Quasi-one-dimensional spin nematic states and their excitations

Oleg Starykh, University of Utah



Leon Balents, KITP, UCSB

APS March meeting, San Antonio, March 3, 2015

Outline

- Very brief intro
- emergence of composite orders from competing interactions

- Nematic *vs* SDW in $LiCuVO_4$

  ✓ spin nematic: “magnon superconductor”

  ✓ collinear SDW: “magnon charge density wave”

- Volborthite kagome antiferromagnet
- experimental status - magnetization plateau
- Nematic, SDW and ***more***
- *Field theory of the Lifshitz point*
- Conclusions

Emergent nematic (Ising) order parameters

VOLUME 64, NUMBER 1 PHYSICAL REVIEW LETTERS 1 JANUARY 1990

Ising Transition in Frustrated Heisenberg Models

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Corporate Research Science Laboratories, Exxon Research and Engineering Company, Annandale, New Jersey 08801

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Serin Physics Laboratory, Rutgers University, P.O. Box 849, Piscataway, New Jersey 08854

$$\sigma = \vec{N}_1 \cdot \vec{N}_2 = \pm 1$$

VOLUME 91, NUMBER 17 PHYSICAL REVIEW LETTERS week ending 24 OCTOBER 2003

Ising Transition Driven by Frustration in a 2D Classical Model with Continuous Symmetry

Cédric Weber,[1,2] Luca Capriotti,[3] Grégoire Misguich,[4] Federico Becca,[5] Maged Elhajal,[1] and Frédéric Mila[1]

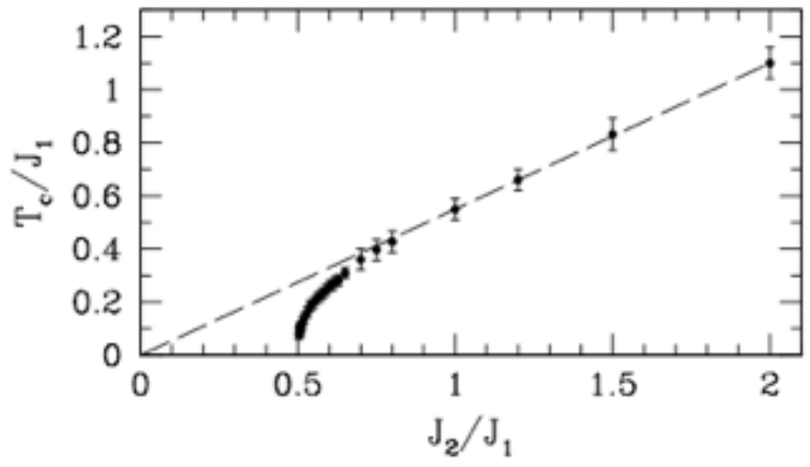


FIG. 4. Monte Carlo results for the critical temperature T_c as a function of the frustrating ratio J_2/J_1. The line is an extrapolation of the large J_2 data down to $J_2 = 0$ (see text).

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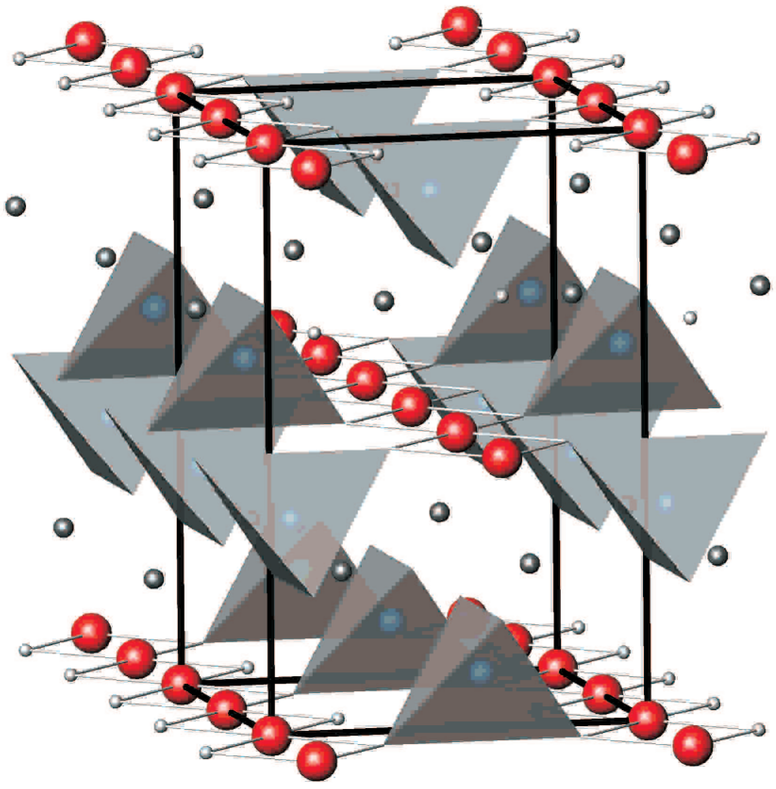
– experimental status - magnetization plateau

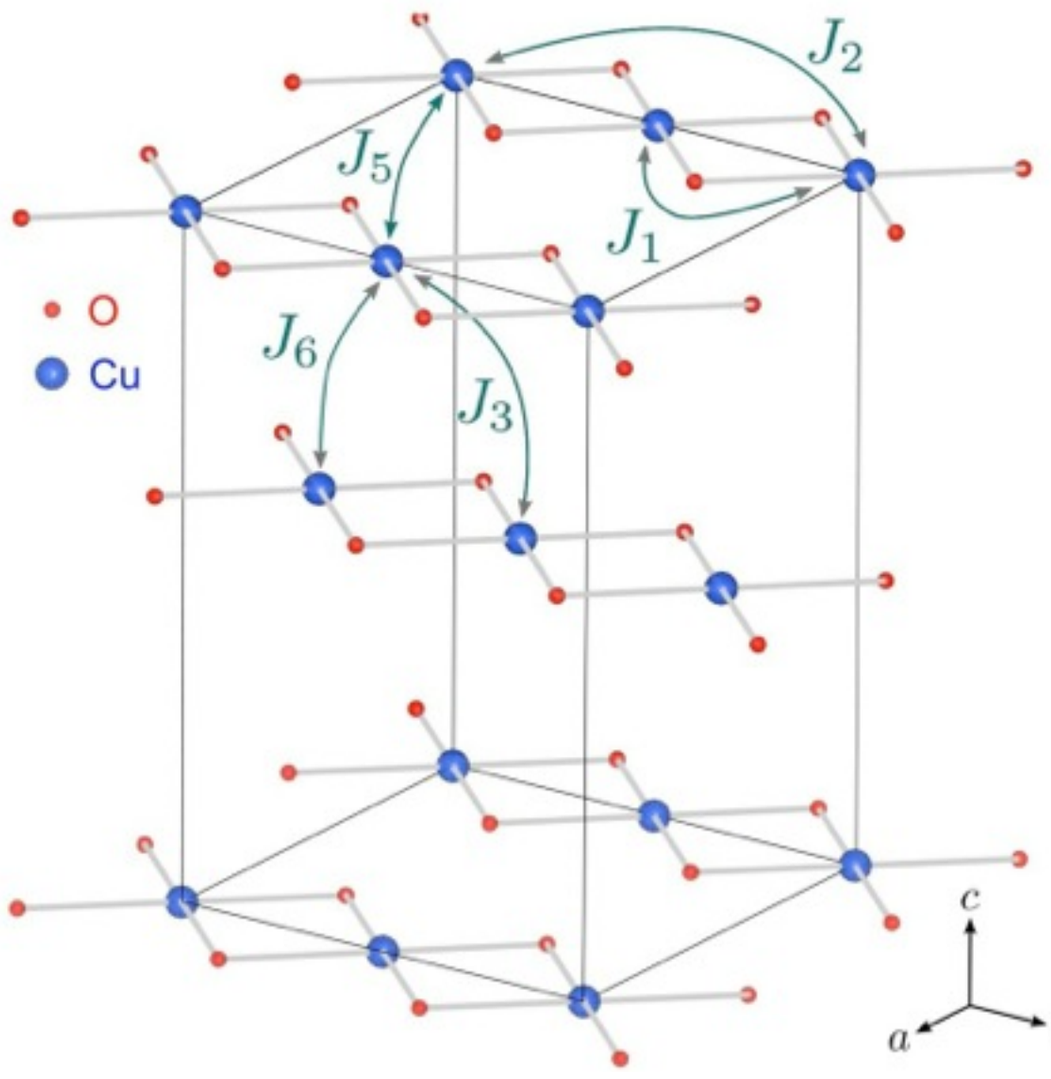
– Nematic, SDW and ***more***

– *Field theory of the Lifshitz point*

- Conclusions

$LiCuVO_4$: magnon superconductor?





estimates:

$J_1 = -1.6$ meV

$J_2 = 3.9$ meV (subject of **active** debates)

$J_5 = -0.4$ meV

High-field analysis: condensate of bound magnon pairs

$$\langle S^+ \rangle = 0 \qquad \langle S^+ S^+ \rangle \neq 0$$

Ferromagnetic $J_1 < 0$ produces attraction in real space

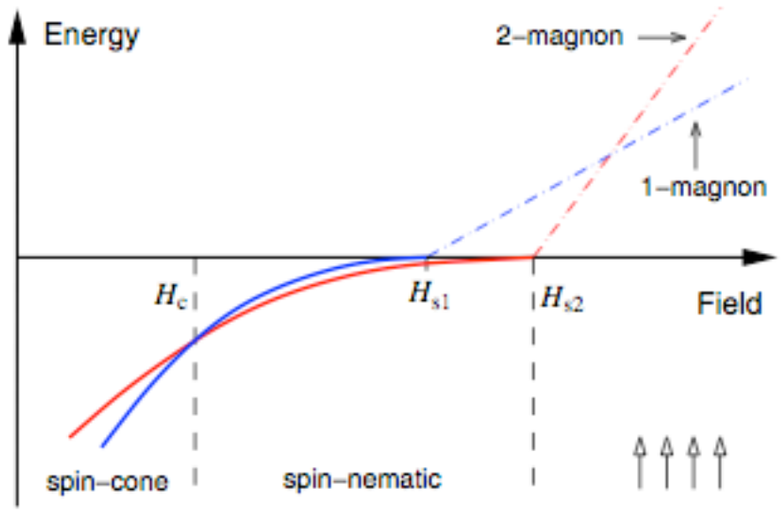


Fig. 1: (Color online) Energy-field diagram for a frustrated quantum magnet close to the saturation field. Dot-dashed lines show lowest one- and two-magnon states. Solid lines represent the ground-state energy for the one-magnon (spin-cone) and the two-magnon (spin-nematic) condensate.

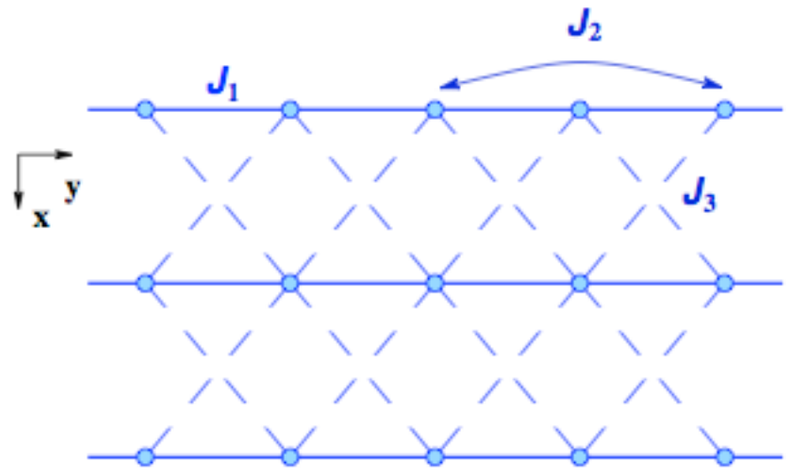
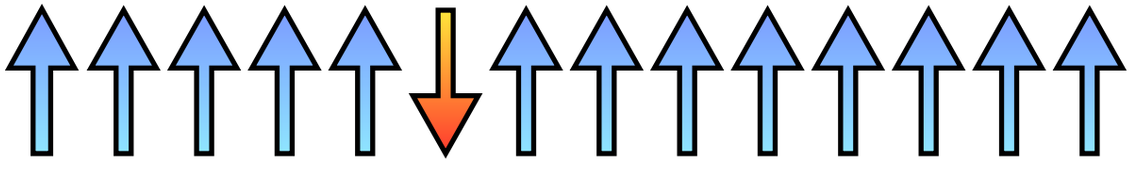


Fig. 2: (Color online) Two-dimensional array of copper ions in $LiCuVO_4$ with principal exchange couplings.

