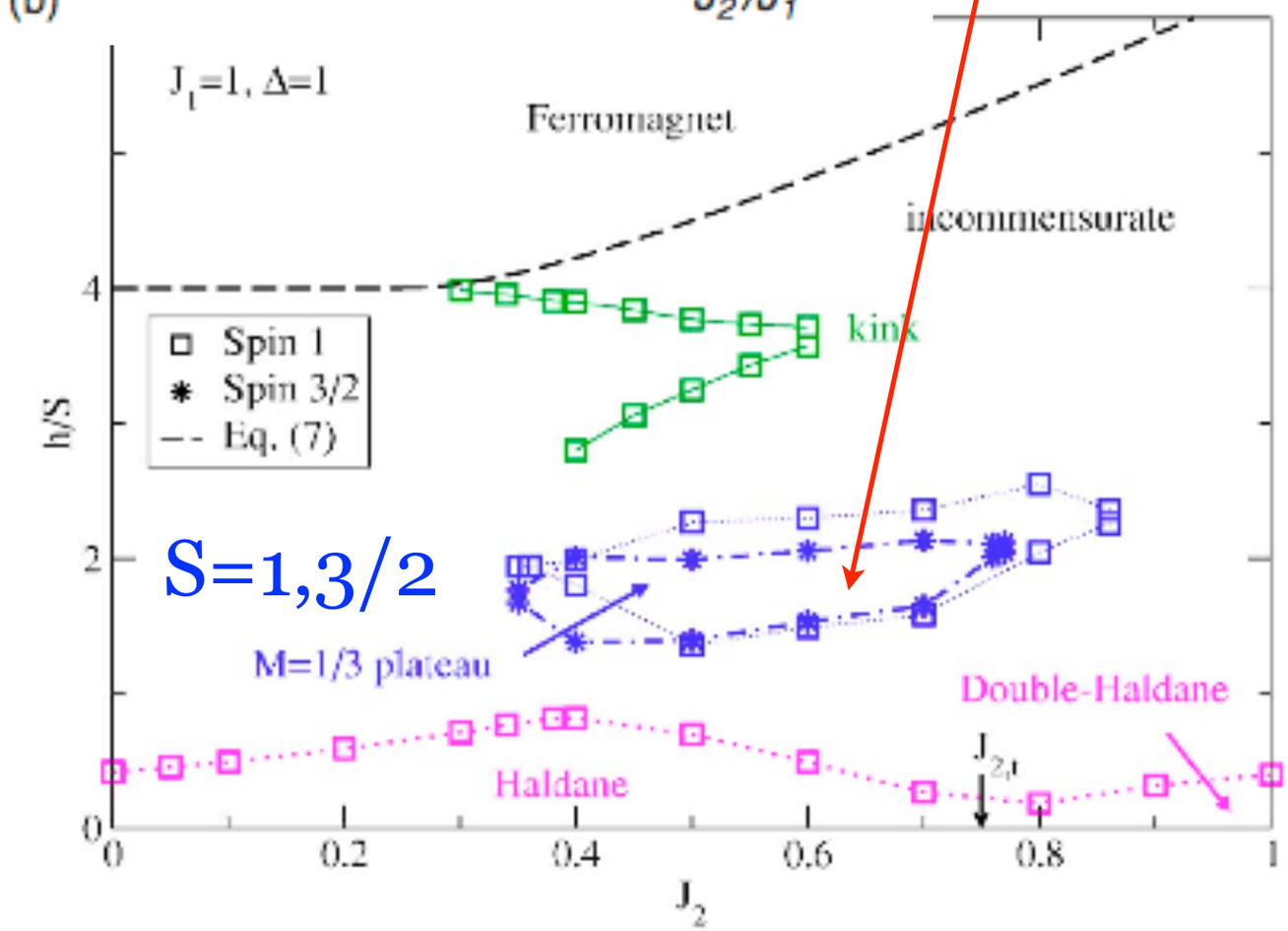
Plateau width increases with T

Theory: Magnetization plateau in one dimensional J_1-J_2 chain (zig-zag ladder)



Okunishi, Tonegawa JPSJ (2003)
Hikihara et al PRB (2010)

M=1/3 plateau



Heirich-Meisner et al PRB (2007)

agrees with Oshikawa, Yamanaka, Affleck argument (PRL 2007):

p S (1 - M) = integer

p = period, S = spin,
M = magnetization:
M=1/3, p=3
possible for all S

Theory: Connections with interacting bosons

- surprisingly complex phase diagram of spatially anisotropic triangular lattice antiferromagnet
- connections with interacting boson system

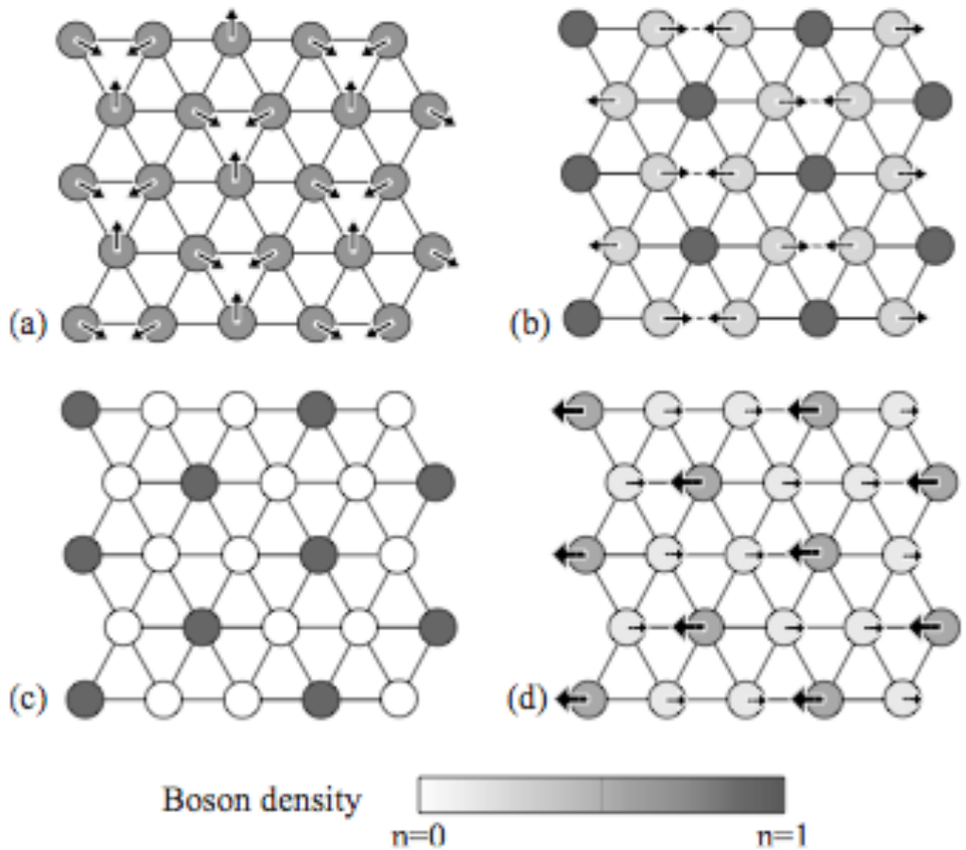
– Superfluids (XY order)

– Mott insulators

– Supersolids

Andreev, Lifshitz 1969

Nikuni, Shiba 1995
Heidarian, Damle 2005
Wang et al 2009
Jiang et al 2009
Tay, Motrunich 2010



theory
proposals of
liquid-like
plateau states

Magnetization Plateaus of $SrCu_2(BO_3)_2$ from a Chern-Simons Theory

G. Misguich,[1] Th. Jolicoeur,[2] and S. M. Girvin[3,4]
[1]*Service de Physique Théorique, CE Saclay, 91191 Gif-sur-Yvette, France*
[2]*Département de Physique, LPMC-ENS, 24 rue Lhomond, 75005 Paris, France*
[3]*Department of Physics, Indiana University, Bloomington, Indiana 47405-7105*
[4]*Institute for Theoretical Physics, UCSB, Santa Barbara, California 93106-4030*
(Received 21 February 2001; published 13 August 2001)

The antiferromagnetic Heisenberg model on the frustrated Shastry-Sutherland lattice is studied by a mapping onto spinless fermions carrying one quantum of statistical flux. Using a mean-field approximation these fermions populate the bands of a generalized Hofstadter problem. Their filling leads to the magnetization curve. For $SrCu_2(BO_3)_2$ we reproduce plateaus at $1/3$ and $1/4$ of the saturation moment and predict a new one at $1/2$. Gaussian fluctuations of the gauge field are shown to be massive at these plateau values.

PHYSICAL REVIEW B **75**, 144411 (2007)

Critical spin liquid at $\frac{1}{3}$ magnetization in a spin-$\frac{1}{2}$ triangular antiferromagnet

Jason Alicea[1] and Matthew P. A. Fisher[2]
[1]*Physics Department, University of California, Santa Barbara, California 93106, USA*
[2]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
(Received 10 February 2007; revised manuscript received 23 February 2007; published 10 April 2007)

Although magnetically ordered at low temperatures, the s
remarkable spin dynamics that strongly suggest proximity
proximate spin liquid may also occur in an applied magneti
spin correlations of this material. Specifically, we explore a
antiferromagnet at $\frac{1}{3}$ magnetization from a dual vortex pe
phase described by quantum electrodynamics in (2+1)-di
number of nontrivial predictions follow for the dynamical s
phase, which can be tested via inelastic neutron scattering.
magnetization plateaus can be captured within our approa
gapless solid phase in a weakly ordered up-up-down stat
minima in the excitation spectrum in the regime of lattice a

Wannier Permanent Wave Functions for Featureless Bosonic Mott Insulators on the 1/3-Filled Kagome Lattice

S. A. Parameswaran,[1] Itamar Kimchi,[1] Ari M. Turner,[2] D. M. Stamper-Kurn,[1] and Ashvin Vishwanath[1]
[1]*Department of Physics, University of California, Berkeley, California 94720, USA*
[2]*Institute for Theoretical Physics, University of Amsterdam, Science Park 904, P.O. Box 94485, 1090 GL Amsterdam, Netherlands*
(Received 26 October 2012; published 19 March 2013)

We study Bose-Hubbard models on tight-binding, non-Bravais lattices, with a filling of one boson per unit cell—and thus fractional *site* filling. We discuss situations where no classical bosonic insulator, which is a product state of particles on independent sites, is admitted. Nevertheless, we show that it is possible to construct a quantum Mott insulator of bosons if a trivial band insulator of *fermions* is possible at the same filling. The ground state wave function is simply a permanent of exponentially localized Wannier orbitals. Such a Wannier permanent wave function is featureless in that it respects all lattice symmetries and is the unique ground state of a parent Hamiltonian that we construct. Motivated by the recent experimental demonstration of a kagome optical lattice of bosons, we study this lattice at $1/3$ site filling. Previous approaches to this problem have invariably produced either broken-symmetry states or topological order. Surprisingly, we demonstrate that a featureless insulator is a possible alternative and is the exact ground state of a local Hamiltonian. We briefly comment on the experimental relevance of our results to ultracold atoms as well as to $1/3$ magnetization plateaus for kagome spin models in an applied field.

Outline

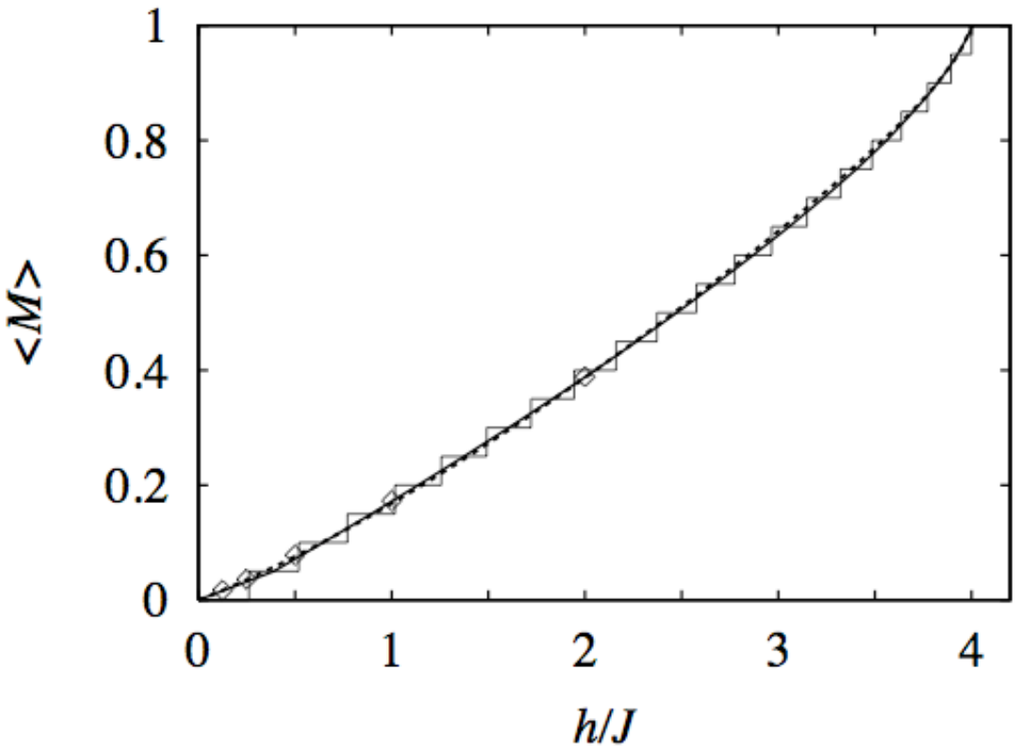
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★ Conclusions

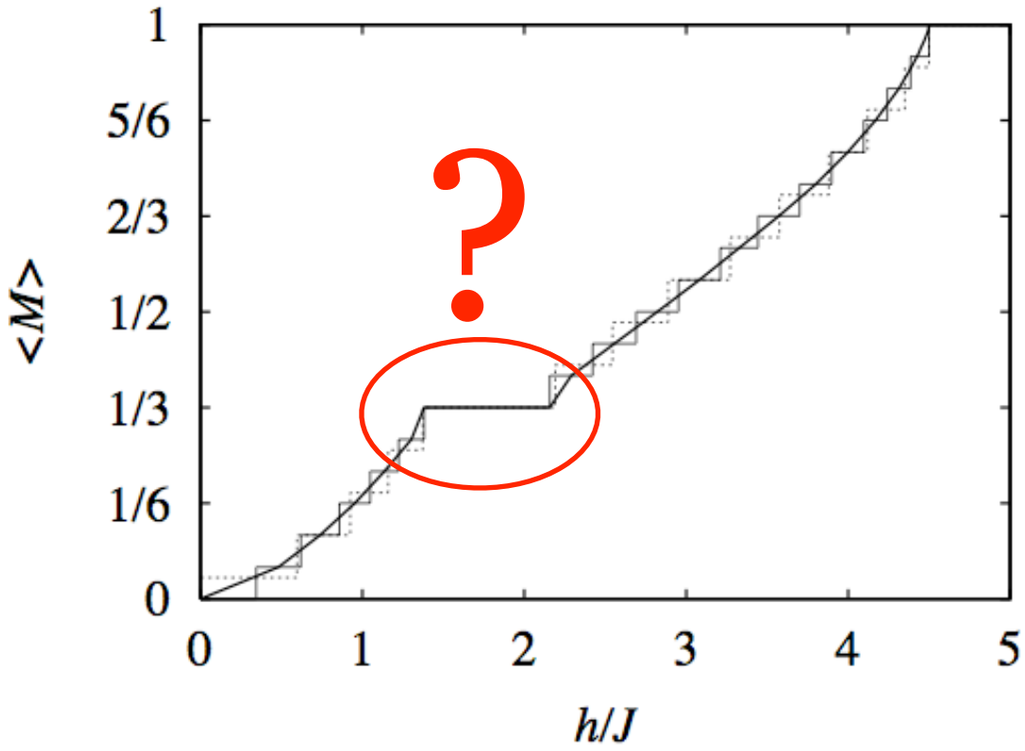
Magnetization curve and **plateau**

Kawamura, Miyashita: classical Heisenberg model+Monte Carlo, 1985
Chubukov, Golosov: quantum large-S, 1991

square



triangular



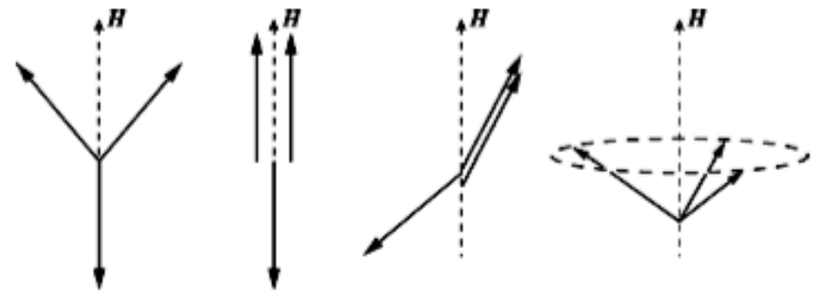
A. Honecker , J. Schulenburg and J. Richter (2004)

Classical isotropic triangular AFM in magnetic field

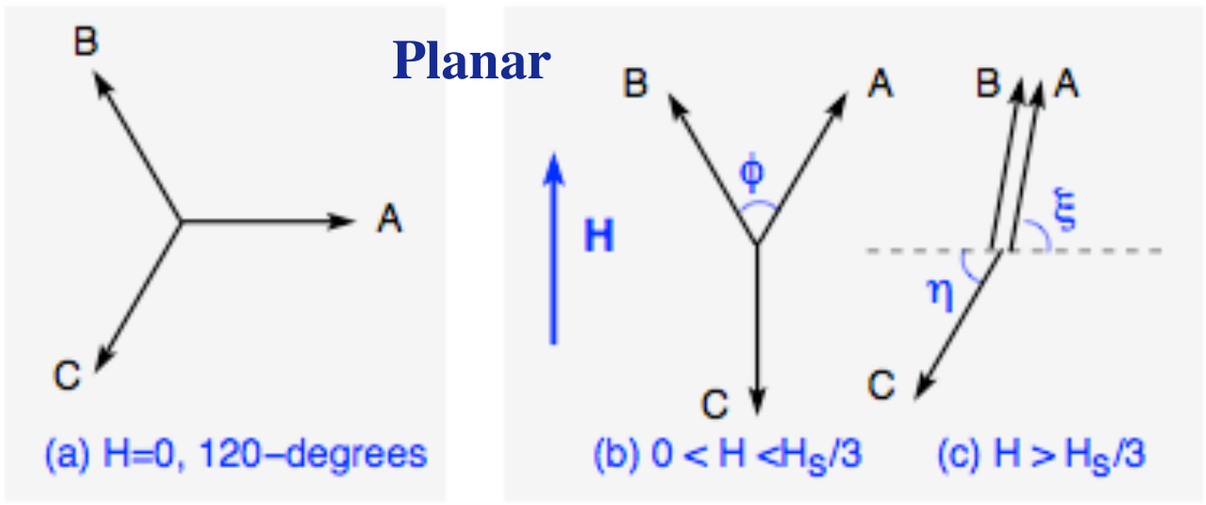
- No field: spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

