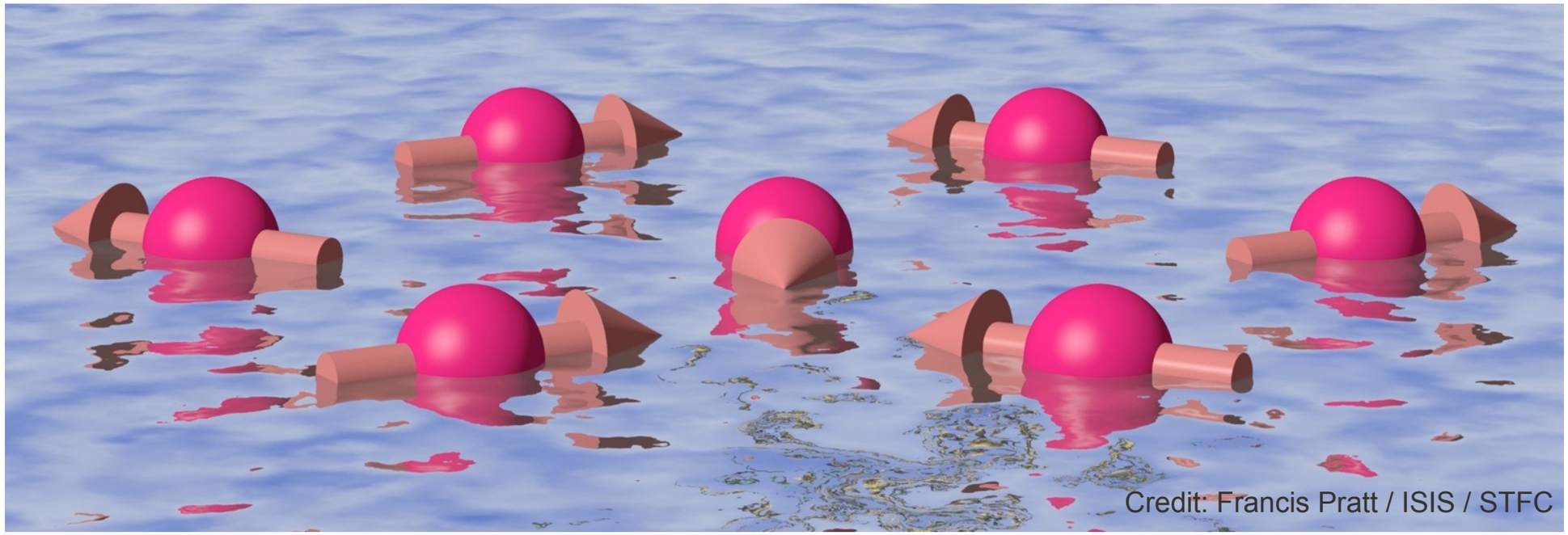
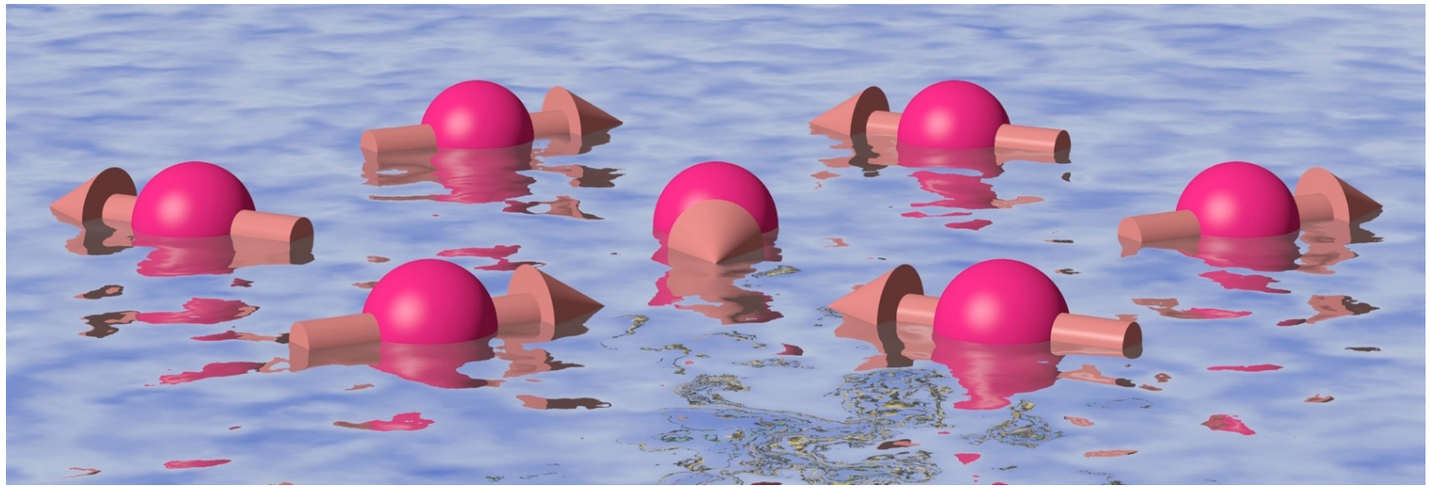
Frustration-driven multi-magnon condensates



Oleg Starykh
University of Utah

RECENT PROGRESS IN LOW-DIMENSIONAL QUANTUM MAGNETISM LDQM2016
5-16 September 2016, EPFL, Lausanne

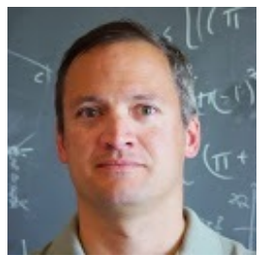
Why do we do it?



Immense potential
for practical applications



Collaborators



Leon Balents, KITP, UCSB



Andrey Chubukov, FTPI, U Minnesota



Jason Alicea, Caltech

General motivation: **Exotic** *but* ordered phases



composite order parameter

$$\mathcal{O}^{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j) = \tfrac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \tfrac{1}{3}\delta^{\alpha\beta}\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

LE JOURNAL DE PHYSIQUT

TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

J. VILLAIN

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(*Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977*)

Spin nematics

A. F. Andreev and I. A. Grishchuk

Institute of Physics Problems, USSR Academy of Sciences
(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

Outline

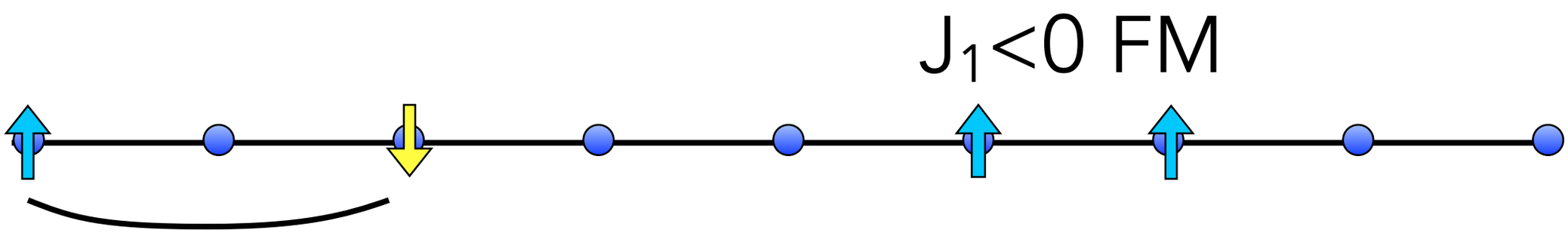
- Experimental evidence: Frustrated ferromagnets and magnon binding
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

magnon "BCS" superconductor

exotic superconductor of magnons

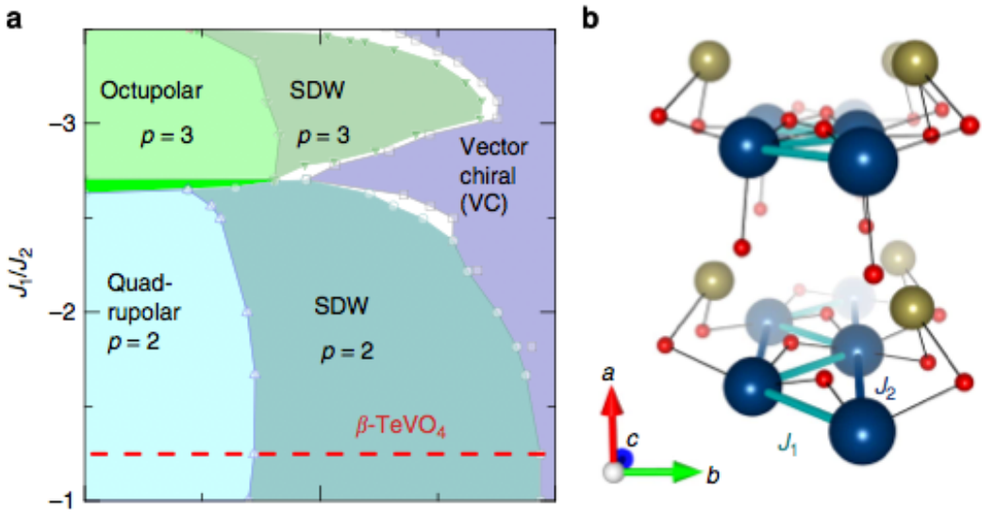
New system: Frustrated ferromagnet

1d S=1/2 chain



J_2>0 AF

$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



$\beta - \mathrm{TeVO_4}$

Pregelj *et al.*, Nat.Comm.2015

Compound	J_1, J_2 (K)	∠ Cu-O-Cu (deg)	T_N (K)	H_s (T)
Li_2ZrCuO_4[12, 13]	−151, 35	94.1	6.4	-
$Rb_2Cu_2Mo_3O_{12}$[14, 15]	−138, 51	89.9, 101.8	< 2	14
		91.9, 101.1		
$PbCuSO_4(OH)_2$[16–18]	−100, 36	91.2, 94.3	2.8	5.4
$LiCuSbO_4$[19]	−75, 34	89.8, 95.0	< 0.1	12
		92.0, 96.8		
$LiCu_2O_2$[20–22]	−69, 43	92.2, 92.5	22.3	110
$LiCuVO_4$[23–31]	−19, 44	95.0	2.1	44.4
$NaCuMoO_4(OH)$	−51, 36	92.0, 103.6	0.59	26

K. Nawa *et al*, arXiv:1409.1310

Complex field-induced states in Linarite $PbCuSO_4(OH)_2$ with a variety of high-order exotic SDW_p states

PRL 2016

B. Willenberg[1,2], M. Schäpers[3], A.U.B. Wolter[3], S.-L. Drechsler[3], M. Reehuis[2], J.-U. Hoffmann[2], B. Büchner[3,4], A.J. Studer[5], K.C. Rule[5], B. Ouladdiaf[6], S. Süllow[1], and S. Nishimoto[3,4]

multi-polar states with $9 \geq \mathbf{p} \geq 2$

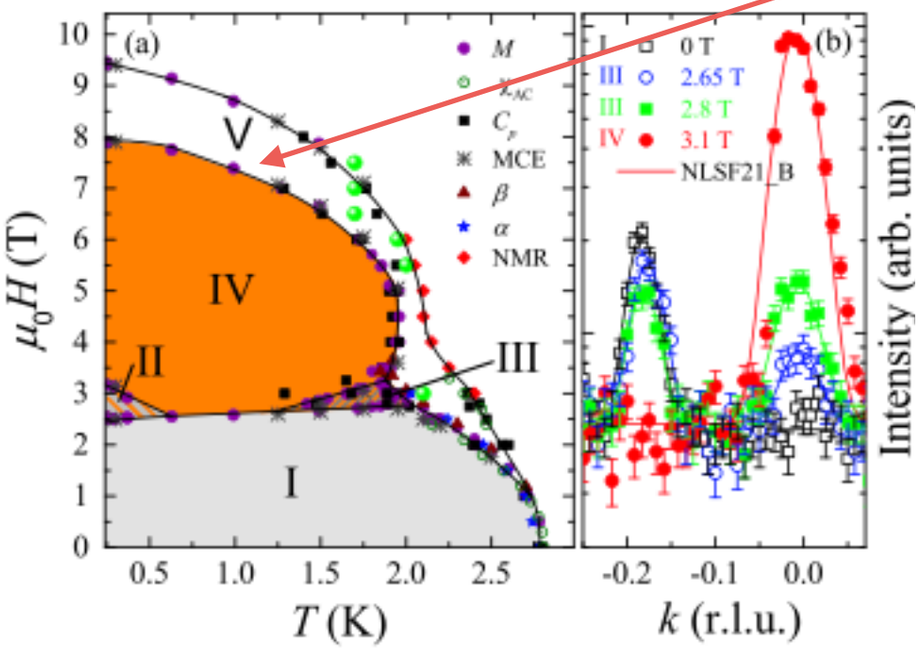


FIG. 1. (color online) (a) Phase diagram of linarite with $H \parallel b$. For a description of the phases I to V see text. Transition temperatures into phase V are depicted as red diamonds for the first time. Green balls indicate (H,T) points of Fig. 3. (b) Neutron scattering scans along k at 1.7 K as function of magnetic field, crossing from phase I via III into IV. Solid lines: Gaussians fits for peak position determination.

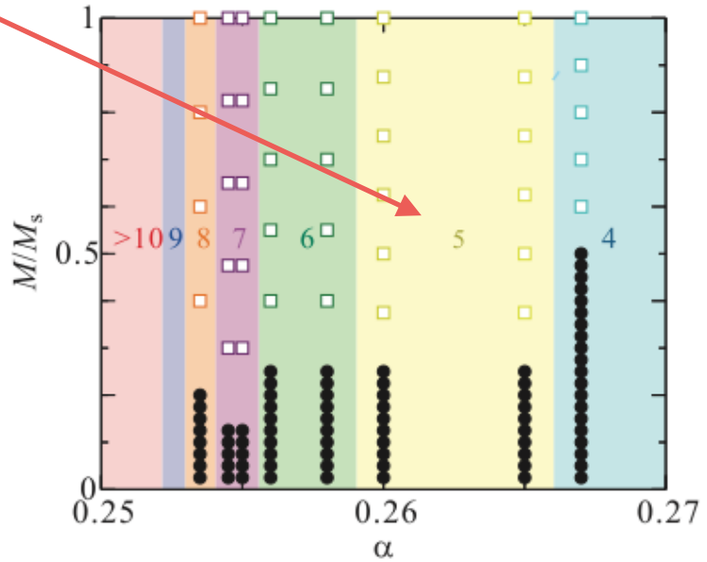
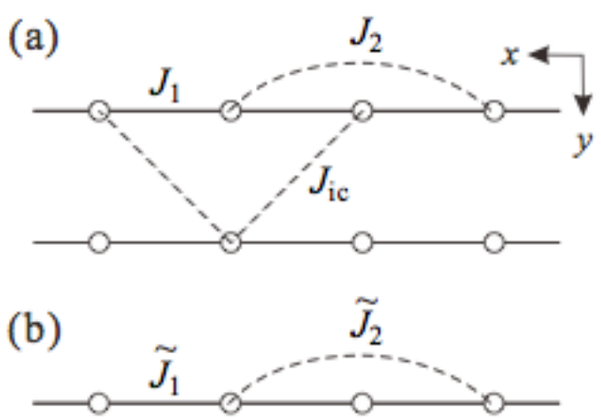


FIG. S5. (color online) Phase diagram showing dominant p-magnon bound states near the ferromagnetic critical point $\alpha_{eff} = 1/4$. The numbers from 4 to 10 are estimated $p(\equiv p_{\rm full})$ values in the fully polarized state. The largest binding energy is given by $p = p_{\rm full}$ ($p = 1$) by open squares (filled circles).

$$\alpha = J_{2,\mathrm{eff}} / |J_{1,\mathrm{eff}}|$$



One-Third Magnetization Plateau with a Preceding Novel Phase in Volborthite

H. Ishikawa,[1] M. Yoshida,[1] K. Nawa,[1] M. Jeong,[2] S. Krämer,[2] M. Horvatić,[2] C. Berthier,[2] M. Takigawa,[1] M. Akaki,[1] A. Miyake,[1] M. Tokunaga,[1] K. Kindo,[1] J. Yamaura,[3] Y. Okamoto,[1,4] and Z. Hiroi[1]

[1]*Institute for Solid State Physics, The University of Tokyo, Kashiwa, Chiba 277-8581, Japan*

[2]*Laboratoire National des Champs Magnétiques Intenses, LNCMI-CNRS (UPR3228), UJF, UPS and INSA, B.P. 166, 38042 Grenoble Cedex 9, France*

[3]*Materials Research Center for Element Strategy, Tokyo Institute of Technology, Yokohama 226-8503, Japan*

[4]*Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan*

(Received 23 November 2014; published 1 June 2015)

Huge 1/3 magnetization plateau !

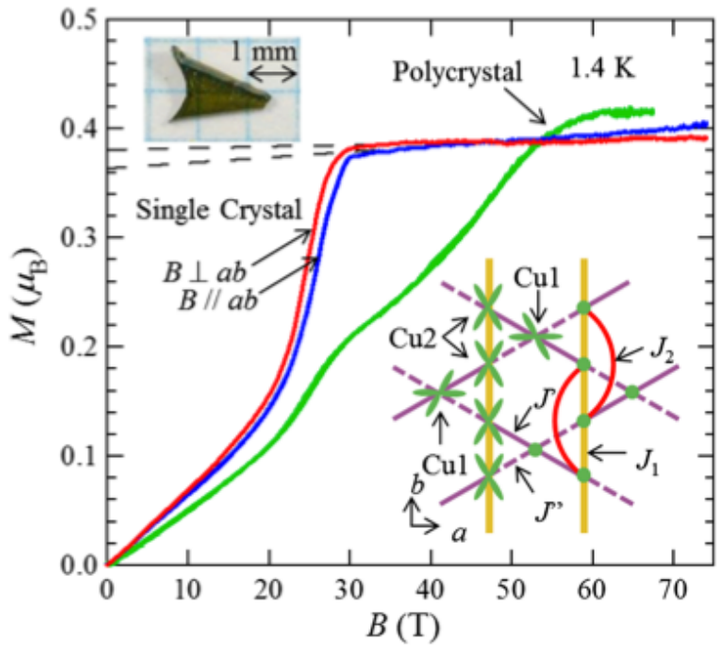


FIG. 1 (color online). Magnetization curves of volborthite measured at 1.4 K on two piles of single crystals in magnetic fields perpendicular (red line) and parallel (blue line) to the *ab* plane, and on a polycrystalline sample (green line) [32]. Shown also are a typical single crystal of volborthite (upper left) and the arrangement of Cu d_{x2-y2} orbitals projected onto the *ab* plane in the low-temperature $P2_1/a$ structure (lower right). J_1 and J_2 represent the NN and NNN interactions in the Cu2 spin chains, respectively. J' and J'' represent the NN interactions between Cu1 and Cu2 spins.

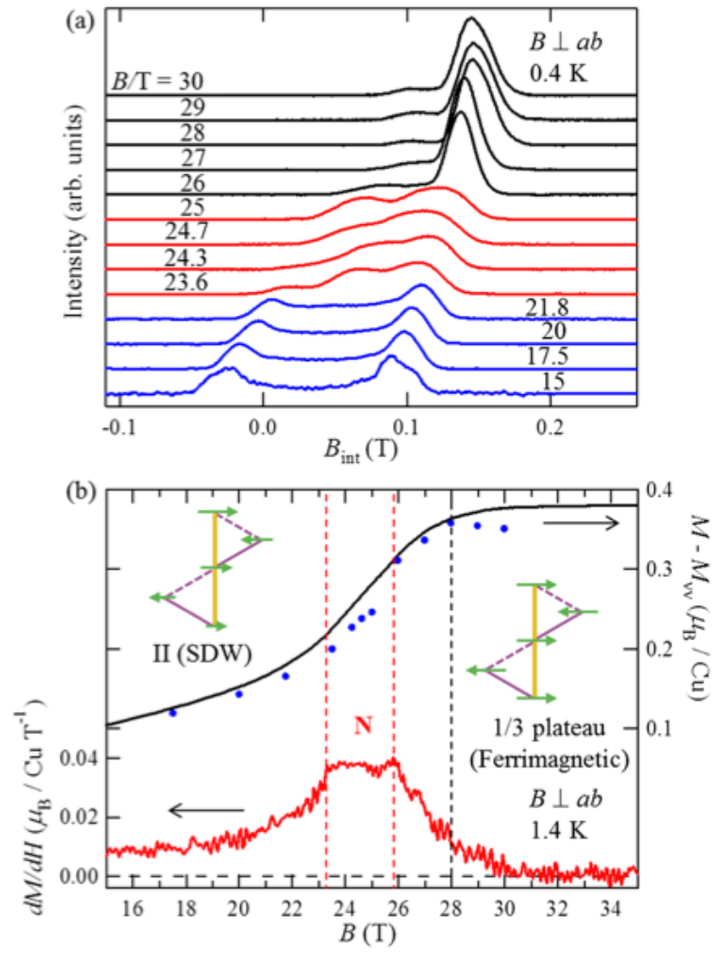


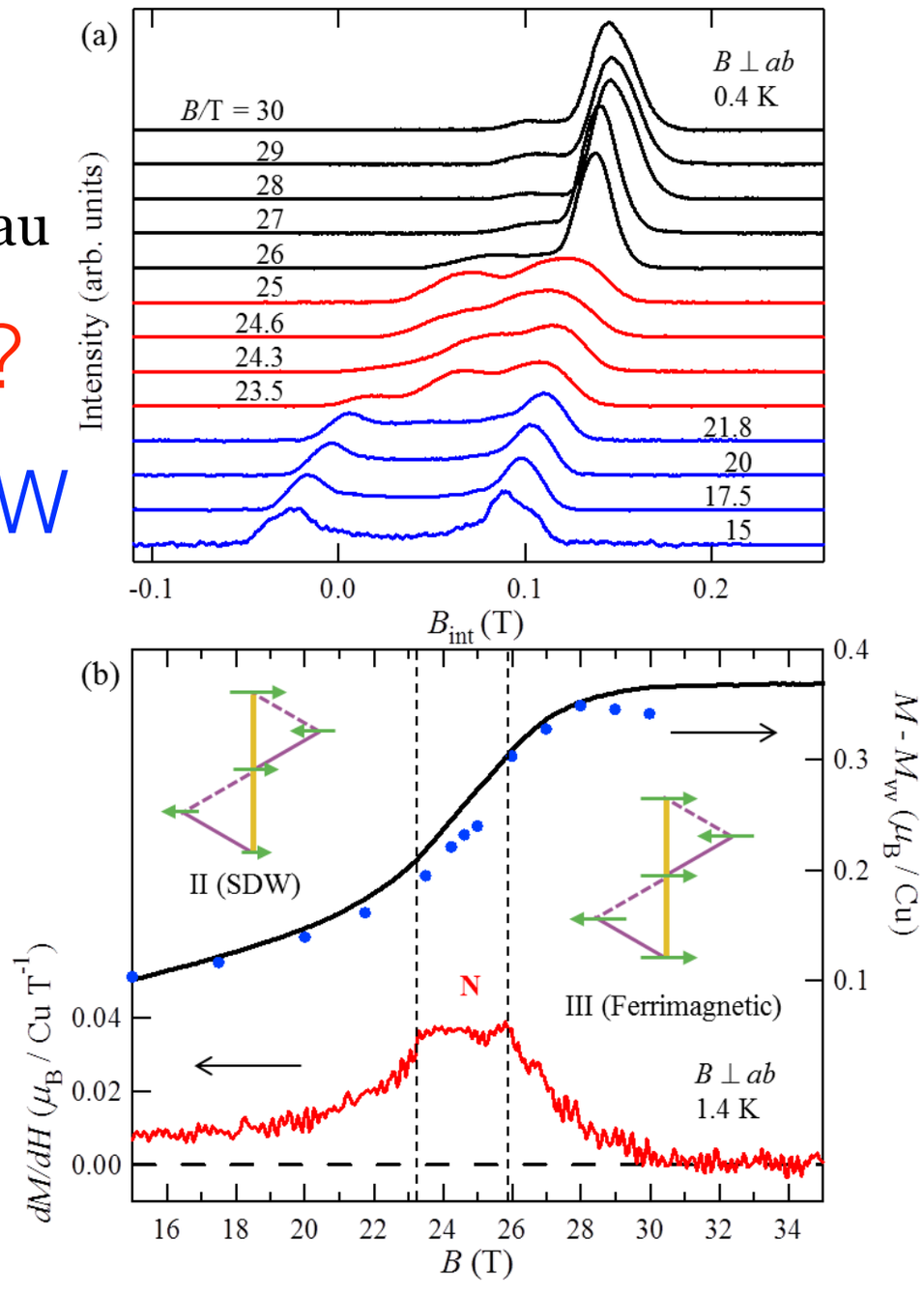
FIG. 2 (color online). (a) ^{51}V NMR spectra measured on a single-domain piece of a crystal in magnetic fields applied perpendicular to the *ab* plane at $T = 0.4$ K. The labeled fields correspond to $B = \nu_0/\gamma(B_{\rm int} = 0)$. (b) Magnetization curve of single crystals (top, black line) and its field derivative (bottom) in $B \perp ab$ at 1.4 K after the subtraction of the Van Vleck paramagnetic magnetization ($M_{\rm VV}$). Magnetization deduced from the center of the gravity of the NMR spectra is also plotted (top, blue circles).