Chubukov 1991
Kecke et al 2007
Kuzian and Drechsler 2007
Hikihara et al 2008
Sudan et al 2009
Zhitomirsky and Tsunetsugu 2010

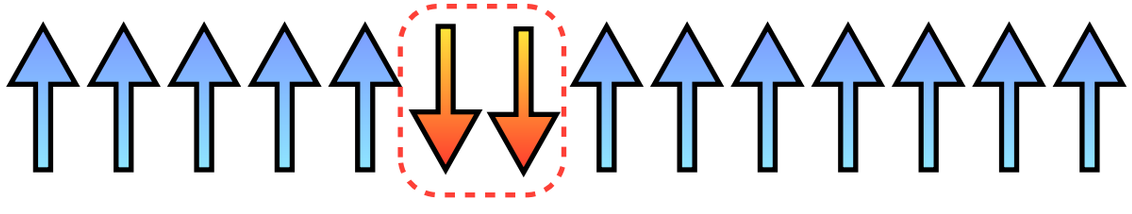
Magnon binding

1-magnon

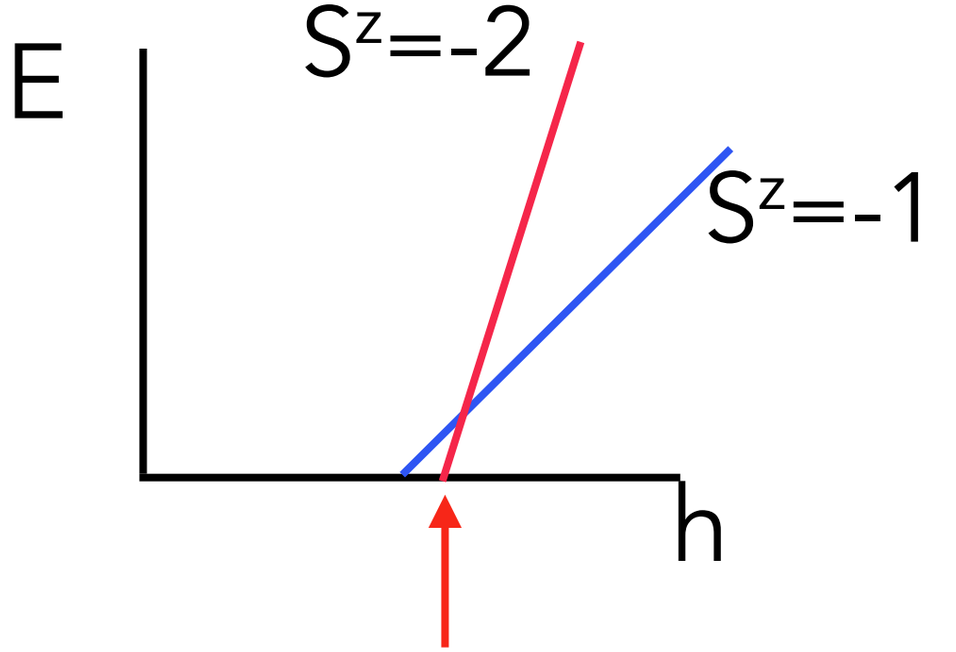


$E\text{-}E_{FM} = \varepsilon_1 + h$

2-magnon bound state



$E\text{-}E_{FM} = \varepsilon_2 + 2h$



$\varepsilon_2 < 2\,\varepsilon_1$: "molecular" bound state

Formation of molecular fluid

For d>1 at T=0 this is a molecular BEC

= true spin nematic

Hidden order

No dipolar order $\langle S_i^+ \rangle = 0$

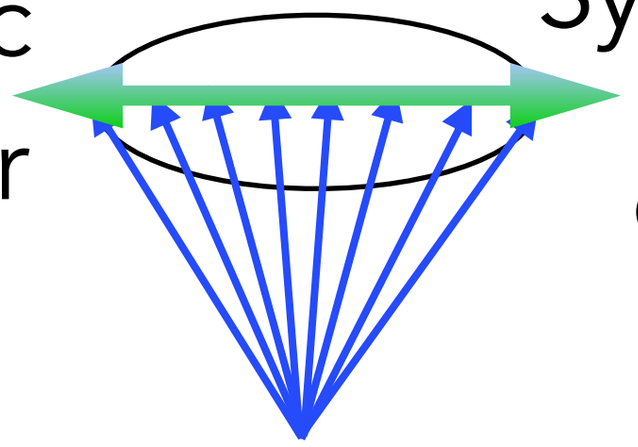
$\langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi}$ S^z=1 gap

Nematic order $\langle S_i^+ S_{i+a}^+ \rangle \neq 0$

Magnetic quadrupole moment

Symmetry breaking U(1) → Z_2

nematic director



can think of a fluctuating fan state

$LiCuVO_4$ experiment: collinear SDW along **B**

Hagiwara, Svistov et al, 2011

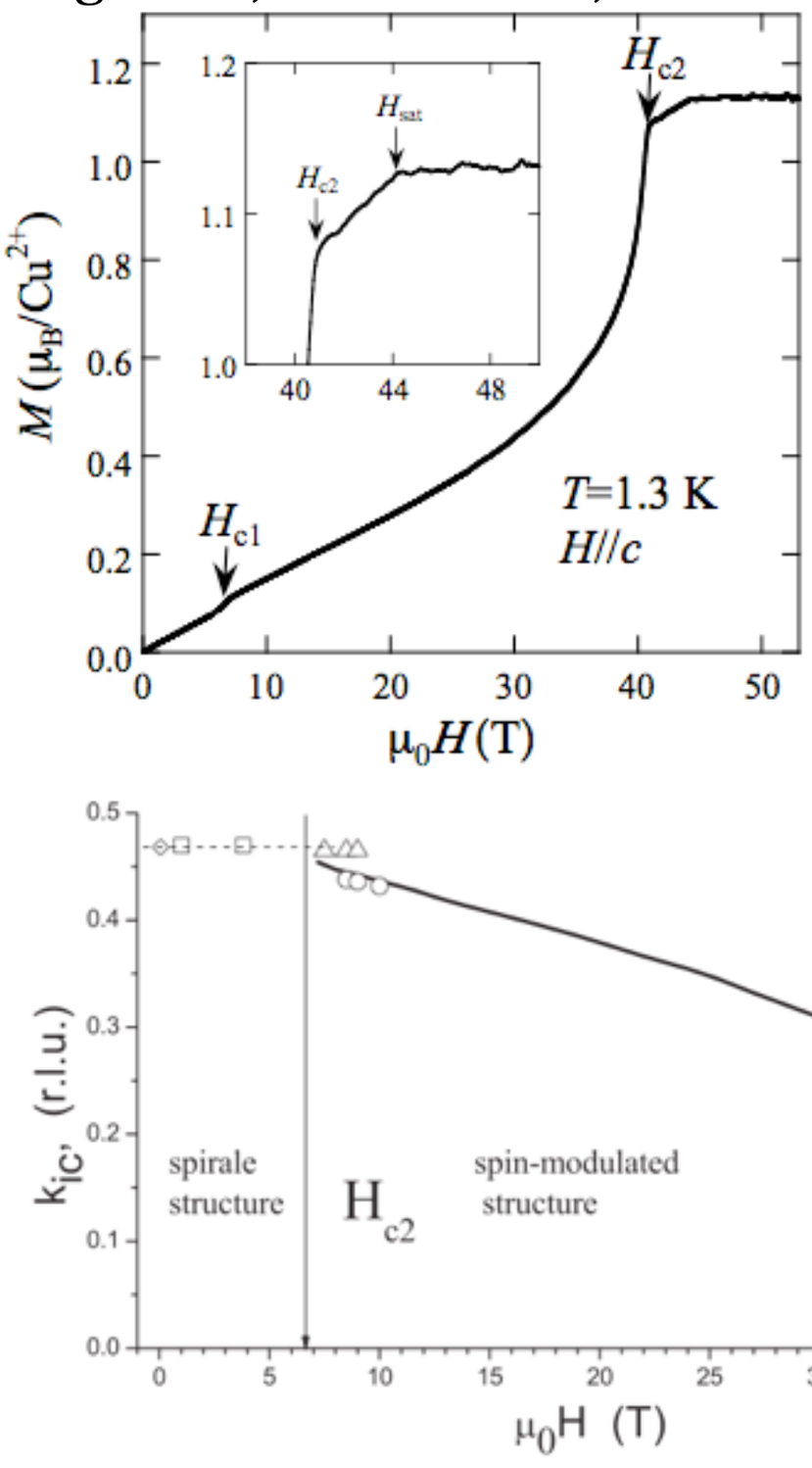
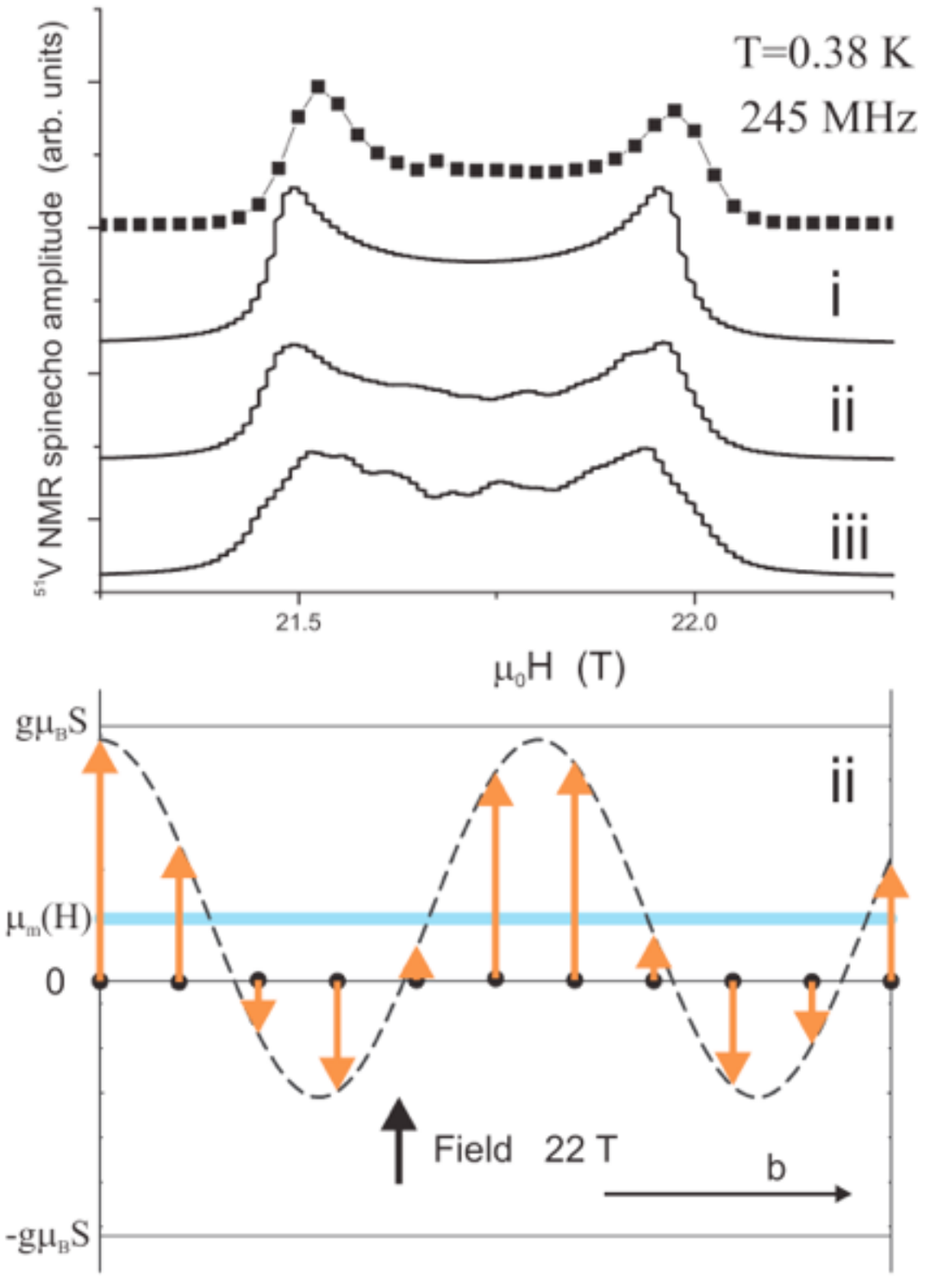


FIG. 2. Field dependence of the incommensurate wave vector k_{ic} for applied magnetic fields **H** ∥ **c** in $LiCuVO_4$. The open symbols

Buttgen et al 2012, 2014

PHYSICAL REVIEW B **85**, 214421 (2012)



Evidence of a Bond-Nematic Phase in $LiCuVO_4$

M. Mourigal,[1,2] M. Enderle,[1] B. Fåk,[3] R. K. Kremer,[4] J. M. Law,[4,*] A. Schneidewind,[5] A. Hiess,[1,†] and A. Prokofiev[6,7]

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, $\Delta S = 1$
NSF = no spin flip, $\Delta S = 0$