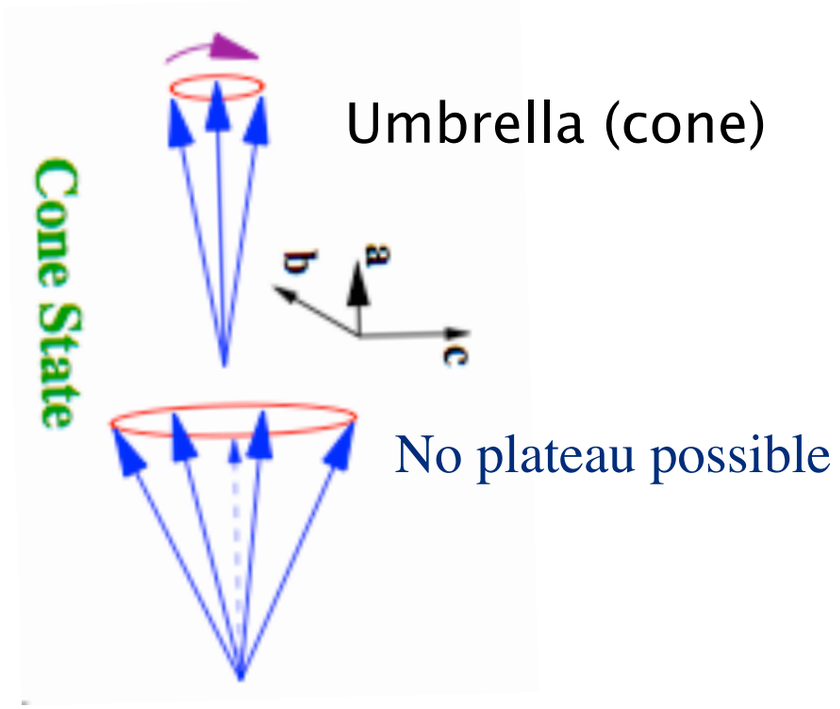
$$H = J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \sum_i \vec{h}\cdot\vec{S}_i$$

$$H = \frac{1}{2}J\sum_{\triangle}\Big(\sum_{i\in\triangle}\vec{S}_i - \frac{\vec{h}}{3J}\Big)^2$$

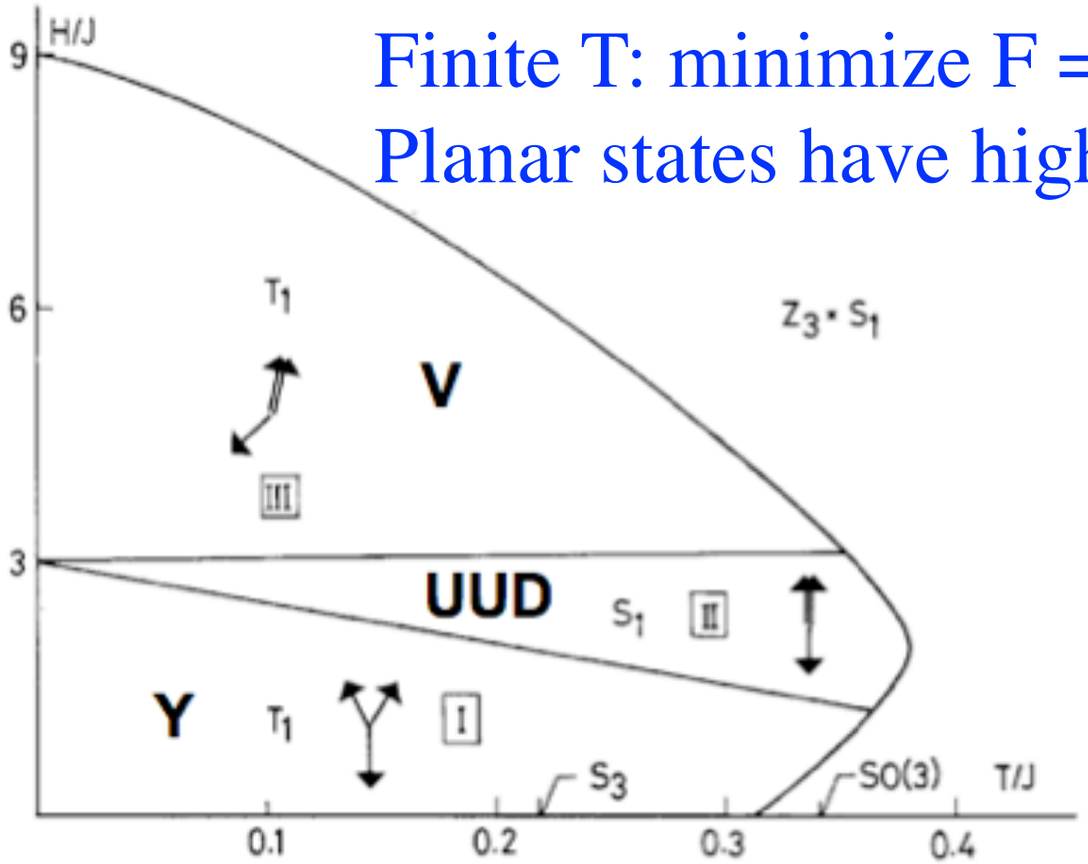


- all states with $\vec{S}_{i1}+\vec{S}_{i2}+\vec{S}_{i3} = \dfrac{h}{3J}$ form the lowest-energy manifold
  - 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about **h**)





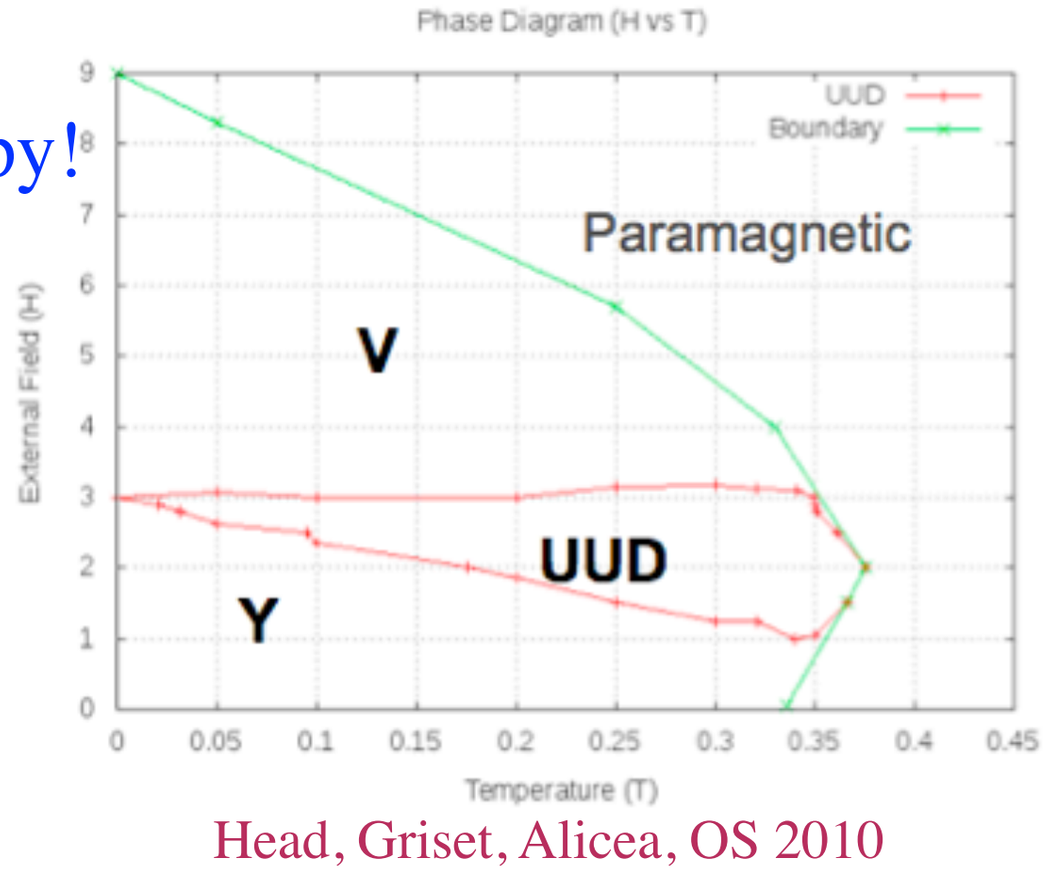
Phase diagram



H. Kawamura, S. Miyashita:
JPSJ (1985)

Finite T: minimize F = E - T S

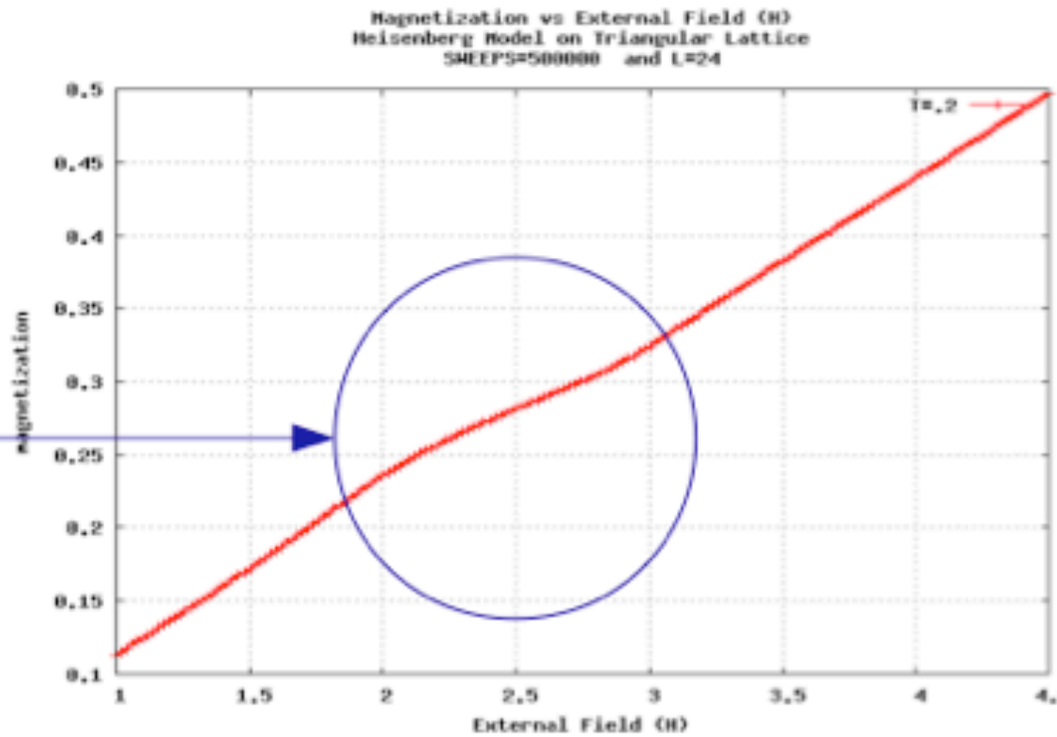
Planar states have higher entropy!



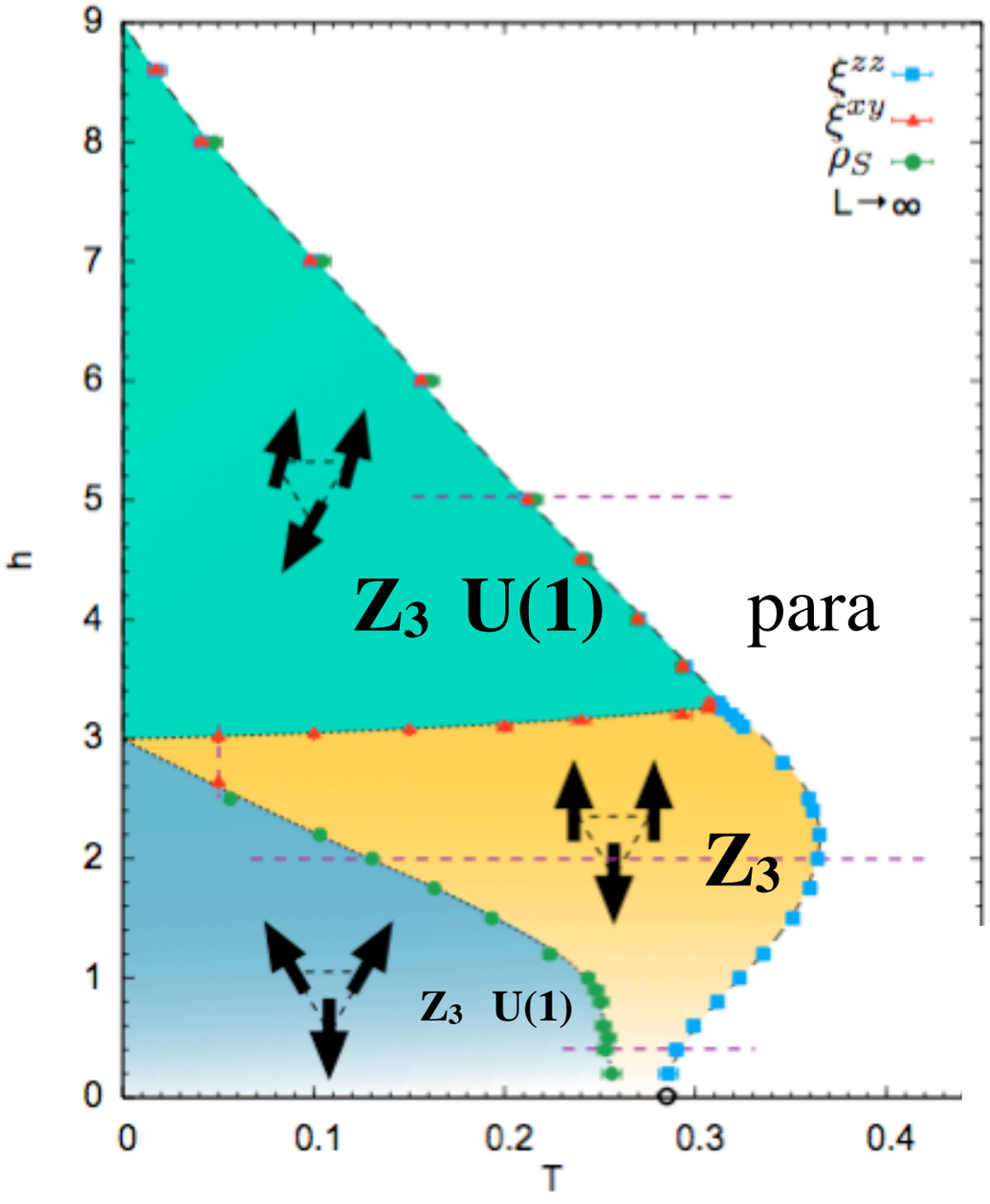
Head, Griset, Alicea, OS 2010

Entropic Selection:

- Planar states favored by thermal fluctuations
- UUD state around m=1/3 resulting in quasi-plateau



Phase diagram of the classical model



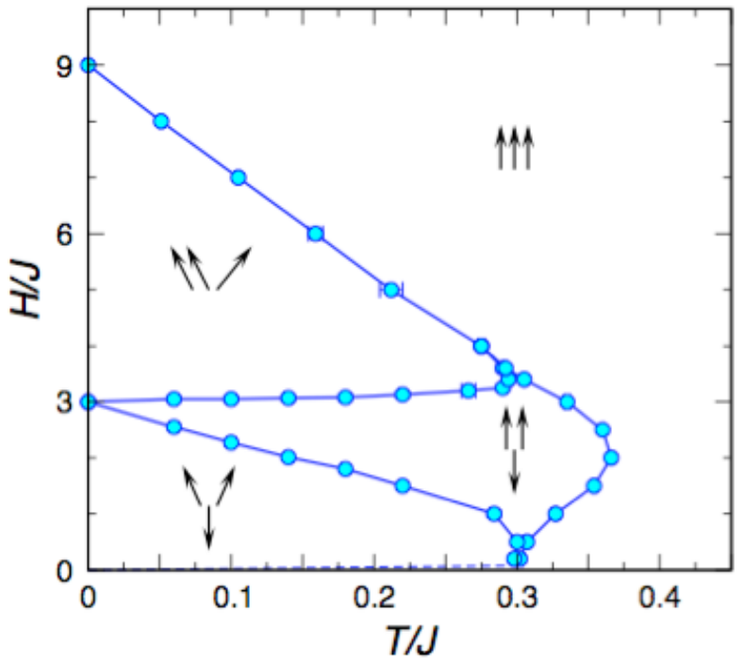
Seabra, Momoi, Sindzingre, Shannon 2011

why so stable?

Collinear UUD state does not break U(1) symmetry (rotations about field axis).

Hence, no gapless Goldstone modes.

All spin excitations are gapped.



Gvozdikova, Melchy, Zhitomirsky 2010

Quantum fluctuations, S >> 1, T=0.

J' = **J**: Quantum fluctuations select co-planar and collinear phases

J. Phys.: Condens. Matter 3 (1991) 69–82. Printed in the UK

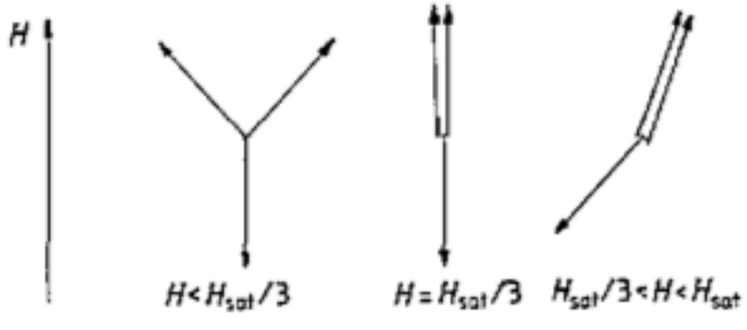
UUD plateau is due to interactions between spin waves

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

A V Chubukov and D I Golosov
Institute for Physical Problems, USSR Academy of Sciences, 117334 ul. Kosygina 2, Moscow, USSR

Received 9 February 1990

Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and *XY* quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.