Phase diagram

1/3 plateau

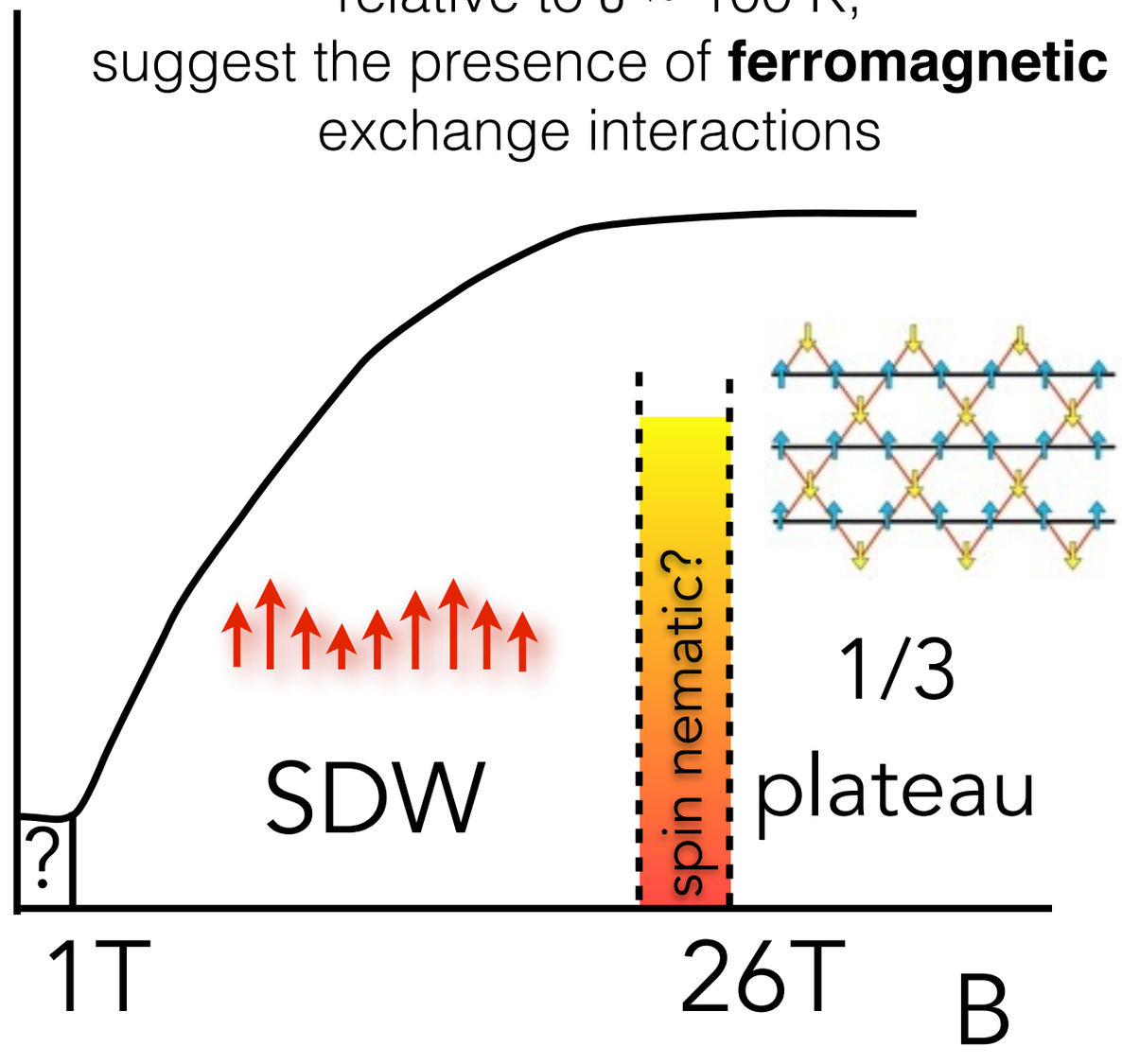
N?

SDW



H. Ishikawa *et al*, PRL 2015

small plateau's onset field of 27 Tesla,
relative to J ~ 100 K,
suggest the presence of **ferromagnetic** exchange interactions



may be a spin nematic??

But the model keeps changing…

1/3 plateau

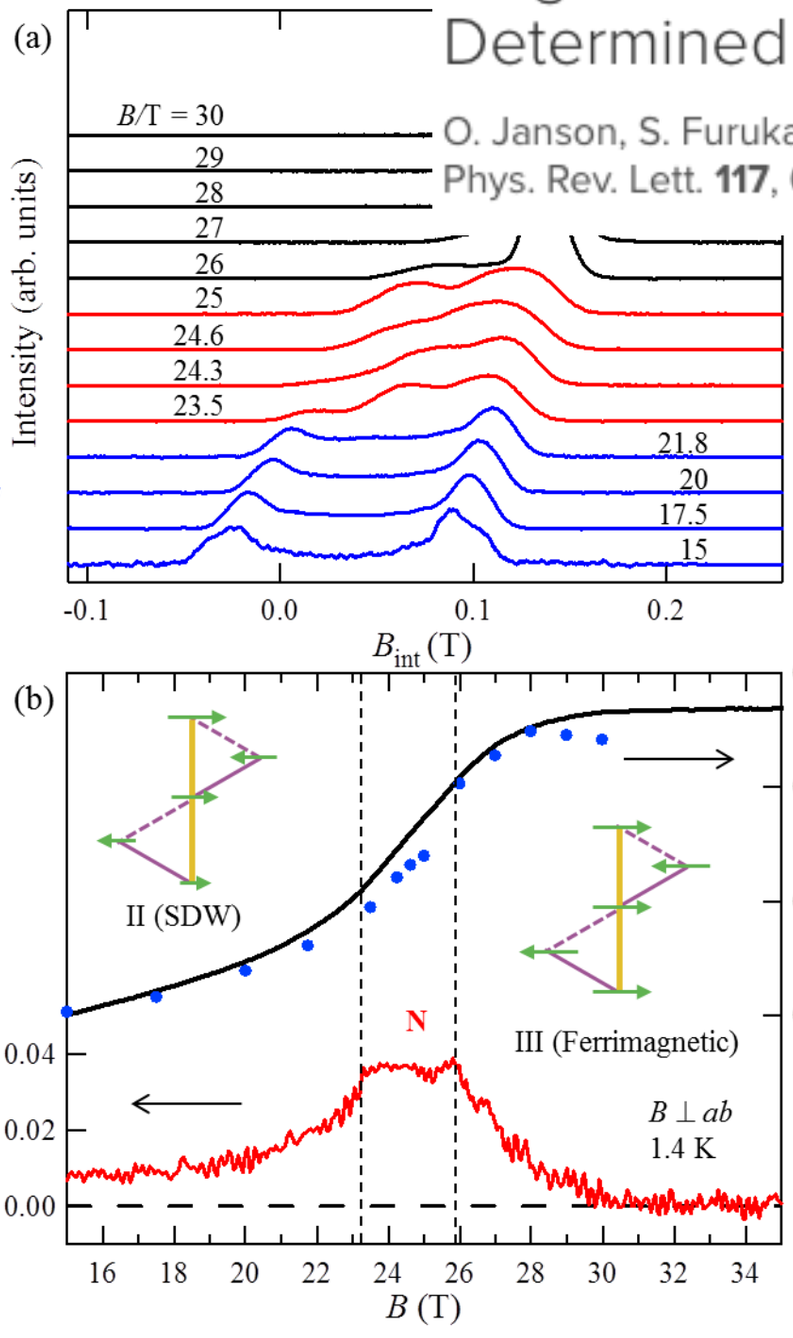
N?

SDW

Magnetic Behavior of Volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ Determined by Coupled Trimers Rather than Frustrated Chains

O. Janson, S. Furukawa, T. Momoi, P. Sindzingre, J. Richter, and K. Held

Phys. Rev. Lett. **117**, 037206 – Published 12 July 2016



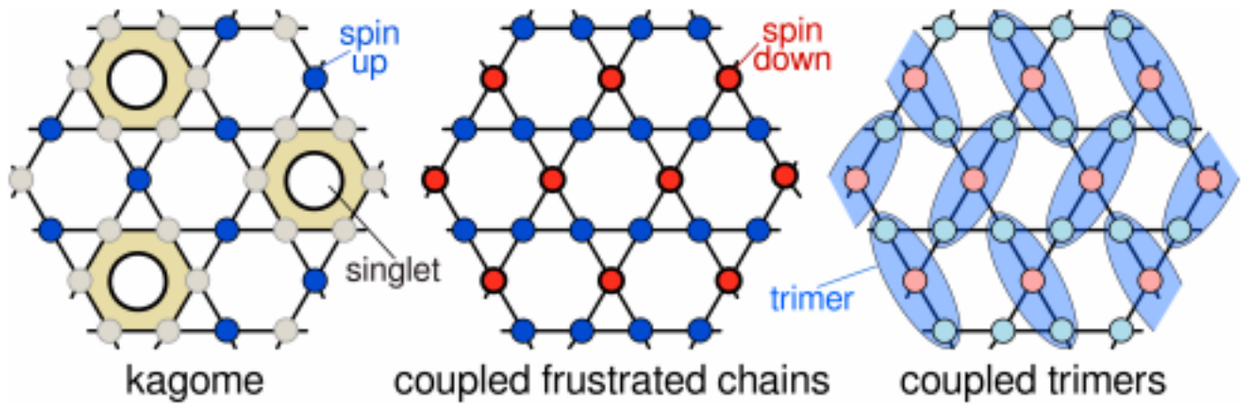


FIG. 1. (Color online) The structure of the $\frac{1}{3}$-magnetization plateau in the kagome model (KHM), coupled frustrated chains (CFC) model from Ref. [23], and the J-J'-J_1-J_2 model.

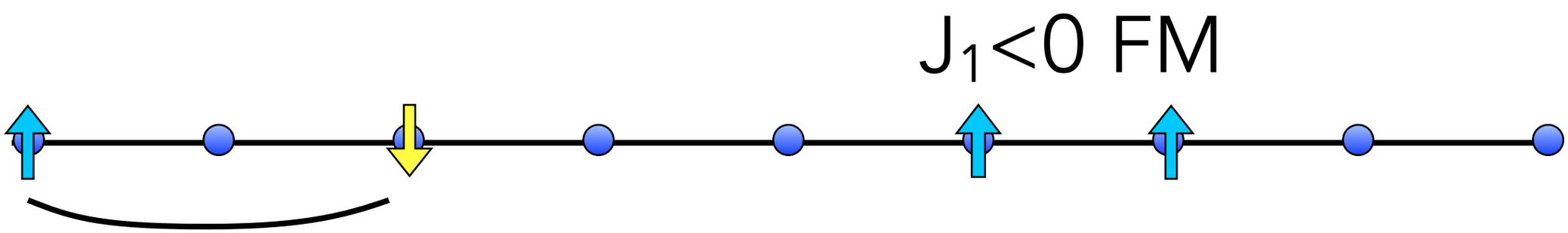
H. Ishikawa *et al*, PRL 2015

Outline

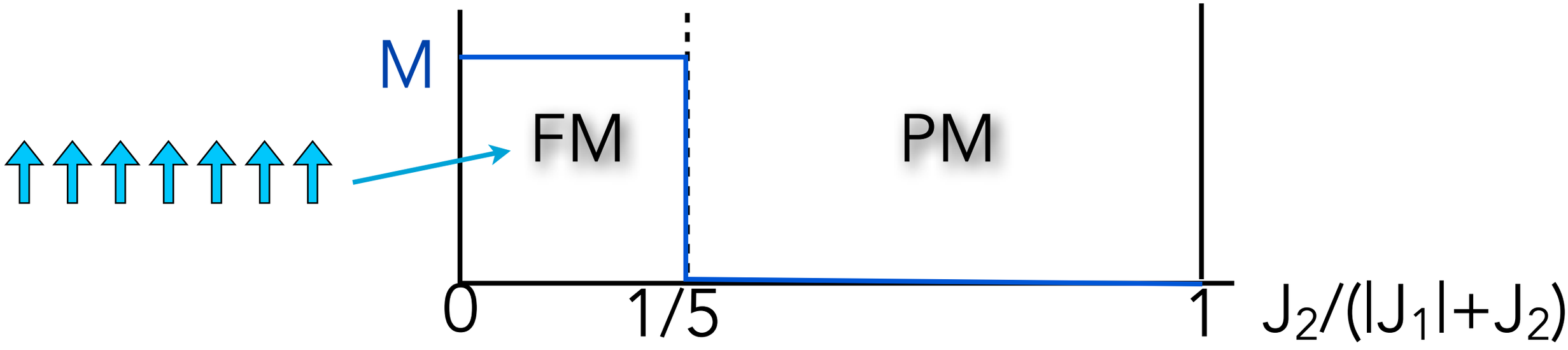
- Experimental evidence: Frustrated ferromagnets and magnon binding
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

Frustrated ferromagnet

1d S=1/2 chain

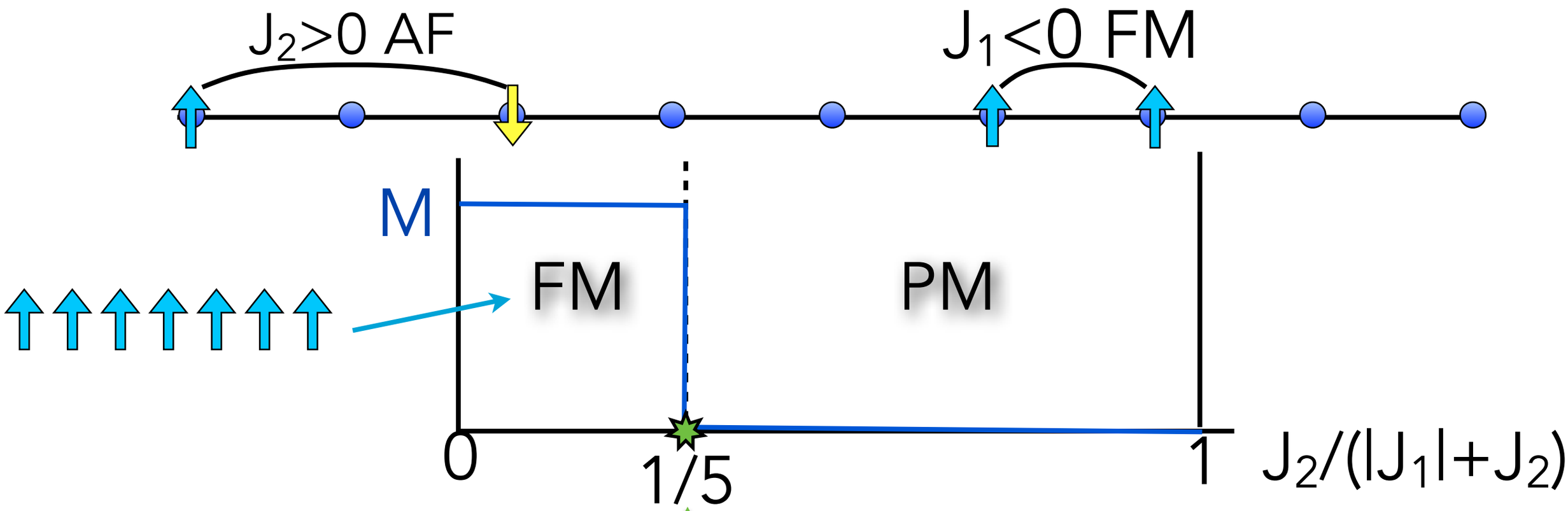


$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

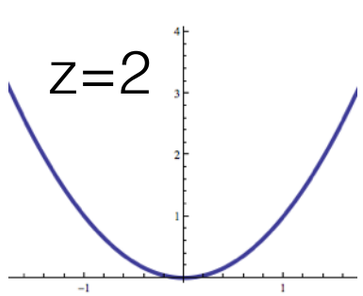


Frustrated ferromagnet

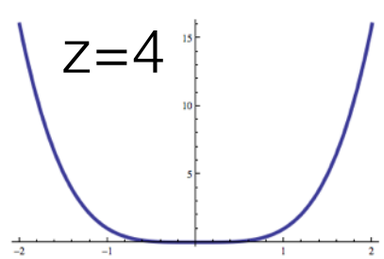
1d S=1/2 chain $$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



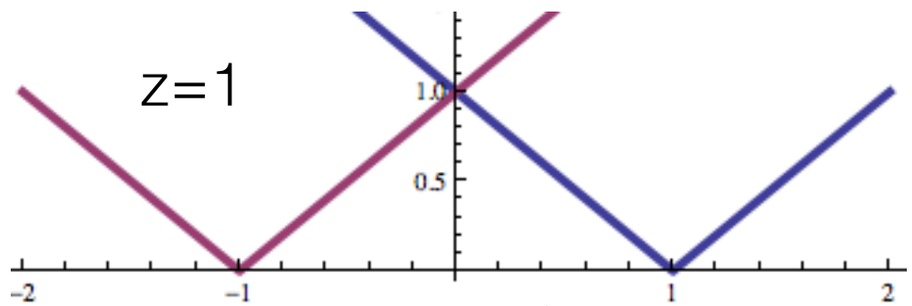
(spin-wave) dispersion



$\omega \sim k^2$

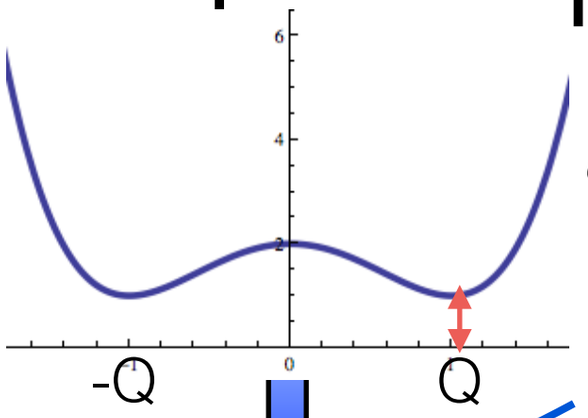


$\omega \sim k^4$

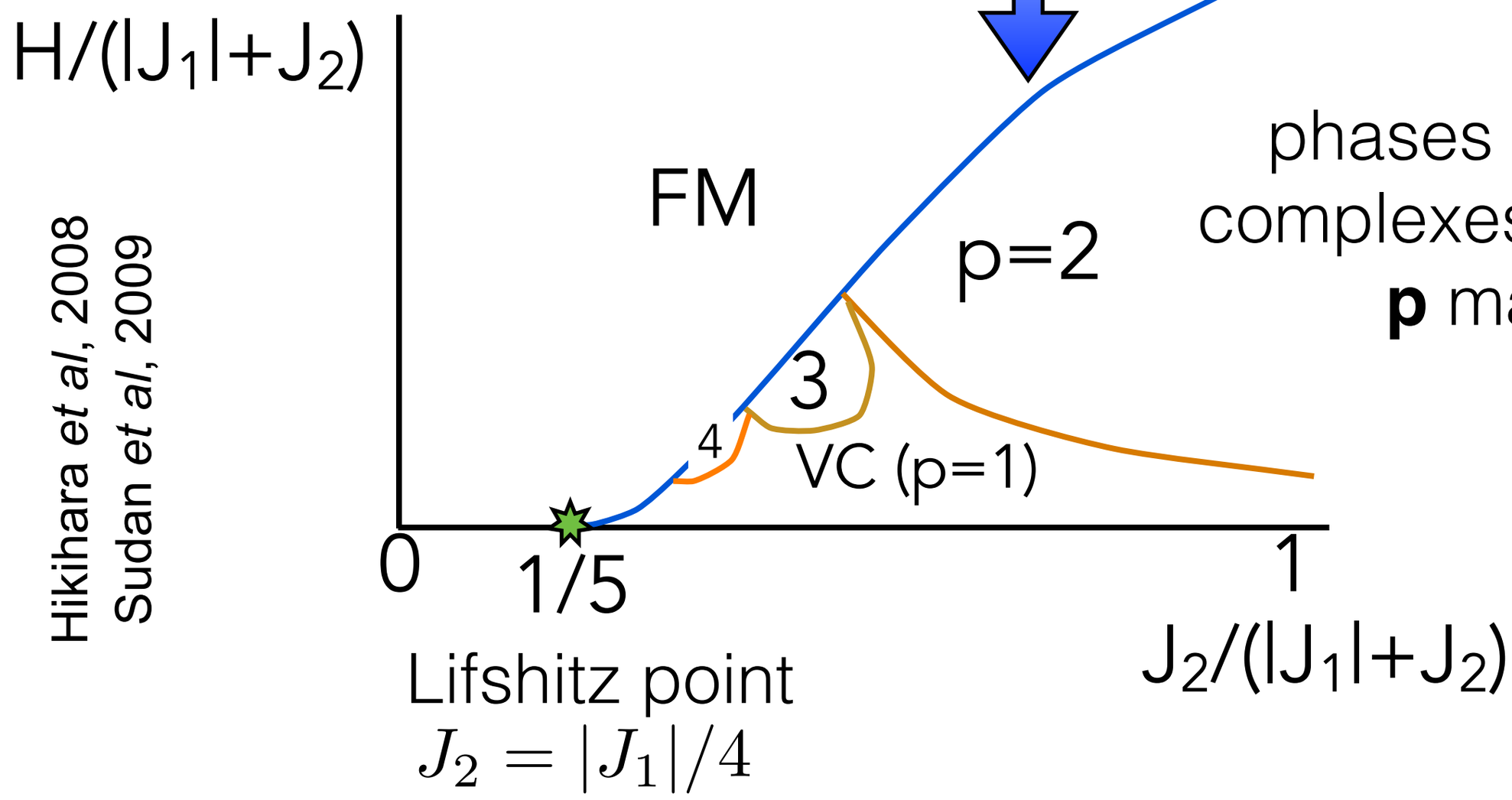


$\omega \sim (4J_2 - |J_1|)^{3/2} |k - K_i|$

Approaching from fully polarized state — Multipolar phases



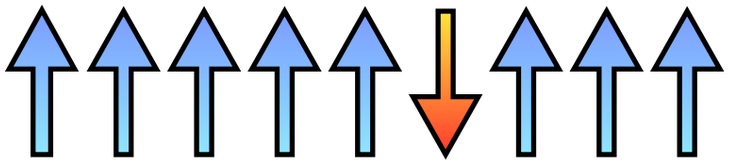
$$\omega \sim (k^2 - Q^2)^2 - (h_{\rm sat} - h)$$

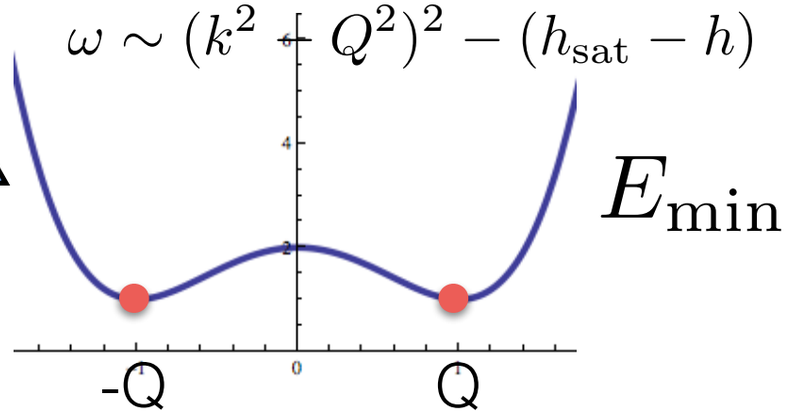


phases with bound complexes made out of **p** magnons

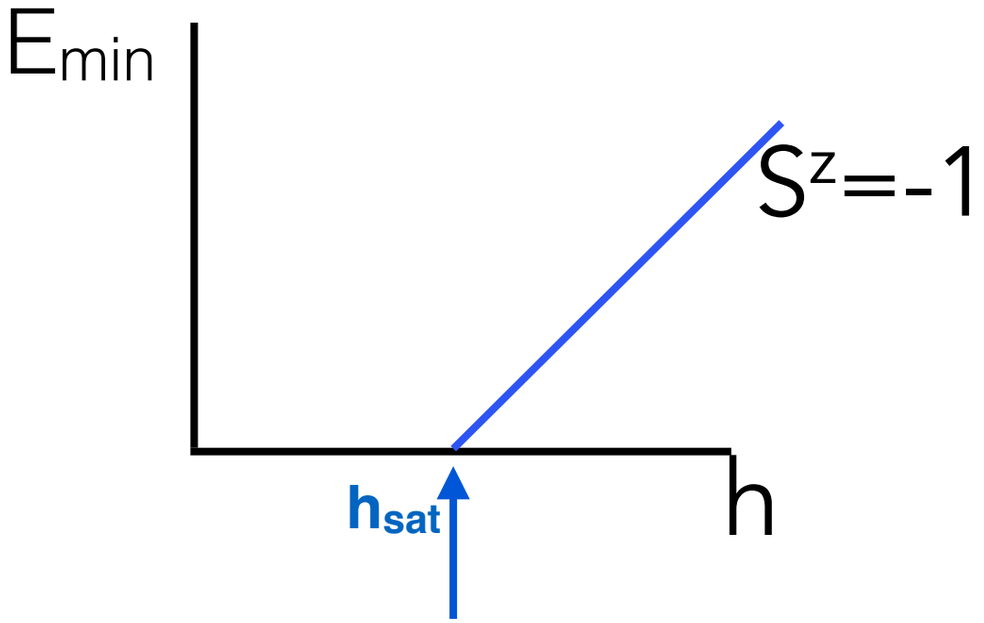
Magnon BEC

1-magnon





$$E_{\rm min} = \epsilon_1(k) + h$$



Condensation:

$$\langle a_k^+ \rangle = \sqrt{N}\ \Psi_Q\ \delta_{k,Q}$$

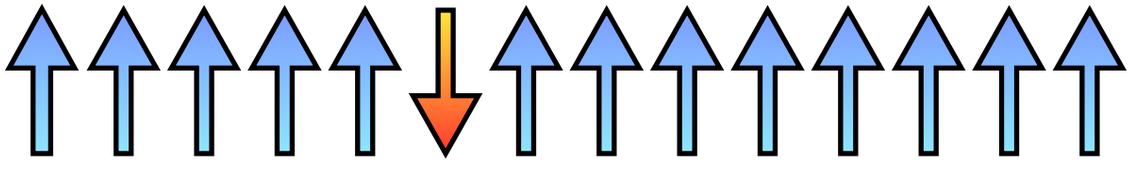
$$\langle S_n^- \rangle \sim \Psi_Q\ e^{iQx_n}$$