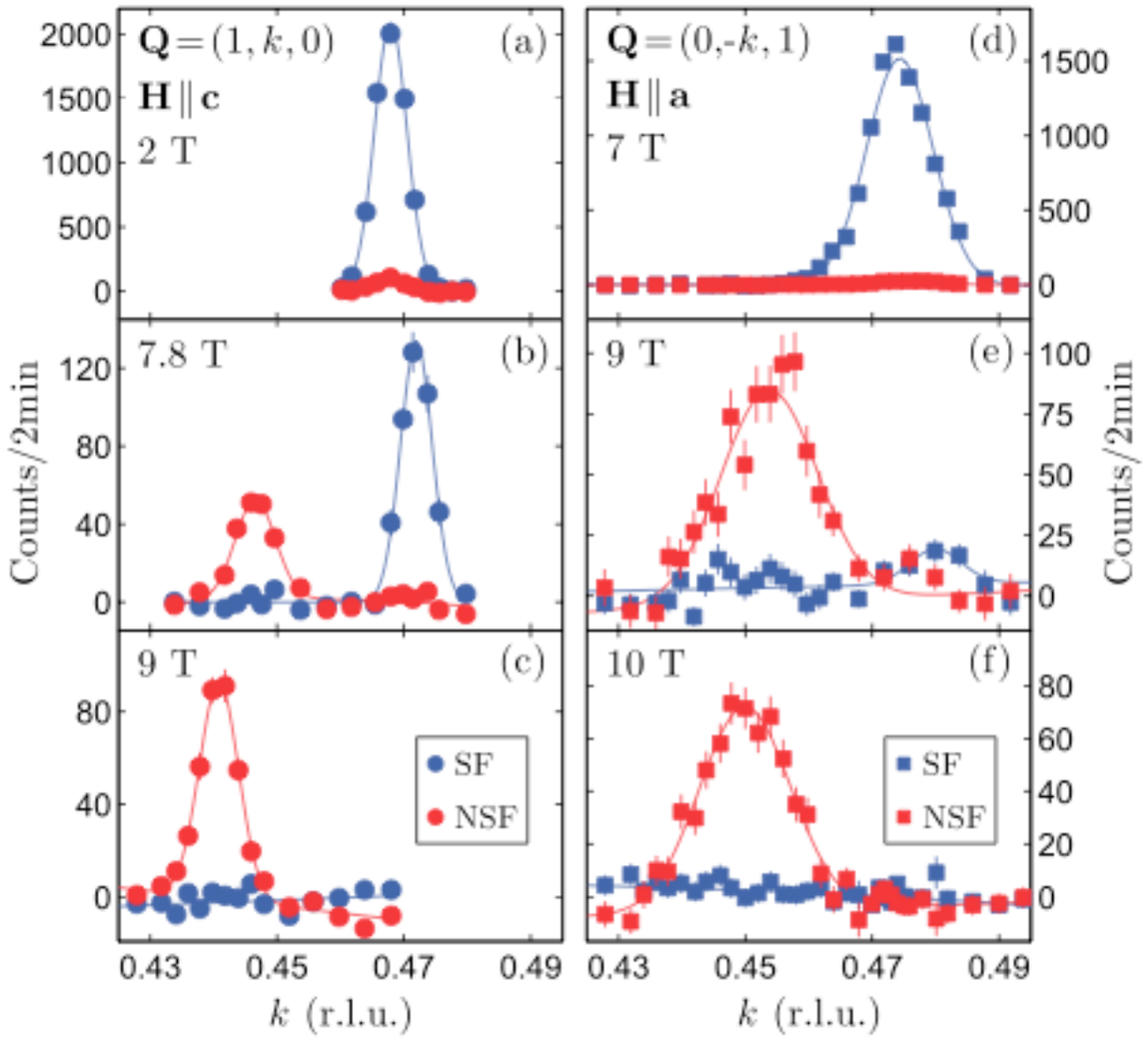
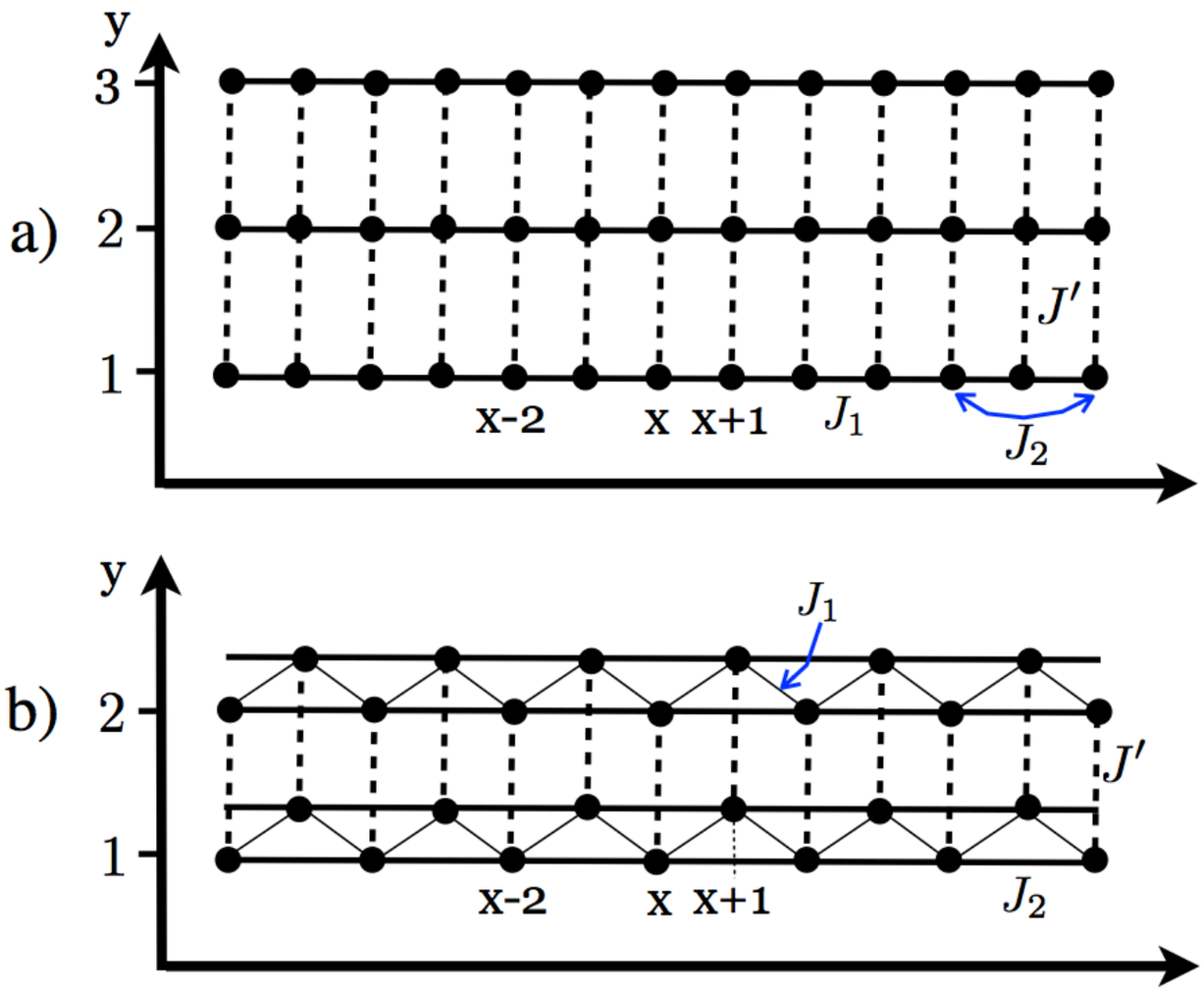
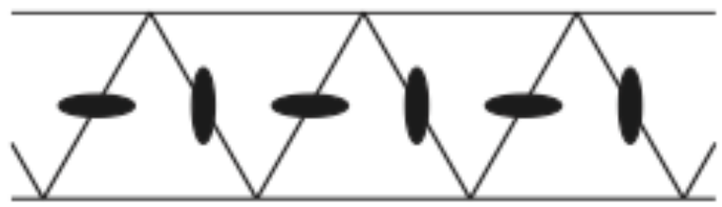


FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $\mathbf{Q} = (1, k_{IC}, 0)$ with $\mathbf{H}\|\mathbf{c}$ [left panels, (a)–(c)] and $\mathbf{Q} = (0, -k_{IC}, 1)$ with $\mathbf{H}\|\mathbf{a}$ [right panels, (d)–(f)].

1d J_1-J_2 chain is only *quasi*-spin-nematic



$J_1 < 0$ (ferro)
$J_2 > 0$, $J' > 0$ (afm)
in magnetic field



- No true condensation [U(1) breaking] in d=1.
- Inter-chain interaction is crucial for establishing symmetry breaking in d=2.
- Need to study weakly coupled “superconducting” chains

Sato et al 2013
Starykh and Balents 2014

Inter-chain interaction $H_{\text{inter-chain}} = \sum_y \int dx\ \vec{S}_y \cdot \vec{S}_{y+1} \sim \sum_y \int dx\ S_y^+ S_{y+1}^- + S_y^z S_{y+1}^z$

Superconducting analogy: single-particle (magnon) tunneling between magnon superconductors is strongly suppressed at low energy (below the single-particle gap)

$$H_{\text{inter}}^{\perp} = \sum_y \int dx\ J' \langle S_y^+(x) S_{y+1}^-(x+1) \rangle_{\text{nematic ground state}} \to 0$$

Superconducting analogy: fluctuations generate two-magnon (***Josephson coupling***) tunneling between chains. They are generically weak, ~ $J_1(J'/J_1)^2 << J'$, but responsible for a true **two-dimensional nematic order**

$$H_{\text{nem}} \sim (J'^2/J_1) \sum_y \int dx\ [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

$$T_y^+(x) \sim S_y^-(x) S_y^-(x+1)$$

At the same time, density-density inter-chain interaction does not experience any suppression. It drives the system toward a **two-dimensional collinear SDW order**.

$$S_y^z = M - 2n_{\text{pair}} = M - \tilde{A}_1 e^{i\frac{\sqrt{2}\pi}{\beta}\varphi_y^+(x)} e^{ik_{\text{sdw}}x}$$

$$H_{\text{inter-chain}}^z = H_{\text{sdw}} \sim J' \sum_y S_y^z S_{y+1}^z \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi_y^+ - \varphi_{y+1}^+)]$$

Away from the saturation, **SDW** is more relevant [and stronger, via $J' >> (J')^2/J_1$] than the **nematic interaction**: **coupled** 1d **nematic chains order in a** 2d **SDW state.**

Simple scaling

$$H_{\rm nem} \sim (J'^2/J_1) \sum_y \int dx \; [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

- describes kinetic energy of magnon pairs, linear in magnon pair density $n_{\rm pair}$

$$H^z_{\rm inter-chain} = H_{\rm sdw} \sim J' \sum_y S^z_y S^z_{y+1} \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi^+_y - \varphi^+_{y+1})]$$

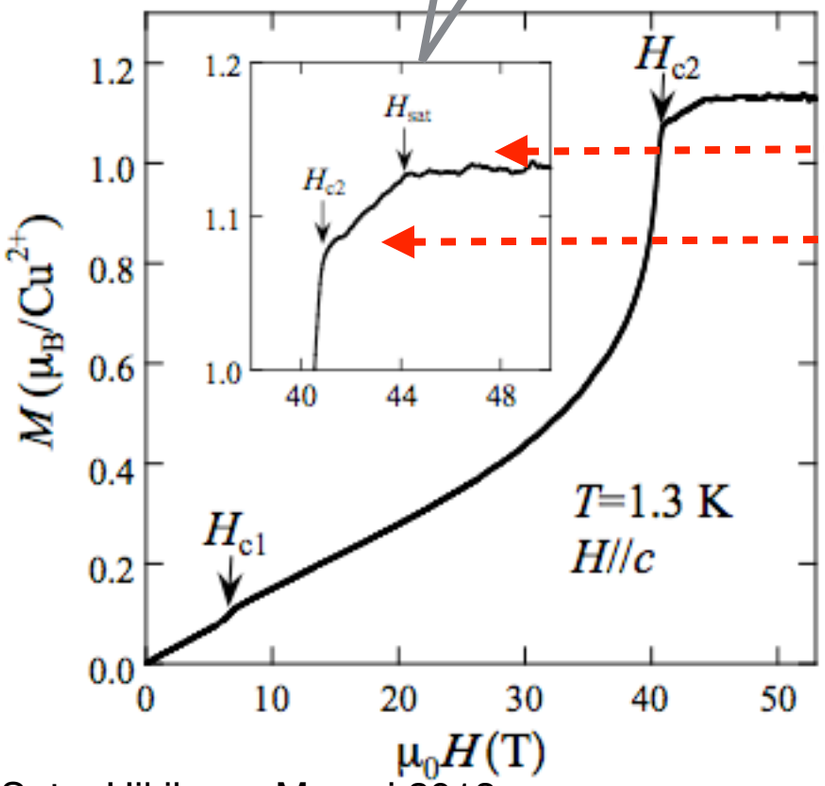
- describes potential energy of interaction between magnon pairs on neighboring chains, quadratic in magnon pair density $n_{\rm pair}$

- Competition $\frac{(J')^2}{J_1} n_{\rm pair} \sim J' n^2_{\rm pair}, \;\; \text{hence} \boxed{n^*_{\rm pair} \sim J'/J_1}$

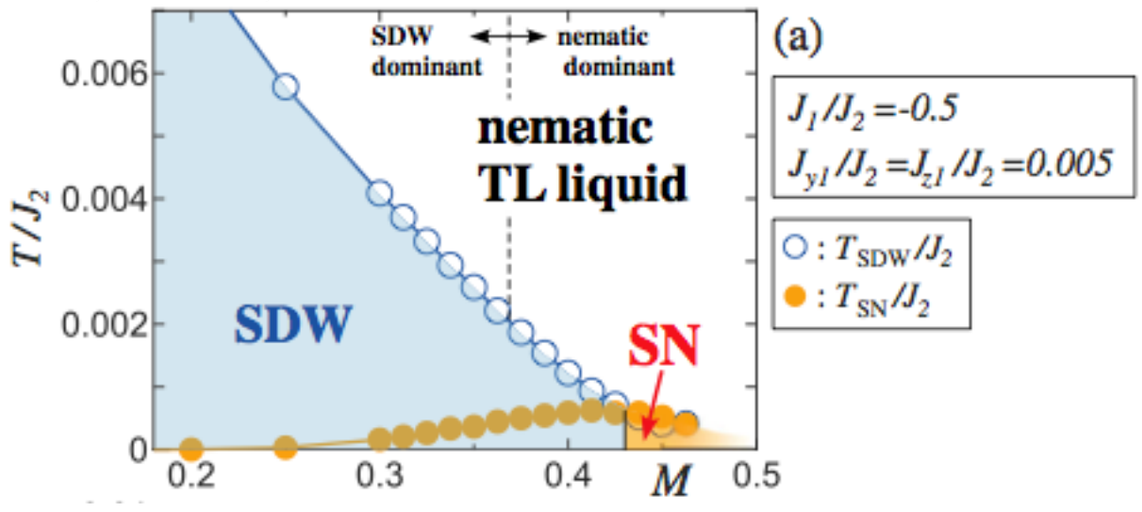
- Hence:
- Spin Nematic *near saturation*, for $n_{pair} < n^*_{pair}$
- SDW for $n_{pair} > n^*_{pair}$