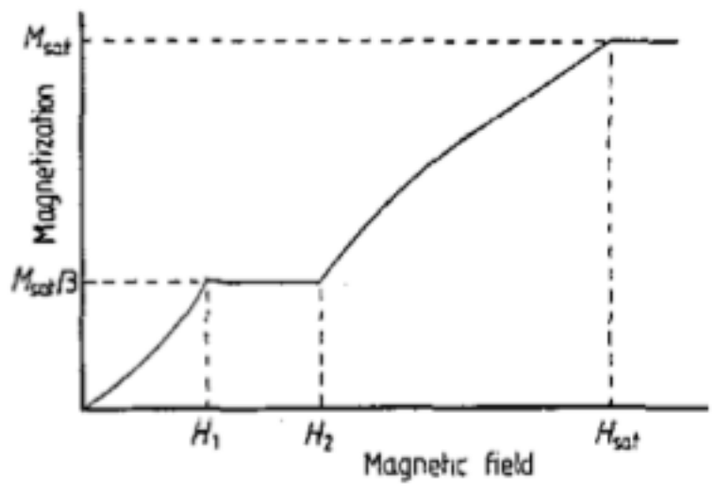


Figure 3. The anticipated behaviour of longitudinal magnetization in 2D Heisenberg AFM on a triangular lattice. The plateau on the magnetization curve results from the stabilization of the collinear phase in the finite region of magnetic fields due to zero-point motion.

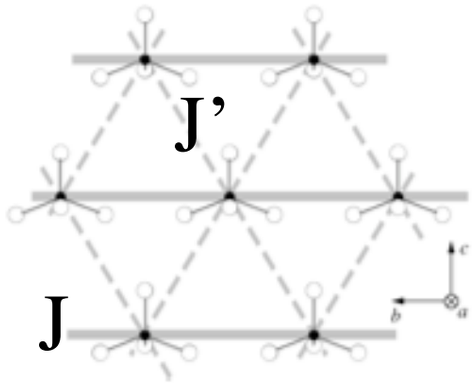
$$h_{c2} - h_{c1} = (0.6/2S)\, h_{sat}$$

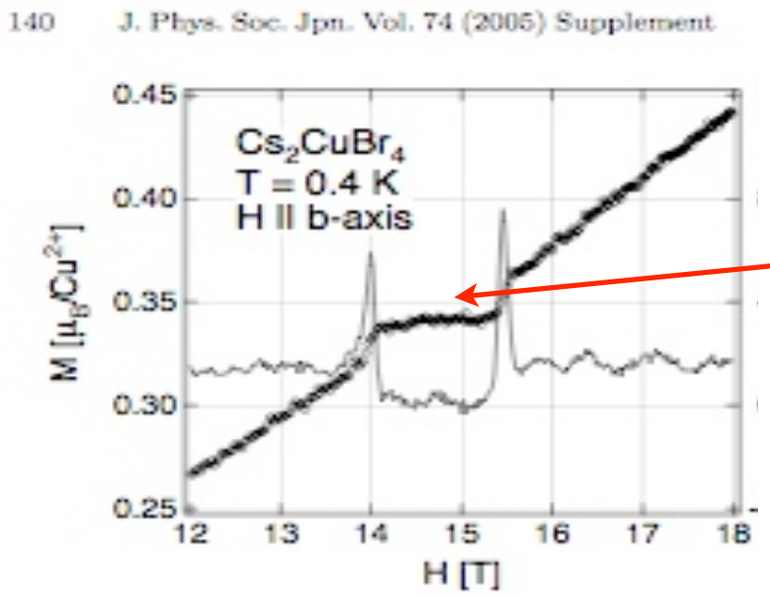
Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_2$ in the vicinity of $H_{sat}/3$.

Exp: M=1/3 magnetization plateau in Cs_2CuBr_4

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.75**

S=1/2





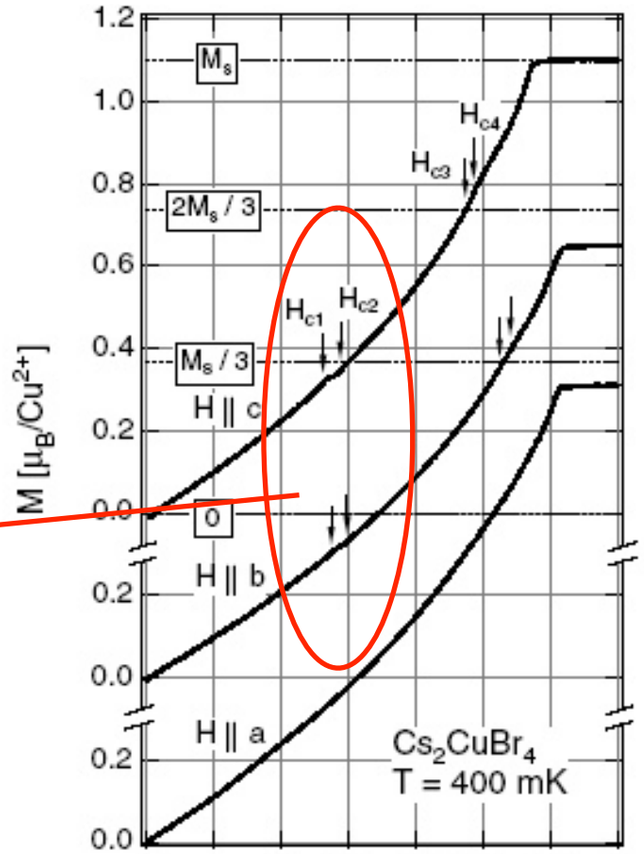
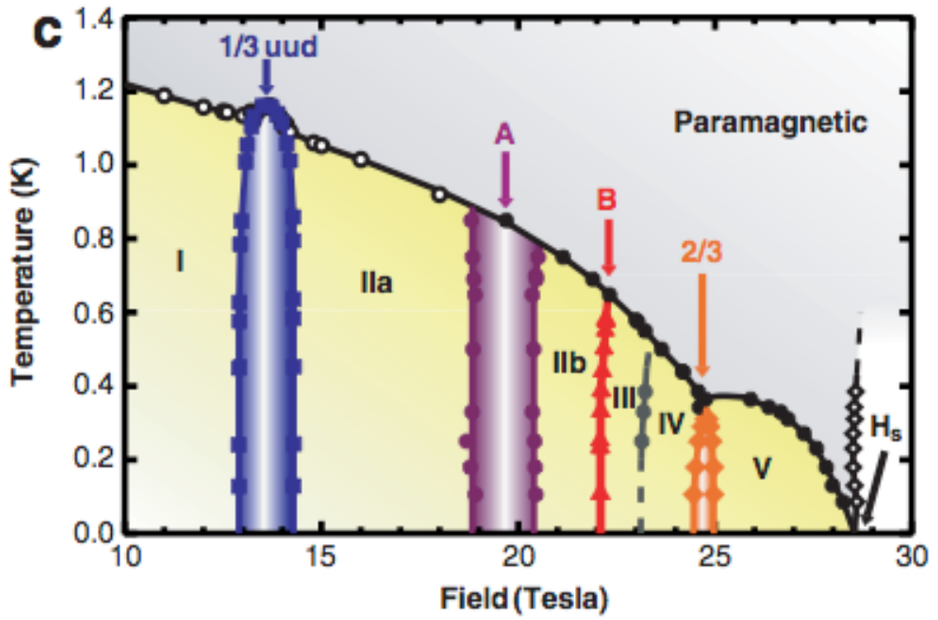


Fig. 8. The magn
measured at T =

★ first observation of "**up-up-down**" state in spin-1/2 triangular lattice antiferromagnet

★ total of **9 phases --** instead of 3 expected!

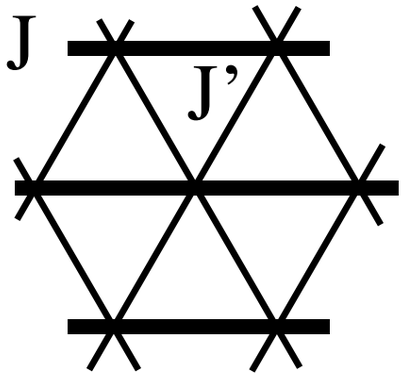


Important: the lattice is strongly anisotropic, J'/J = 0.75

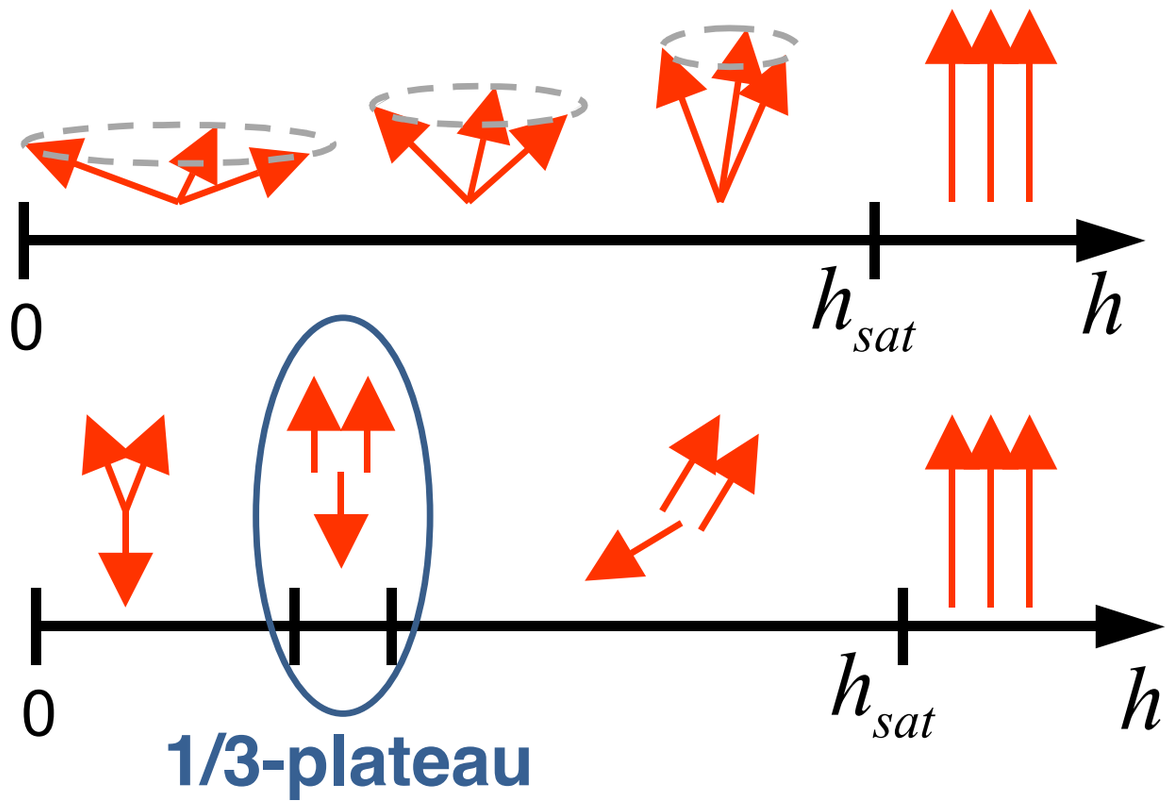
Spatially anisotropic model: classical prediction *fails*

T=0

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$S = \infty$



$J' \neq J$

$S = \frac{1}{2}$

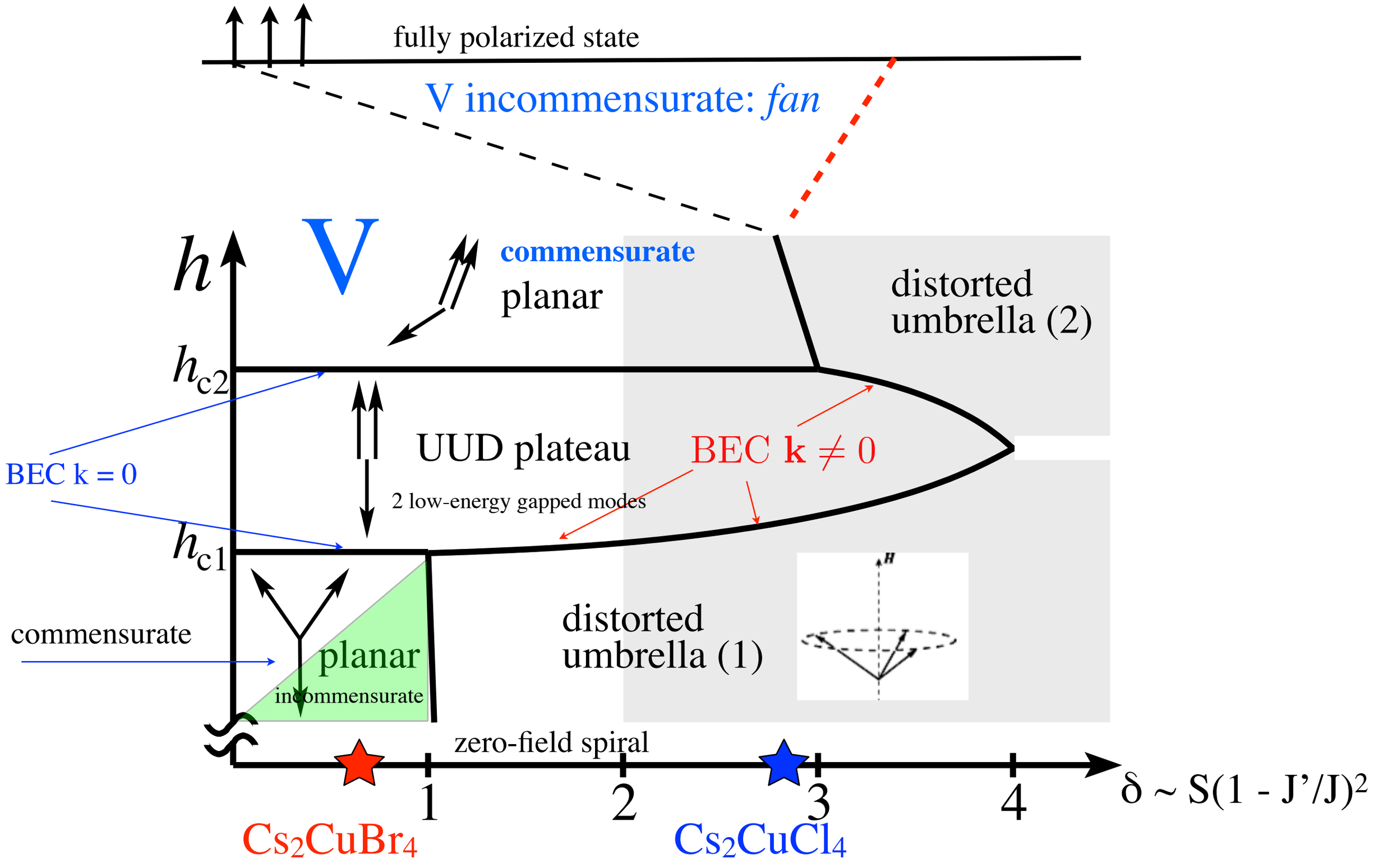
Umbrella state:
favored classically;
energy gain $(J\text{-}J')^2/J$

Planar states: favored by quantum fluctuations;
energy gain J/S

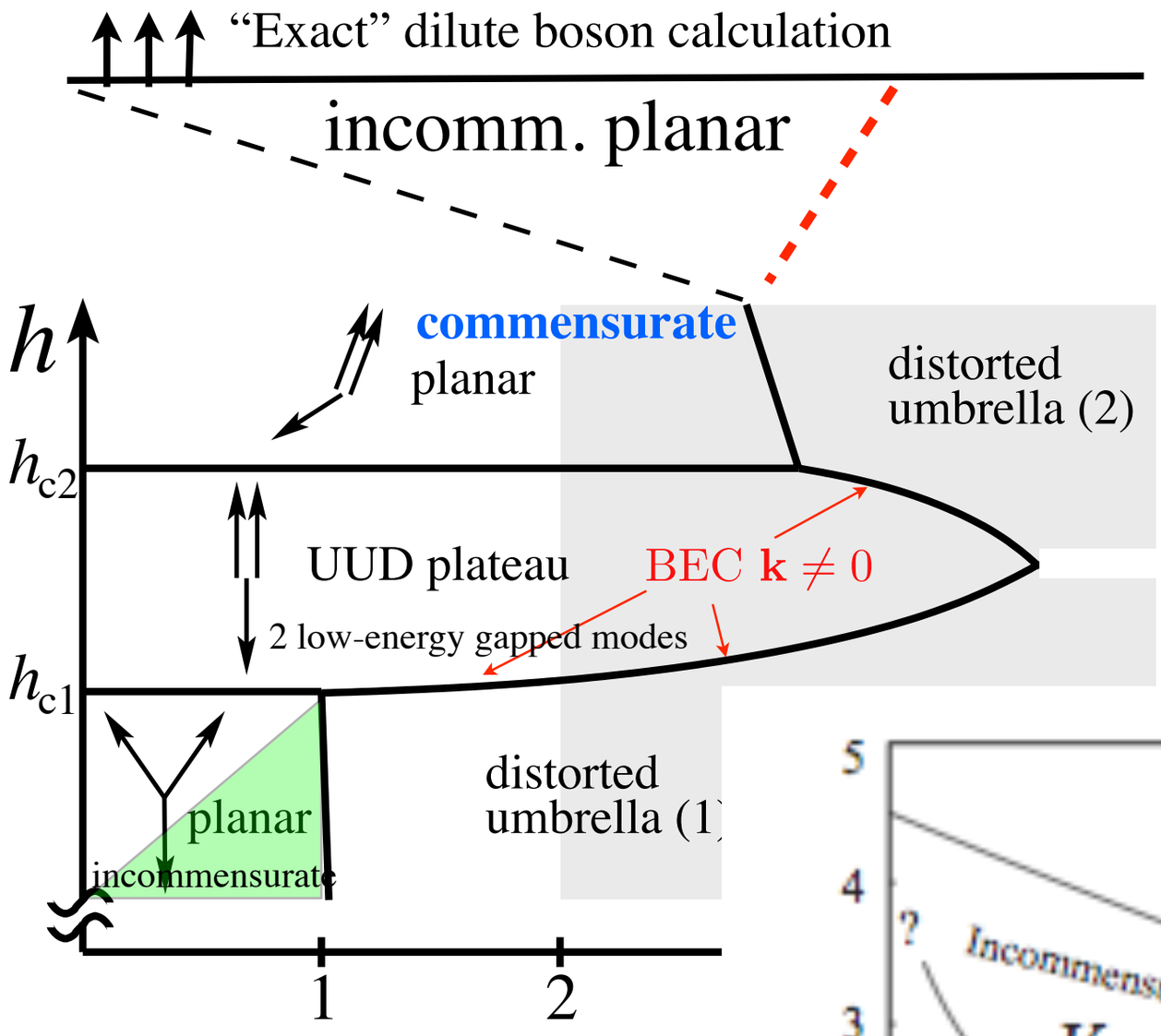
The competition is controlled by dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

Our *semiclassical* approach: treat spatial anisotropy (J–J') as a perturbation to **interacting** spin waves