Magnon (quasi-)BEC

$$h_{\rm sat} = \frac{S(4J_2 - |J_1|)^2}{4J_2}$$

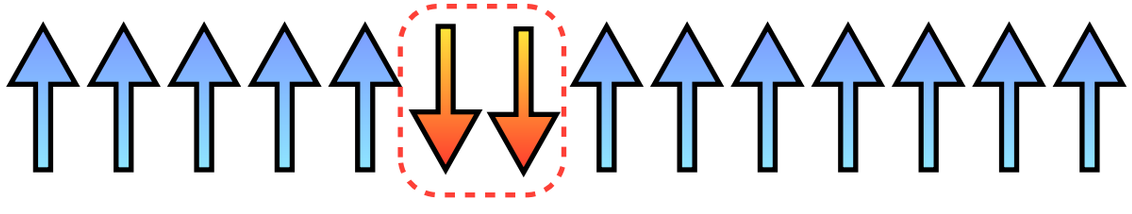
Today: condensation of *magnon pairs*

1-magnon

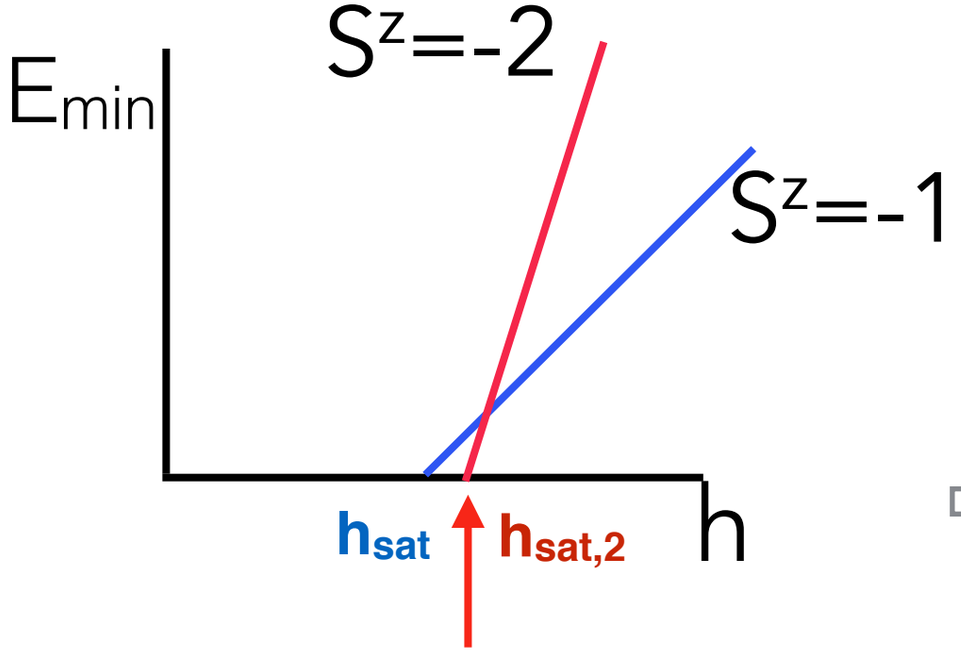


$$E_{\min} = \epsilon_1(k) + h$$

2-magnon bound state



$$E_{\min} = \Omega_2(k) + 2h$$



"molecular" bound state

$$\mathrm{Min}\{\Omega_2\} = 2\epsilon_1(Q) - \Delta_{\mathrm{bound}}$$

$$\Rightarrow h_{\mathrm{sat},2} = h_{\mathrm{sat}} + \Delta_{\mathrm{bound}}/2$$

Formation of molecular fluid

For d>1 at T=0 this is a molecular BEC

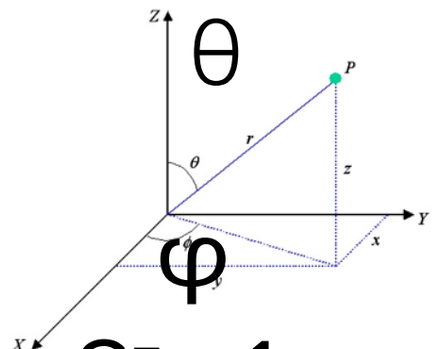
= true spin nematic

Hidden order

No dipolar order

$$\langle S_n^- \rangle = 0$$

$$\langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi}$$



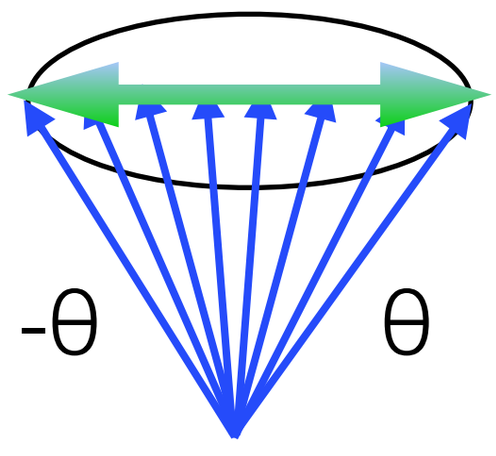
S^z=1 gap

Nematic order

$$\langle S_n^- S_{n+a}^- \rangle = \Phi \neq 0$$

$$\langle S_n^- S_m^- \rangle = \langle S_n^x S_m^x - S_n^y S_m^y - i(S_n^x S_m^y + S_n^y S_m^x) \rangle \sim \sin^2\theta(\cos 2\varphi - i \sin 2\varphi)$$

nematic director



Magnetic quadrupole moment

Symmetry breaking U(1) → Z_2

think of a fluctuating fan state:

φ is constant, while θ fluctuates

in the interval (θ_0, - θ_0)

Condensate of bound magnon pairs

$$\langle S_n^- \rangle = 0 \qquad \langle S_n^- S_{n+a}^- \rangle = \Phi \neq 0$$

Ferromagnetic $J_1 < 0$ produces attraction in real space

quasi-1d

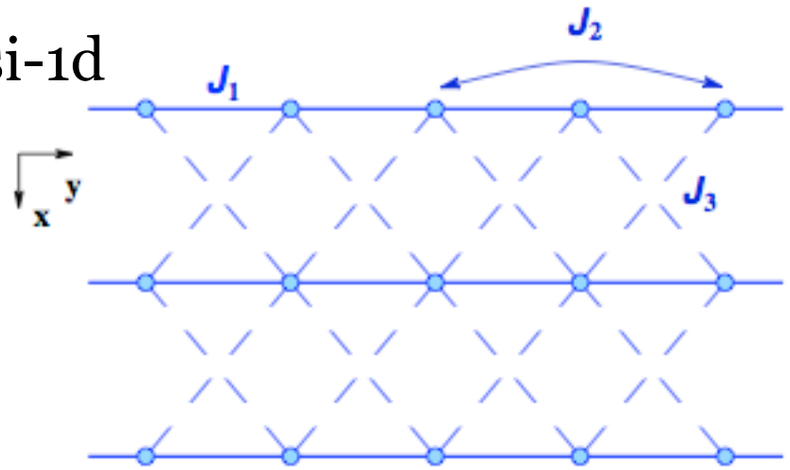


Fig. 2: (Color online) Two-dimensional array of copper ions in $LiCuVO_4$ with principal exchange couplings.

Chubukov 1991
Kecke et al 2007
Kuzian and Drechsler 2007
Hikihara et al 2008
Sudan et al 2009
Zhitomirsky and Tsunetsugu 2010

2d

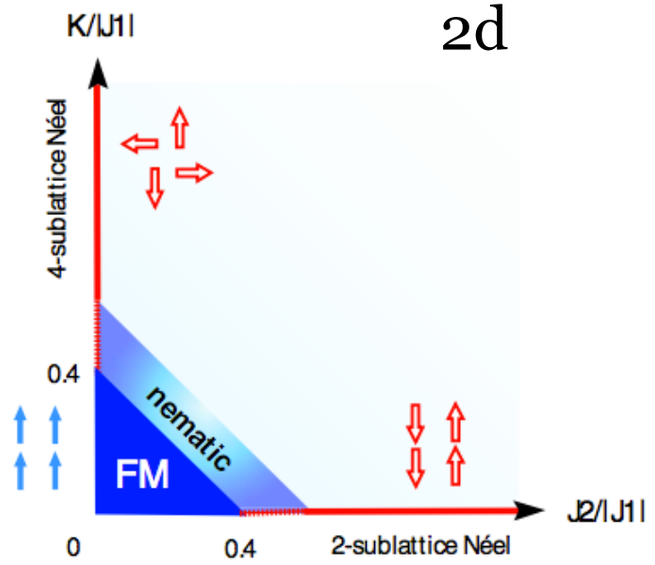
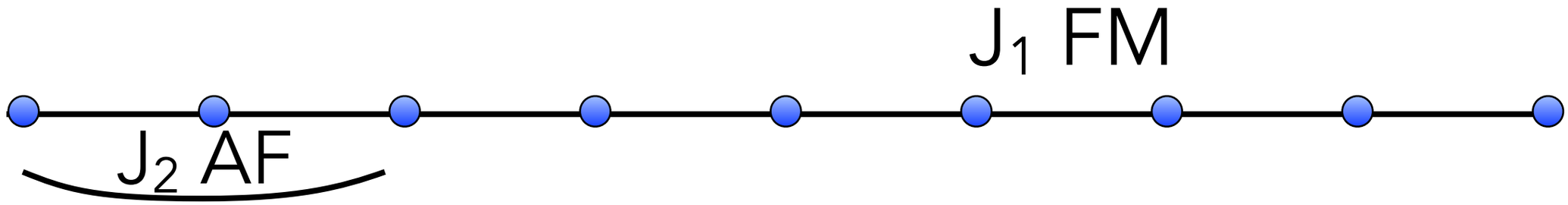


FIG. 1 (color online). Phase diagram for the spin-1/2 J_1-J_2-K Heisenberg model on a square lattice, in the parameter range where $J_1 < 0$ is FM and both $J_2 \geq 0$ and $K \geq 0$ are AF. Solid lines on the $J_2/|J_1|$ and $K/|J_1|$ axes denote known Néel phases. A single nematic phase interpolates between the FM occurring for large FM J_1 and these two, different, AF ordered states.

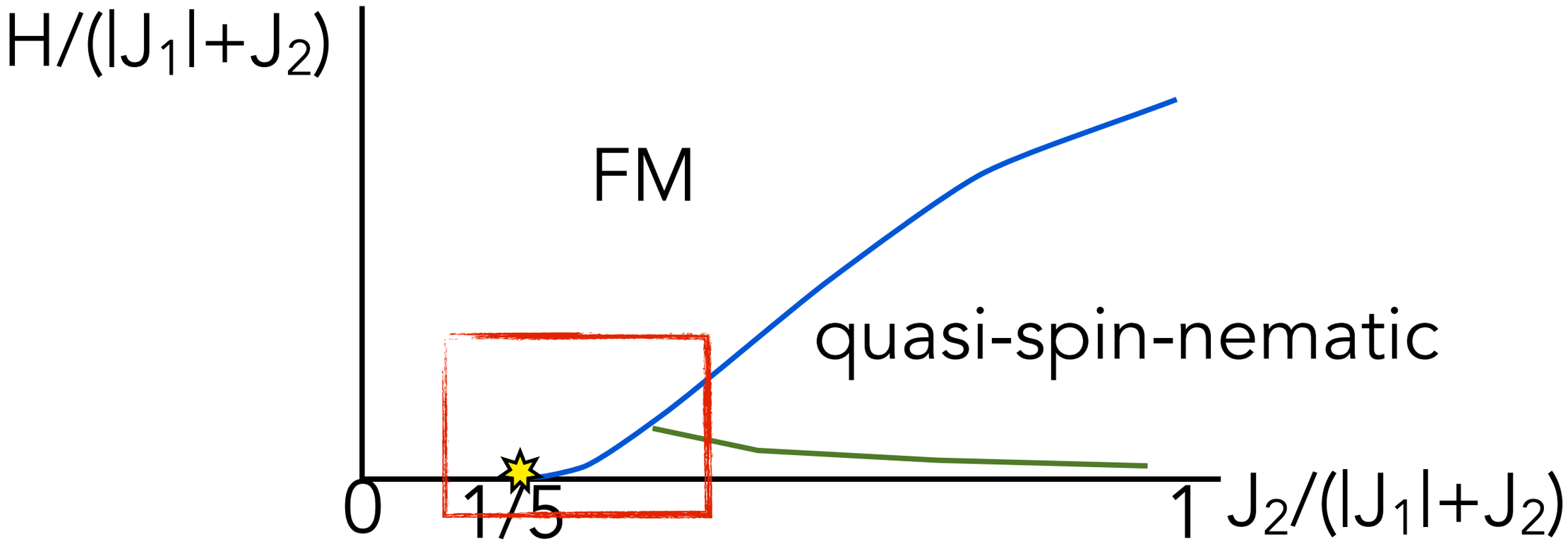
Shannon, Momoi, Sindzingre PRL 2006

Spin chain numerics

Frustrated ferromagnetic chain



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



PHYSICAL REVIEW B **74**, 020403(R) (2006)

Frustrated ferromagnetic spin-$\frac{1}{2}$ chain in a magnetic field: The phase diagram and thermodynamic properties

F. Heidrich-Meisner,[1,2] A. Honecker,[3,4] and T. Vekua[5,6]

We compute the ground-state energies $E_0(S^z_{\text{total}}, h=0)$ in subspaces labeled by S^z_{total} on chains with periodic boundary conditions (PBC) using the Lanczos algorithm. The ground-state energies of substantially larger chains with open boundary conditions (OBC) are calculated with DMRG. Typically, we keep up to $m=400$ states in our DMRG calculations. Then, we include the Zeeman term and obtain the field-dependent ground-state energies

$$E_0(S^z_{\text{total}}, h) = E_0(S^z_{\text{total}}, h=0) - hS^z_{\text{total}}. \quad (4)$$

The magnetization curves are constructed by solving the equations $E_0(S^z_{\text{total}}, h_{\text{step}}) = E_0(S^z_{\text{total}}+s, h_{\text{step}})$, which define those magnetic fields at which the magnetization increases from $M = S^z_{\text{total}}/(NS)$ to $M' = (S^z_{\text{total}}+s)/(NS)$. Steps larger than $s=1$ may occur.

s=1/2

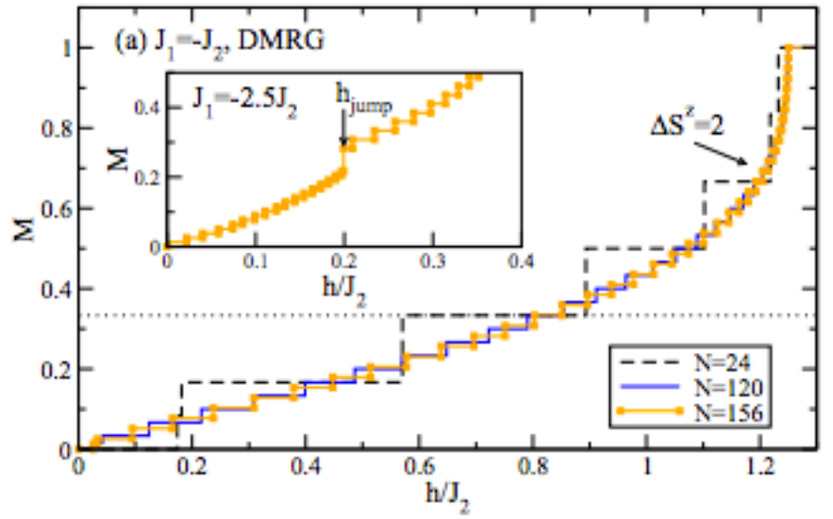


FIG. 1: (Color online) (a), main panel (inset): Magnetization curve $M(h)$ for $J_1 = -J_2$ ($J_1 = -2.5\,J_2$). The horizontal dotted line marks $M = 1/3$. (b): $M(h)$ for $J_1 = -3\,J_2$. (c): Magnetic phase diagram of the frustrated FM chain. The dotted line (with stars) marks the first-order transition between the EO phase and the $\Delta S^z = 1$ region, while the line $h = h_t$ (dashed, triangles) separates the $\Delta S^z = 1$ region from the $\Delta S^z \geq 3$ part. Uncertainties of the transition lines, e.g. due

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Vector chiral and multipolar orders in the spin-$\frac{1}{2}$ frustrated ferromagnetic chain in magnetic field

Toshiya Hikihara,[1] Lars Kecke,[2,3] Tsutomu Momoi,[2] and Akira Furusaki[2]

s=1/2

TABLE I. Number of magnons p and total momentum k of the multimagnon bound states which become gapless at the saturation field.

Parameter range	p	k
$-2.669<J_1/J_2<0$	2	π
$-2.720<J_1/J_2<-2.669$	2	$\pi\pm\delta$ $(\delta>0)$
$-3.514<J_1/J_2<-2.720$	3	π
$-3.764<J_1/J_2<-3.514$	4	π
$-3.888<J_1/J_2<-3.764$	5	π
$-3.917<J_1/J_2<-3.888$	6	π
$-4<J_1/J_2<-3.917$	7	π

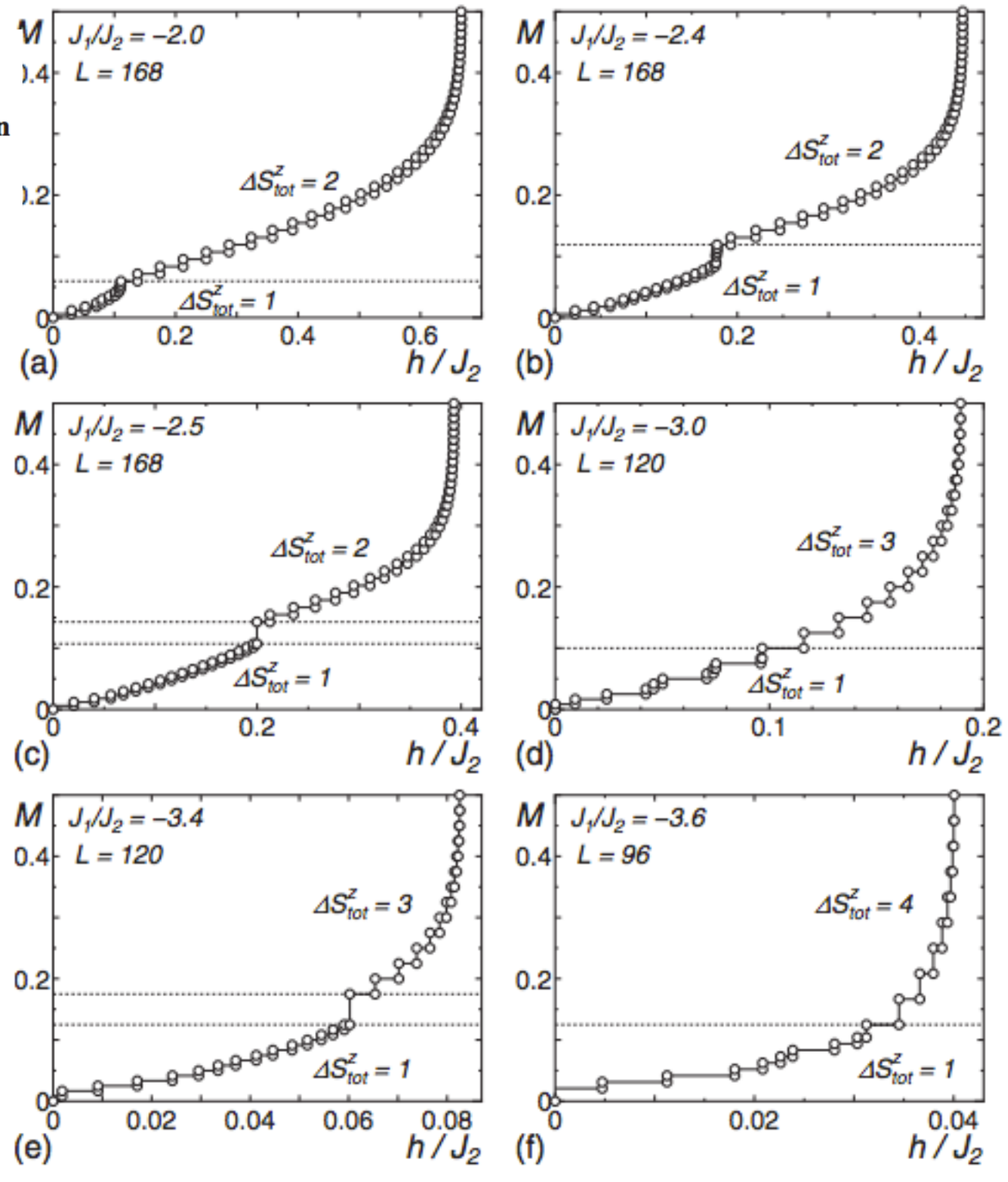


FIG. 9. Magnetization curves for (a) $J_1/J_2=-2.0$, (b) $J_1/J_2=-2.4$, (c) $J_1/J_2=-2.5$, (d) $J_1/J_2=-3.0$, (e) $J_1/J_2=-3.4$, and (f) $J_1/J_2=-3.6$. The dotted lines represent the boundaries of the regions of $\Delta S^z_{\rm tot}=1$ and $\Delta S^z_{\rm tot}\geq 2$.