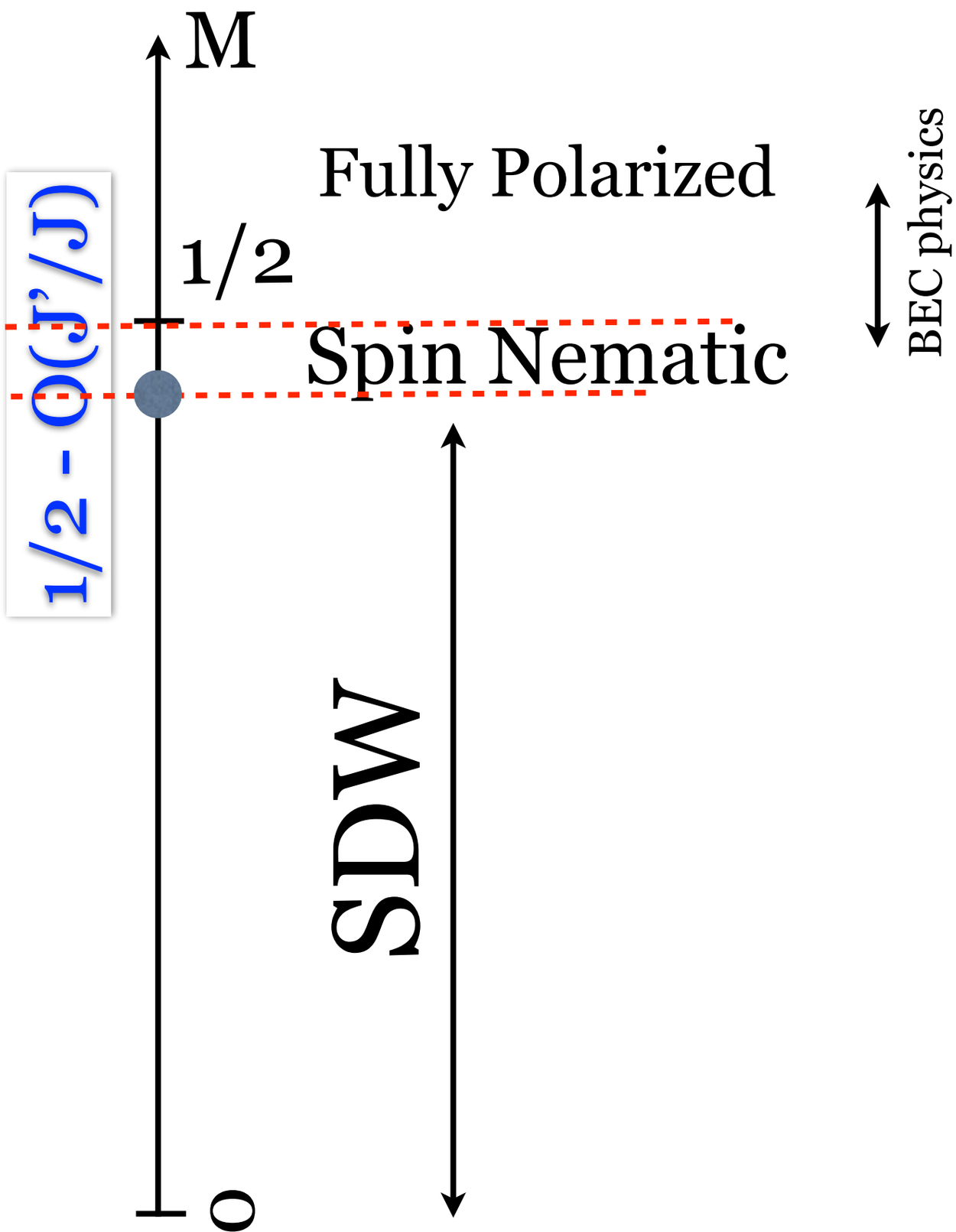
T=0 schematic phase diagram of weakly coupled **nematic** spin chains

Cautionary remark:
maybe impurity effect



cf: Sato, Hikihara, Momoi 2013





Intermediate Summary

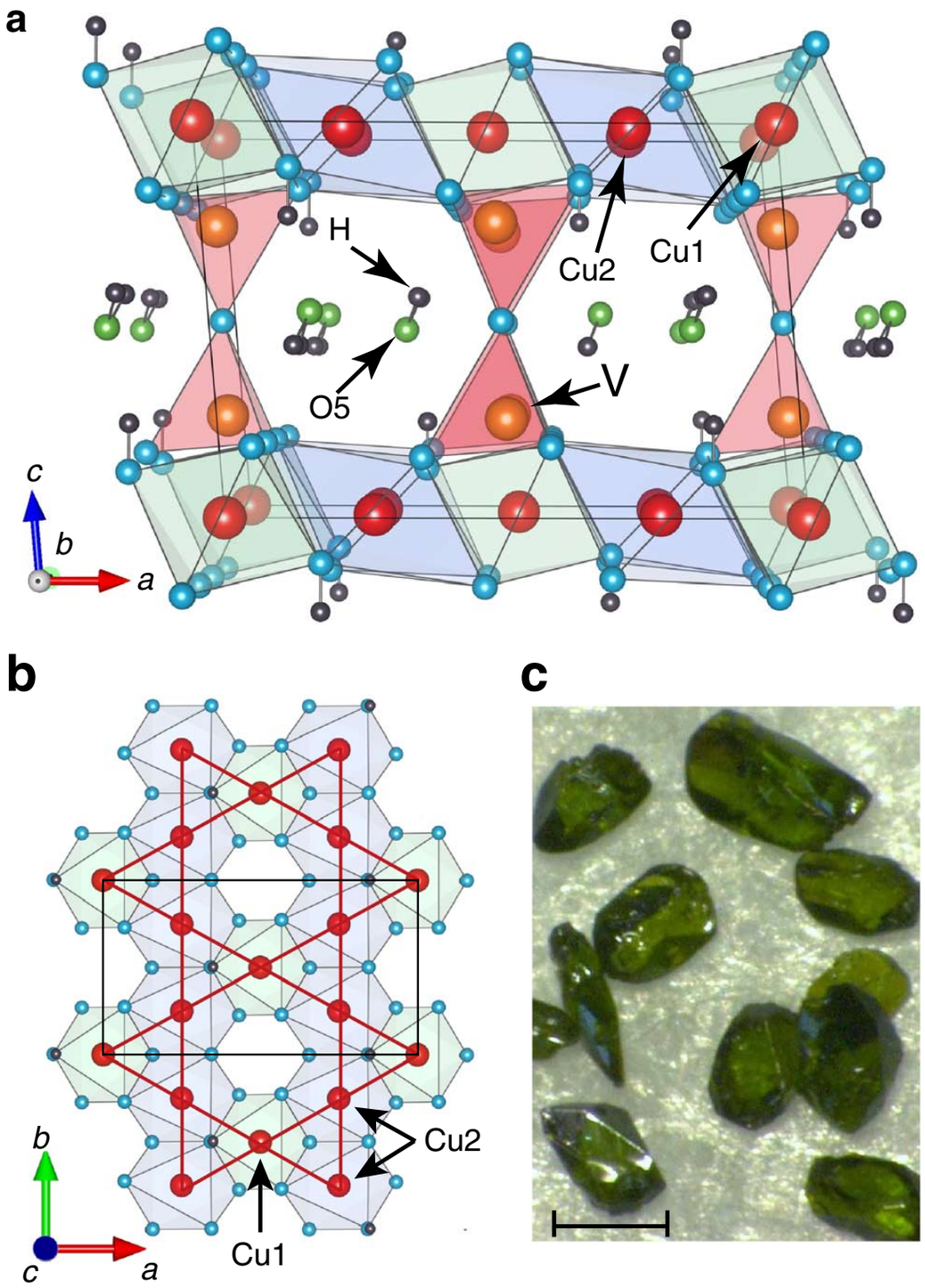
- Interesting magnetically ordered states: SDW and Spin Nematic
  - Gapped $\Delta S=1$ excitations (no usual spin waves!)
  - 2d Nematic very near the full saturation
  - 2d SDW from nematic chains, occupies most of the phase diagram

toy problem of "magnon high T_c"

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Volborthite





Volborthite's timeline

Formula: $Cu_3(V_2O_7)(OH)_2 \cdot 2H_2O$

System: Monoclinic **Colour:** Olive-green, ...

Hardness: 3½

Name: Named after Alexander von Volborth (1800–1876), Russian paleontologist, who first noted the mineral.

A secondary mineral found in the oxidized zones of vanadium-bearing hydrothermal deposits.

At least two different monoclinic space-group variants (C2/m, C2/c) seem to be stable at ambient temperature.

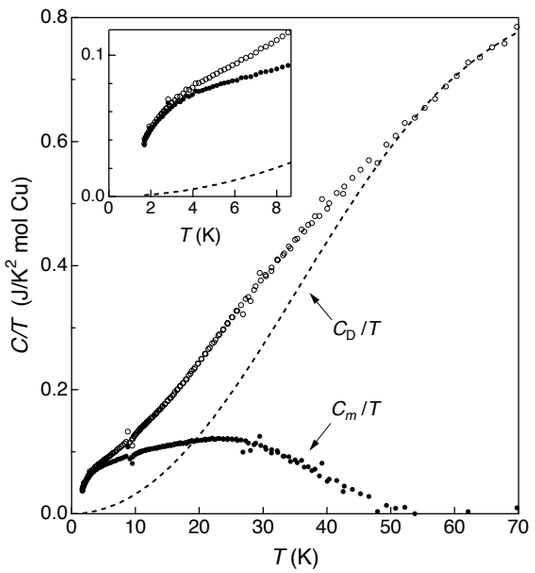
Visually similar to vésigniéite.

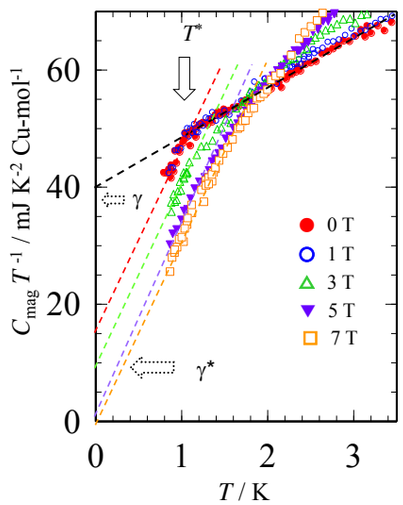
2001 quantum spin liquid?!

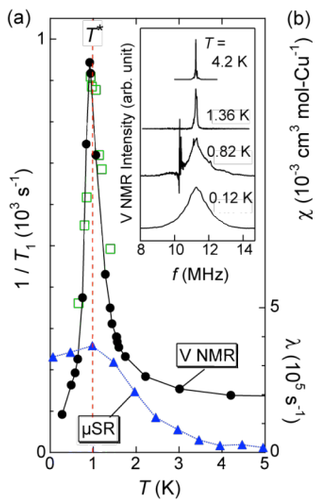
2009 impurity ordering at low T? magnetization steps?

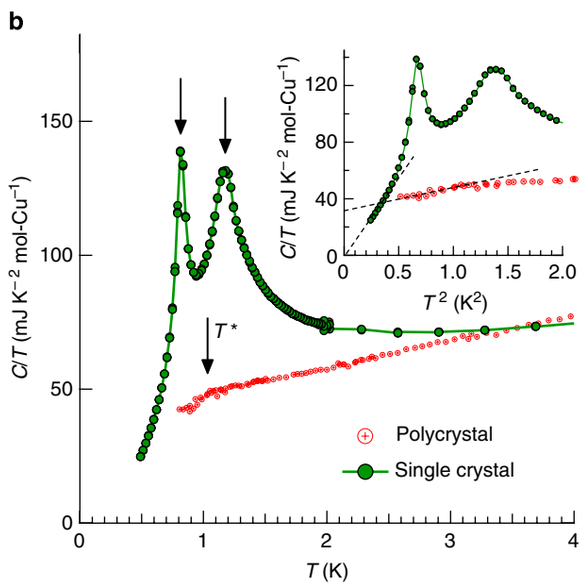
2012 magnetic order !

2014 magnetization plateau









time = material quality

2014: huge plateau!

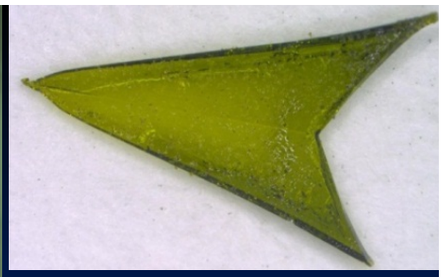
H. Ishikawa…M.Takigawa…Z.Hiroi, unpublished, 2014

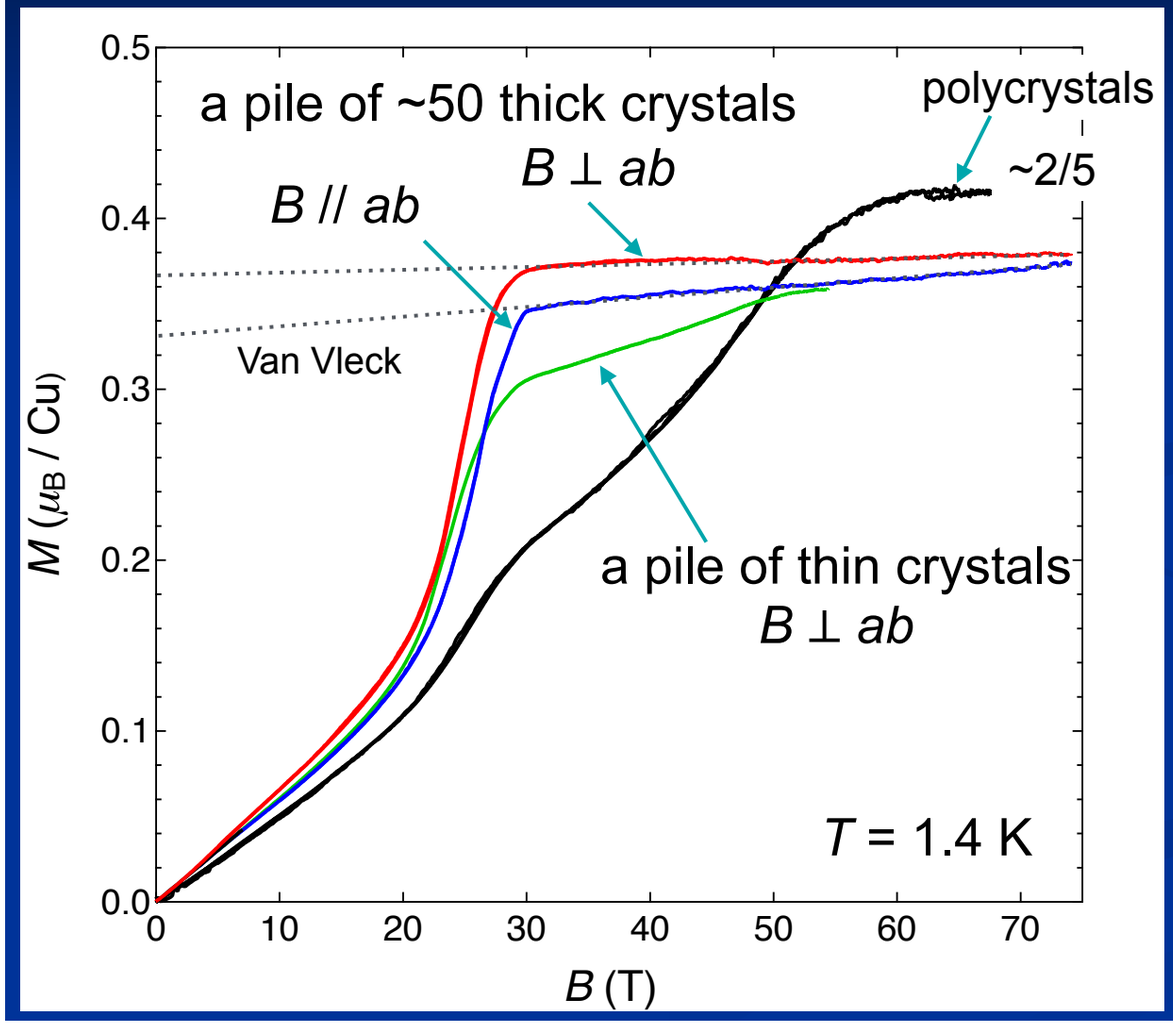
High-field magnetization

more different *MH* curves in a pile of 50
large “thick” arrowhead-shaped crystals

30 days growth







Huge 1/3 plateau!