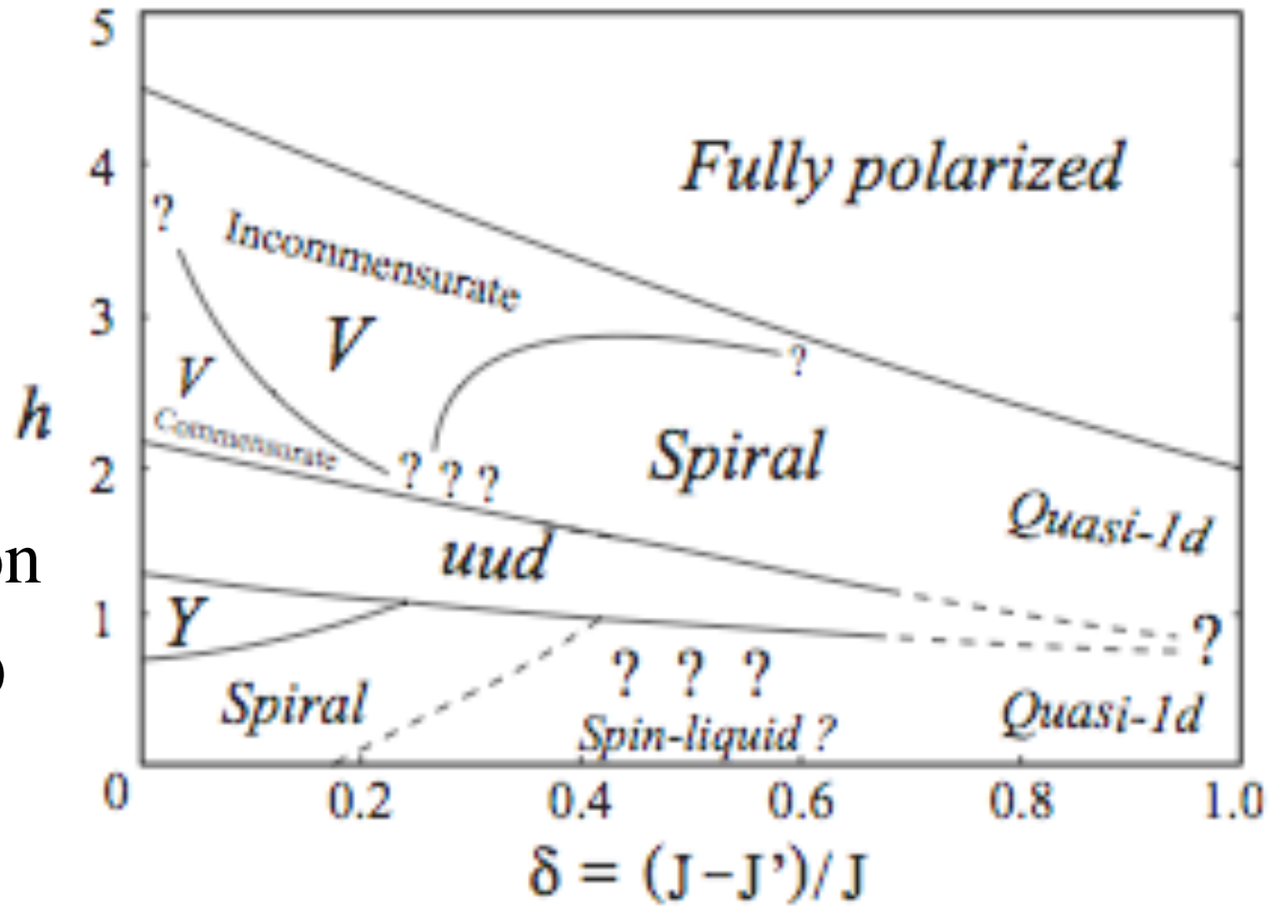


Alicea, Chubukov, Starykh PRL 102, 137201 (2009)



Variational wave function calculation

Tay, Motrunich PRB 2010

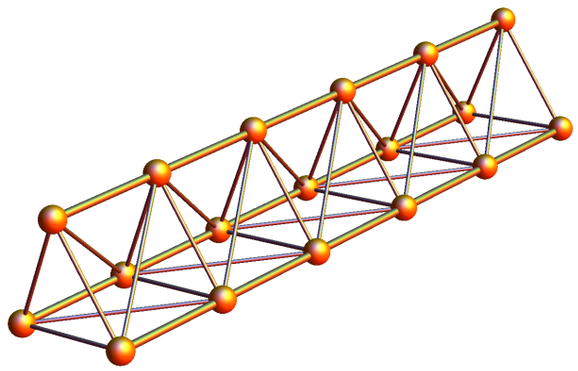


Outline

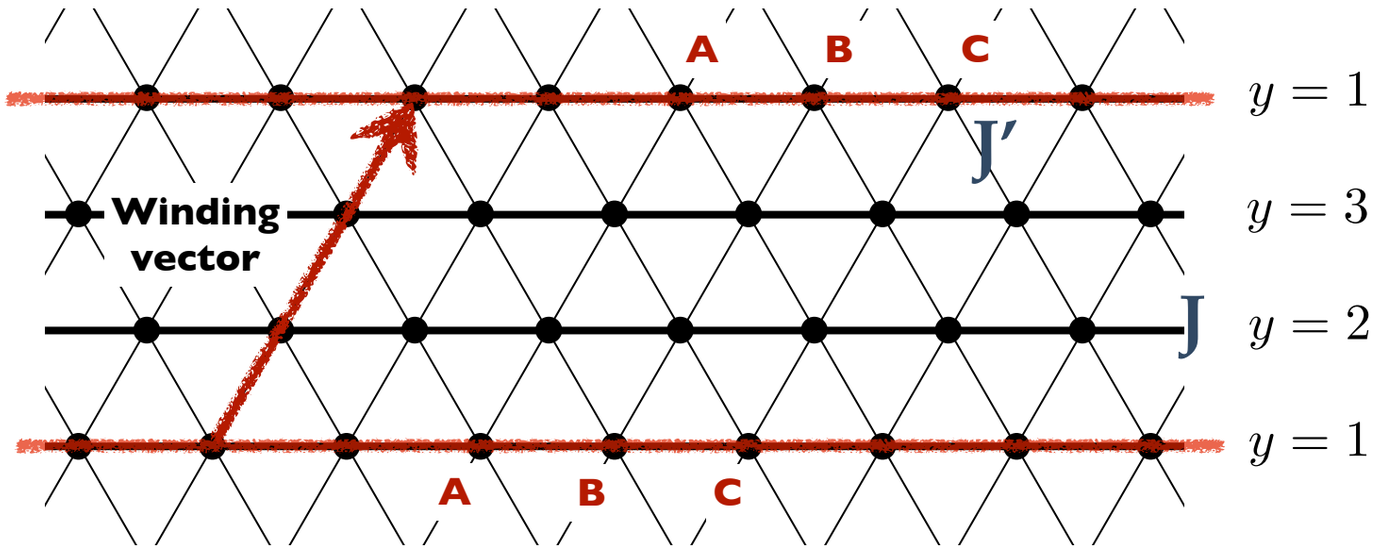
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★ Conclusions

S=1/2 Model







$y = 1$

$y = 3$

$y = 2$

$y = 1$

Periodic boundary conditions along y in numerical studies

- Hamiltonian

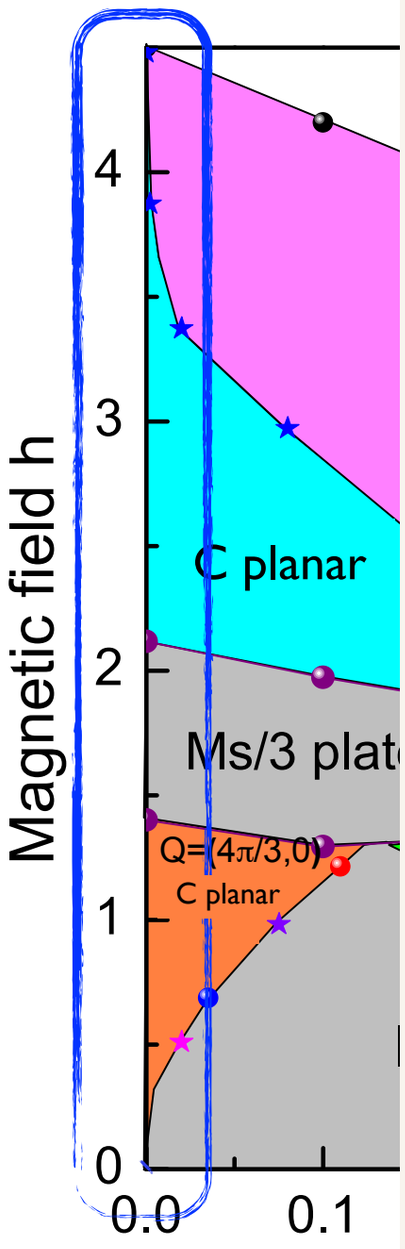
$$H = \sum_{x,y} J S_{x,y} \cdot S_{x+1,y} + J' \left(S_{x,y} \cdot S_{x,y+1} + S_{x,y} \cdot S_{x-1,y+1} \right)$$

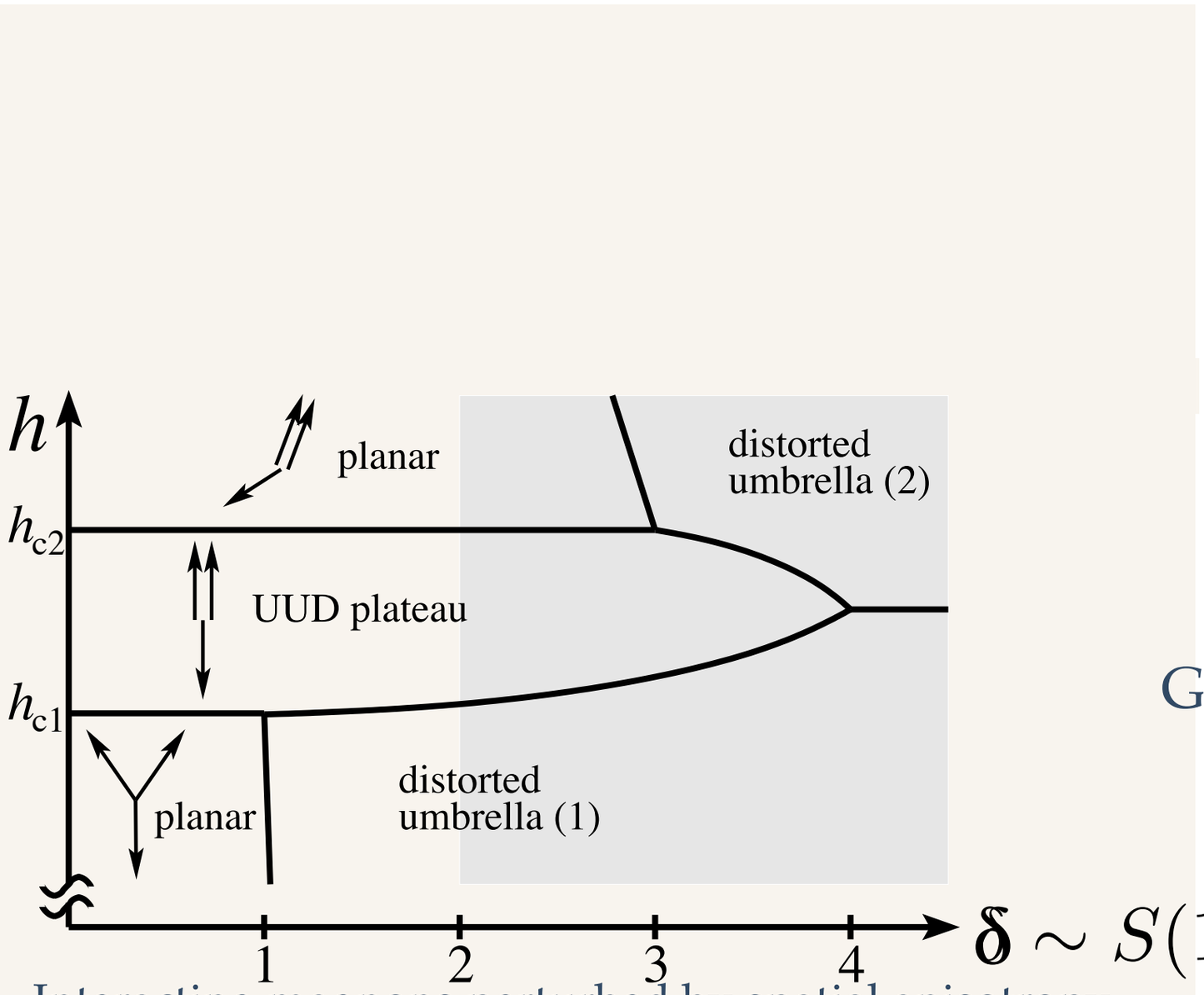
$$-h \sum_{x,y} S^z_{x,y}$$

Cs_2CuCl_4: $J'/J = 0.34$

Cs_2CuBr_4: $J'/J = 0.5\text{-}0.7$

Isotropic case: quantum fluctuations select co-planar states (agrees with semi-classical S>> 1 calculations)



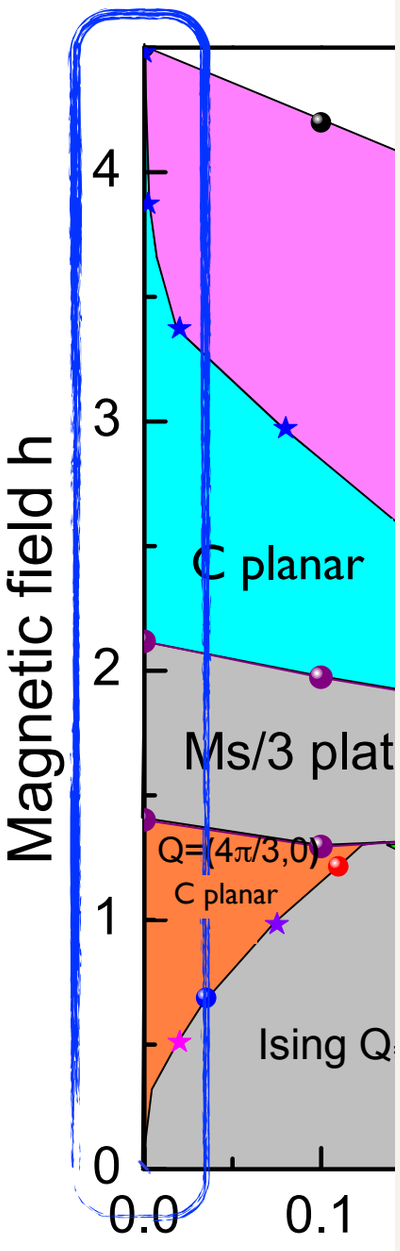


Interacting magnons perturbed by spatial anisotropy

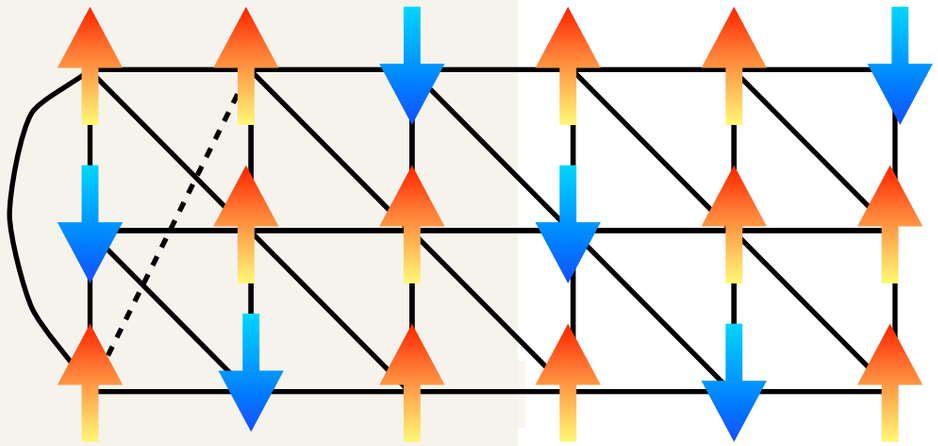
isotropic case:
Chubukov+Golosov, 1991;
Alicea, Chubukov, Starykh, 2008

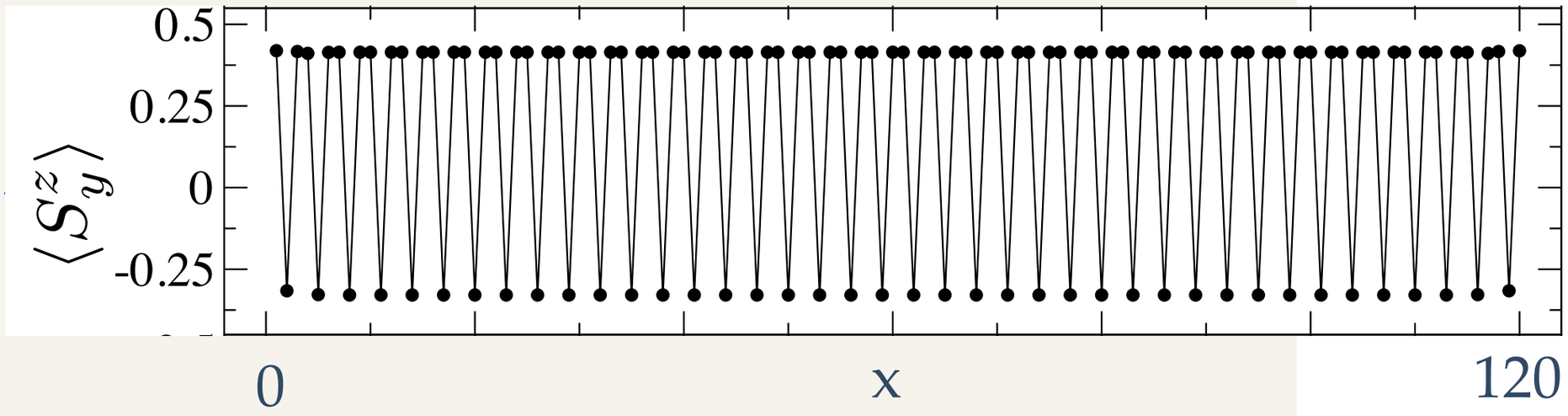
UUD preserves U(1) symmetry. Gapped spin waves