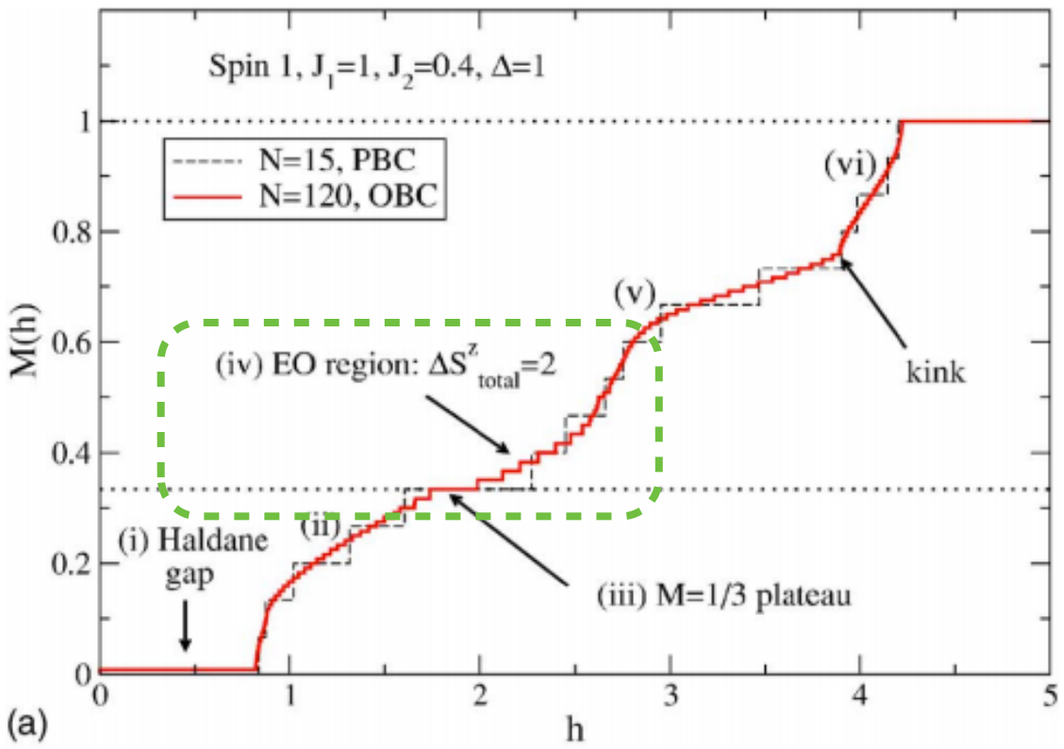
PHYSICAL REVIEW B **75**, 064413 (2007)

Universal emergence of the one-third plateau in the magnetization process of frustrated quantum spin chains

F. Heidrich-Meisner,[1] I. A. Sergienko,[1] A. E. Feiguin,[2] and E. R. Dagotto[1]

$$H = \sum_i \left[\sum_{n=1,2} J_n \left\{ \frac{1}{2}(S_i^+ S_{i+n}^- + S_i^- S_{i+n}^+) + \Delta S_i^z S_{i+n}^z \right\} - h S_i^z + D(S_i^z)^2 \right],$$

s=1,3/2,2



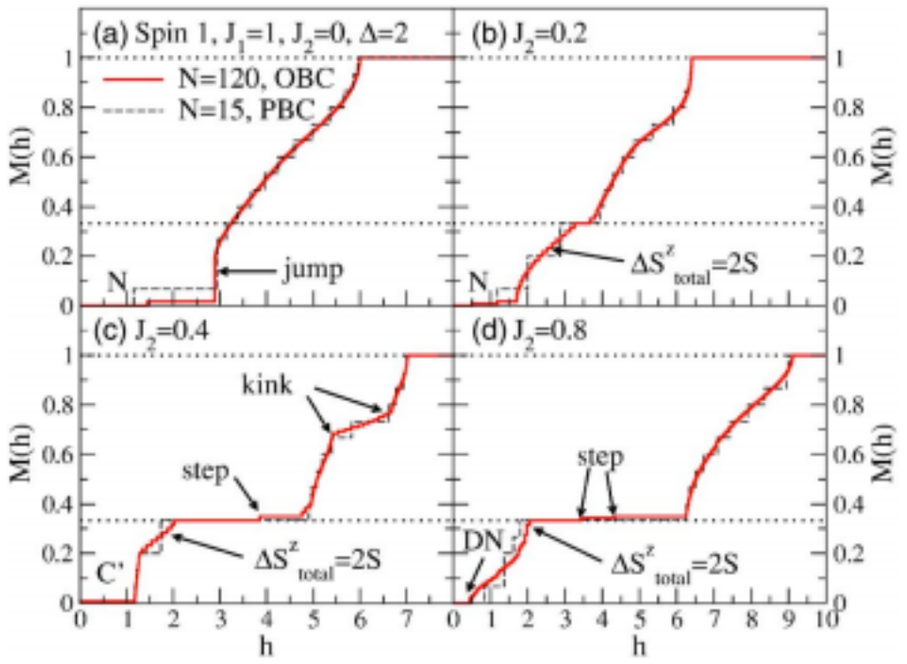


FIG. 11. (Color online) Magnetization curves of frustrated spin-1 chains with an anisotropic exchange ($\Delta=2$) for (a) $J_2=0$, (b) $J_2=0.2$, (c) $J_2=0.4$, and (d) $J_2=0.8$. DMRG results (straight lines) are for $N=60$ sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase **N**, canted phase **C**, double-Néel

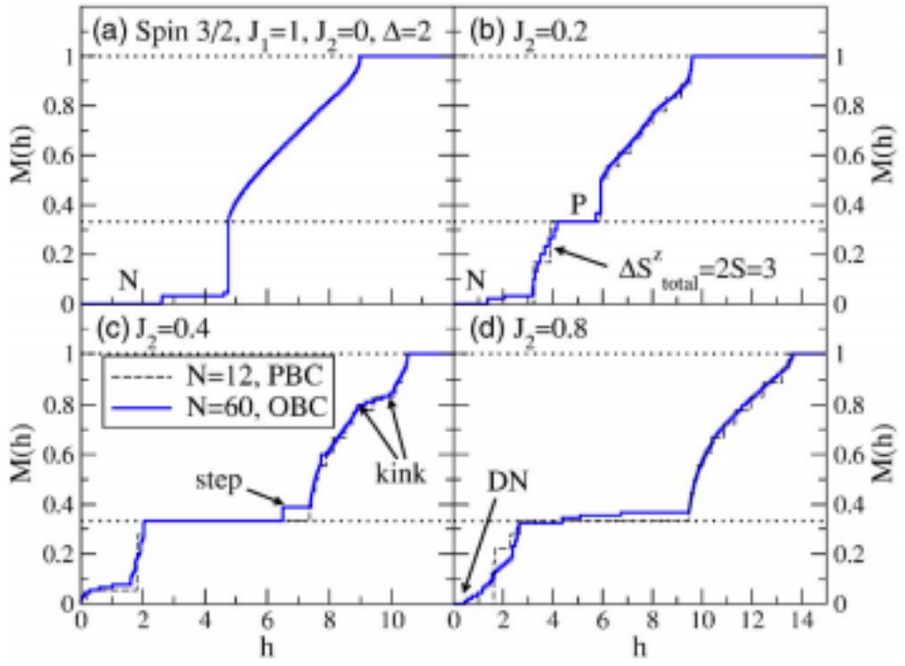
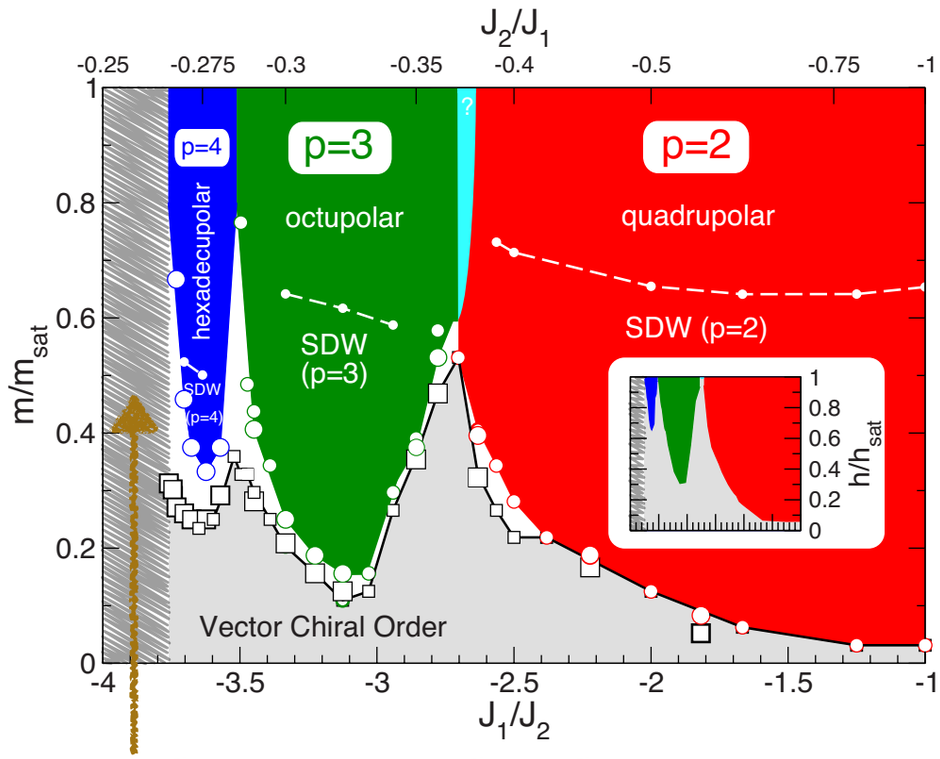


FIG. 12. (Color online) Magnetization curves for frustrated spin-3/2 chains with an anisotropic exchange ($\Delta=2$) for (a) $J_2=0$, (b) $J_2=0.2$, (c) $J_2=0.4$, and (d) $J_2=0.8$. DMRG results (straight lines) are for $N=60$ sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase **N**, canted phase **C**, double-Néel phase **DN**.

Numerics: nematicity and 1st order seem connected?

1d frustrated chain

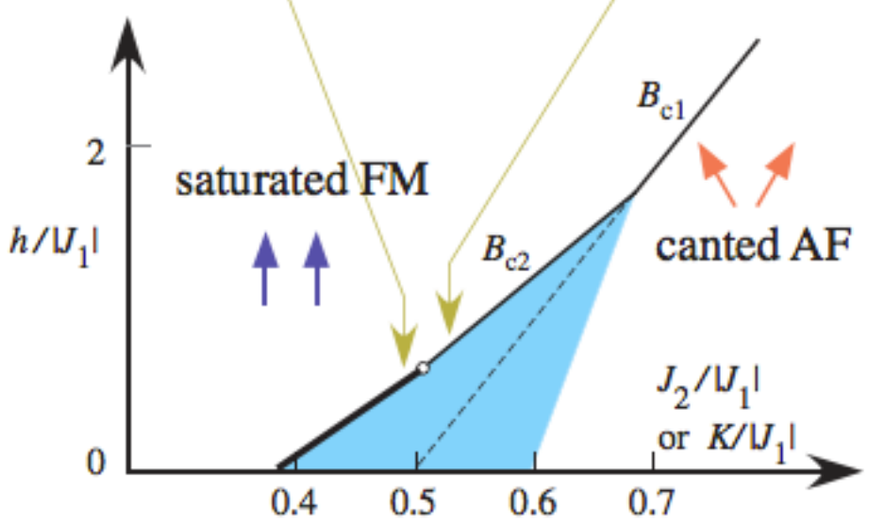


Sudan et al, 2009

1st order

2nd order

2d frustrated square lattice



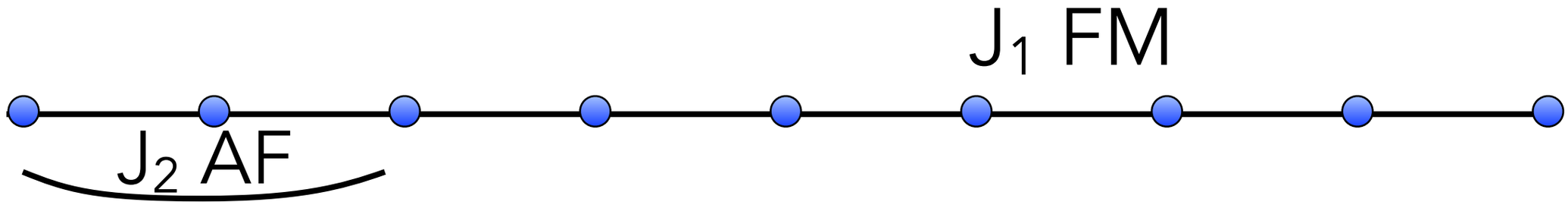
Shannon, Momoi, Sindzingre PRL 2006

Outline

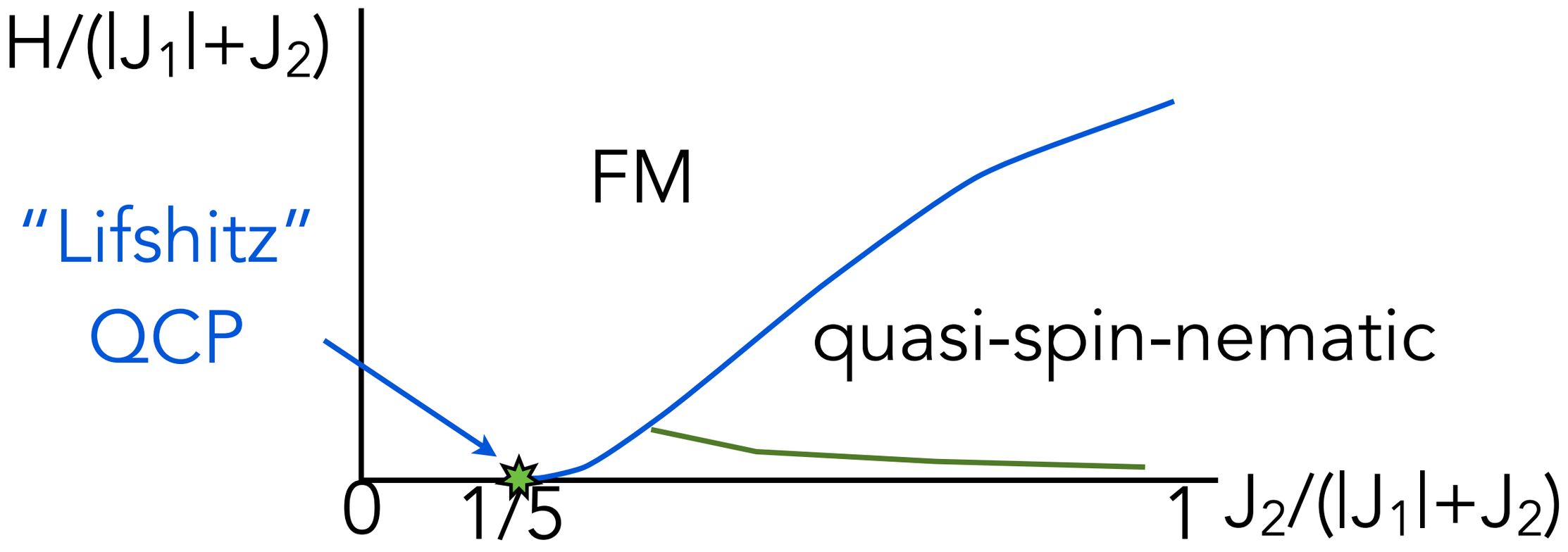
- Experimental evidence: Frustrated ferromagnets and magnon binding
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

Quantum Lifshitz point

Frustrated ferromagnetic chain



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



Lifshitz Point

Balents, Starykh PRL 2016

- Unusual QCP: order-to-order transition
- Effective action - NL**σ**M for unit vector **m**

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$\mathcal{A}_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}$ Berry phase term

tunes QCP $\delta \propto |J_1| - 4J_2$

two symmetry allowed interactions at $O(q^4)$

All properties near Lifshitz point obey "one parameter universality" dependent upon u/K ratio

Lifshitz Point

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau$$

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Large parameter:
saddle point!

$$v = \frac{u}{K} \qquad \overline{h} = \frac{hK}{\delta^2}$$

Lifshitz point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

v derives from quantum fluctuations

Need it be positive?

$$\hat{m} \cdot \hat{m} = 1 \quad \Rightarrow \quad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}|$$

Theory is stable for v>-1

In fact, v<0

- Semiclassical large s limit: $v = -\frac{3}{2s}$
- s=1/2 estimate: $v_{s=1/2} = -\frac{9}{(2+\sqrt{7})^2} \approx -0.42$

Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

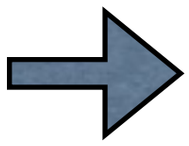
$$\hat{m} = \begin{pmatrix} |\Psi|\cos(qx+\phi) \\ \pm|\Psi|\sin(qx+\phi) \\ \sqrt{1-|\Psi|^2} \end{pmatrix}$$



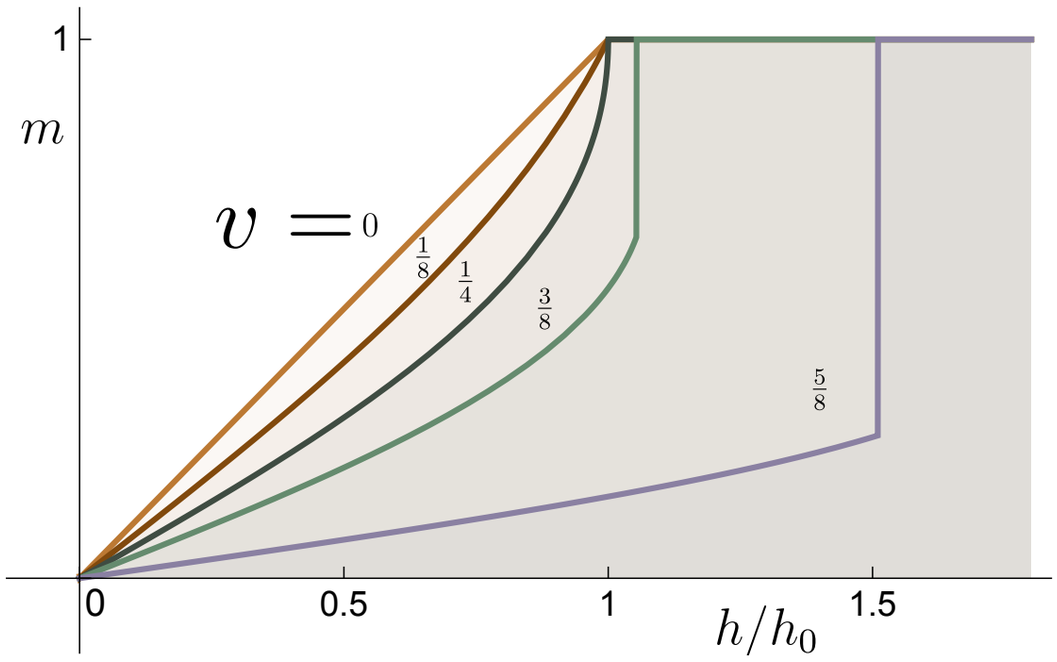
cone state

Obtain

q, Ψ versus h, v, δ



many physical quantities



Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

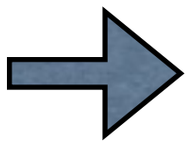
$$\hat{m} = \begin{pmatrix} |\Psi|\cos(qx+\phi) \\ \pm|\Psi|\sin(qx+\phi) \\ \sqrt{1-|\Psi|^2} \end{pmatrix}$$



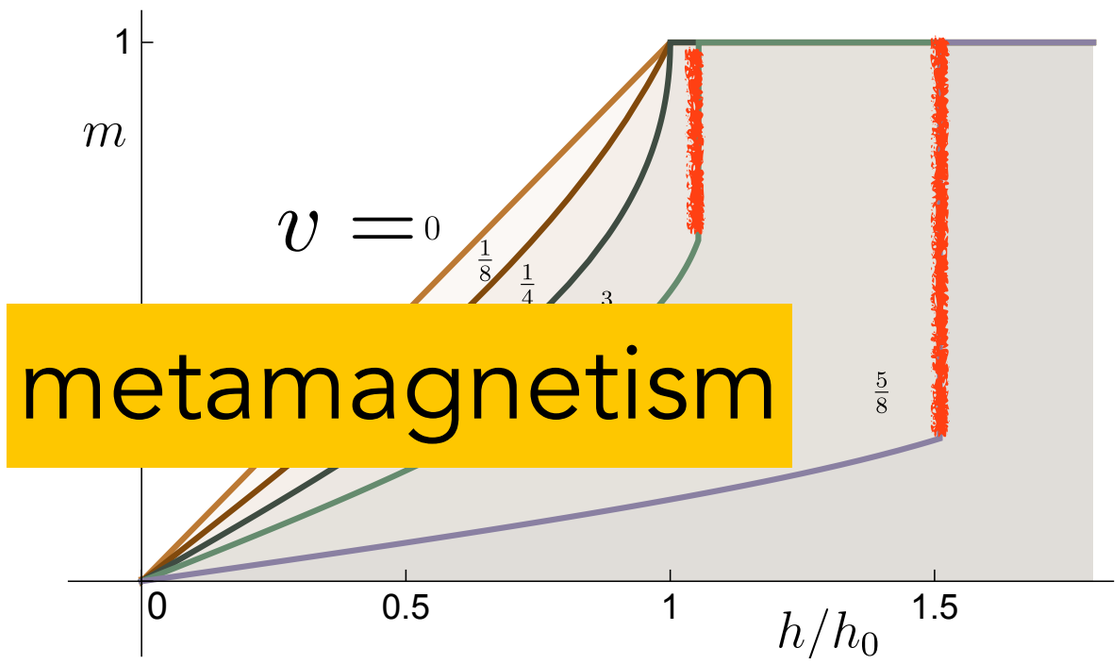
cone state

Obtain

q, Ψ versus h, v, δ



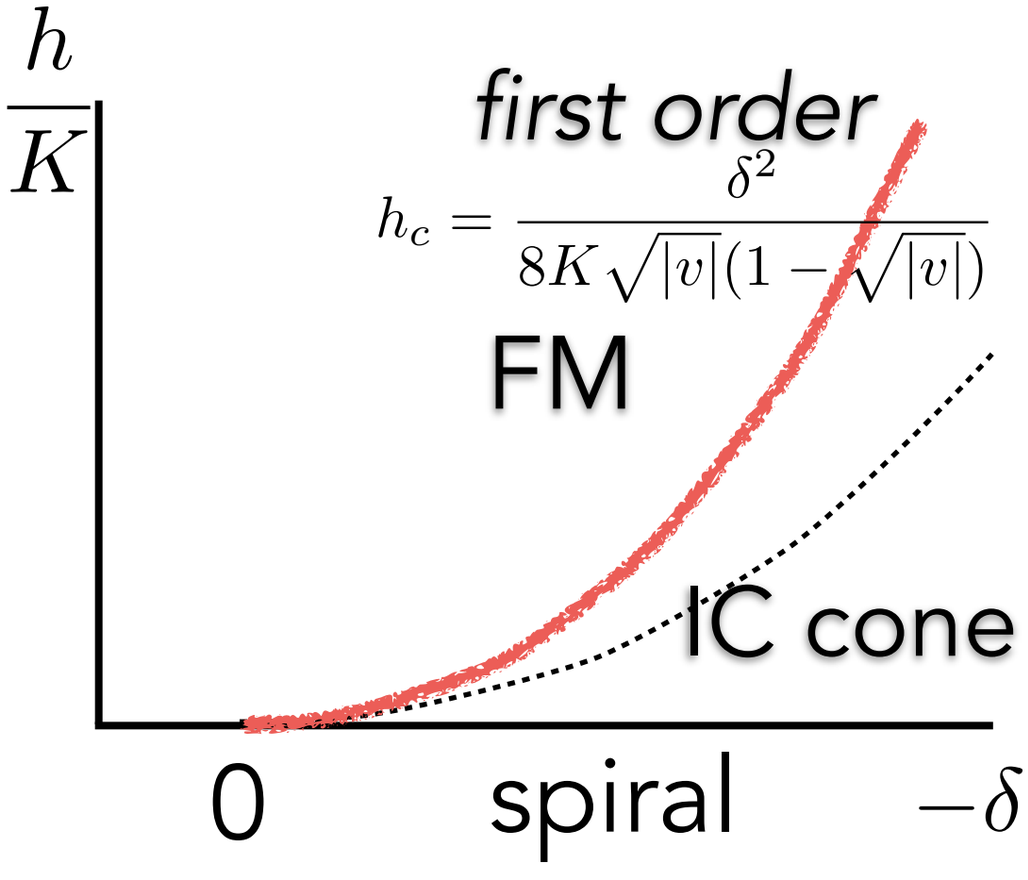
many physical quantities



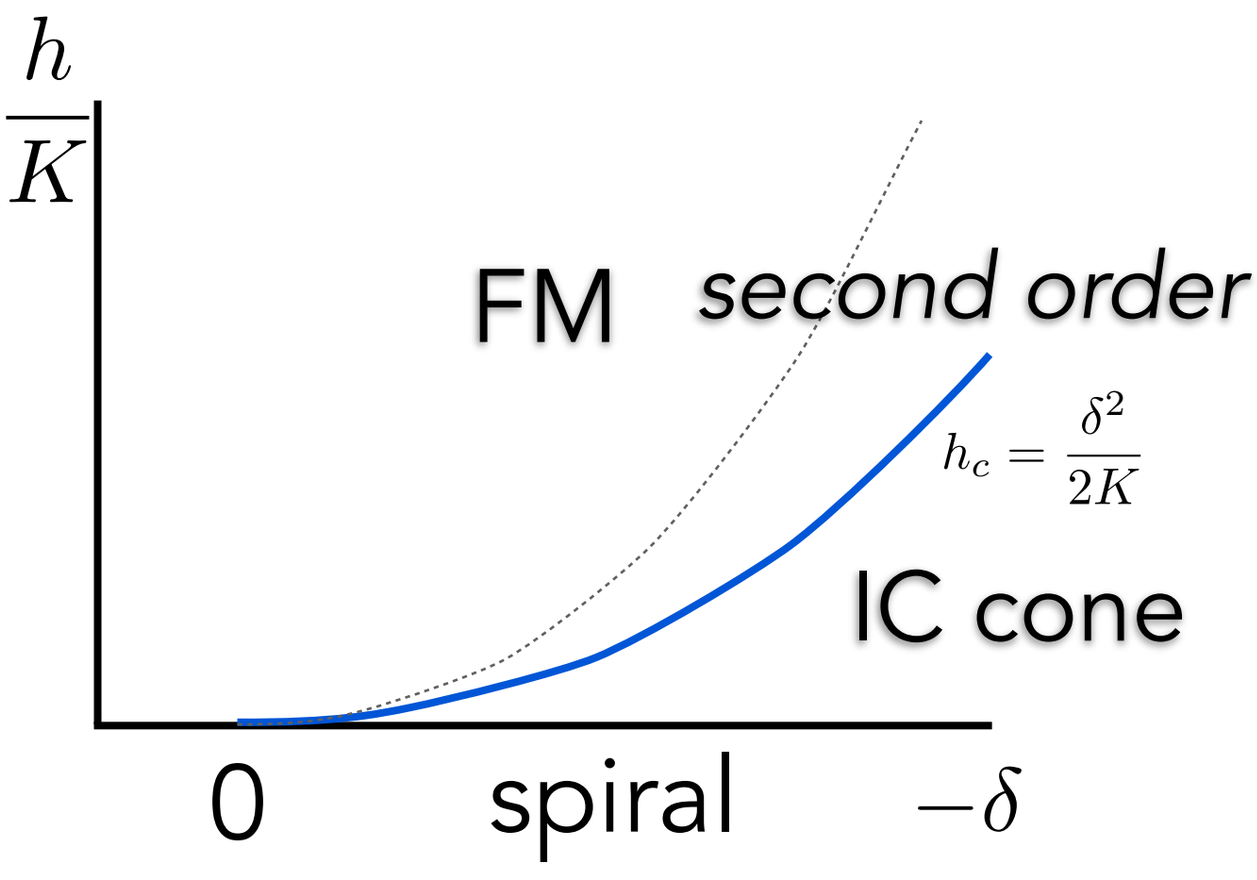
metamagnetism

Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$



$-1 < v < -1/4$



$v > -1/4$

Note: at saddle point level there is no scale for **δ**

Saddle point predicts 1st order transition for S < 6 !