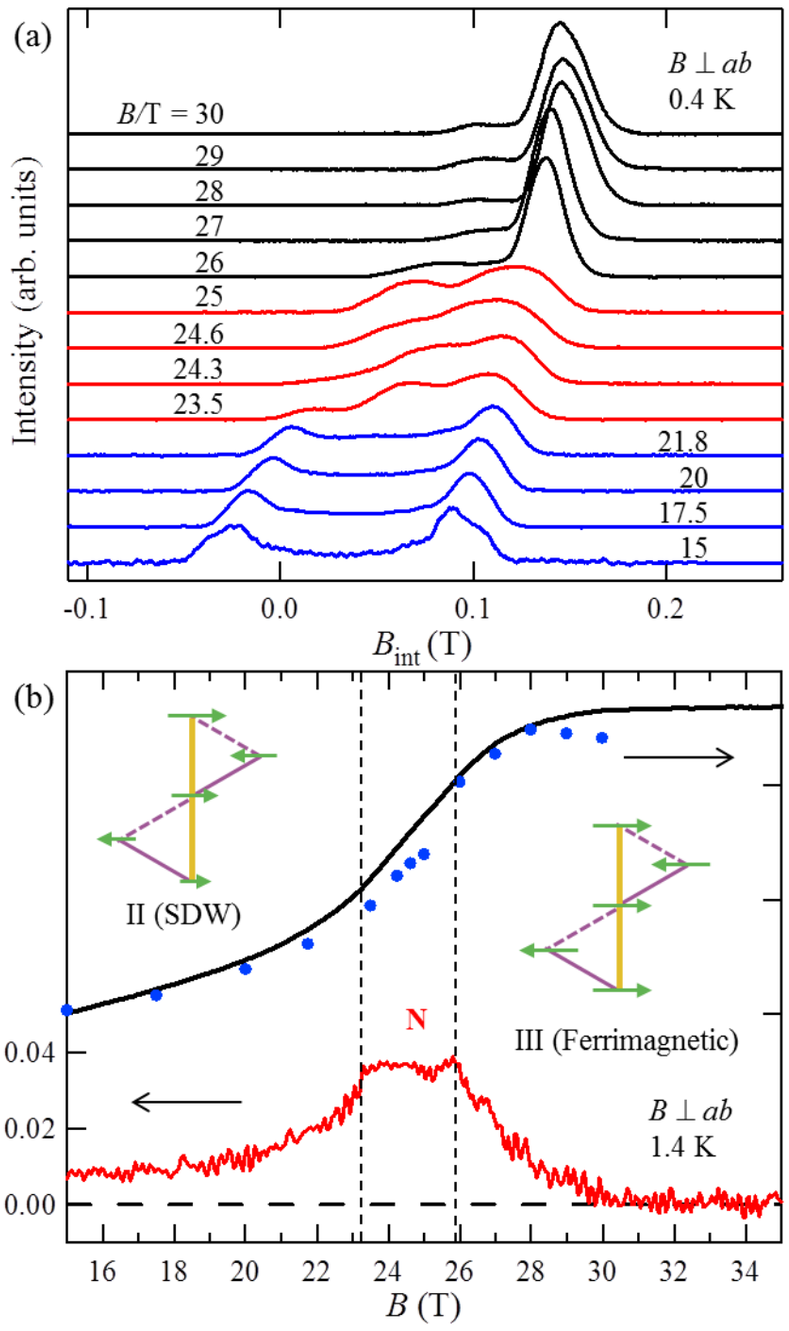
further optical meas.
@ Takeyama lab
It survives over 120 T!

Kagome plateau or ferrimagnetic state?

coupled to lattice, but already distorted

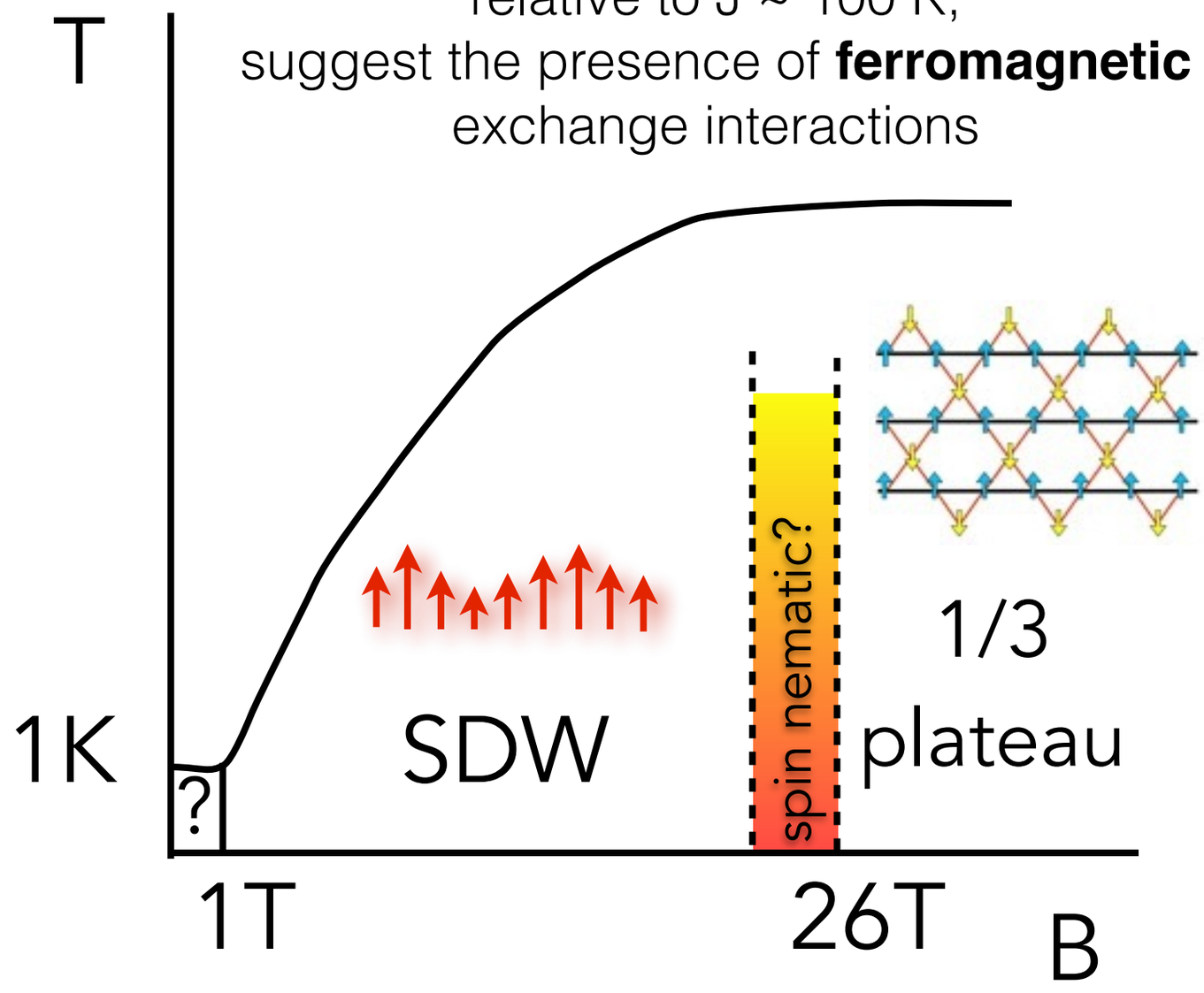
high-field mag. meas.
@ Tokunaga & Kindo labs

Phase diagram



H. Ishikawa *et al*,
unpublished

small plateau's onset field of 27 Tesla,
relative to J ~ 100 K,
suggest the presence of **ferromagnetic**
exchange interactions



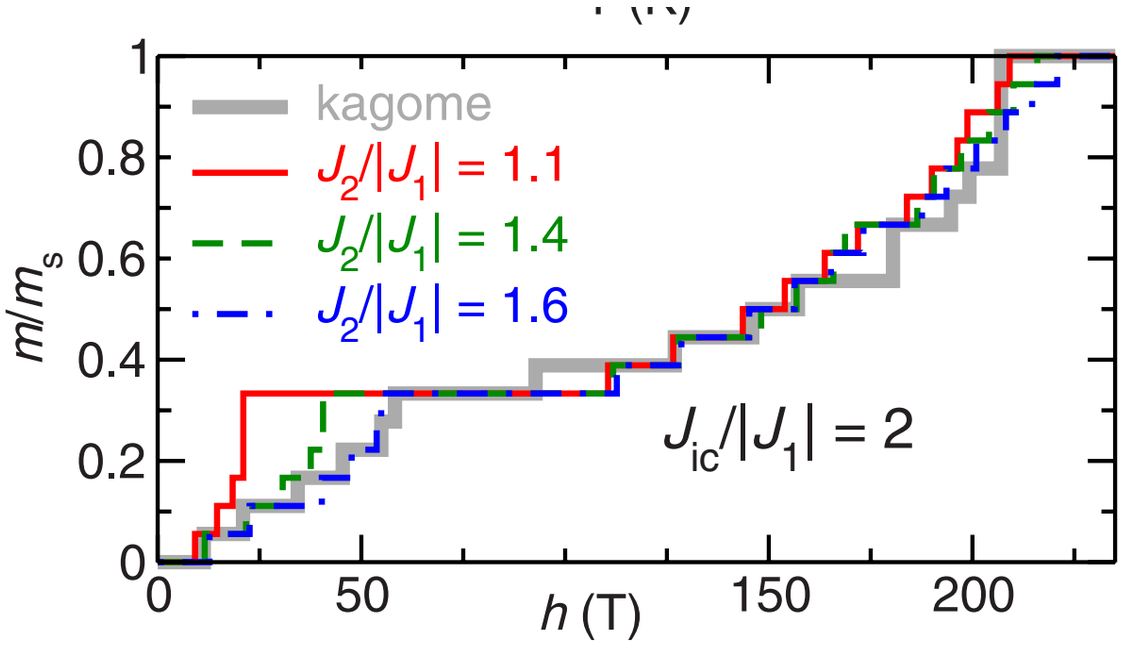
may be a spin nematic??

Frustrated ferromagnetism

PHYSICAL REVIEW B **82**, 104434 (2010)

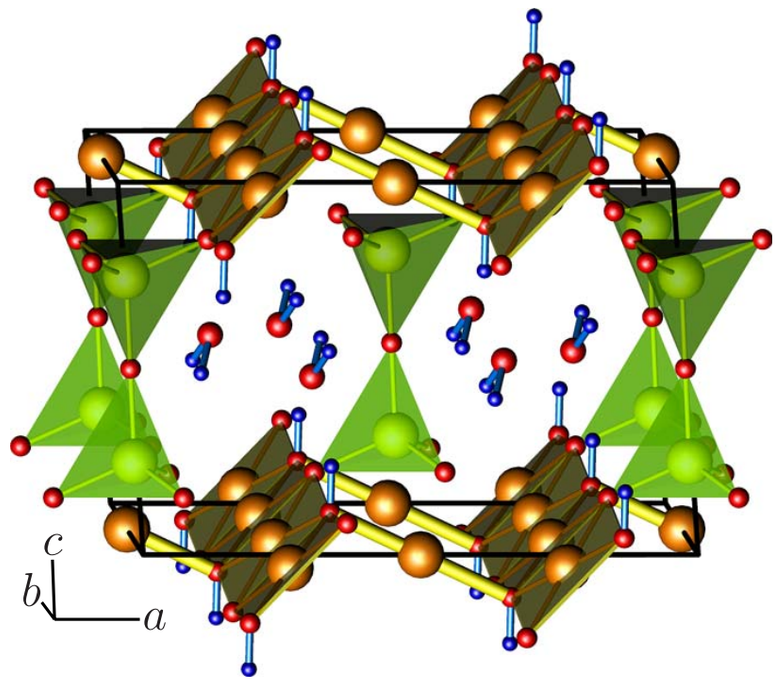
Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