Plateau phase



$M/M_s=1/3$ plateau:
uud state

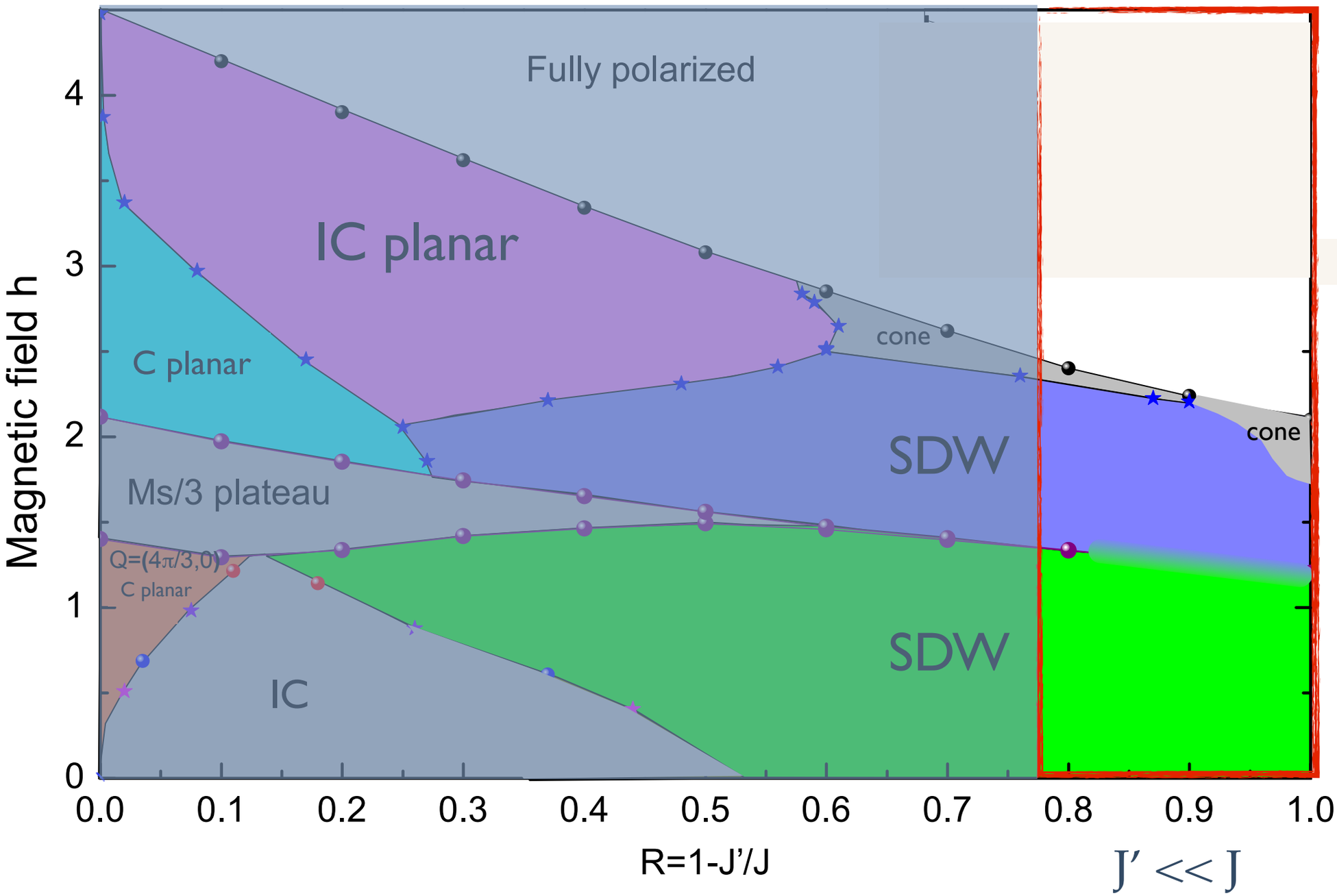




$R = 0.2$

$R = 1 - J'/J$

Weakly coupled chains



weakly coupled chains

$J' \ll J$

$R = 1 - J'/J$

Ideal J-J' model in magnetic field

OS, Balents 2007

- Two important couplings for h>0
- Quantum phase transition between SDW and Cone states

Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2

dim $1+2\pi R^2$: 2 -> 3/2

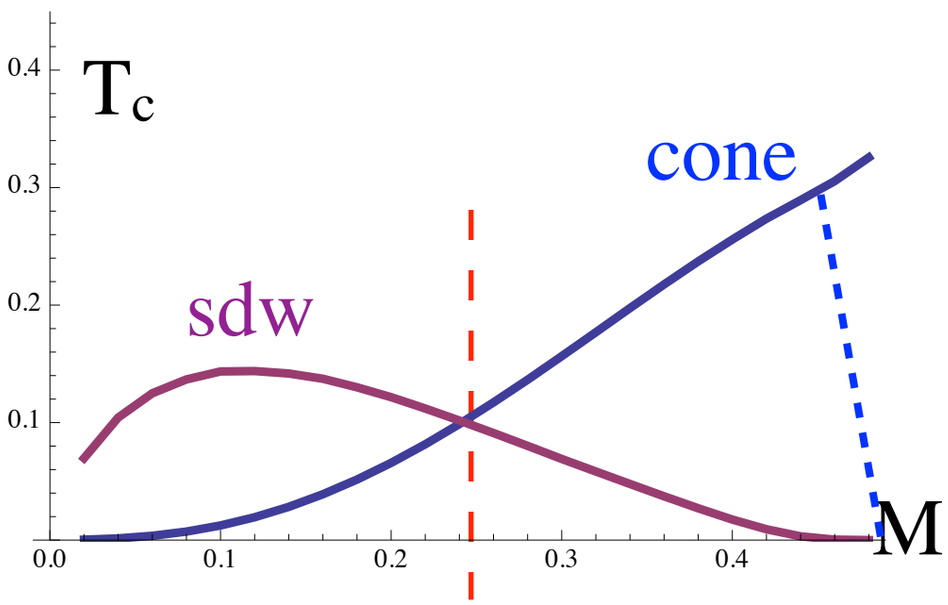
$k_{F\downarrow} - k_{F\uparrow} = 2\delta = 2\pi M$

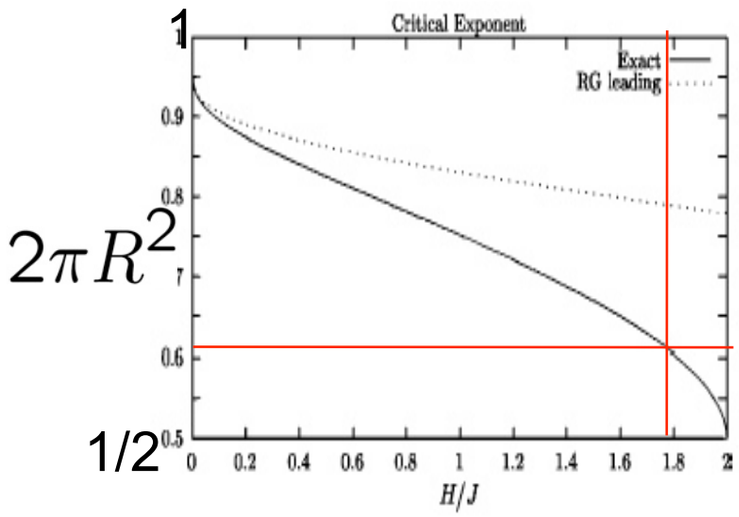
"collinear" SDW

spiral "cone" state

- "Critical point": $1+2\pi R^2 = 1/2\pi R^2$ gives at M = 0.3

$$2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$$



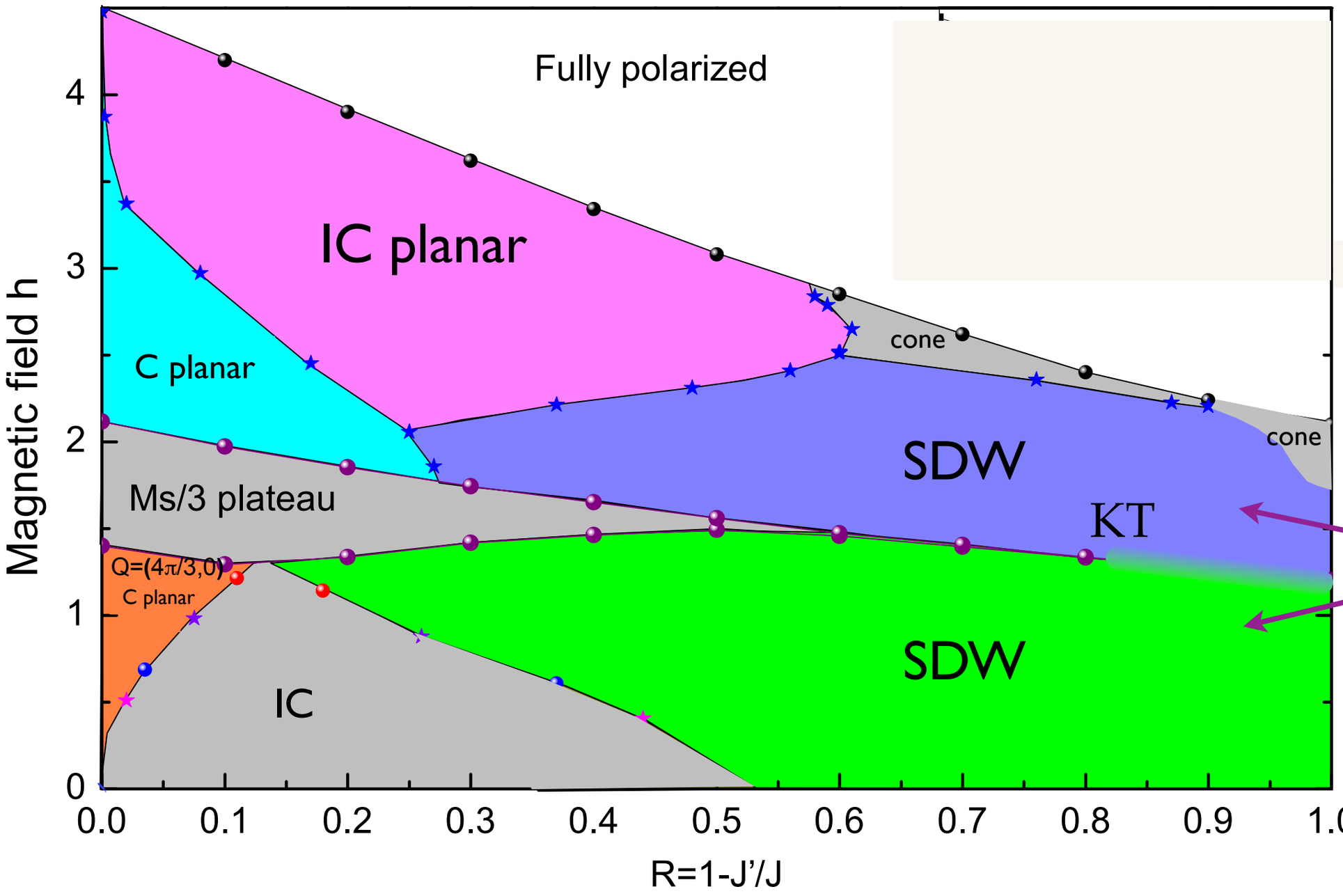


also: Kolezhuk, Vekua 2005

h/h_{sat}

3-leg ladder: dominant SDW phase

Ru Chen, Hyejin Ju, Hongchen Jiang, OS, Leon Balents (2012)



SDW phase agrees with analytical predictions for $J' \ll J$

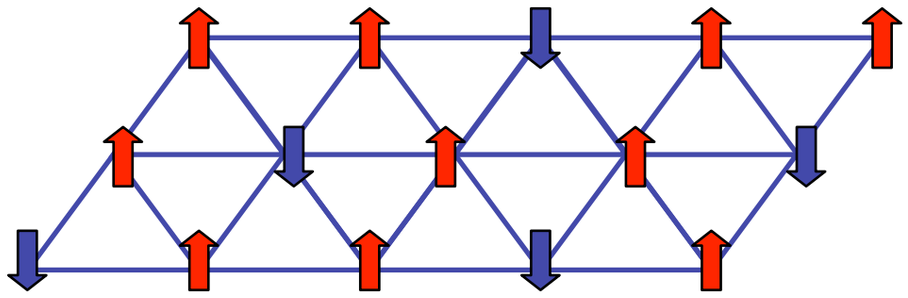
J-J’ model: magnetization plateaux via commensurate locking of **SDW**

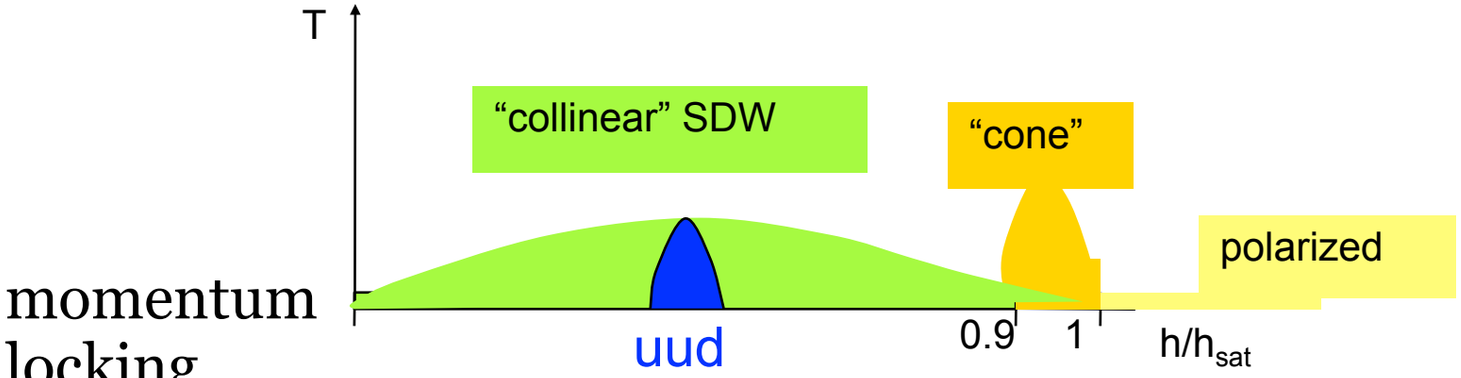
- “Collinear” SDW state *locks* to the lattice at low-T
  - -“irrelevant” (1d) umklapp terms become relevant once SDW order is present (when *commensurate*): multiparticle umklapp scattering

  - -strongest locking is at M=1/3 M_{sat}

    ✓Observed in **Cs_2CuBr_4** (Ono 2004, Tsuji 07, Fortune 09)

    - ***down***-spins at the centers of hexagons





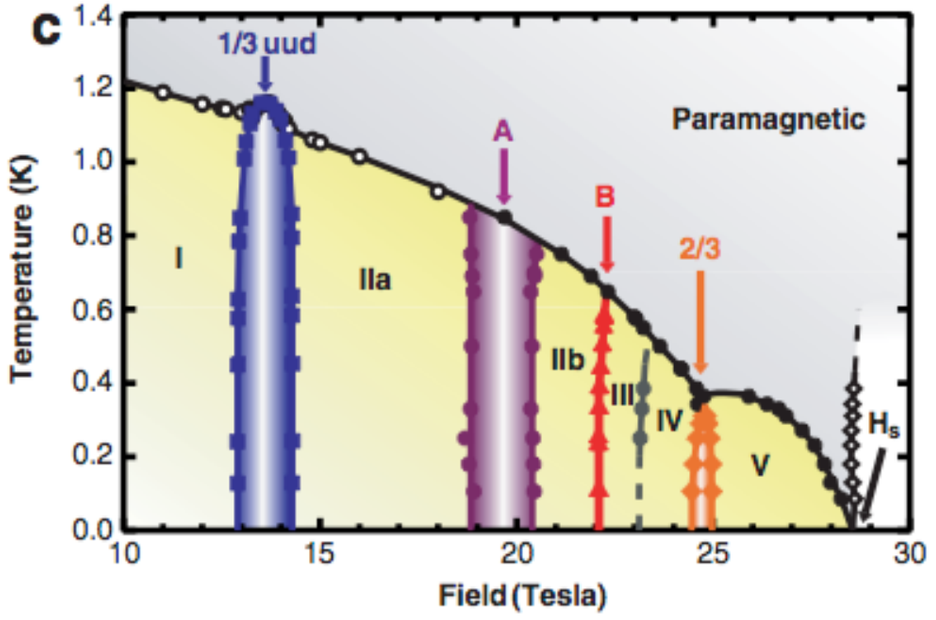
$$\left(\Psi_R^\dagger \Psi_L\right)^n \rightarrow (\pi - 2\delta)n = 2\pi m \rightarrow \boxed{2M = 1 - 2m/n}$$

n	3	4	5	5	6
m	1	1	1	2	1
2M	1/3	1/2	3/5	1/5	2/3

naively thinking

Cs_2CuBr_4 Fortune et al 2009

1/3 2/3



Plateau more carefully

OS, Katsura, Balents PRB 2010

$$M^{(n,m)} = \frac{1}{2}\left(1 - \frac{2m}{n}\right)$$

- Umklapp must respect triangular lattice symmetries
  - translation along chain direction
  - translation along diagonal
  - spatial inversion

$$\phi_y(x) \to \phi_y(x+1) - R(\pi - 2\delta)$$
$$\phi_y(x) \to \phi_{y+1}(x+1/2) - R(\pi - 2\delta)/2$$
$$\phi_y(x) \to \pi R - \phi_y(-x)$$

$$H^{(n)}_{umk} = \sum_y \int dx\, t_n \cos[\frac{n}{R}\phi_y] \quad \textbf{and} \quad n = m \ (mod\ 2)$$

same parity condition

- n-th plateau width (in field) $width \sim \left(J'/J\right)^{n^2/(4(4\pi R^2-1))}$