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Resonances in a dilute gas of magnons and metamagnetism of isotropic frustrated ferromagnetic spin chains

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[3]*Institut für Theoretische Physik, Georg-August-Universität Göttingen, D-37077 Göttingen, Germany*
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(Received 22 November 2011; published 14 December 2011)

We show that spin-S chains with SU(2)-symmetric, ferromagnetic nearest-neighbor and frustrating antiferromagnetic next-nearest-neighbor exchange interactions exhibit metamagnetic behavior under the influence of an external magnetic field for small S, in the form of a first-order transition to the fully polarized state. The corresponding magnetization jump increases gradually starting from an S-dependent critical value of exchange couplings and takes a maximum in the vicinity of a ferromagnetic Lifshitz point. The metamagnetism results from resonances in the dilute magnon gas caused by an interplay between quantum fluctuations and frustration.

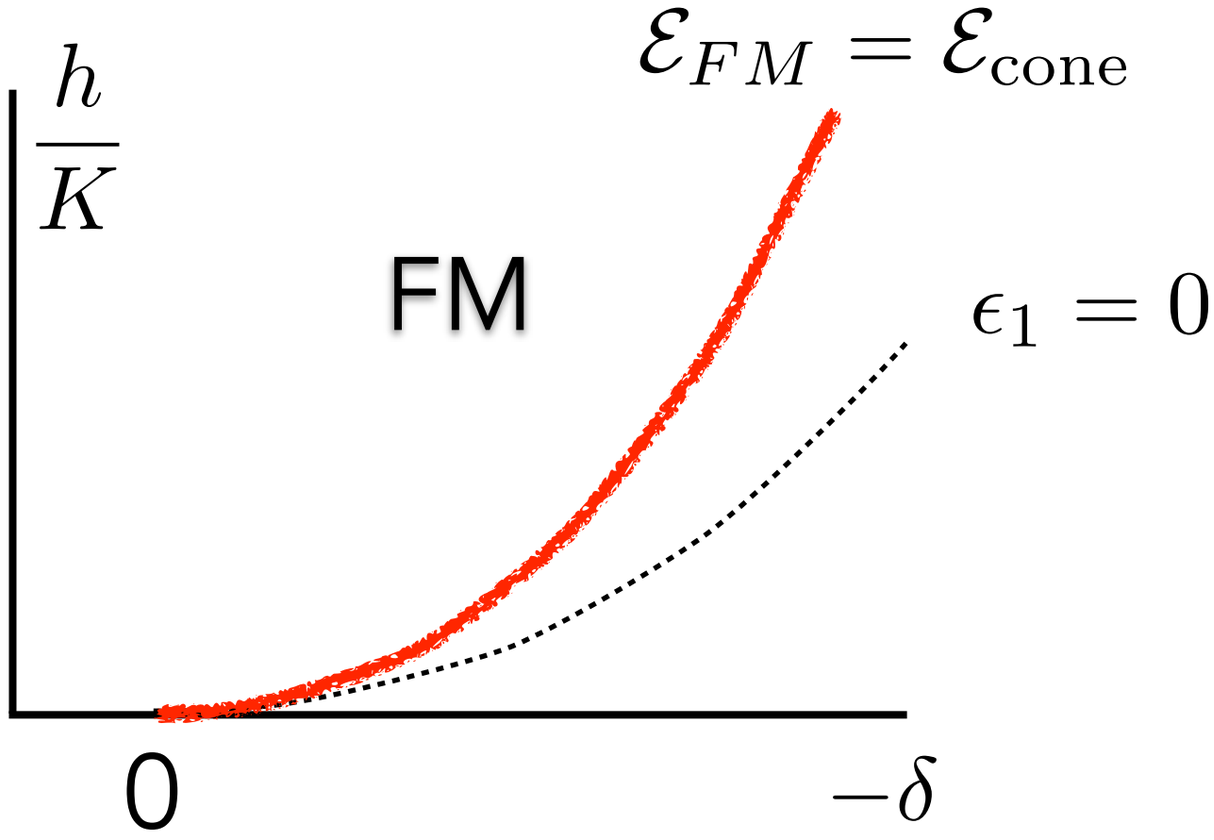
$$v = -\frac{3}{2s}$$ ⇨ find $S_{cr}=6$

PHYSICAL REVIEW B **85**, 064420 (2012)

Frustrated spin chains in strong magnetic field: Dilute two-component Bose gas regime

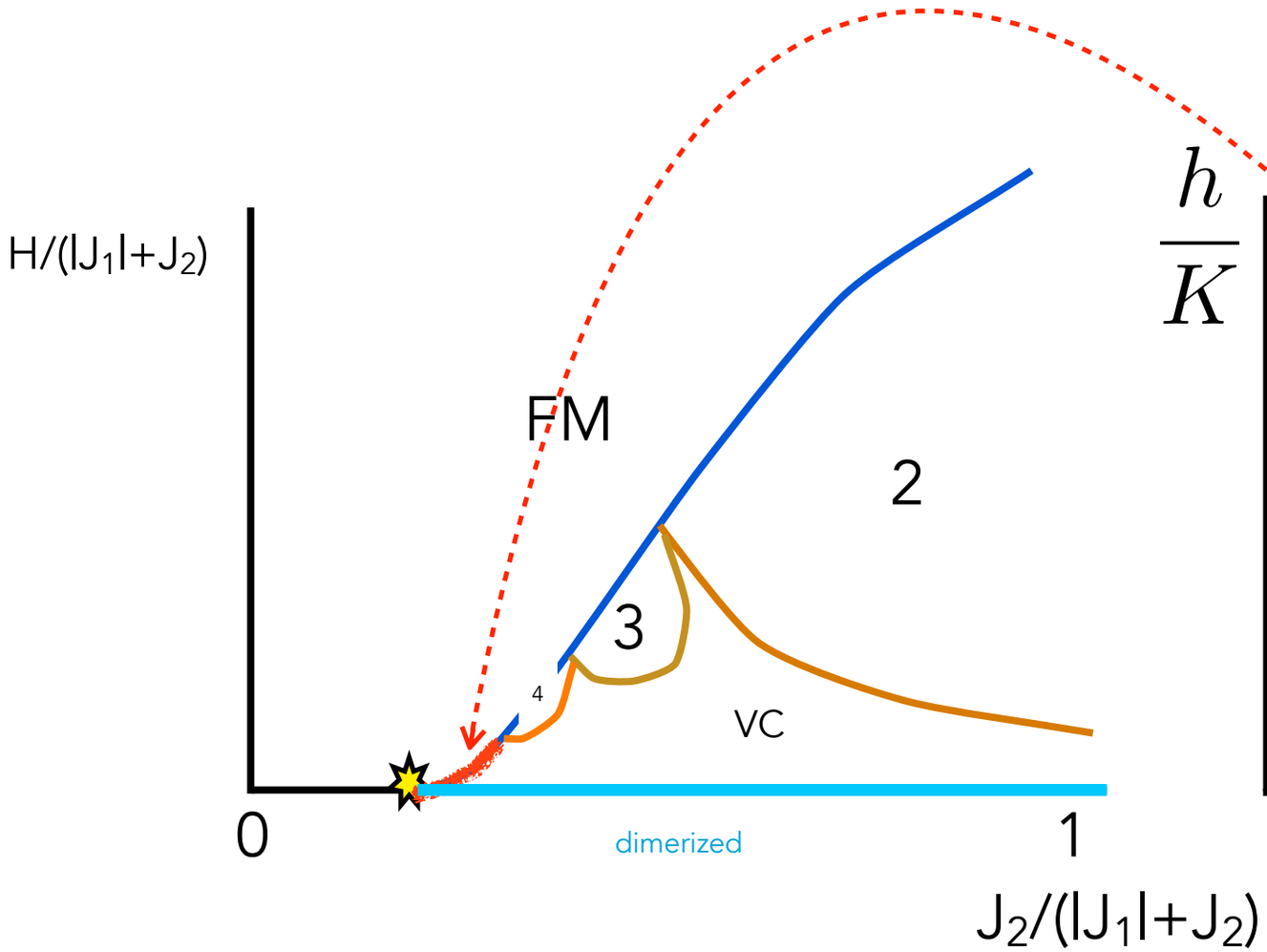
A. K. Kolezhuk,[1,2] F. Heidrich-Meisner,[3] S. Greschner,[4] and T. Vekua[4]
[1]*Institute of Magnetism, National Academy of Sciences and Ministry of Education, 36-b Vernadskii av., 03142 Kiev, Ukraine*
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(Received 24 December 2011; published 28 February 2012)

We study the ground state of frustrated spin-S chains in a strong magnetic field in the immediate vicinity of saturation. In strongly frustrated chains, the magnon dispersion has two degenerate minima at inequivalent momenta $\pm Q$, and just below the saturation field the system can be effectively represented as a dilute one-dimensional lattice gas of two species of bosons that correspond to magnons with momenta around $\pm Q$. We present a theory of effective interactions in such a dilute magnon gas that allows us to make quantitative predictions for arbitrary values of the spin. With the help of this method, we are able to establish the magnetic phase diagram of frustrated chains close to saturation and study phase transitions between several nontrivial states, including a two-component Luttinger liquid, a vector chiral phase, and phases with bound magnons. We study those phase transitions numerically and find a good agreement with our analytical predictions.

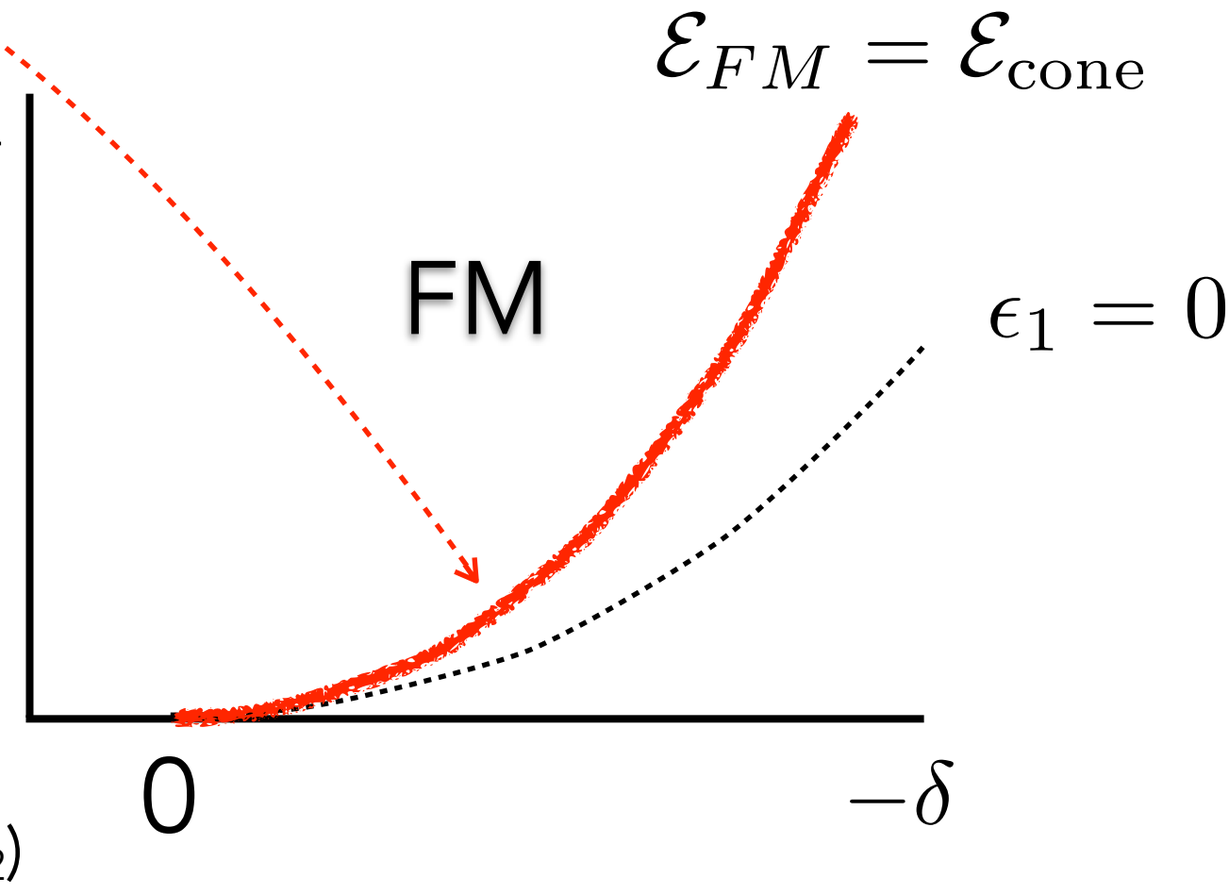


Thus, $S_{\rm cr} = 5$ is the critical value of spin where the metamagnetic behavior vanishes in isotropic chains (for $S > 5$, it exists only in the presence of an easy-axis anisotropy). We would like to note that Ref. 22 reported a slightly different value of $S_{\rm cr} = 6$; this discrepancy is due to the fact that Ref. 22 used just the leading term in the large-S expansion, while our present approach is exact to all orders in $1/S$. Figure 6 illustrates the behavior of $S_{\rm cr}$ as a function of the order of the $1/S$ expansion.

Missing?



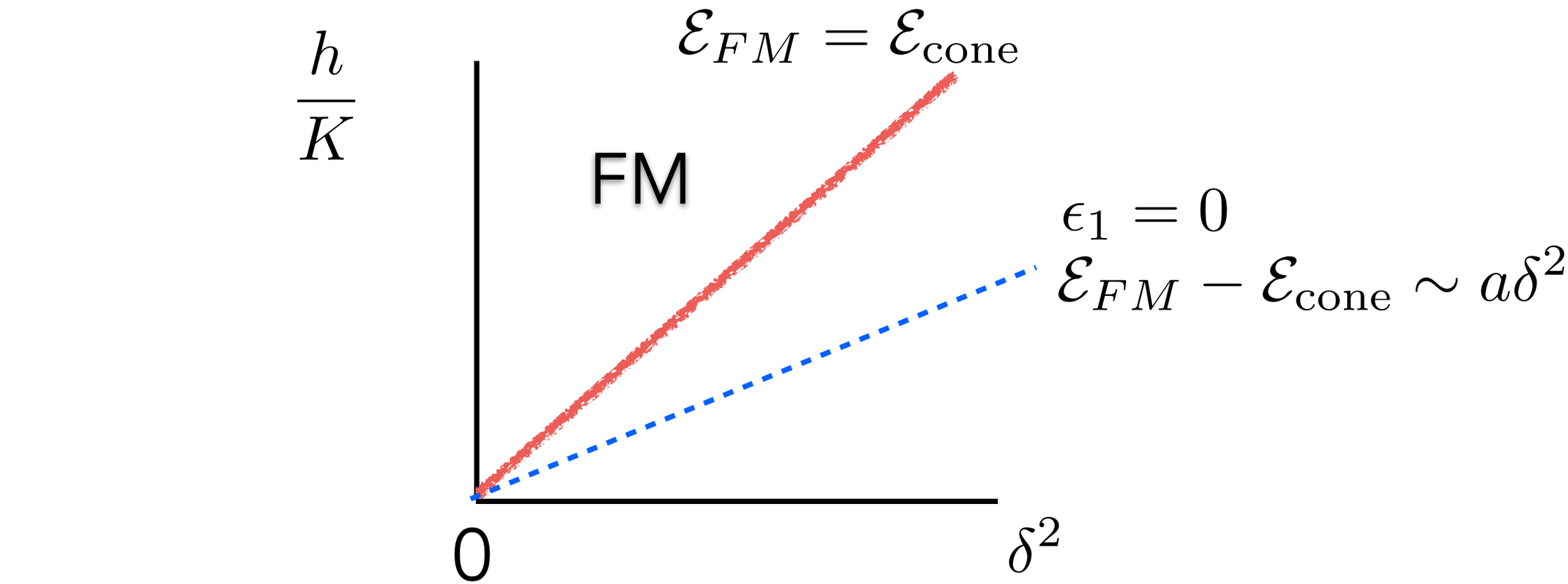
J_1-J_2



saddle point

misses metamagnetic **endpoint**
and multipolar phases

Metamagnetic endpoint?



Quantum corrections penalize E_{cone} but not E_{FM}

$$\sim \mathcal{E} \times \sqrt{\delta/K}$$

$$\Delta\mathcal{E}_{\text{cone}} = +f(v)\delta^{5/2}$$

Quantum corrections

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

transformation to rotating frame $\hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}_{\mathrm{saddle-point}}$

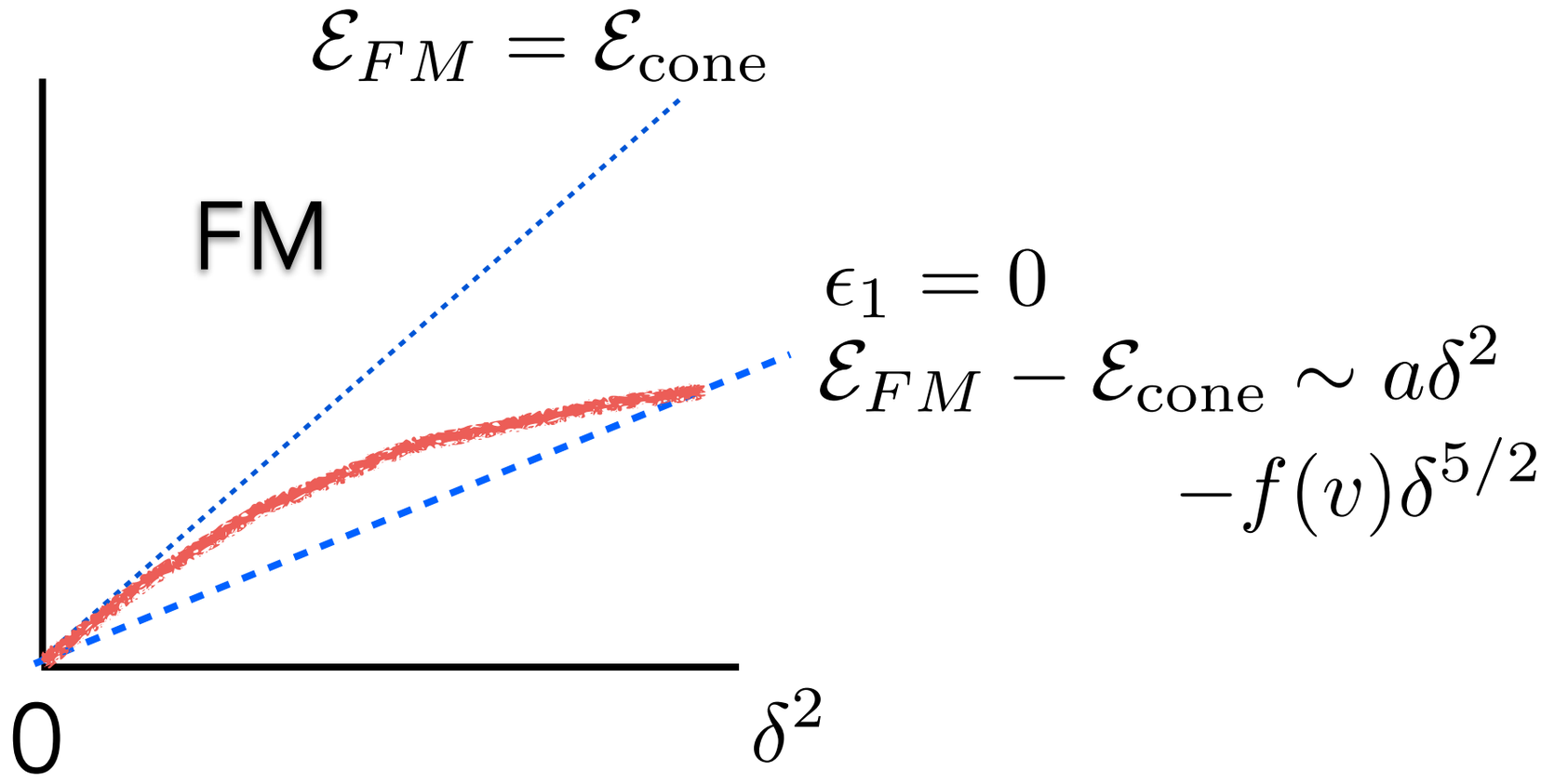
$$\hat{m} = \sqrt{2 - \frac{\bar{\eta}\eta}{s}}[\frac{\bar{\eta} + \eta}{2\sqrt{s}}\hat{e}_1 + i\frac{\bar{\eta} - \eta}{2\sqrt{s}}\hat{e}_2] + (1 - \frac{\bar{\eta}\eta}{s})\hat{e}_3,$$

effective Bogoliubov Hamiltonian

$$S = S_{\mathrm{sp}} + \int dx\, d\tau\ \{\overline{\eta}\partial_\tau \eta + H(\overline{\eta}, \eta)\} \quad + O(\eta^3)$$

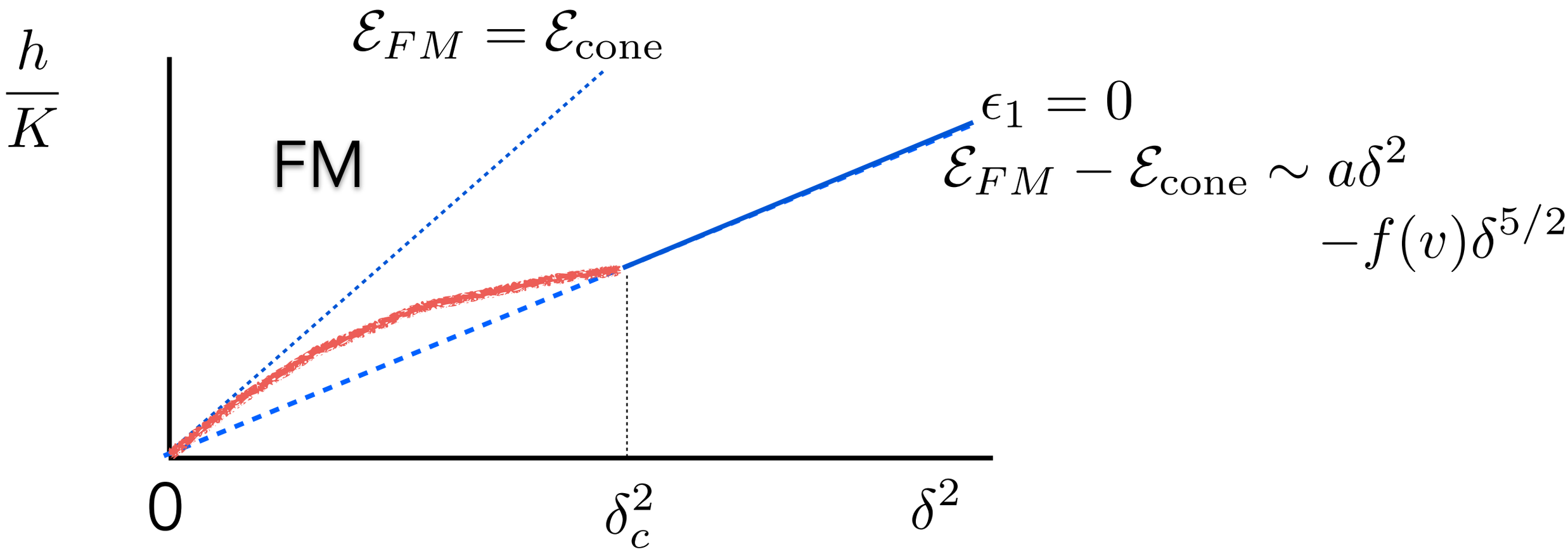
diagonalization gives correction to GS energy

Metamagnetic endpoint?



Corrected first order curve bends slightly downward to intersect second order line

Metamagnetic endpoint?