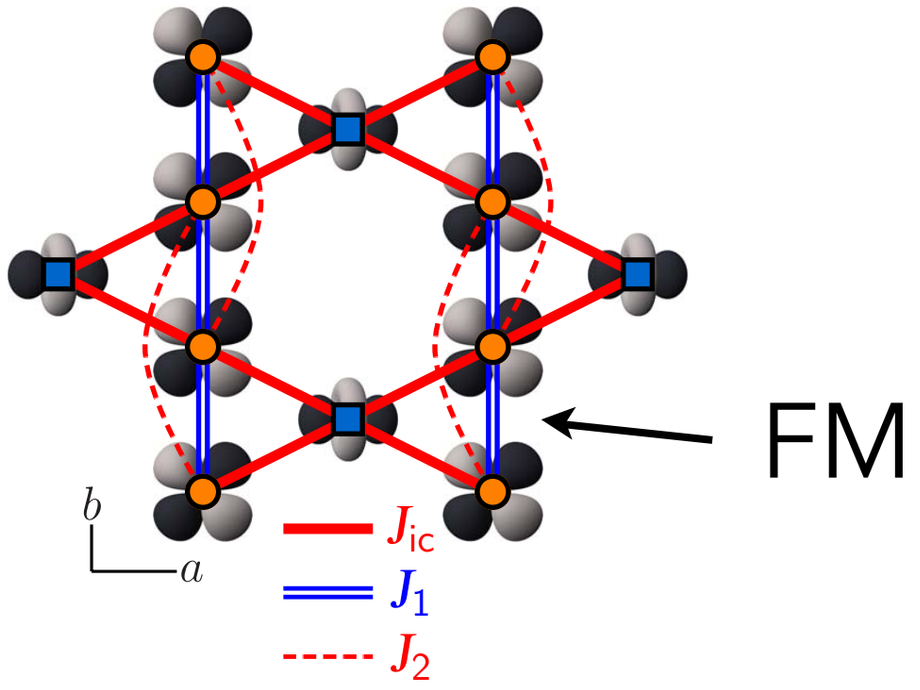
O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Magdeburg, D-39016 Magdeburg, Germany*
[3]*Laboratoire de Physique Théorique de la Matière Condensée, Univ. P. & M. Curie, Paris, France*
(Received 9 August 2010; published 30 September 2010)



DFT gets it right!

$$J_1 < 0, J_2 > 0, J' > 0$$





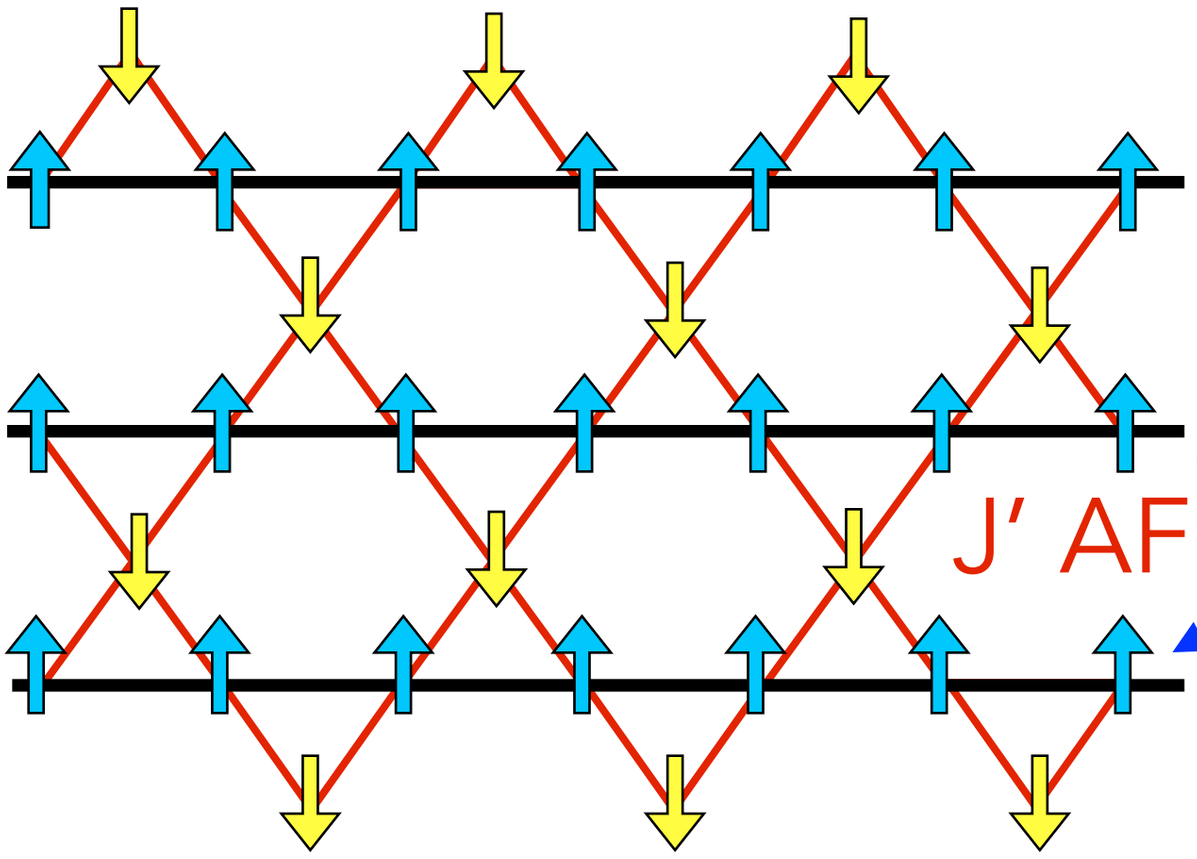
Ferrimagnetic state

PHYSICAL REVIEW B **82**, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
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$$J_1 < 0, J_2 > 0, J' > 0$$



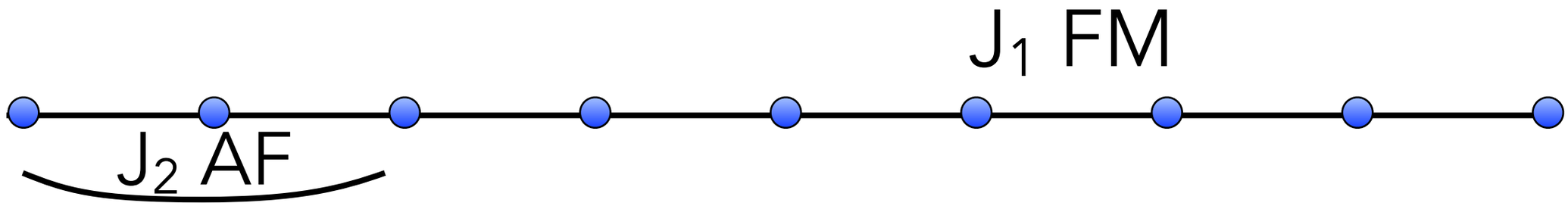
J_1 FM, J_2 AF

J’ AF

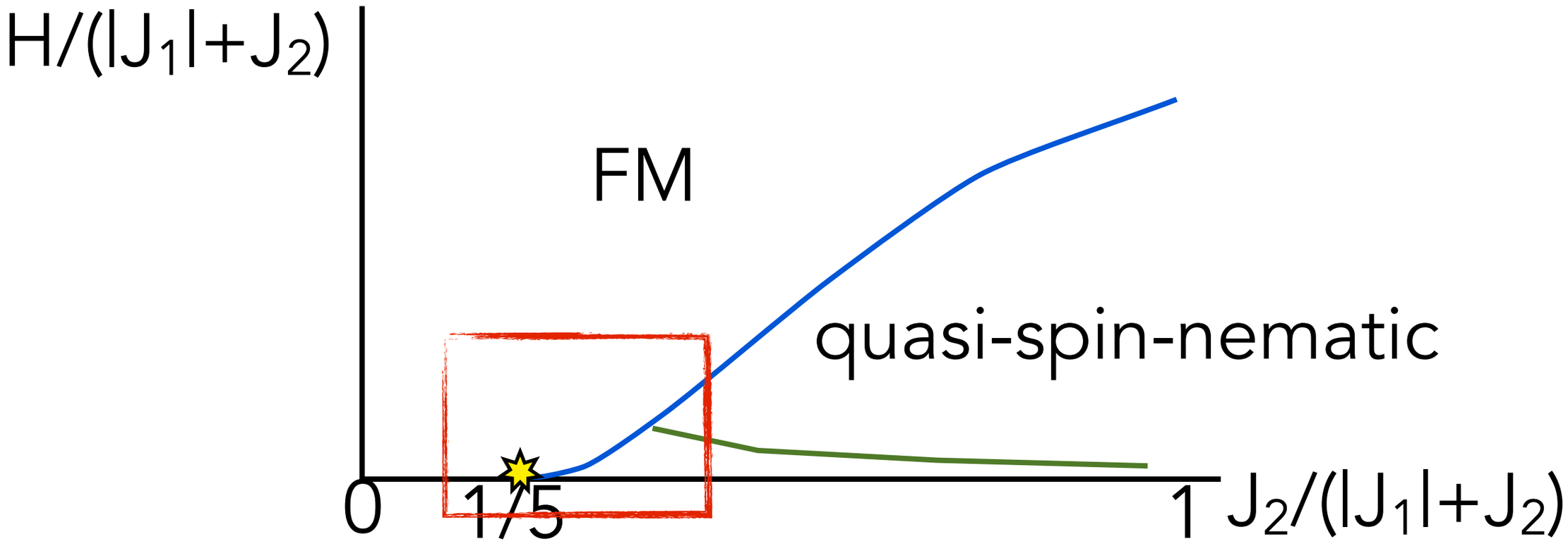
polarized chains?!

Spin chain redux

Frustrated ferromagnetic chain

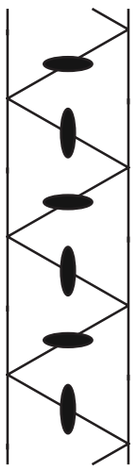


$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



Quasi-1d nematic

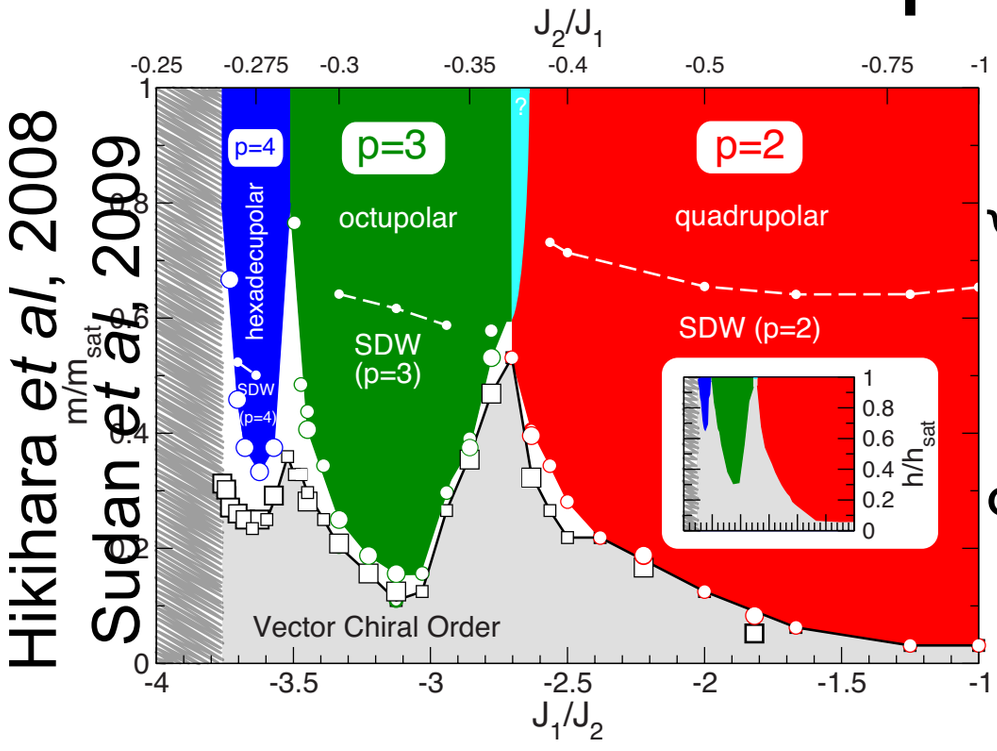
1d J_1-J_2 chain is only *quasi*-spin-nematic



power-law correlations

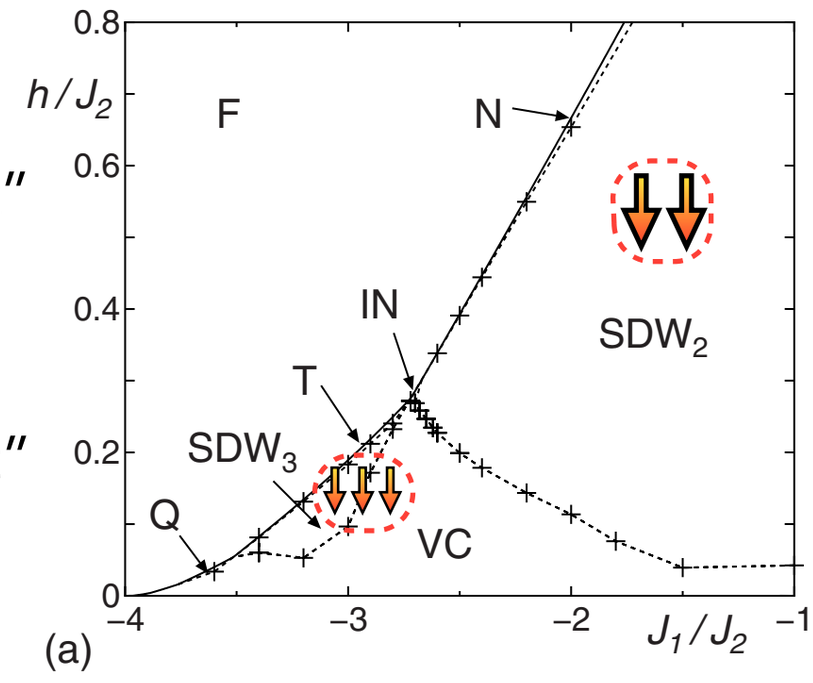
$\Psi \sim (S^+)^2$: spin-nematic

$\phi \sim S^z e^{i q x}$: SDW



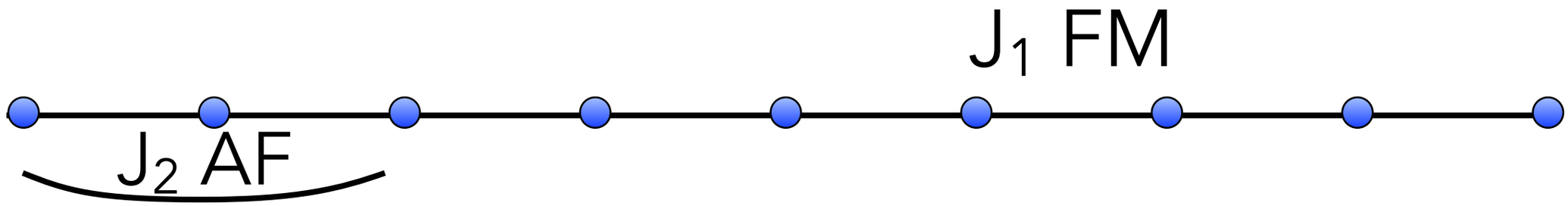
Ψ "dominant"