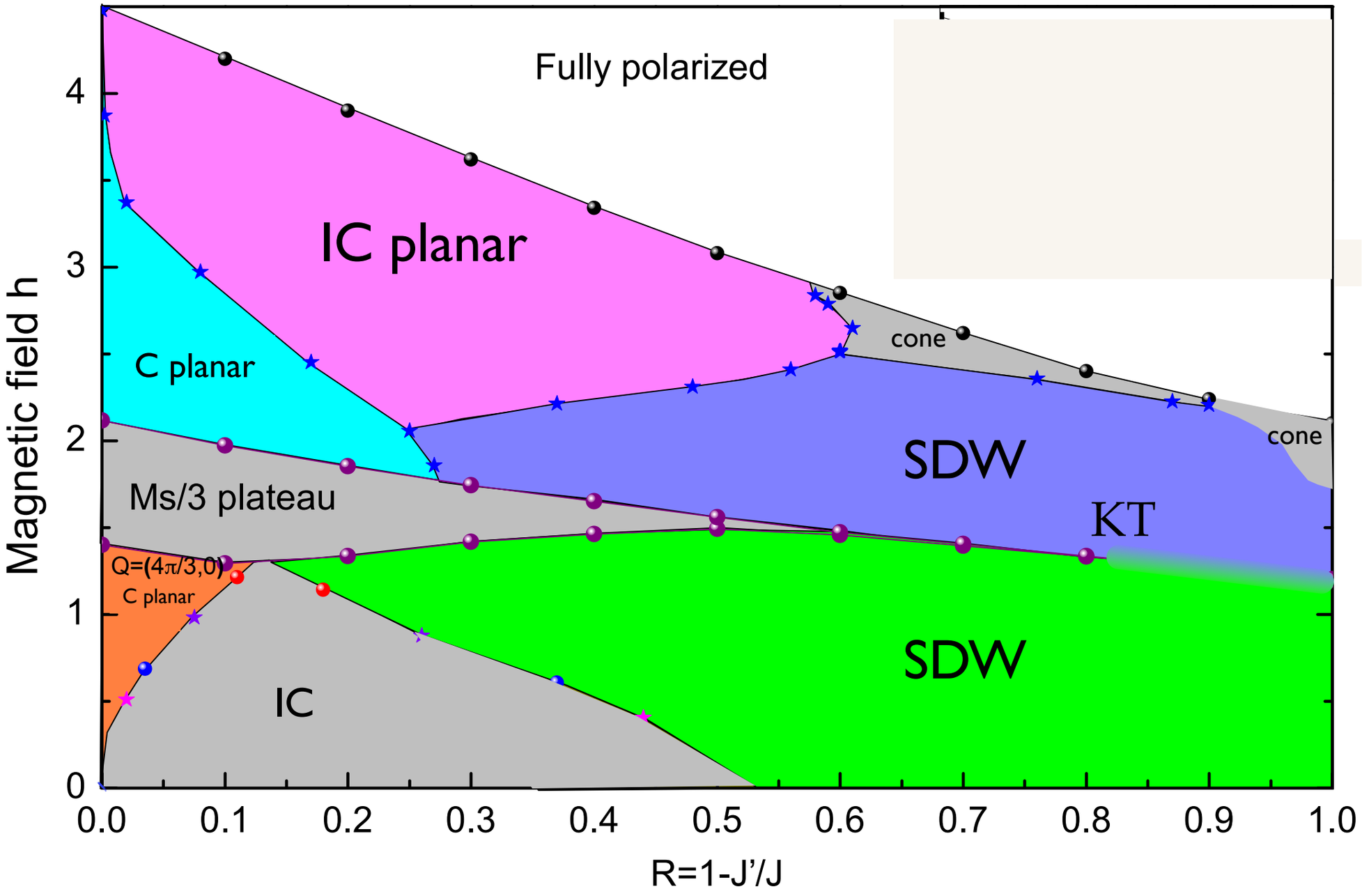
n	3	8	5	10	12
m	1	2	1	4	2
2M	1/3	1/2	3/5	1/5	2/3

large **n** leads to exponential suppression

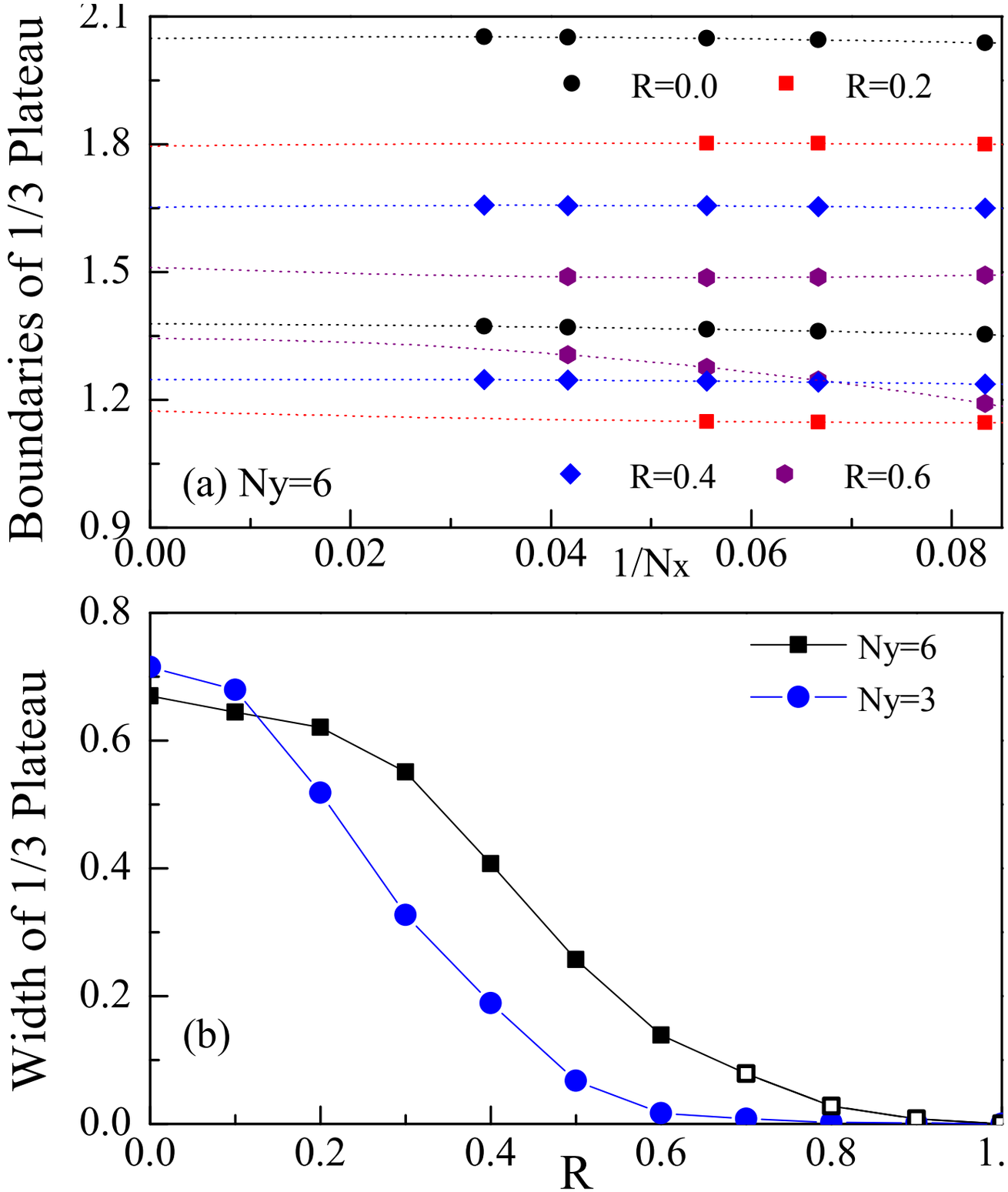
- Ladder: Kosterlitz-Thouless transition to the plateau state @ R=0.7±0.1 (J'/J = 0.3) because umklapp operator becomes irrelevant for J'/J -> 0

Plateau endpoint (3-leg ladder)

- Ladder: Kosterlitz-Thouless transition to the plateau state @ R=0.7±0.1 (J'/J = 0.3) because umklapp operator becomes irrelevant for J'/J -> 0



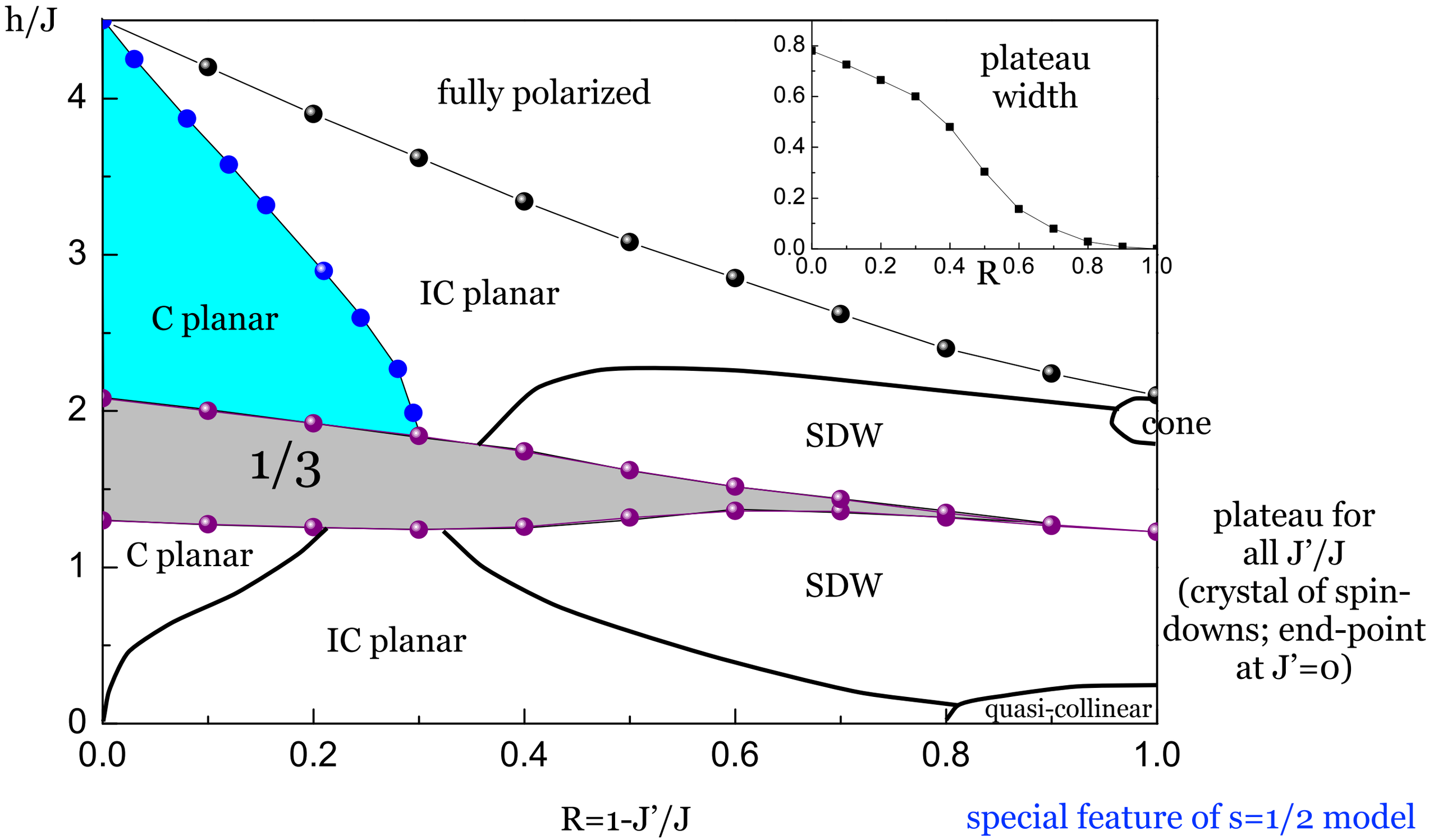
DMRG: From 3- to 6-leg ladder



plateau expands in a wider ladder: stable phase in two dimensions

two-dimensional

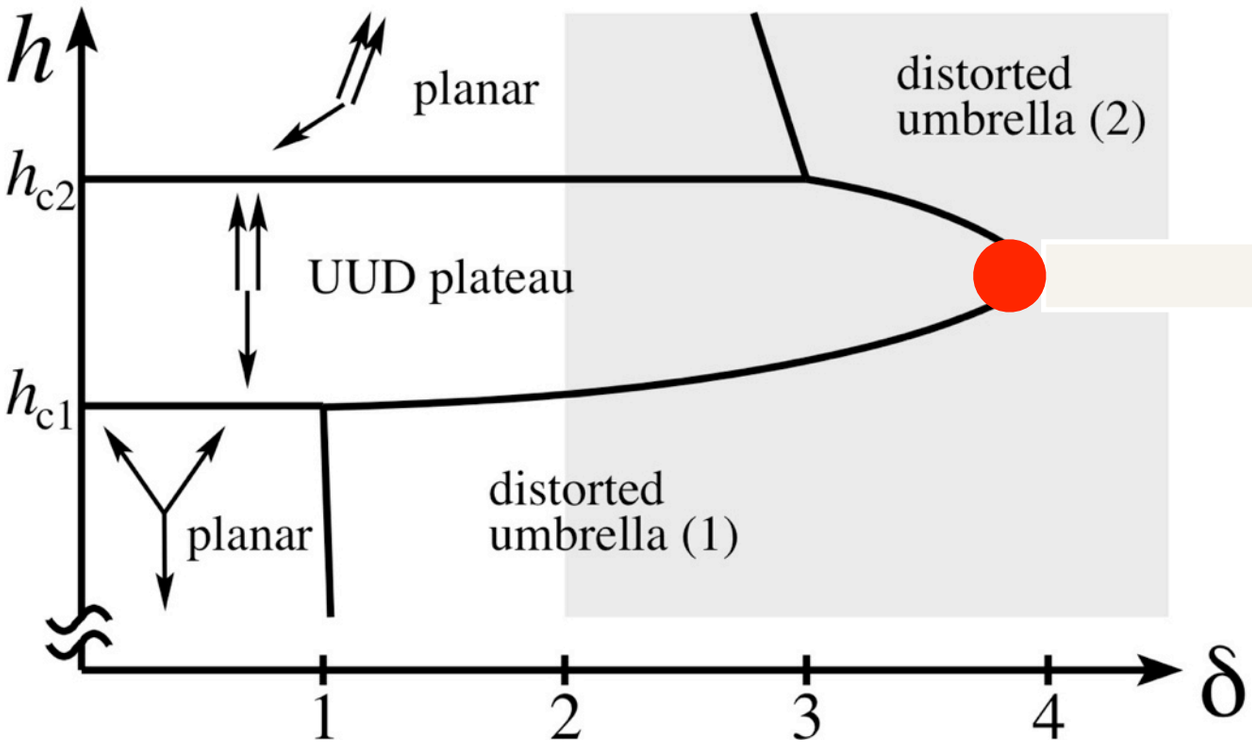
Schematic phase diagram for spin-1/2 triangular lattice AFM



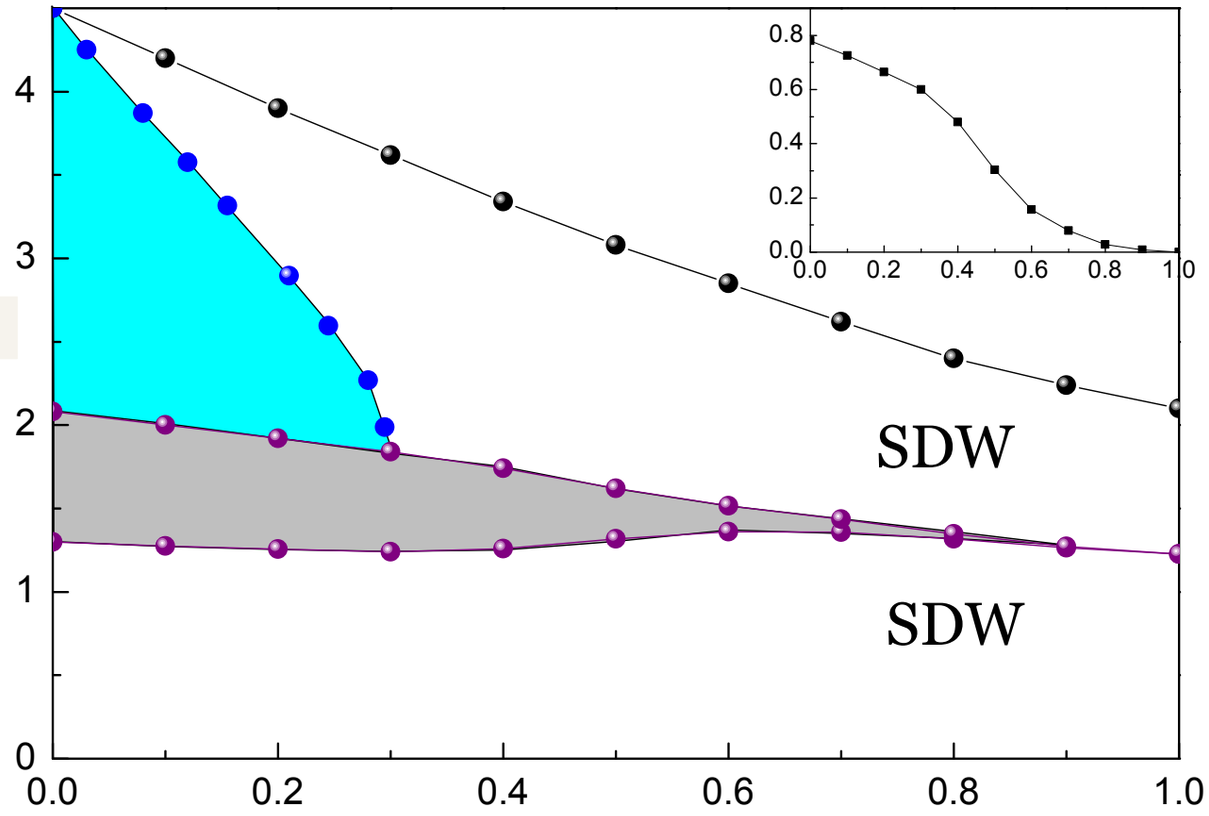
plateau for all J'/J (crystal of spin-downs; end-point at J'=0)

special feature of s=1/2 model

S>>1 ? S=1/2



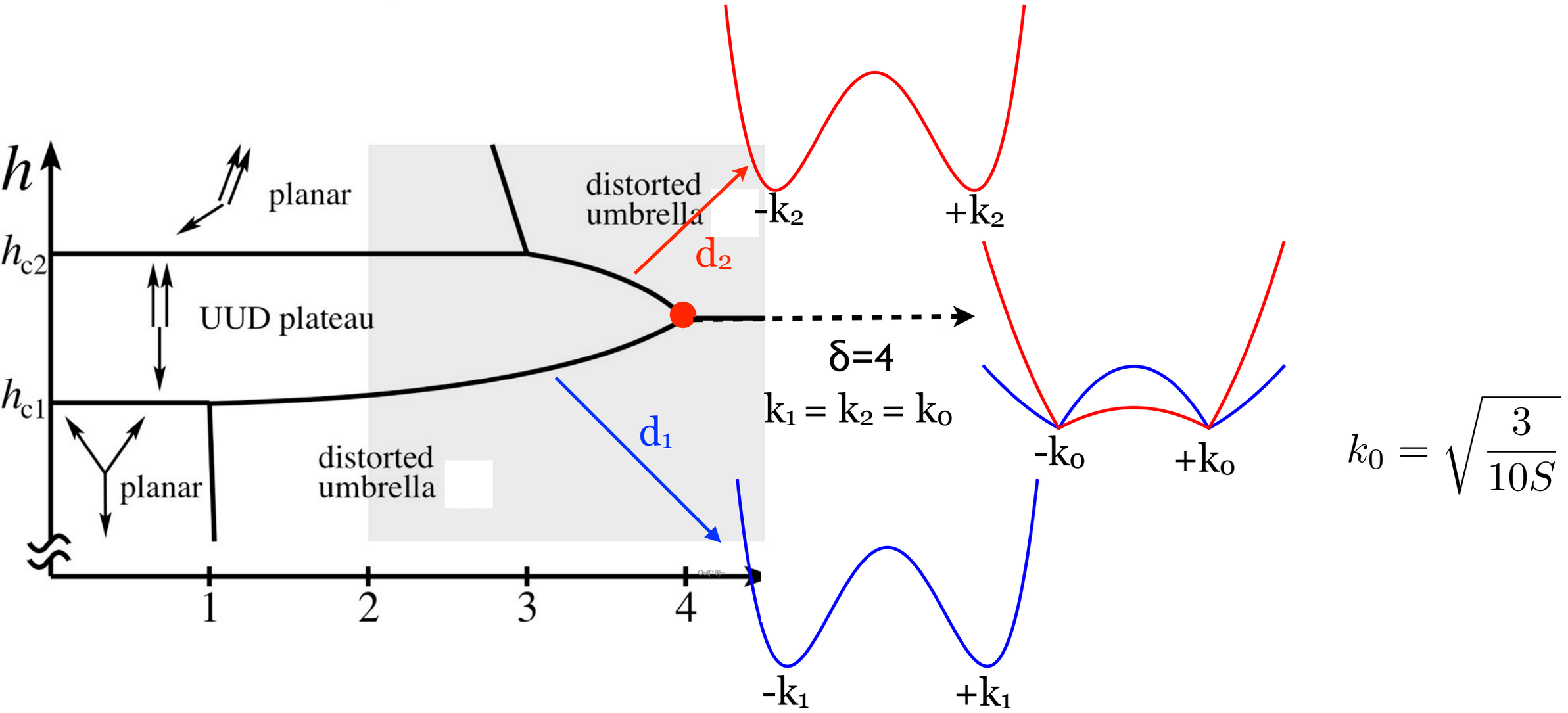
out of plateau transitions -
non-coplanar states



out of plateau transitions -
collinear SDW state

Low-energy excitation spectra near the plateau's end-point

S>>1



$$k_0 = \sqrt{\frac{3}{10S}}$$

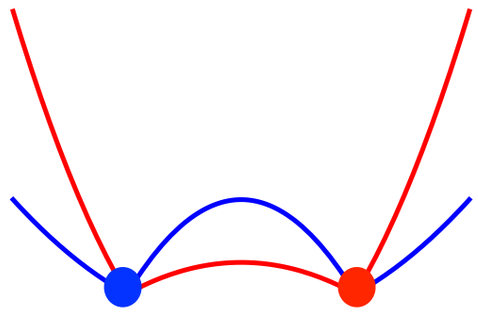
A. Chubukov, OS, PRL 2013

Interaction between plateau's eigen-modes

$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\left(d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}\right) + \text{h.c.}$$