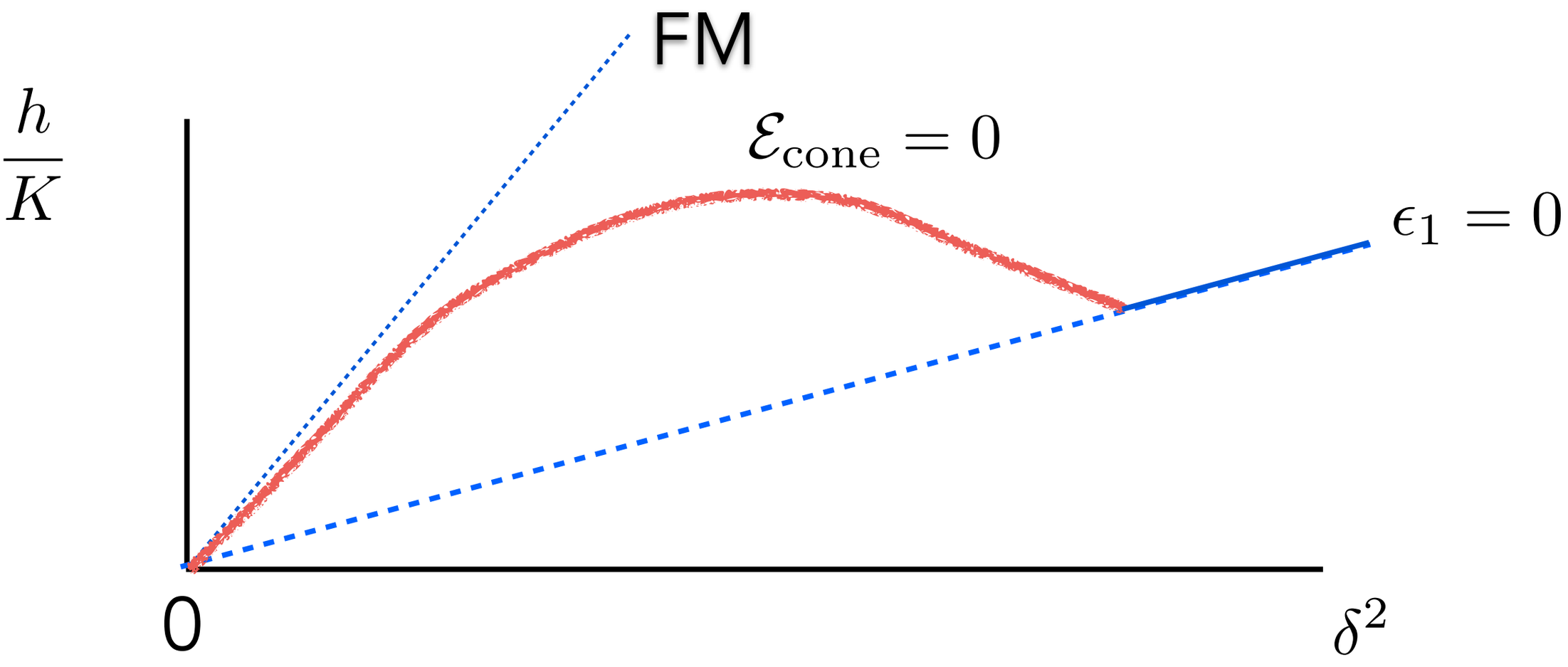
Control? $v = -1/4 - \varepsilon$

$$\mathcal{E}_{FM} - \mathcal{E}_{\rm cone}\big|_{\epsilon_1=0} \sim \varepsilon^3\delta^2 - \varepsilon^2\delta^{5/2}$$

$$\delta_c \sim \varepsilon^2 \ll 1$$

Instabilities

- Choose $E_{FM}=0$



What about multi-particle instabilities?

Low density limit

$$\hat{m}^x + i\hat{m}^y = \left(2 - \overline{\psi}\psi\right)^{1/2} \psi \qquad\qquad \hat{m}^z = 1 - \overline{\psi}\psi$$

Low energy $\psi \sim \psi_1 e^{iqx} + \psi_2 e^{-iqx}$

$$\mathcal{L} \sim \overline{\psi}_a(\partial_\tau + h - \frac{\delta^2}{2K} - 4\delta\partial_x^2)\psi_a$$

$$+\gamma_1[(\overline{\psi}_1\psi_1)^2 + (\overline{\psi}_2\psi_2)^2] + \gamma_2\overline{\psi}_1\psi_1\overline{\psi}_2\psi_2$$

$$\gamma_1 = \frac{\delta^2}{4K}(1+4v) \qquad\qquad \gamma_2 = \frac{\delta^2}{K}(5+4v)$$

$$\sim -\varepsilon\delta^2 < 0 \qquad\qquad \sim +\delta^2$$

Low density limit

$$H = -4\delta \sum_i \frac{\partial^2}{\partial x_i^2} + 2\gamma_1 \sum_{i<j} \delta(x_i - x_j)$$

$$\gamma_1 \sim -\varepsilon\delta^2 < 0$$

attractive delta-function gas!

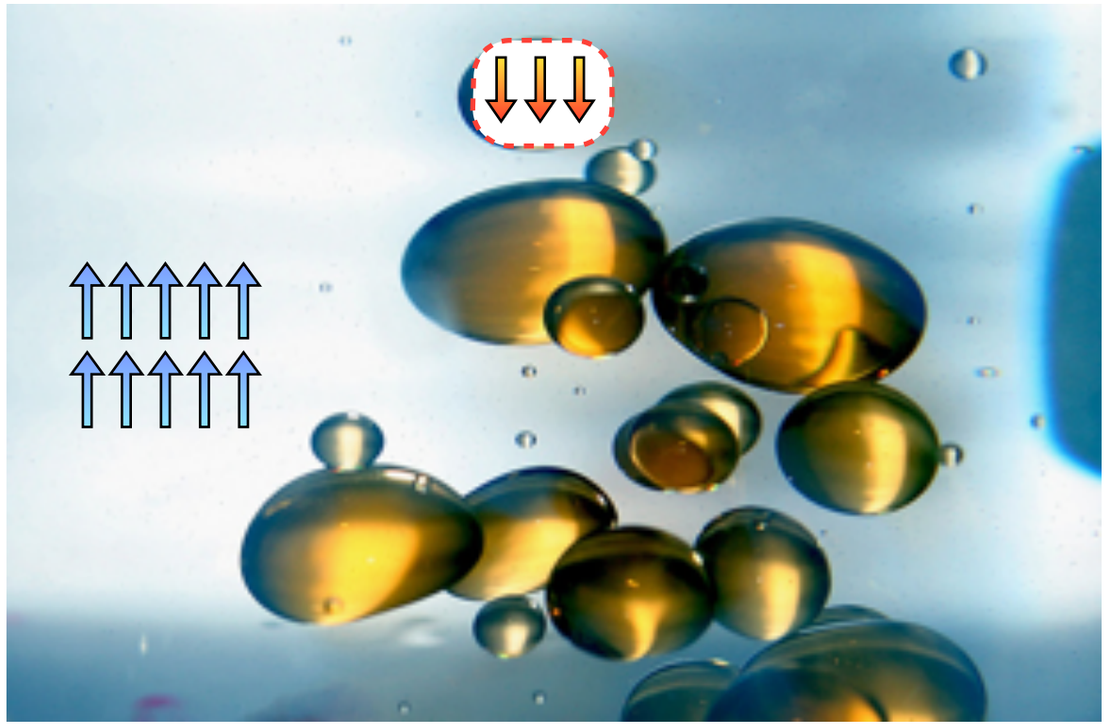
$$\epsilon_n = \epsilon_b \frac{n(n^2-1)}{6} \qquad \epsilon_b = -\frac{\gamma_1^2}{8\delta} = -\frac{\varepsilon^2\delta^3}{8}$$

collapse: bound states have size

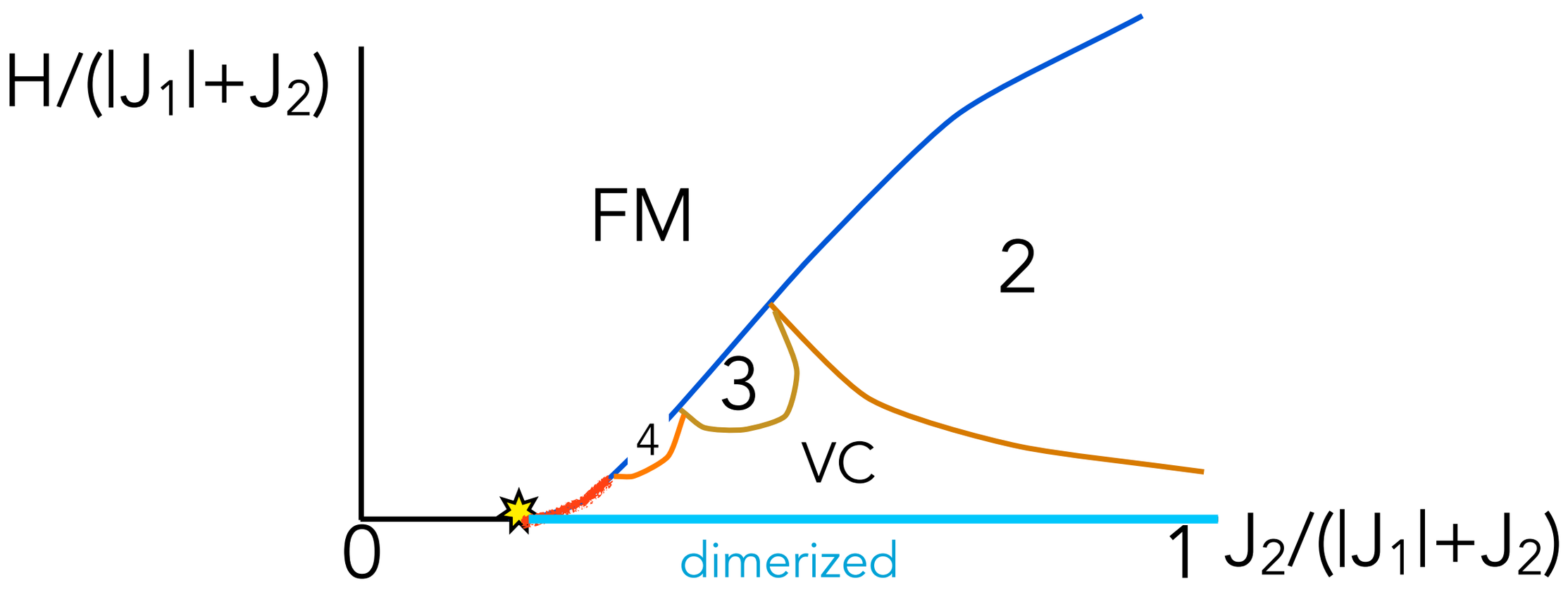
$$\ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon\delta}$$

Why bound states?

- Metamagnetism = first order transition
  - Metastable bubbles of one phase inside the other
- Bubble = multi-magnon bound state



Multipolar phases



expect bubbles of multi-magnon condensates to proliferate near the metamagnetic end-point

A guess

- n magnon bound state

$$\ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon\delta}$$

- Matching to de Broglie wave length

$$\ell_0 \sim \frac{1}{q_0} \sim \frac{1}{\sqrt{\delta}}$$

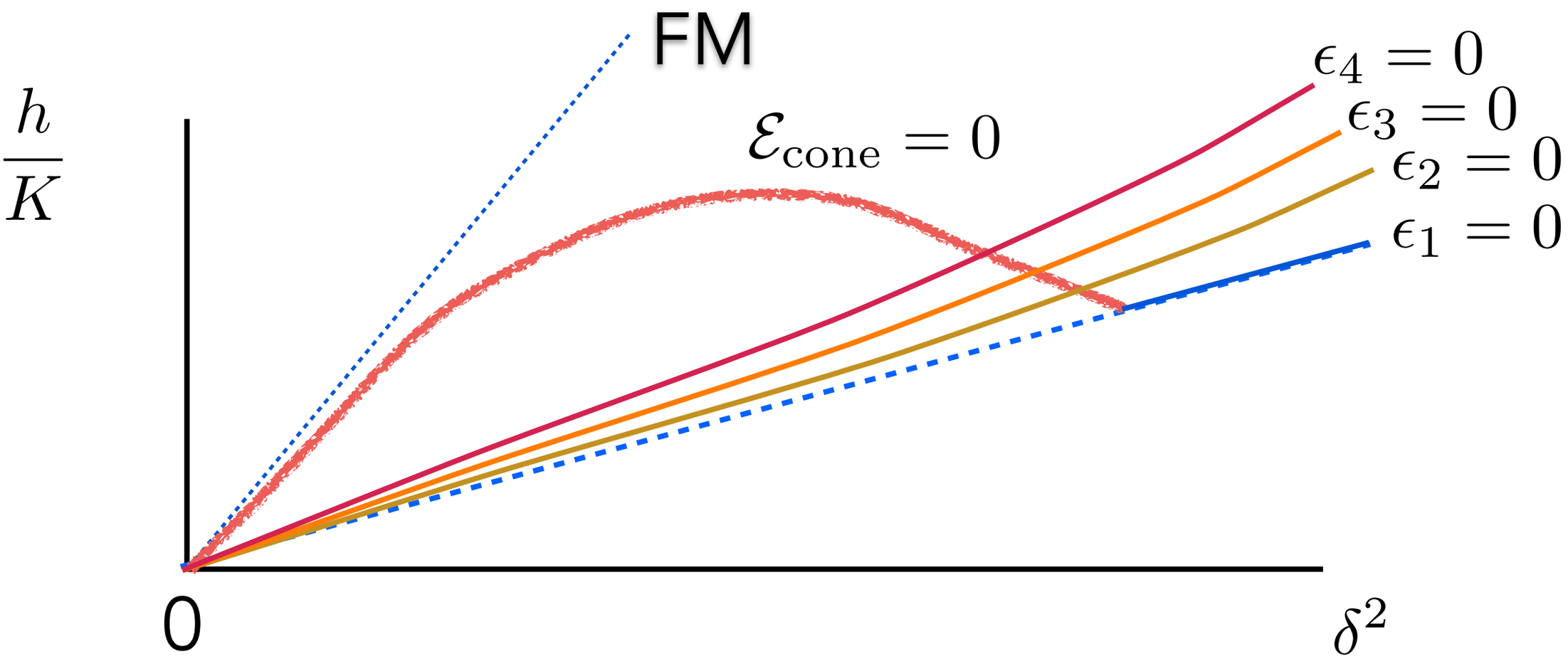
- Suggests maximum bound state

$$n_{\max} \sim \frac{1}{\epsilon\sqrt{\delta}} \sim \frac{1}{\epsilon^2}$$

(at this scale, 3-body interactions enter)

Instabilities

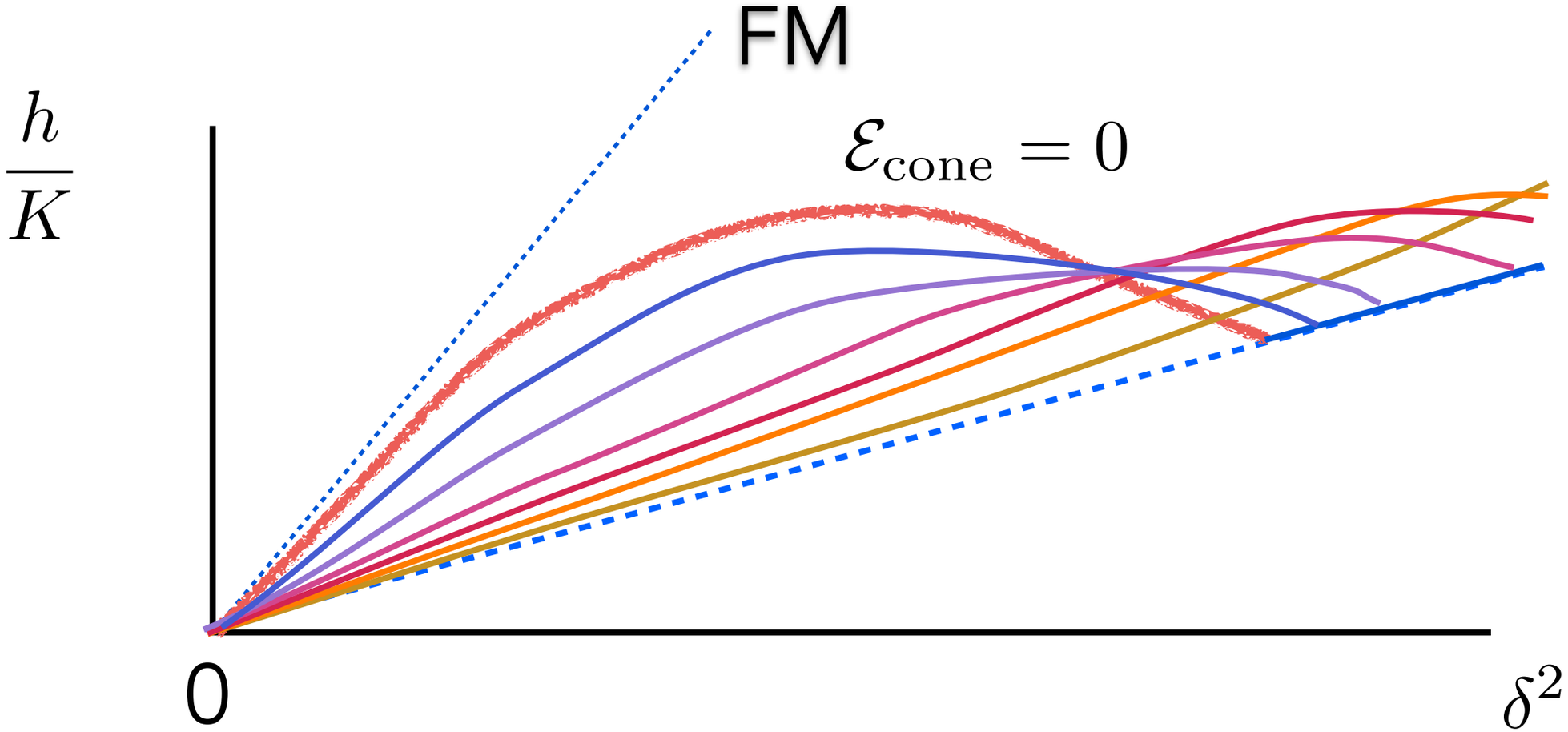
- Choose $E_{FM}=0$



At low density level it appears higher bound state instabilities dominate

Instabilities

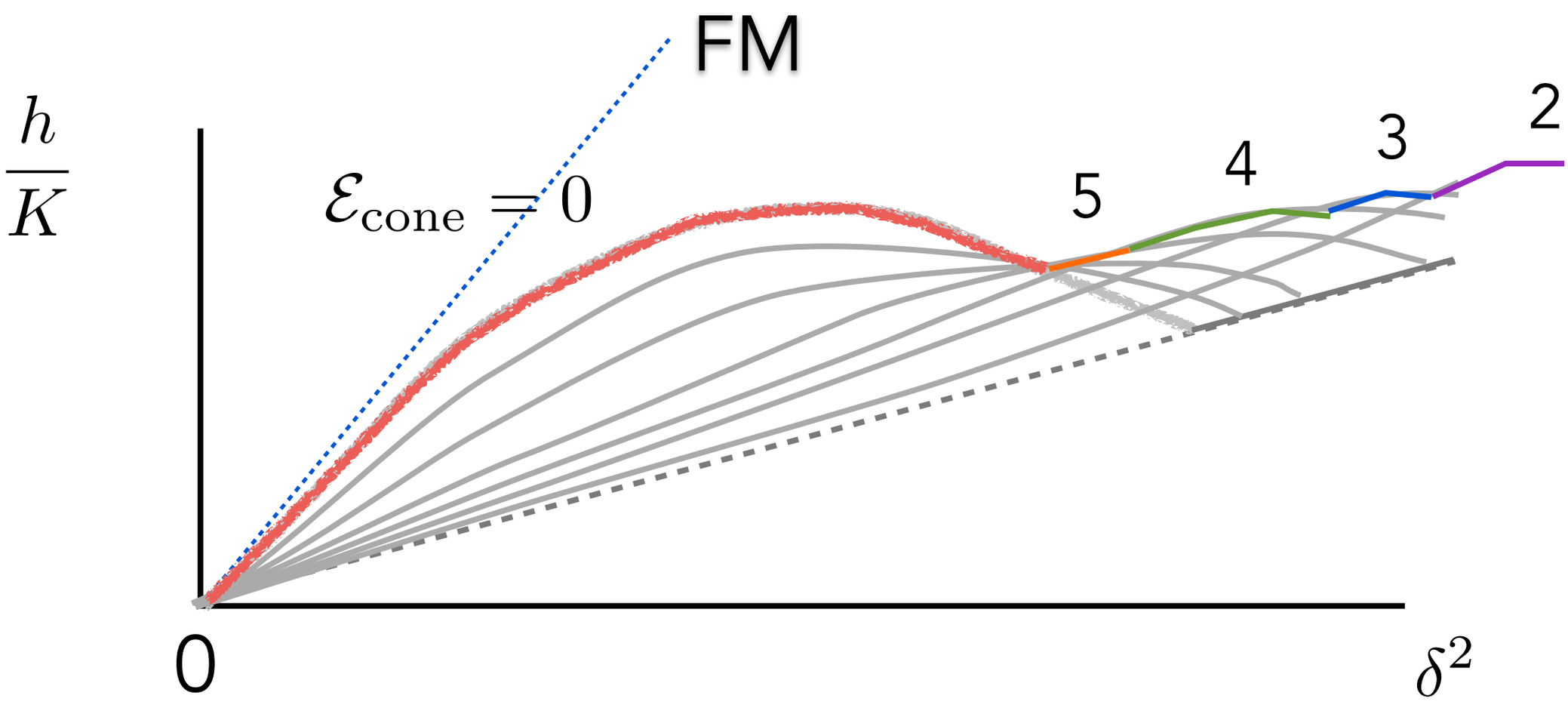
- Choose $E_{FM}=0$



Expect that n-boson bound states bend with increasing n to approach continuum line

Instabilities

- Choose $E_{FM}=0$



Expect that n-boson bound states bend with increasing n to approach continuum line

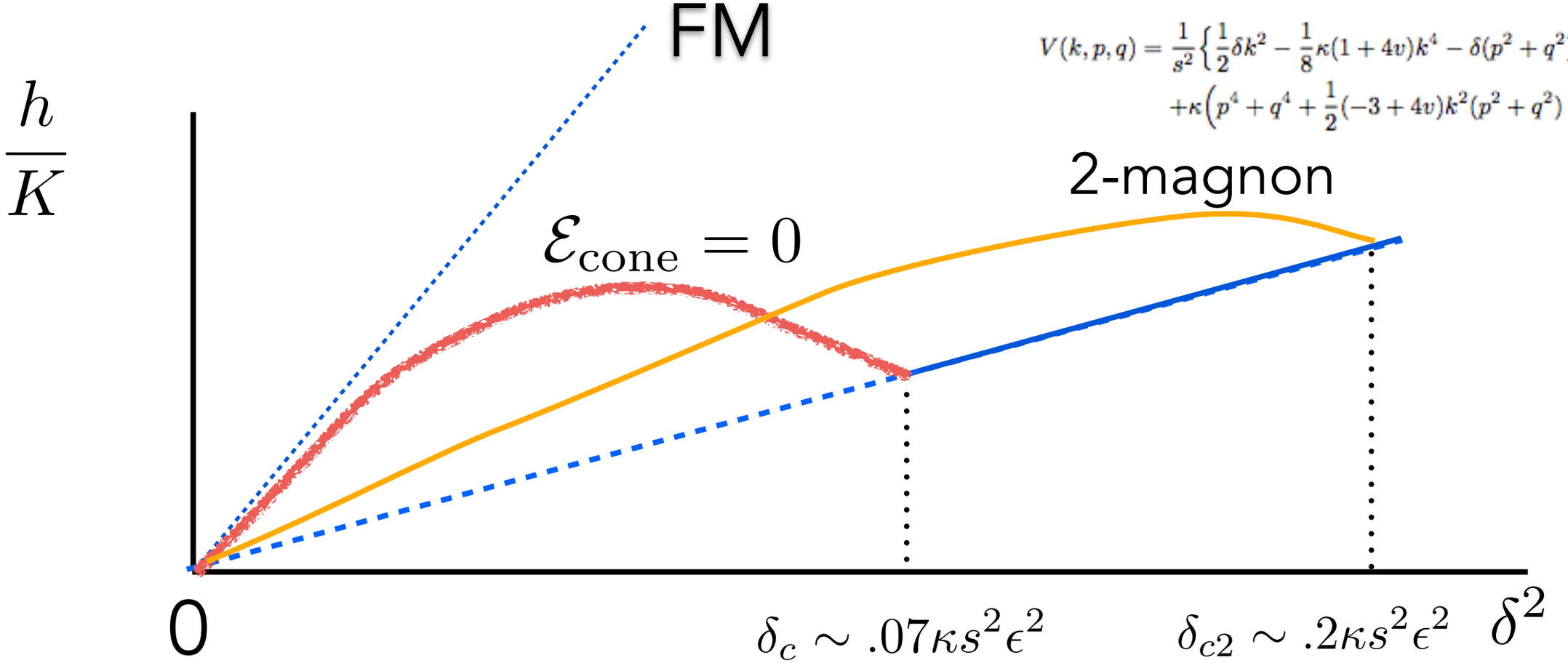
2-magnon check of the proposed scenario

- Compute *exact* 2-magnon energy in QFT

$$H = \sum_k \epsilon_k \bar{\eta}_k \eta_k + \frac{1}{2L} \sum_{kpp'} V(k,p,p') \bar{\eta}_{k/2+p} \bar{\eta}_{k/2-p} \eta_{k/2-p'} \eta_{k/2+p'},$$

$$\epsilon_k = (h + 2\kappa k^4 - 2\delta k^2)/s$$

$$V(k,p,q) = \frac{1}{s^2}\Big\{\frac{1}{2}\delta k^2 - \frac{1}{8}\kappa(1+4v)k^4 - \delta(p^2+q^2) + \\ +\kappa\Big(p^4+q^4+\frac{1}{2}(-3+4v)k^2(p^2+q^2)+4(3-2v)p^2q^2\Big)\Big\}$$