ϕ "dominant"

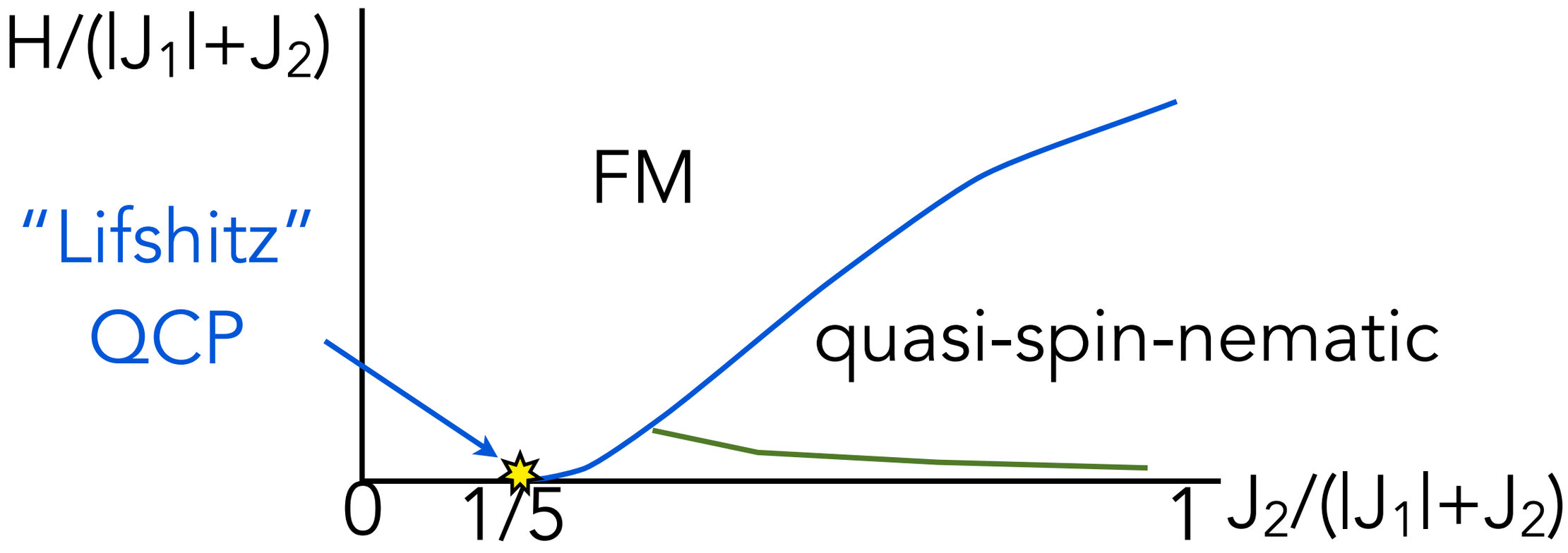


A QCP parent?

Frustrated ferromagnetic chain



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



Lifshitz Point

- Unusual QCP: order-to-order transition
- Effective action - NL**σ**M for unit vector **m**

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$\mathcal{A}_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}$ Berry phase term

tunes QCP $\delta \propto |J_1| - 4J_2$

two symmetry allowed interactions at $O(q^4)$

All properties near Lifshitz point obey "one parameter universality" dependent upon u/K ratio

Lifshitz Point

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau$$

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Large parameter:
saddle point!

$$v = \frac{u}{K} \qquad \overline{h} = \frac{hK}{\delta^2}$$

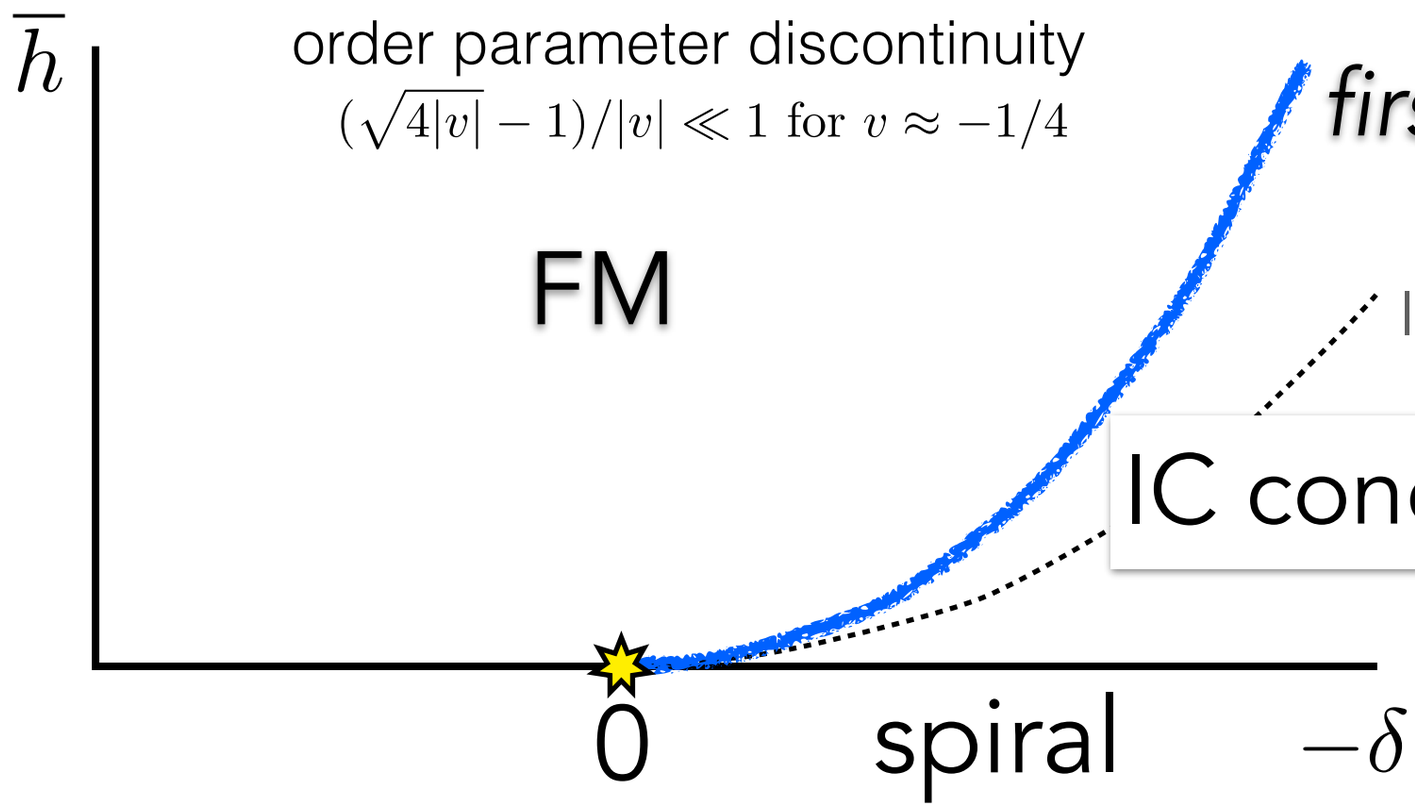
Saddle point

$$S=\sqrt{\frac{K}{\delta}}\int dxd\tau\{is\mathcal{A}_B[\hat{m}]+\mathrm{sgn}(\delta)|\partial_x\hat{m}|^2+|\partial_x^2\hat{m}|^2+v|\partial_x\hat{m}|^4-\overline{h}\hat{m}_z\}$$

-1 < v < -1/4 derives from quantum fluctuations

Large S >>1: v ~ -3/(2S) < 0

S=1/2: v = -5/8



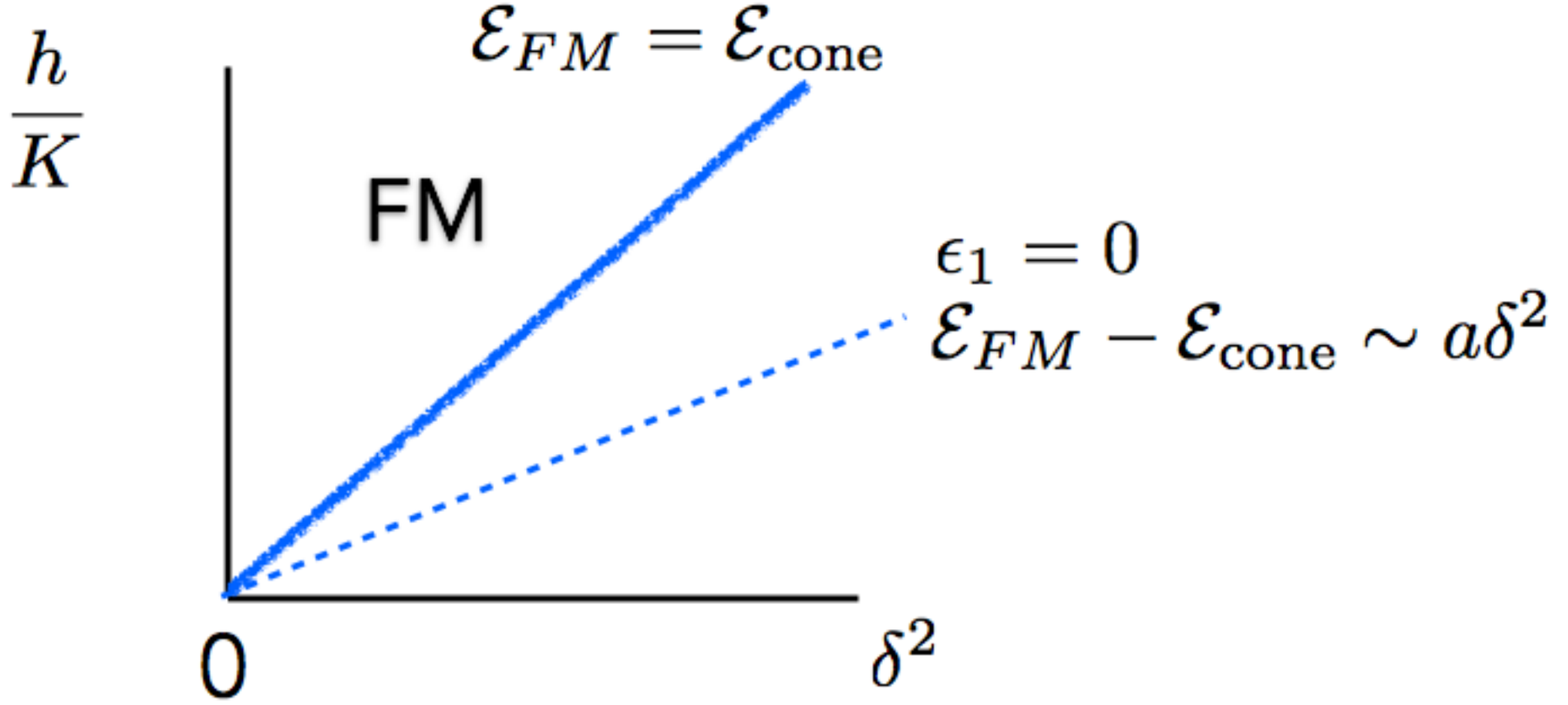
first order $h_c=\dfrac{\delta^2}{8K\sqrt{|v|}(1-\sqrt{|v|})}$

$-1<v<-\frac{1}{4}$

local instability of FM state
(1-magnon condensation)

$$\hat{m}=\begin{pmatrix}|\Psi|\cos(qx+\phi)\\ \pm|\Psi|\sin(qx+\phi)\\ \sqrt{1-|\Psi|^2}\end{pmatrix}$$

Metamagnetic endpoint?



Quantum corrections
penalize E_{cone} but not E_{FM}

$$\Delta\mathcal{E}_{\text{cone}} = +f(v)\delta^{5/2}$$

Metamagnetic endpoint?