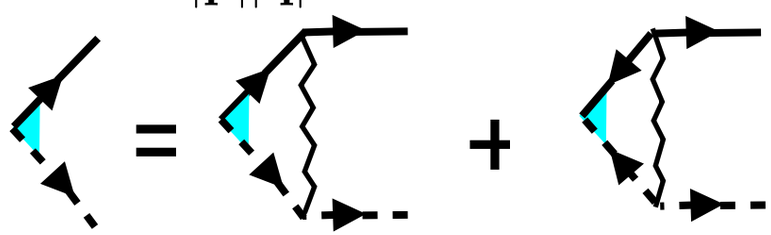
$$\Psi^\dagger_{1,p}\Psi_{2,q} \qquad \Psi^\dagger_{1,p}\Psi^\dagger_{2,q}$$

magnon **pairs** $\Psi_{1/2,p} = d_{1,\pm k_0+p}d_{2,\mp k_0-p}$



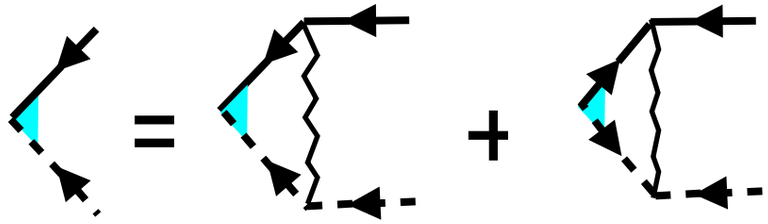
interact strongly near the plateau's end-point δ=4

$$\Phi(p,q) = -3J\frac{k_0}{\sqrt{\mathbf{p}^2+(1-\delta/4)k_0^2}}\frac{k_0}{\sqrt{\mathbf{q}^2+(1-\delta/4)k_0^2}} \to -3J\frac{k_0^2}{|\mathbf{p}||\mathbf{q}|}$$



`Superconducting' solution with *imaginary* order parameter

$$\langle\Psi_{1,p}\rangle = \langle\Psi_{2,q}\rangle \sim i\frac{\Upsilon}{|\mathbf{p}|}$$



no single particle condensate!

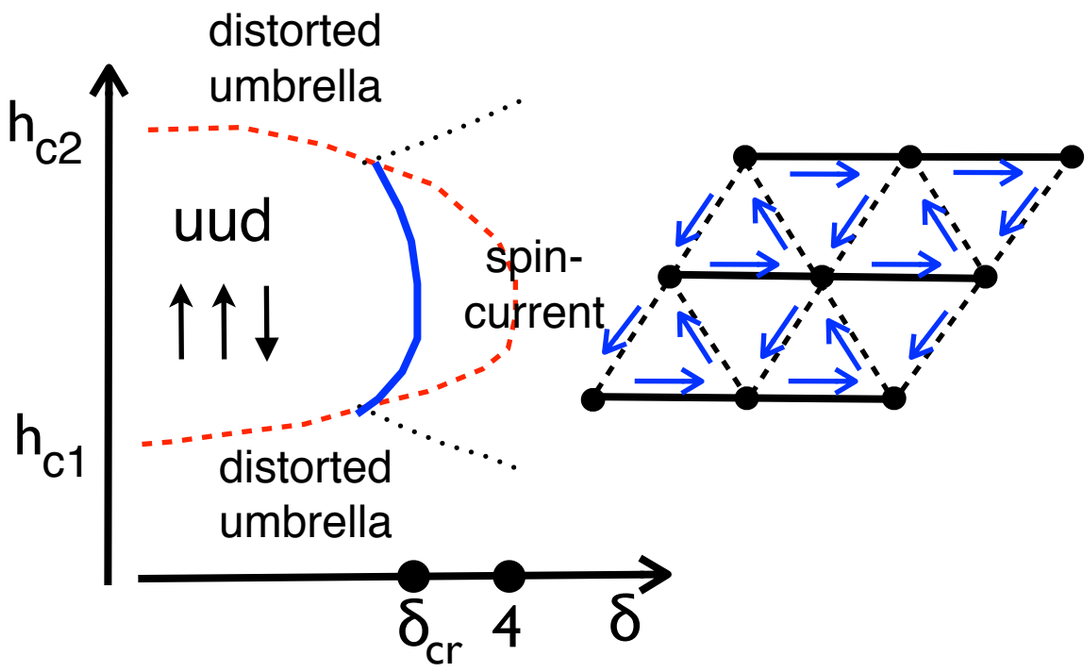
$$\langle d_{1/2,\mathbf{k}}\rangle = 0$$

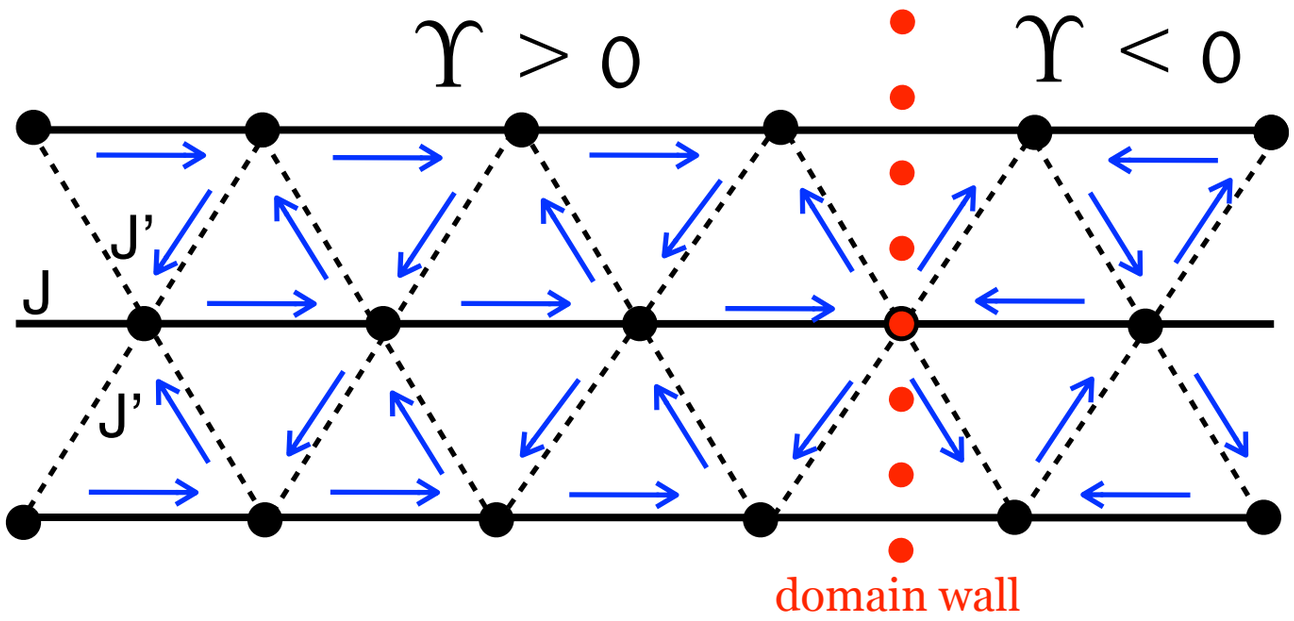
Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

$$1 = \frac{1}{S}\frac{1}{N}\sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2+(1-\delta/4)k_0^2}}$$

A. Chubukov, OS, PRL 2013

Two-magnon condensate = Spin-current state





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$

Finite scalar (and vector) chiralities. Sign Y determines sense of spin-current circulation

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

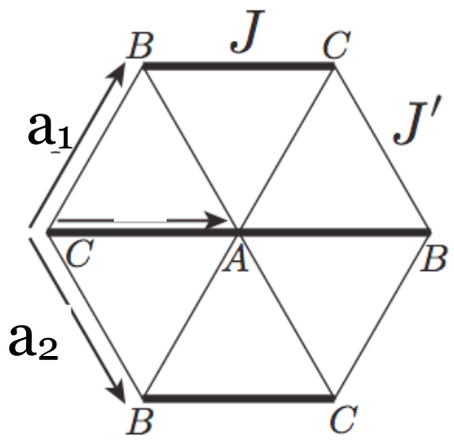
but $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

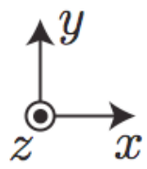
Spontaneously broken Z_2 , in addition to broken Z_3 inherited from the UUD state

A. Chubukov, OS, PRL 2013

Spontaneous generation of Dzyaloshinskii-Moriya interaction

$$\mathcal{H}^{(4)}_{d_1 d_2} \propto \frac{1}{N}\sum_{k\in+k_0}\frac{-ik_0}{\sqrt{(k-k_0)^2+(1-\delta/4)k_0^2}}(d^\dagger_{1,\mathbf{k}}d^\dagger_{2,-\mathbf{k}}-d_{1,\mathbf{k}}d_{2,-\mathbf{k}})\sum_{p\in-k_0}\frac{-ik_0}{\sqrt{(p+k_0)^2+(1-\delta/4)k_0^2}}(d^\dagger_{1,\mathbf{p}}d^\dagger_{2,-\mathbf{p}}-d_{1,\mathbf{p}}d_{2,-\mathbf{p}})$$





continuum limit of DM in triangular lattice

$$\sum_{\mathbf{r}}\hat{z}\cdot\mathbf{S_r}\times(\mathbf{S_{r+a_1}}+\mathbf{S_{r+a_2}})$$

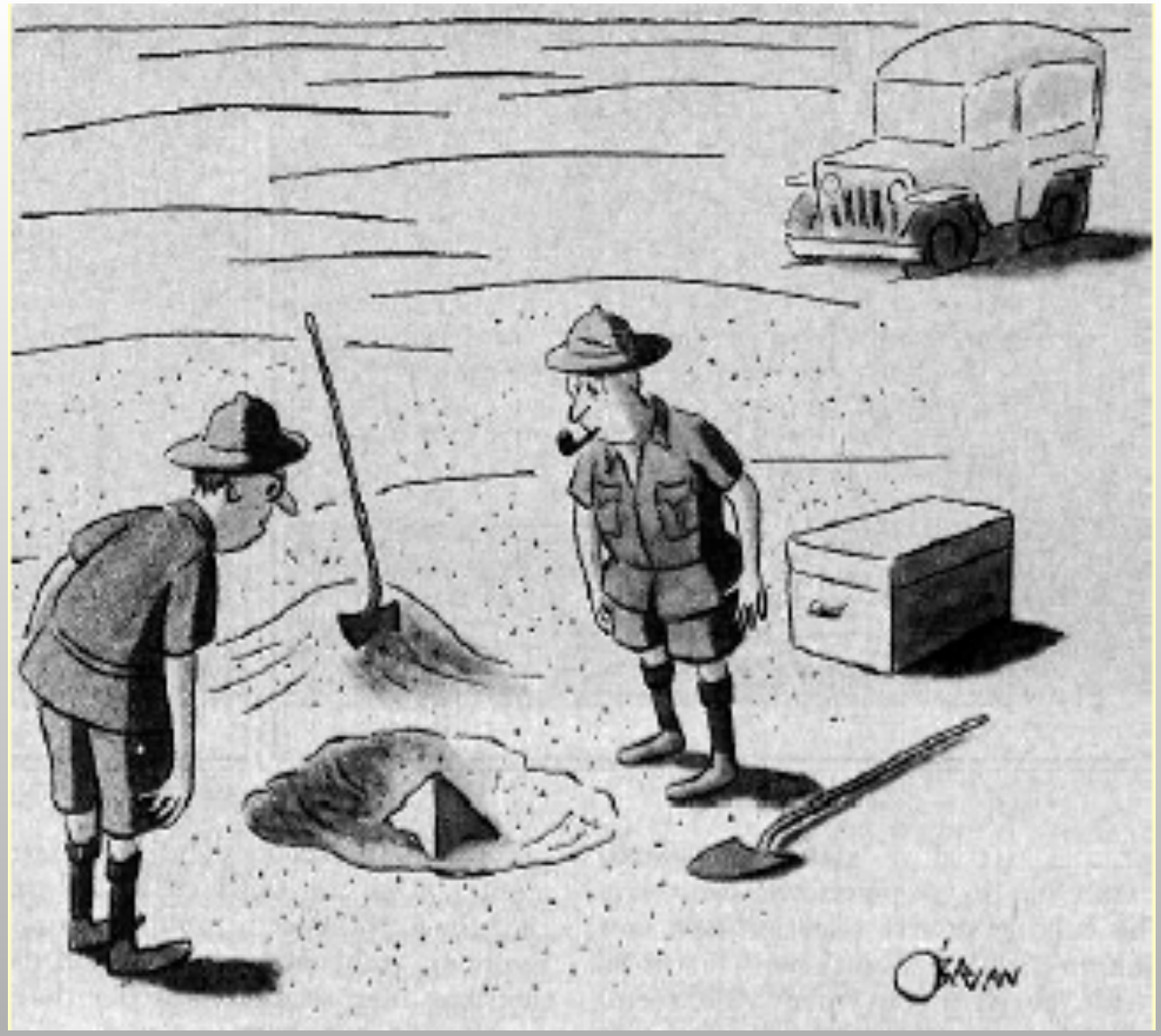
B || z

Mean-field approximation:

$$\mathcal{H}^{(4)}_{d_1 d_2} \to D\sum_k(\frac{k_0}{|k-k_0|}+\frac{k_0}{|k+k_0|})(d^\dagger_{1,\mathbf{k}}d^\dagger_{2,-\mathbf{k}}-d_{1,\mathbf{k}}d_{2,-\mathbf{k}})$$

$$D\sim\Upsilon k_0^2\ln(\frac{4}{4-\delta})$$

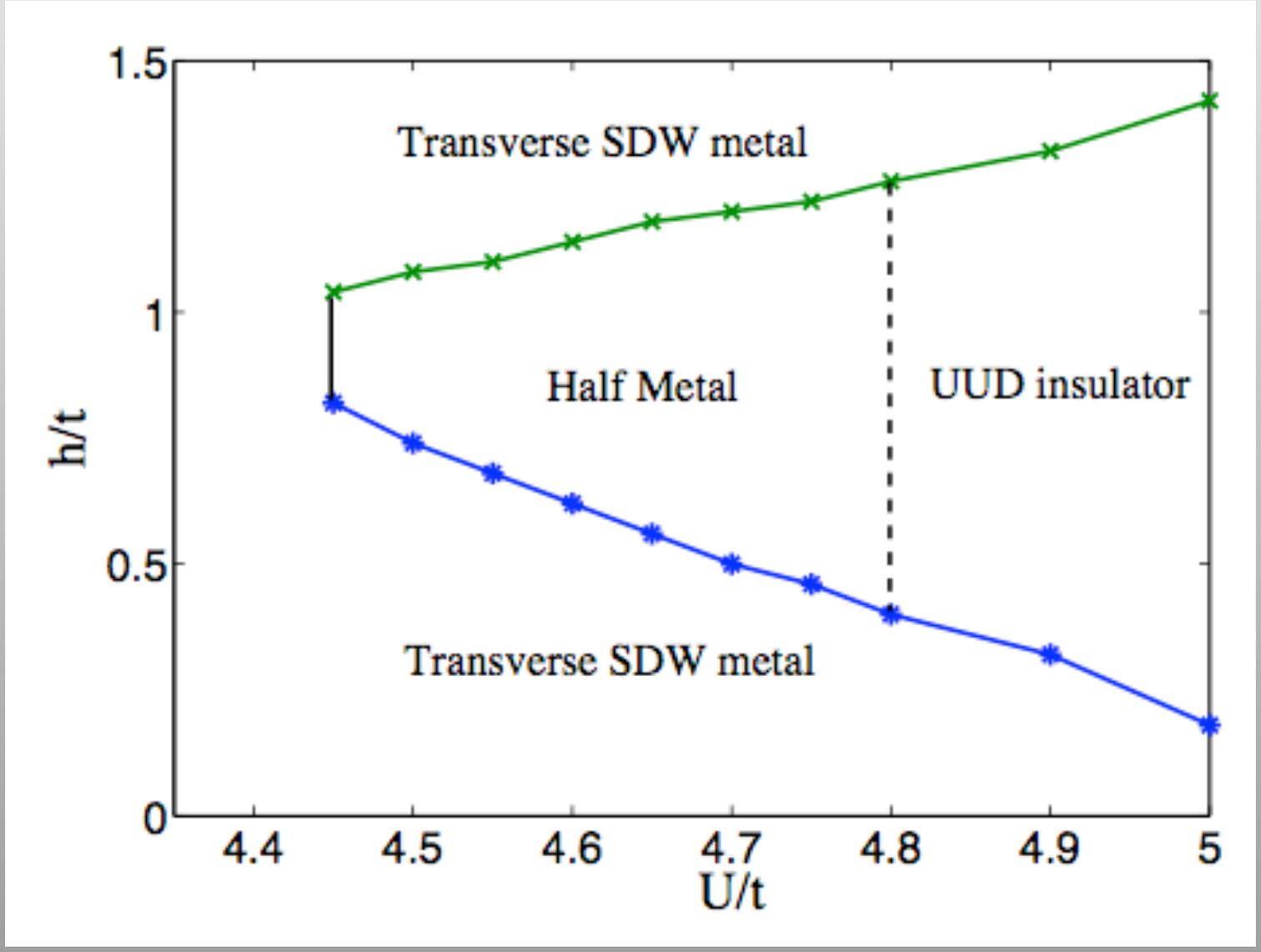
spin currents appear due to **spontaneously generated DM**
(similar to Lauchli et al (PRL 2005) for Heis.+ring exchange model;
also ‘chiral Mott insulator’, Dhar et al, PRB 2013)



"This could be the discovery of the century. Depending, of course, on how far down it goes"

Magnetization plateau in itinerant electron systems

- Heisenberg = large-U Hubbard model
- What is the fate of M =1/3 plateau when U/t is reduced?