Separation of metamagnetism and multipole formation

d>1

$$S = \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

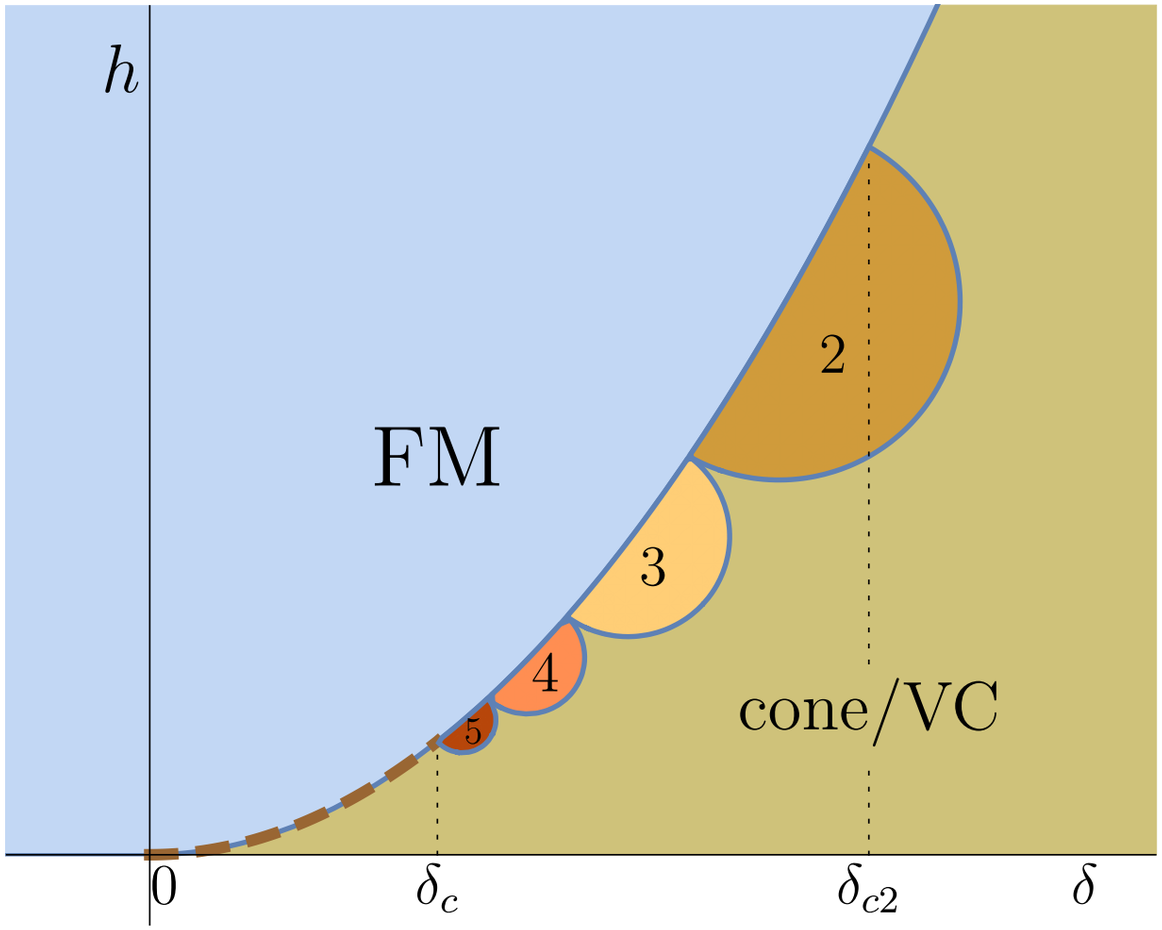
- Rescaling:

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau \qquad y \to \frac{\sqrt{cK}}{\delta} y$$

$$S = \frac{\sqrt{K^d C^{d-1}}}{\delta^{d-1/2}} \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

∴ Similar theory applies in d>1, and very similar conclusions apply

Summary



Lifshitz point is a "parent" of many phases

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

Outline

- Experimental evidence: Frustrated ferromagnets and magnon binding
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

Question

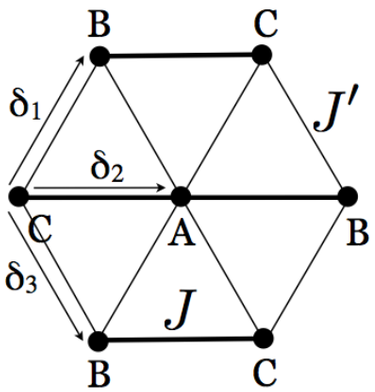
- Is magnon pairing possible in a system with purely repulsive (antiferromagnetic) interactions?

Nematic — superconductor analogy suggests positive answer: Magnon analogue of Kohn-Luttinger mechanism (e.g. pairing due to repulsive interactions)

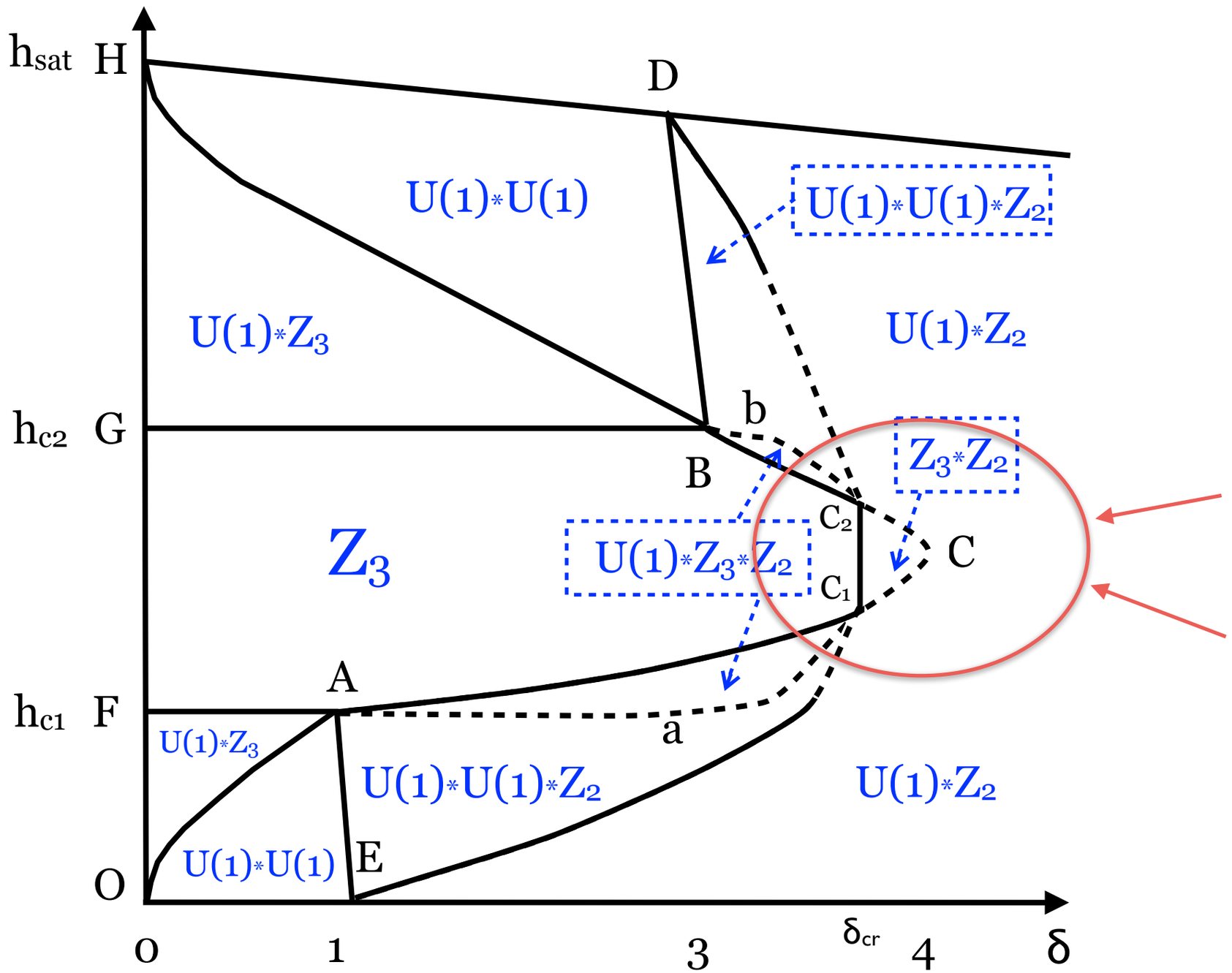
THIS TALK:

Emergent Ising order near the end-point of the 1/3 magnetization plateau

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

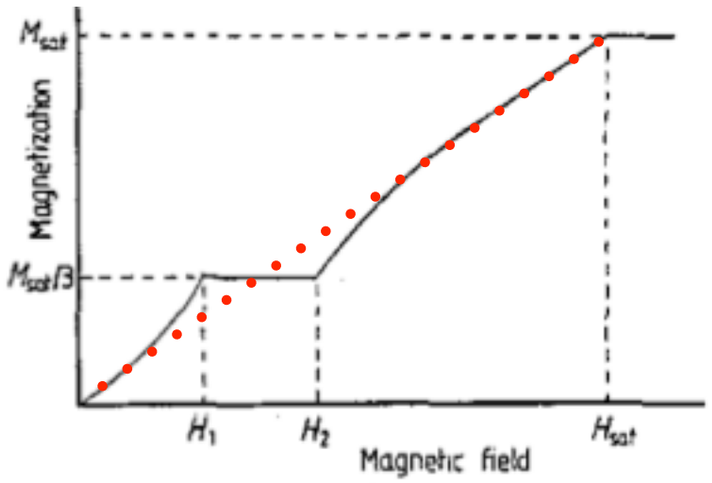


$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$

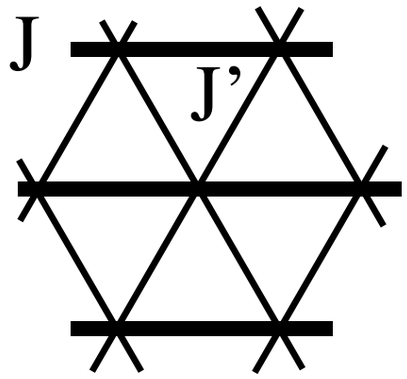


OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

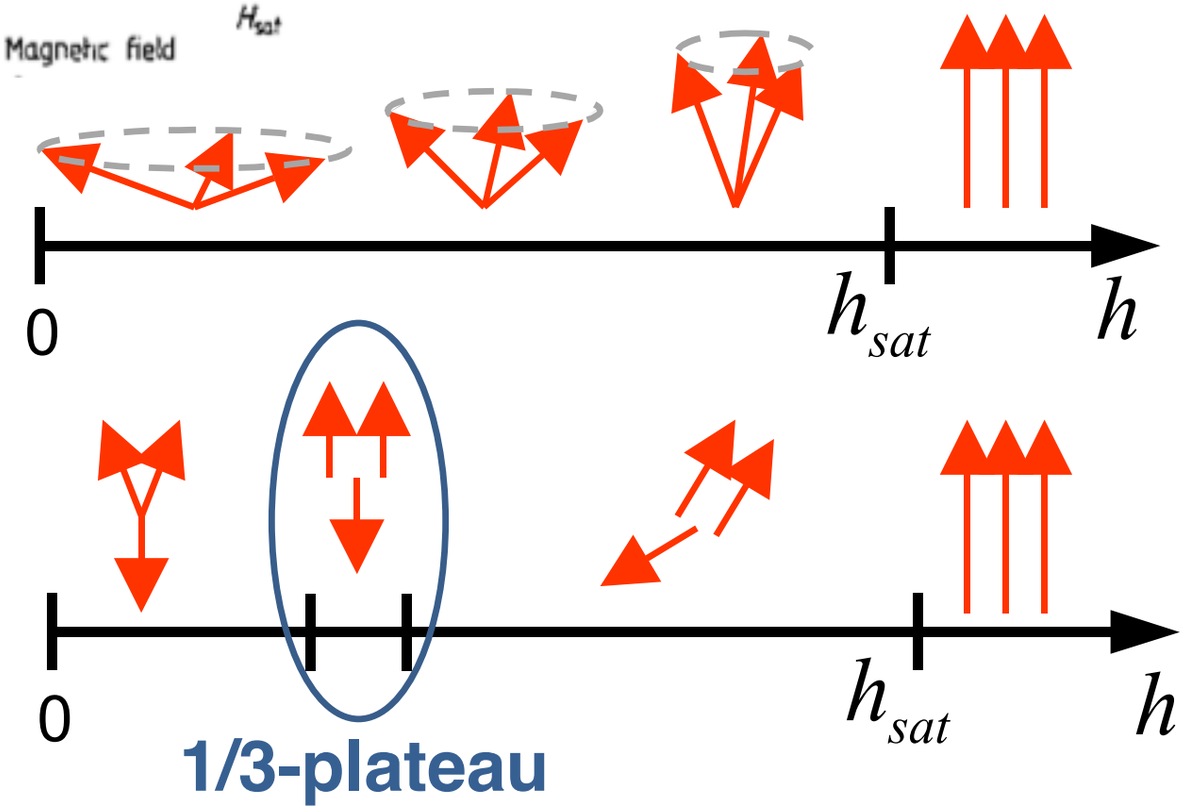
Spatially anisotropic model: classical vs *quantum*



$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$S = \infty$



Umbrella state:
favored classically;
energy gain $(J\text{-}J')^2/J$

$$J' \neq J$$

$$S = \frac{1}{2}$$

Planar states: favored by quantum fluctuations;
energy gain J/S

The competition is controlled by dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

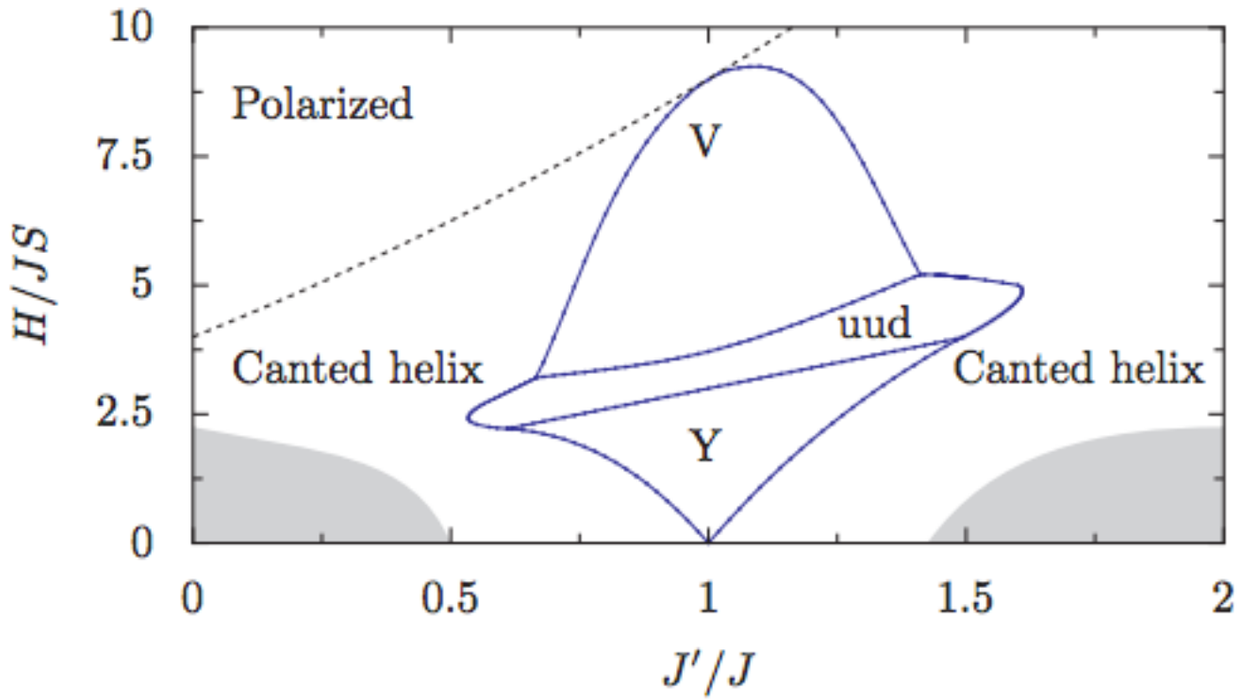


FIG. 4. (Color online) Phase diagram of the spin-1/2 anisotropic triangular lattice in magnetic field. Y and V regions denote three-sublattice planar states. The dashed line is the classical saturation field. The gray shading denotes regions where phases other than the canted helical states may be expected.

PHYSICAL REVIEW B **94**, 075136 (2016)

Semiclassical theory of the magnetization process of the triangular lattice Heisenberg model

Tommaso Coletta,[1] Tamás A. Tóth,[2] Karlo Penc,[3,4] and Frédéric Mila[5]

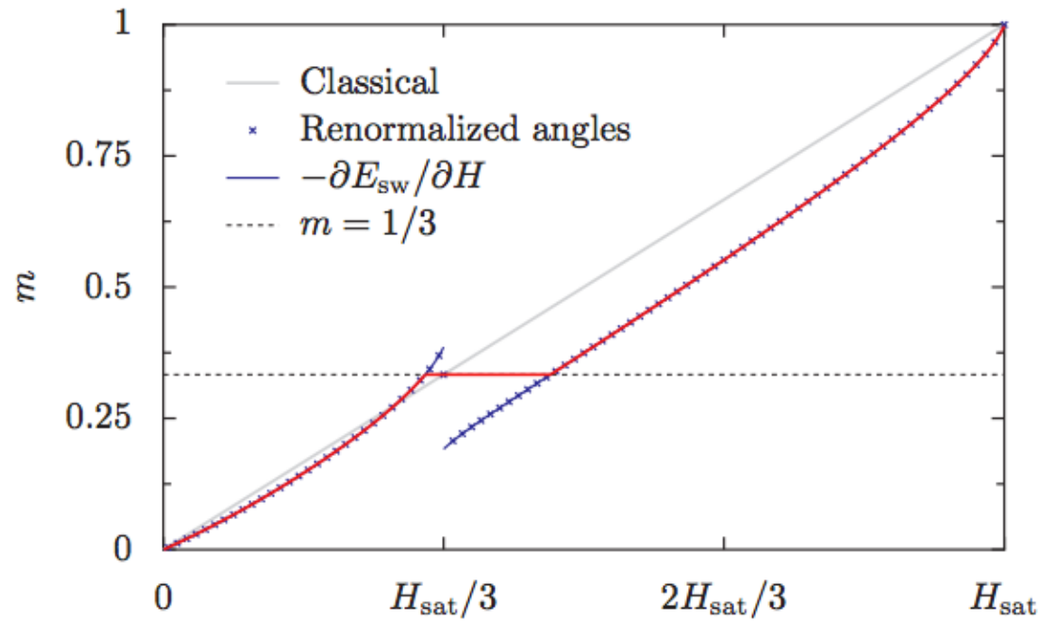
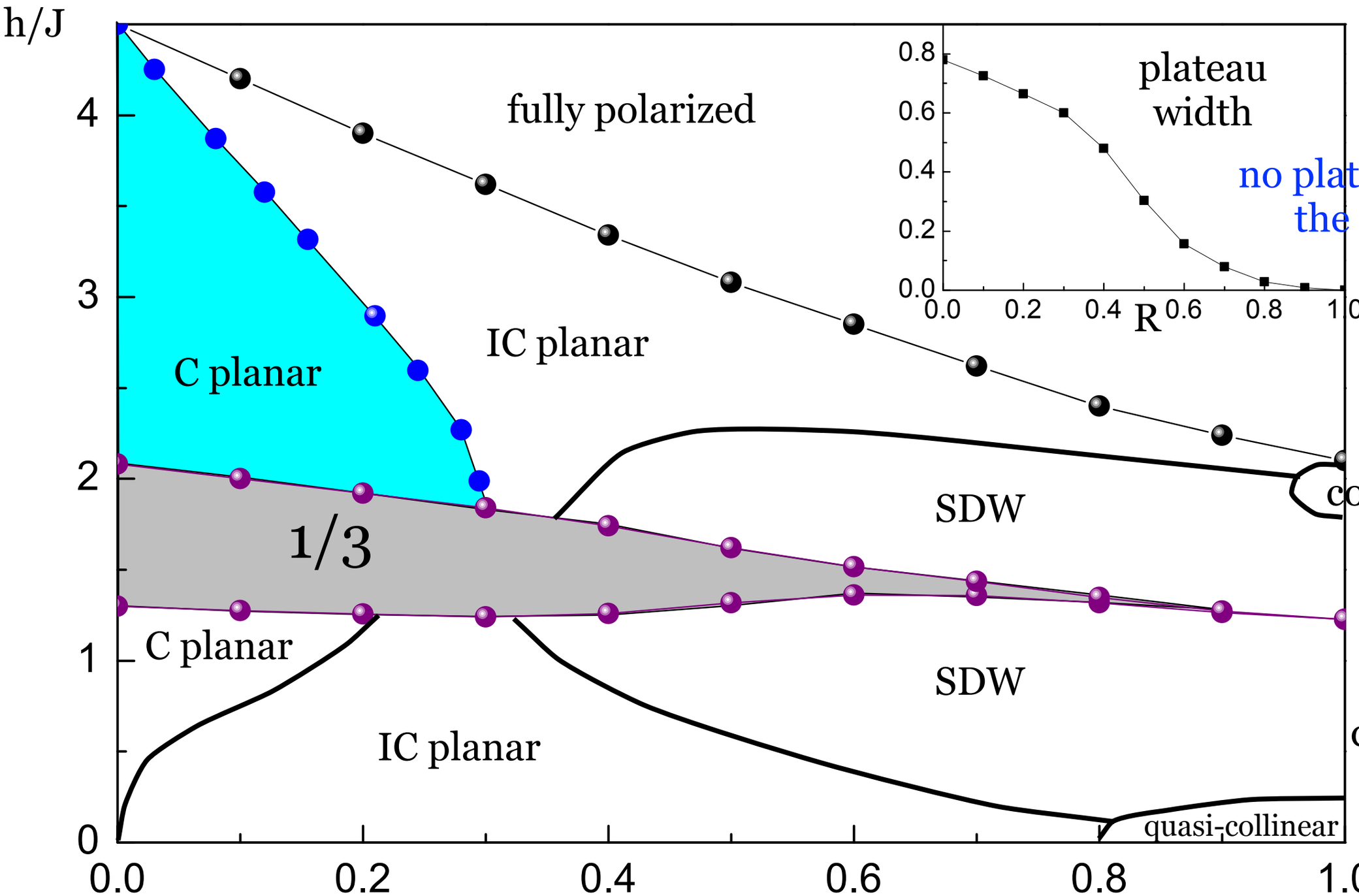


FIG. 3. Plots of the classical magnetization (gray solid line) and of the magnetization including corrections to first order in $1/S$ for $S = 1/2$ (blue curve). The $1/S$ corrections to the magnetization are computed in two equivalent ways: either as the derivative of the energy with respect to the magnetic field (blue curve) or by direct calculation taking into account the renormalization of the spin orientations (crosses). The overall $1/S$ magnetization curve obtained from our phenomenological approach is shown in red.