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$$\hat{m} = \sqrt{2 - n_1^2 - n_2^2}(n_1 \hat{e}_1(x) + n_2 \hat{e}_2(x)) + (1 - n_1^2 - n_2^2)\hat{e}_3(x)$$

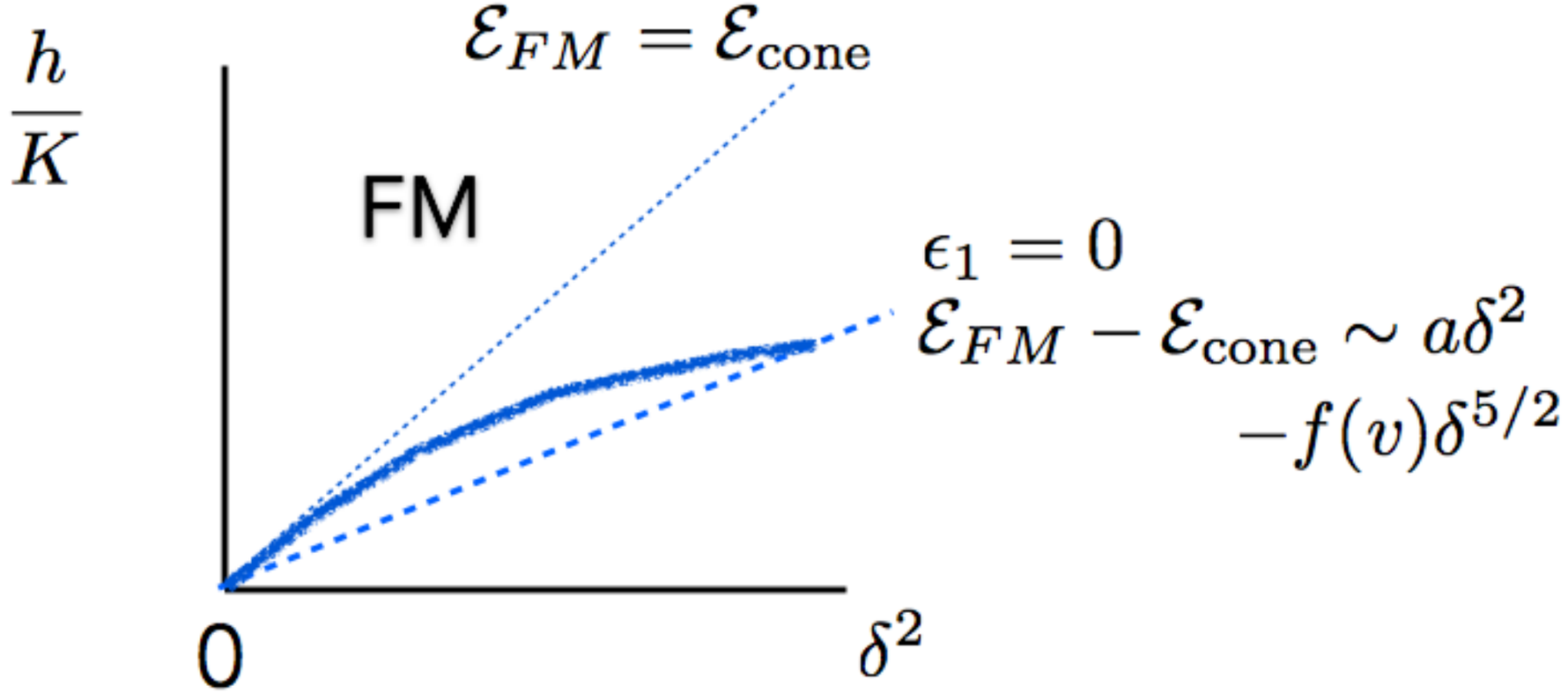
$$\hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}^{sp}(x)$$

$$\eta = n_1 + in_2 \qquad \overline{\eta} = n_1 - in_2$$

$$S = S_{\rm sp} + \int dx\, d\tau\, \{\overline{\eta}\partial_\tau \eta + H(\overline{\eta}, \eta)\} \quad + O(\eta^3)$$

Bogoliubov transformation gives
correction to GS energy

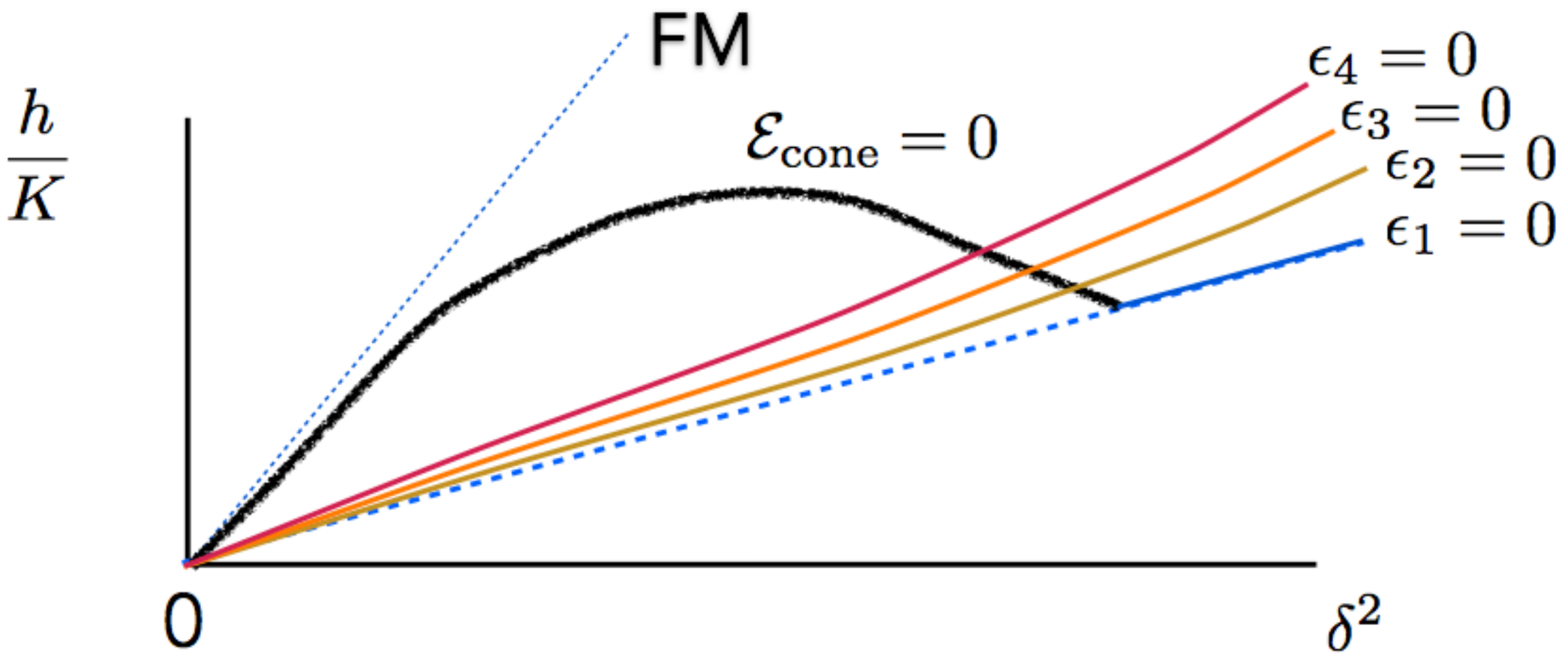
Metamagnetic endpoint?



Corrected first order curve bends slightly downward to intersect second order line

Instabilities

- Choose $E_{FM}=0$

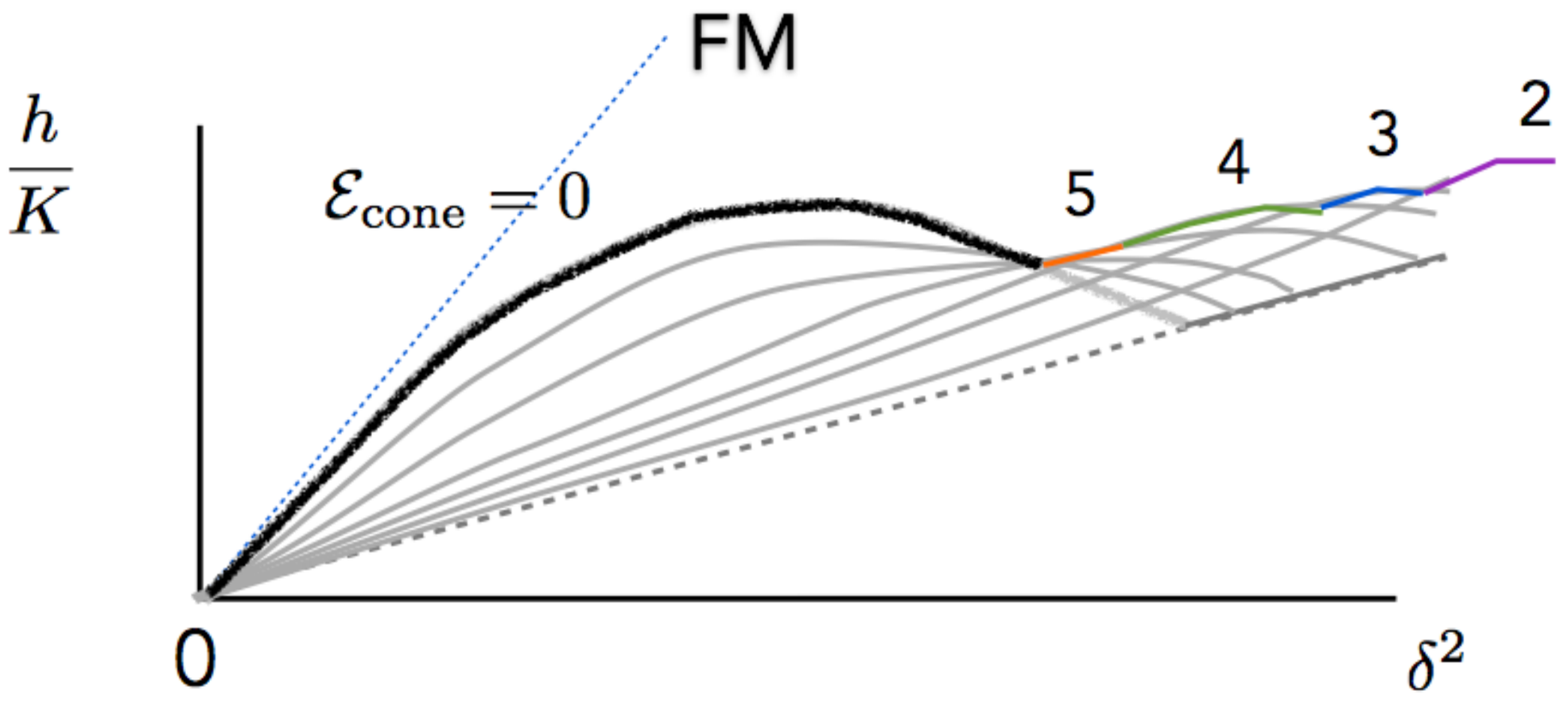


But the bound states cannot get arbitrarily deep - low density approximation is violated

A natural speculation

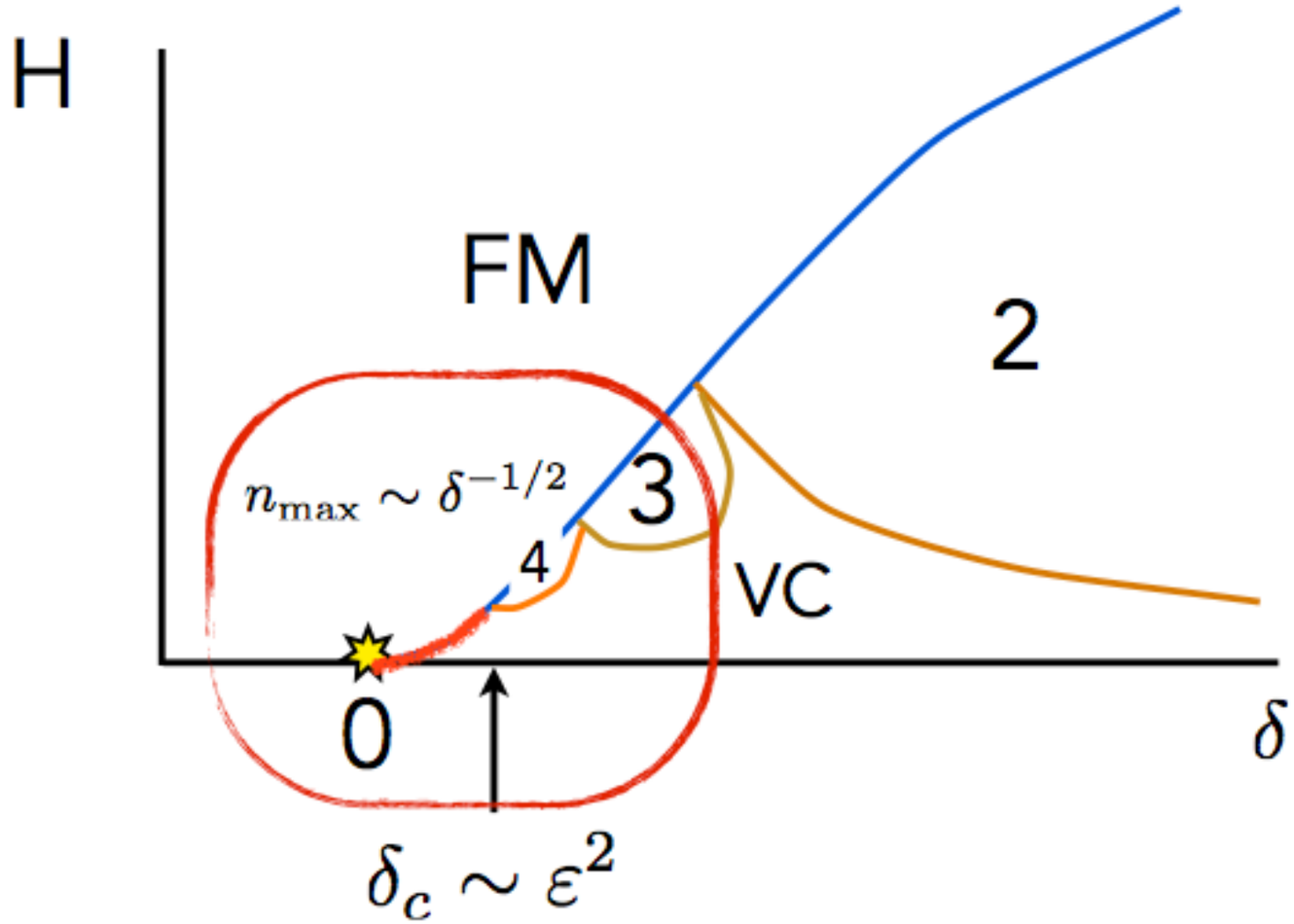
Instabilities

- Choose $E_{FM}=0$



Expect that n-boson bound states bend with increasing n to approach continuum line

Summary



Lifshitz point is a "parent" of many phases

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

Summary

- Spin chains keep showing up in unexpected places

✓ Nematic physics of frustrated ferromagnets

✓ Explored Lifshitz point as a “parent” for multipolar states and metamagnetism

d>1

$$S = \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Rescaling:

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau \qquad y \to \frac{\sqrt{cK}}{\delta} y$$

$$S = \frac{\sqrt{K^d C^{d-1}}}{\delta^{d-1/2}} \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

∴ Similar theory applies in d>1, and very similar conclusions apply