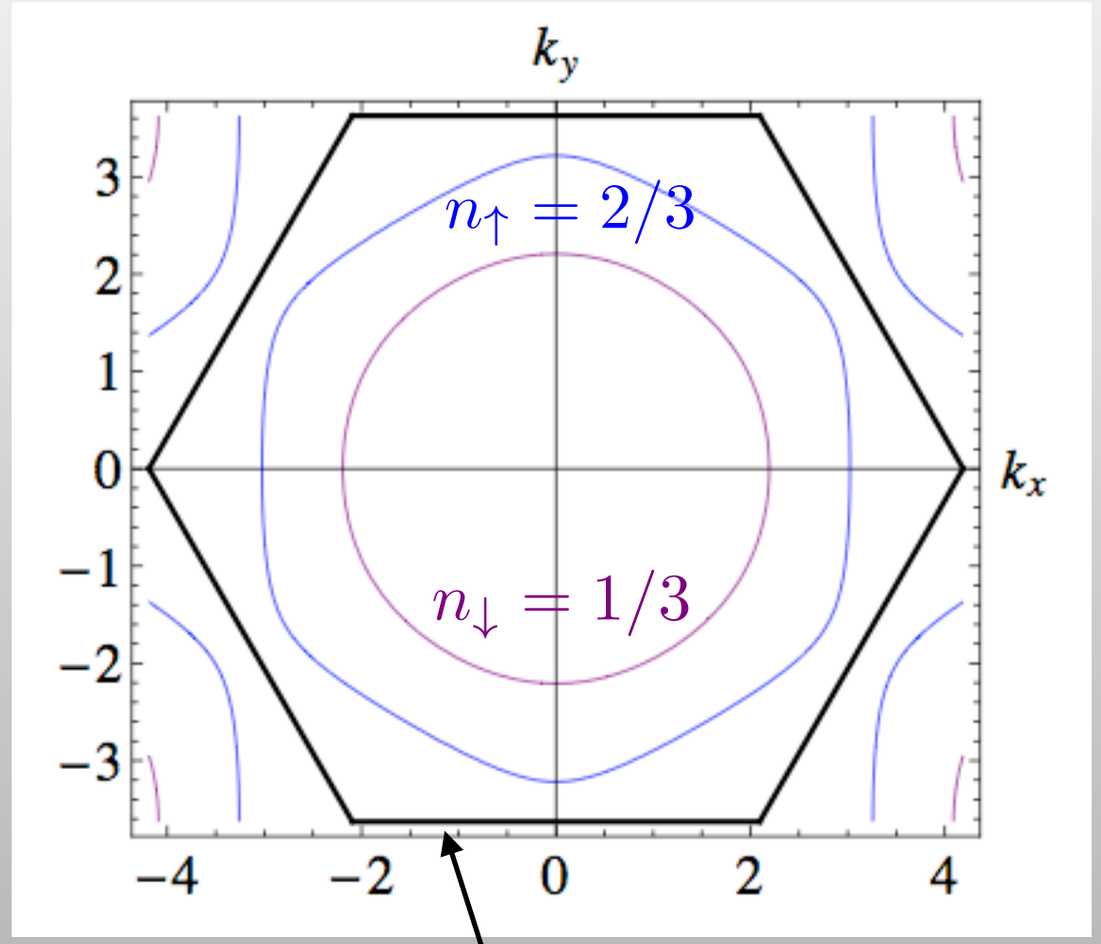


simple mean-field analysis
of the UUD state

Zhihao Hao and OS, PRB 2013

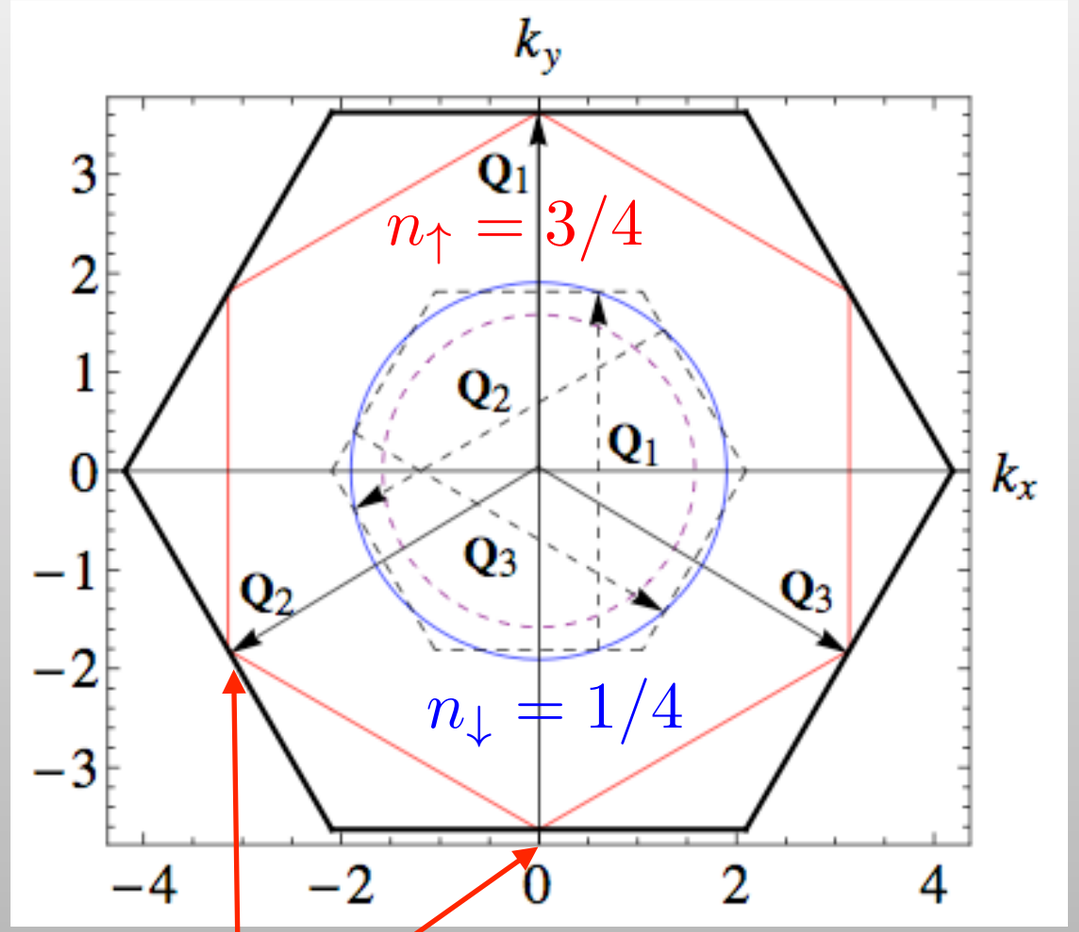
Band structure and geometry, U=0

M=1/3



Brillouin zone of triangular lattice

M=1/2



van Hove points

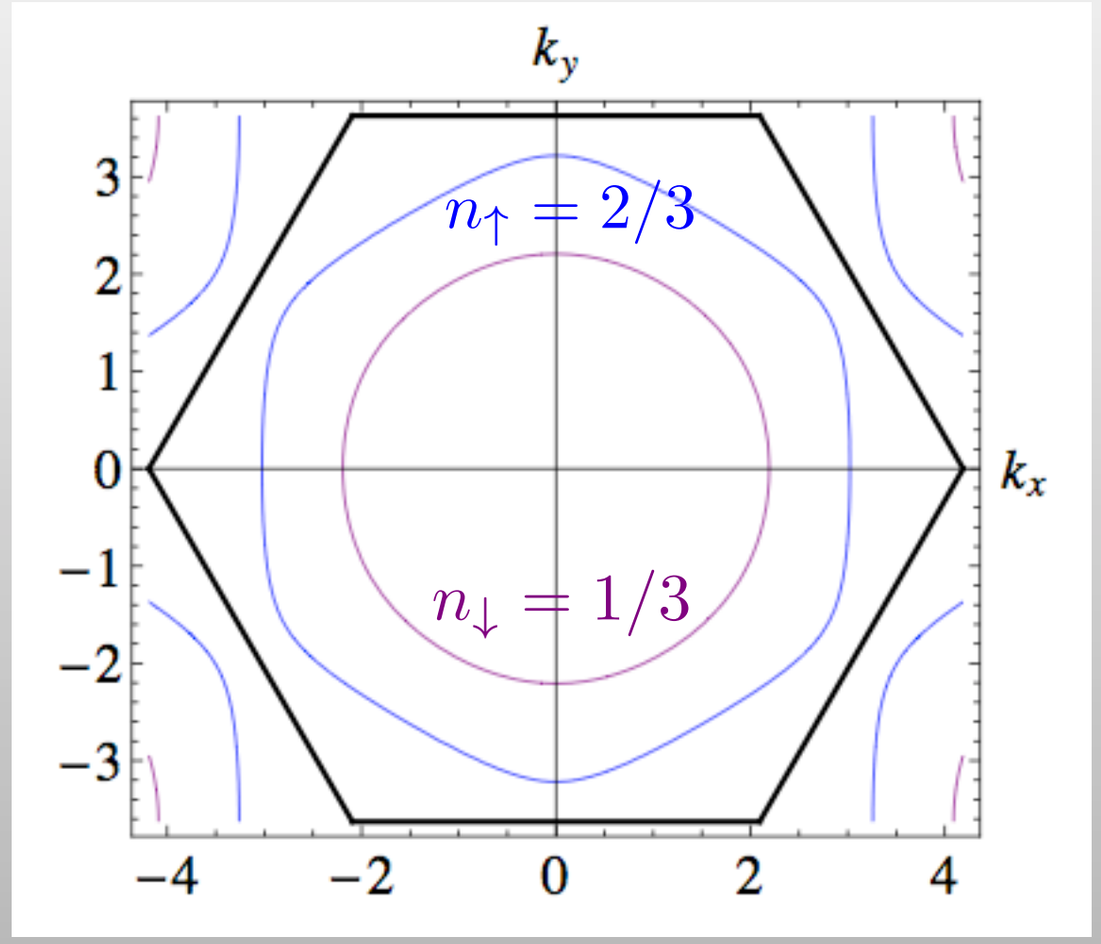
$$n_\uparrow + n_\downarrow = 1$$

$$M/M_{\rm sat} = (n_\uparrow - n_\downarrow)$$

Z. Hao, OS, PRB 2013

Band structure and geometry, U=0

M=1/3

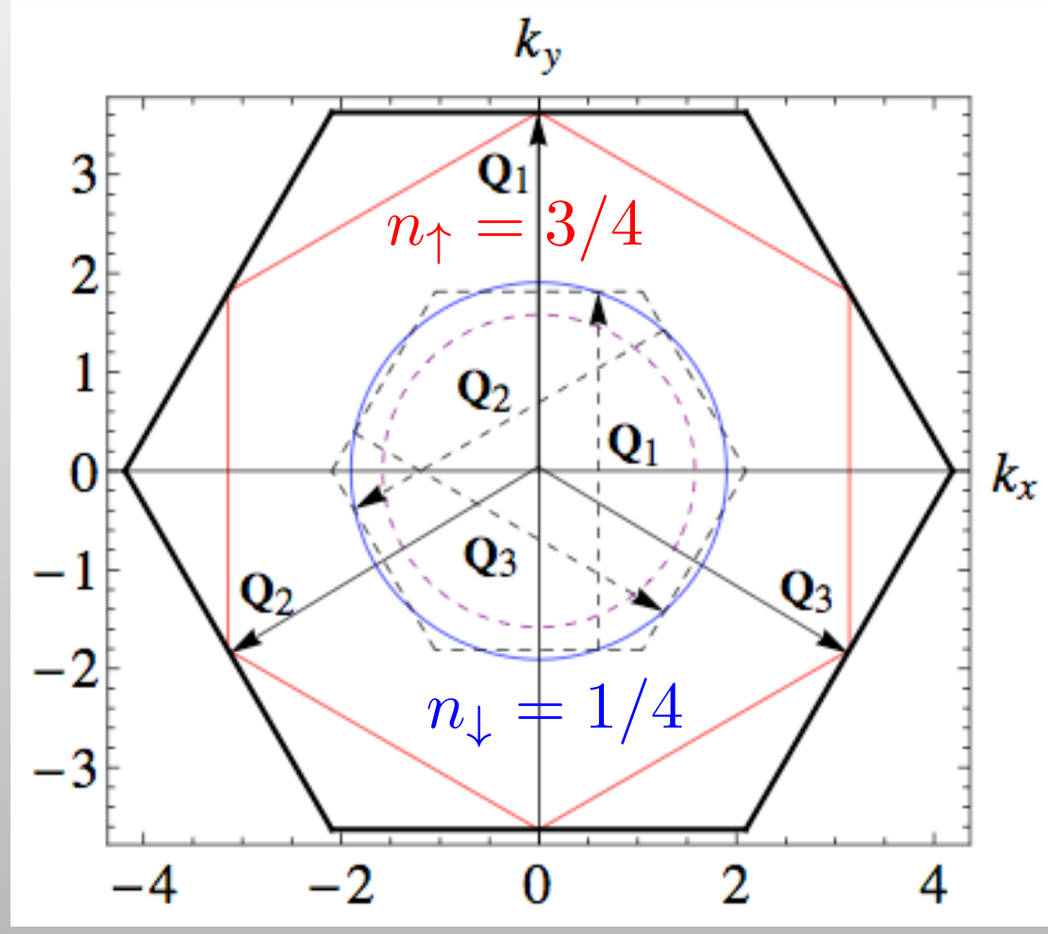


Strong-coupling instability.

Result = **half-metal**.

Magnetization plateau state.

M=1/2



Weak-coupling instability:
arbitrary weak e-e interaction gaps out up-spins Fermi surface completely;
down-spins FS reconstructs but remains gapless.
Result = **half-metal**, electric current is carried by minority (down) spin electrons.
Magnetization plateau state.

Extensions

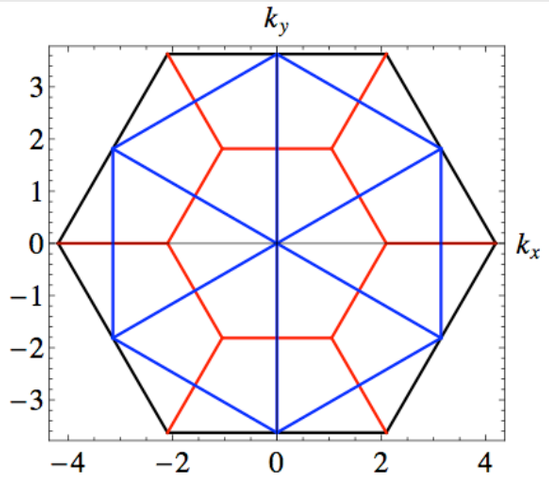
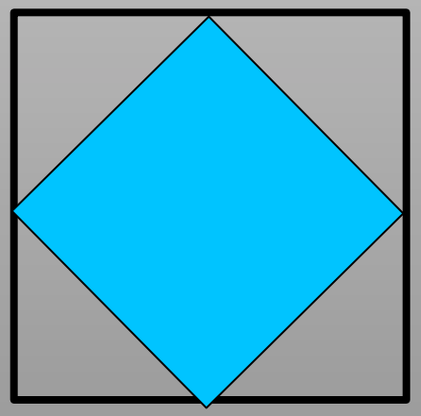


FIG. 1: The Fermi surface at $y_\uparrow = 0$ (the blue lines) under the folded zone scheme. The folded zone (the red hexagons) is four times smaller than the first Brillouin zone of the triangular lattice (the black hexagon).

reconstructed Fermi surface

Other lattices: *almost* 1/2-filled square



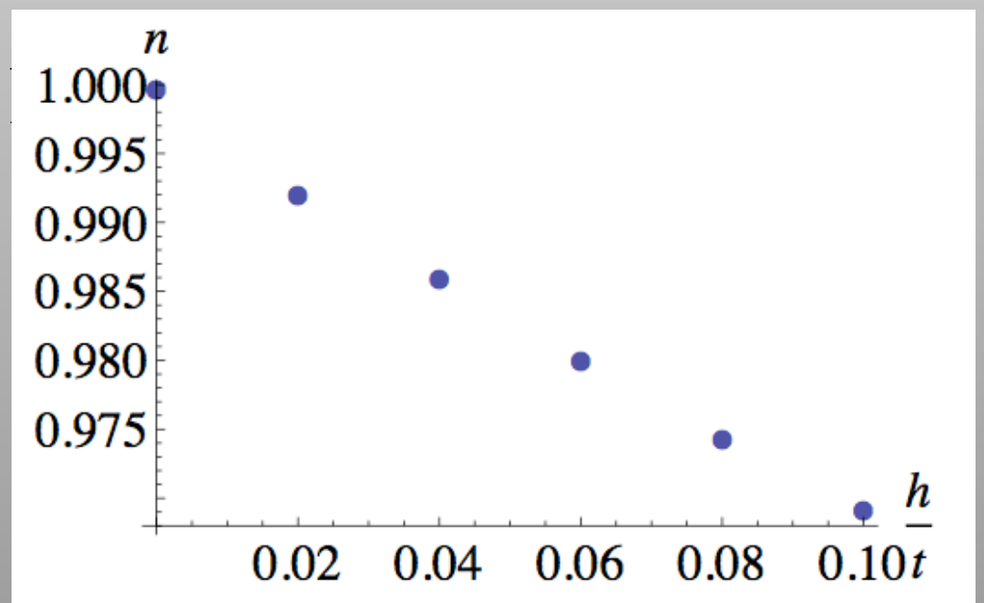


FIG. 3: Electron density n vs magnetic field h for slightly hole-doped square lattice.

Conclusions

- Magnetization plateau are interesting states
  - collinear
  - commensurate
  - highly stable
  - allow access to novel magnetic phases
    - collinear SDW (s=1/2)
    - spin-current nematic (s > 1/2)
    - interaction-induced half-metals (works for many lattice geometries)

Thank you!

07.04.2013