Side remark: spin-**1/2** triangular lattice AFM *is different*



no plateau end-point in the s=1/2 model!

plateau for all J'/J (crystal of spin-downs; end-point at J'=0)

PHYSICAL REVIEW B **87**, 165123 (2013)

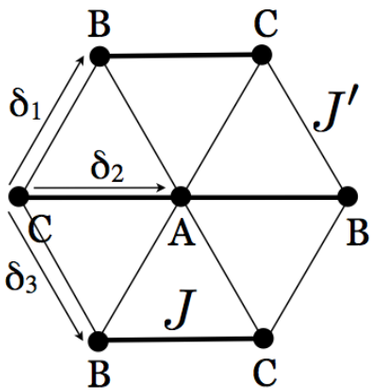
Ground states of spin-$\frac{1}{2}$ triangular antiferromagnets in a magnetic field

Ru Chen,[1] Hyejin Ju,[1] Hong-Chen Jiang,[2] Oleg A. Starykh,[3] and Leon Balents[2]
[1]*Department of Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA*
[2]*Kavli Institute of Theoretical Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA*
[3]*Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA*

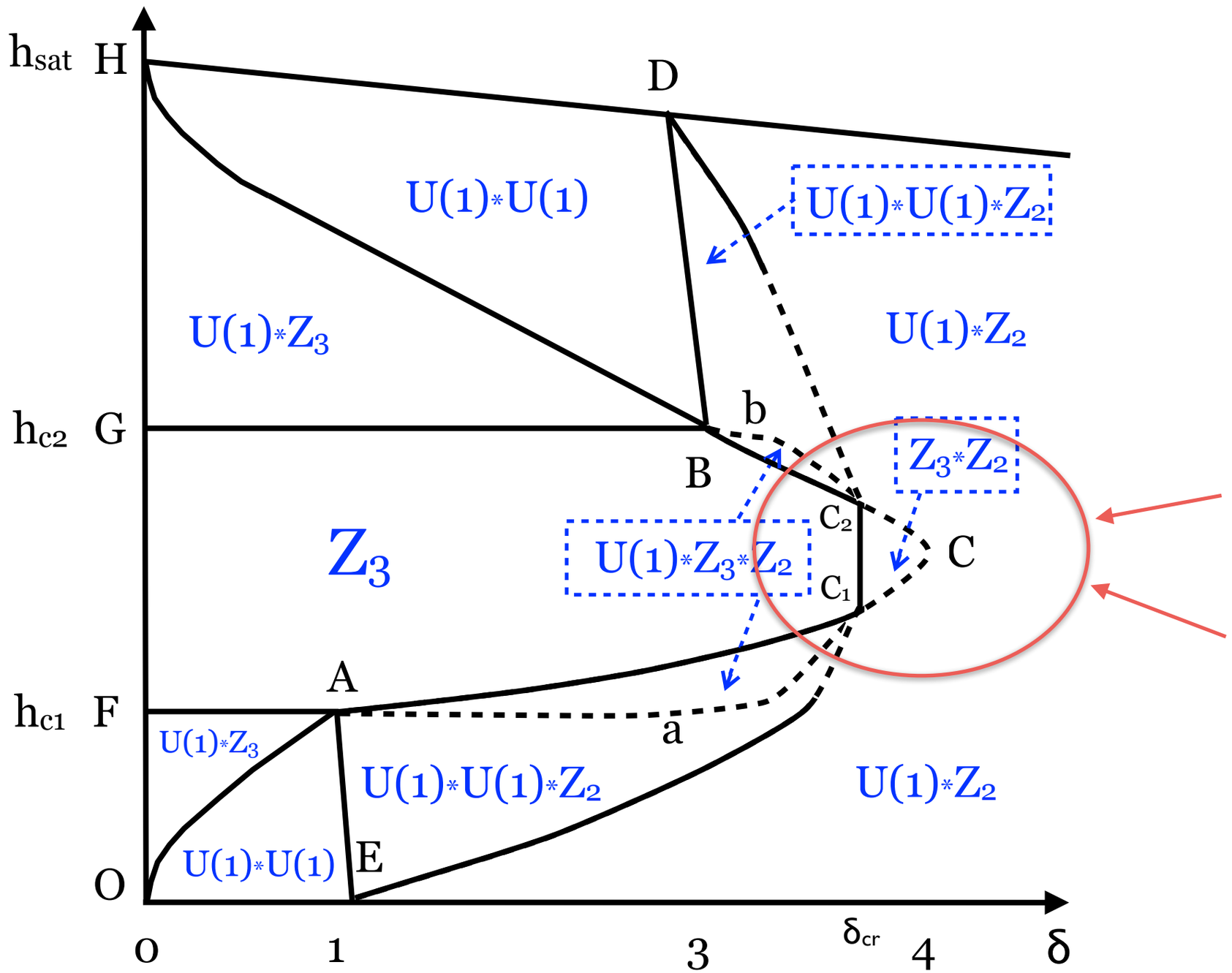
THIS TALK:

Emergent Ising order near the end-point of the 1/3 magnetization plateau

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



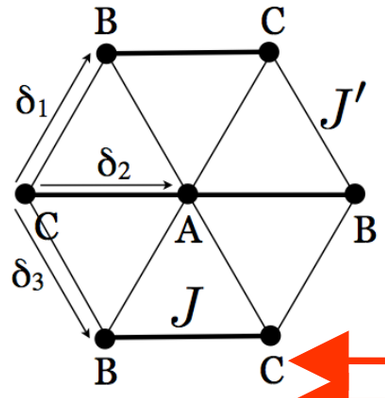
$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at T=0

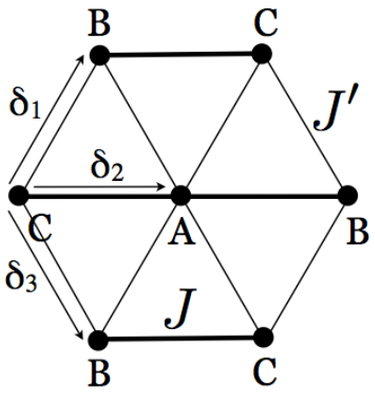
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



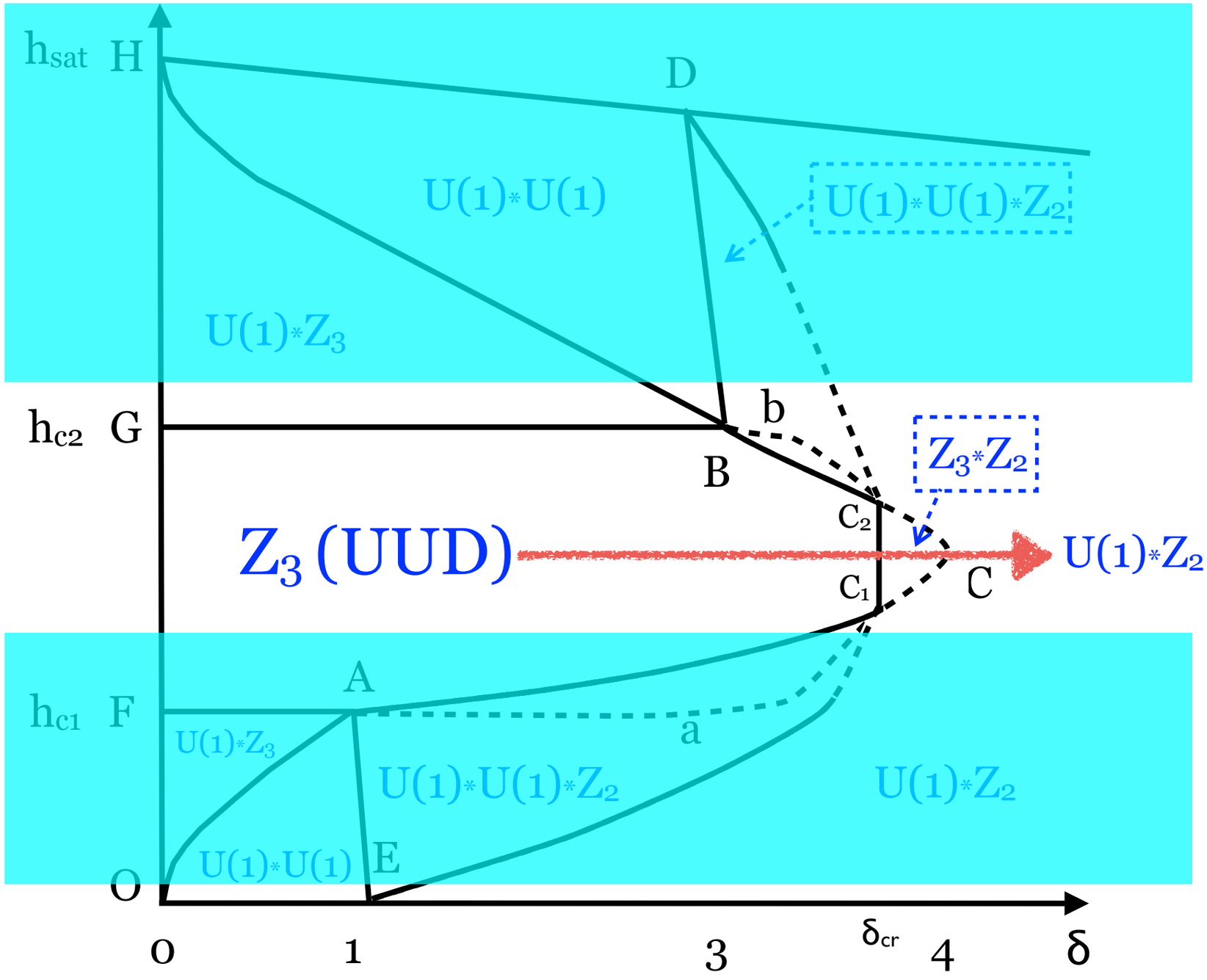
UUD-to-cone phase transition

$$Z_3 \to U(1) \times Z_2 \text{ or } Z_3 \to \text{smth else} \to U(1) \times Z_2 ?$$

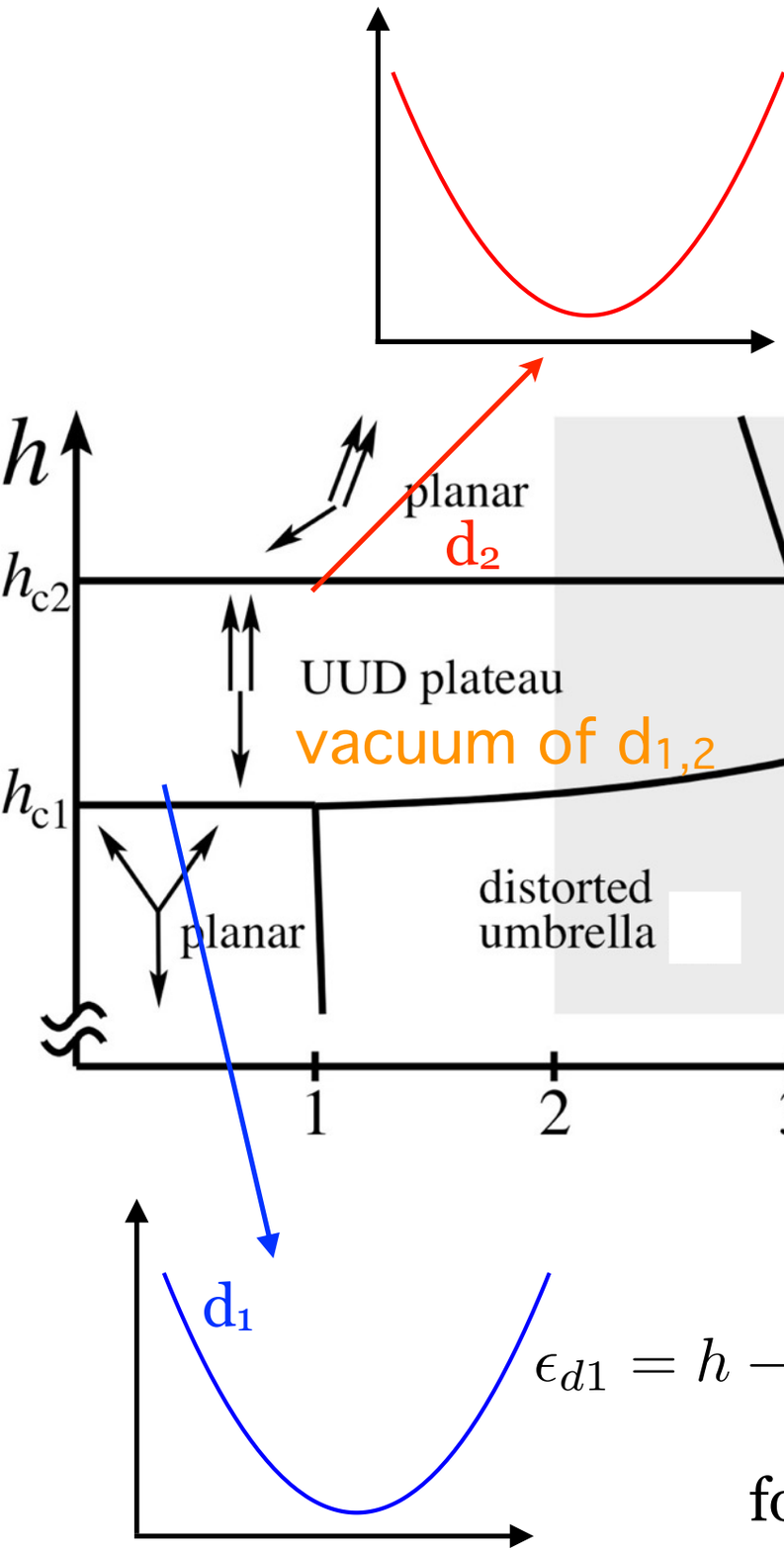
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



Low-energy excitation spectra



$$\epsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4}$$

for δ < 3

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3. Hence, ***very stable***.

$$h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\rm sat} = \frac{0.6}{2S}(9JS)$$

$$\delta = \frac{40}{3} S (1 - J'/J)^2$$

$$\epsilon_{d1} = h - h_{c1} + \frac{3Jk^2}{4}$$

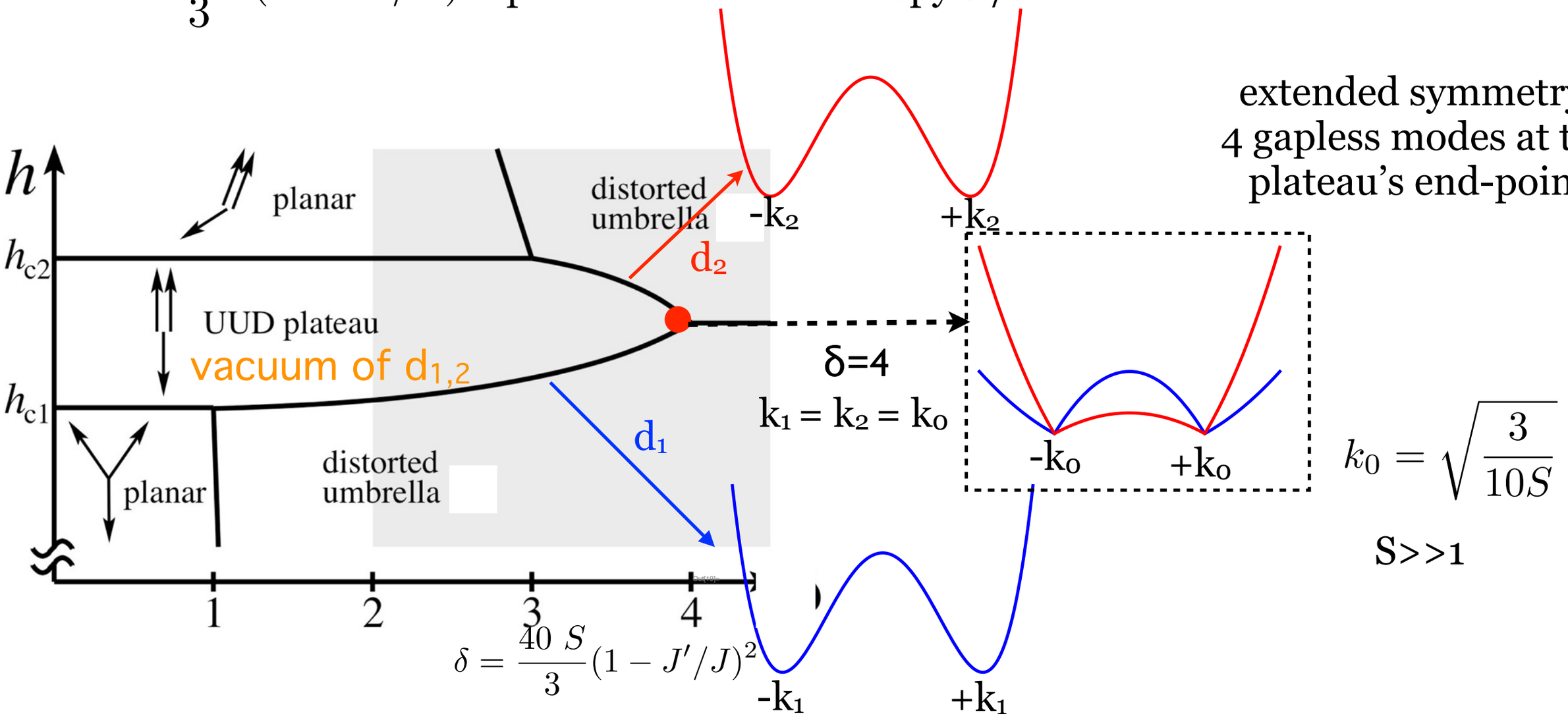
for δ < 1

Bose-Einstein condensation of d_1 (d_2) mode at k =0 leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009

Low-energy excitation spectra near the plateau's end-point

$\delta = \frac{40\ S}{3}(1 - J'/J)^2$ parameterizes anisotropy J'/J



extended symmetry:
4 gapless modes at the plateau's end-point

$k_0 = \sqrt{\frac{3}{10S}}$

S>>1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3.

Alicea, Chubukov, OS PRL 2009

Bosonization of 2d interacting magnons

$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\Big(d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}\Big) + \text{h.c.}$$

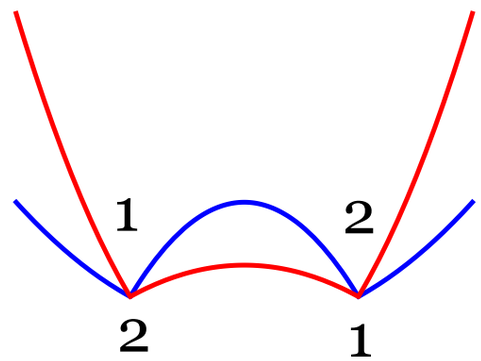
$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

singular magnon interaction

$$\Psi^\dagger_{1,p}\Psi_{2,q} \qquad \Psi^\dagger_{1,p}\Psi^\dagger_{2,q}$$

magnon **pair operators**

$$\begin{cases}\Psi_{1,p} = d_{1,k_0+p}d_{2,-k_0-p}\\ \Psi_{2,p} = d_{1,-k_0+p}d_{2,k_0-p}\end{cases}$$



Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}},\Psi_{2,\mathbf{q}}] = \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}\Big(1 + d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d_{1,\mathbf{k}_0+\mathbf{p}} + d^\dagger_{2,\mathbf{k}_0+\mathbf{p}}d_{2,\mathbf{k}_0+\mathbf{p}}\Big) \to \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state $\langle d^\dagger_1 d_1\rangle_{\text{uud}} = \langle d^\dagger_2 d_2\rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of $\mathbf{d}_{1,2}$ bosons = non-interacting Hamiltonian in terms of $\Psi_{1,2}$ magnon pairs

Chubukov, OS PRL 2013

Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for Ψ - Hamiltonian

$$\langle \Psi^\dagger_{1,\mathbf{p}} - \Psi_{1,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{2,\mathbf{q}} - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi^\dagger_{2,\mathbf{p}} - \Psi_{2,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{1,\mathbf{q}} - \Psi_{1,\mathbf{q}} \rangle$$

`Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

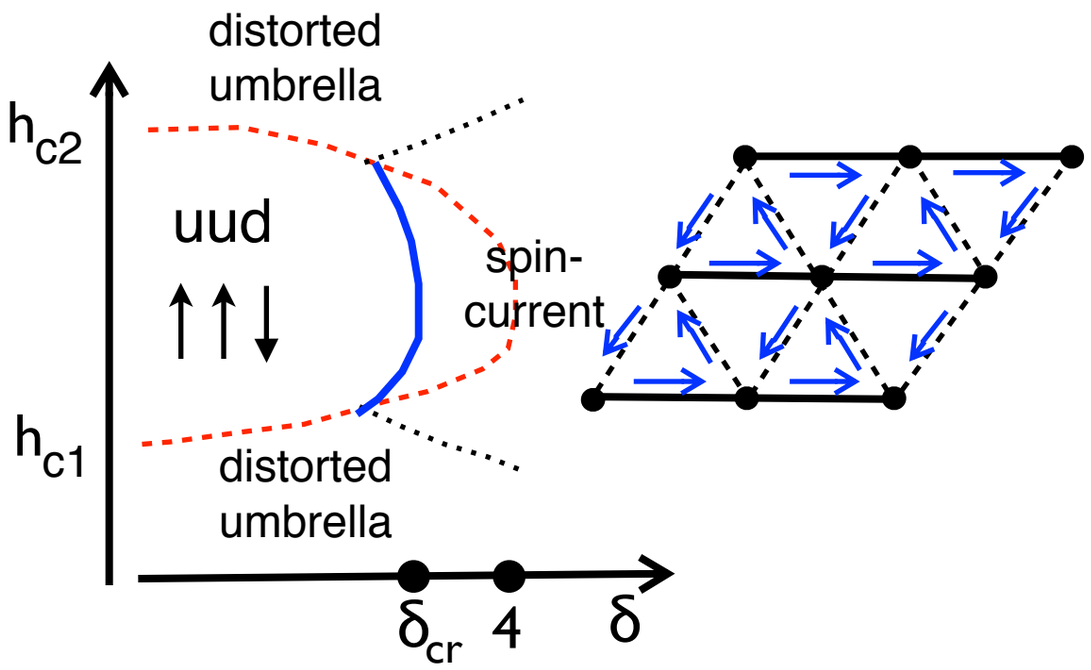
$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1-\delta/4)k_0^2}}$$

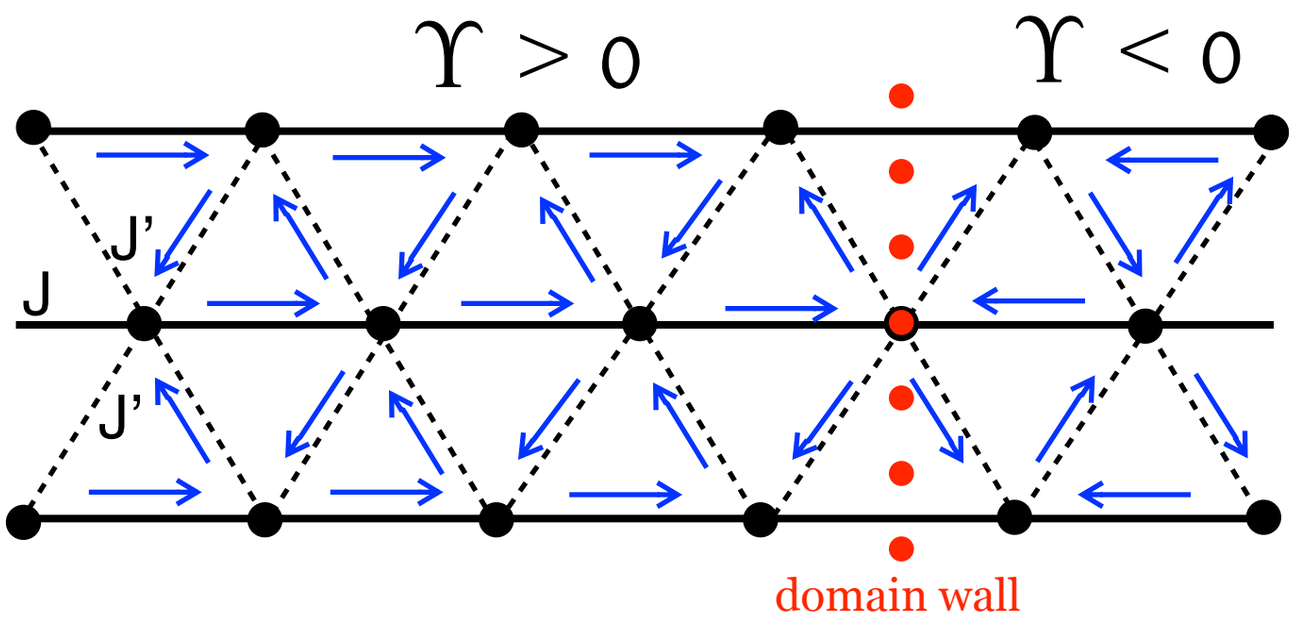
no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

Chubukov, OS PRL 2013

Two-magnon condensate = Spin-current nematic state





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

Finite scalar (and vector) chiralities. Sign of Υ determines sense of spin-current circulation

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

Spontaneously broken Z_2 -- spatial inversion [in addition to broken Z_3 inherited from the UUD state]

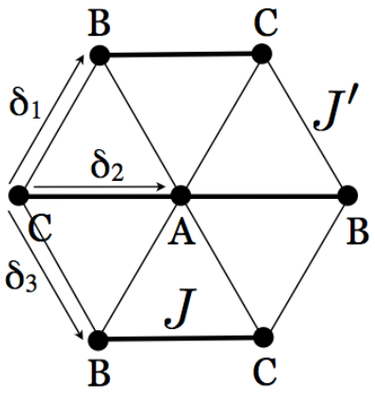
Leads to **spontaneous generation of Dzyaloshisnkii-Moriya interaction**

Chubukov, OS PRL 2013

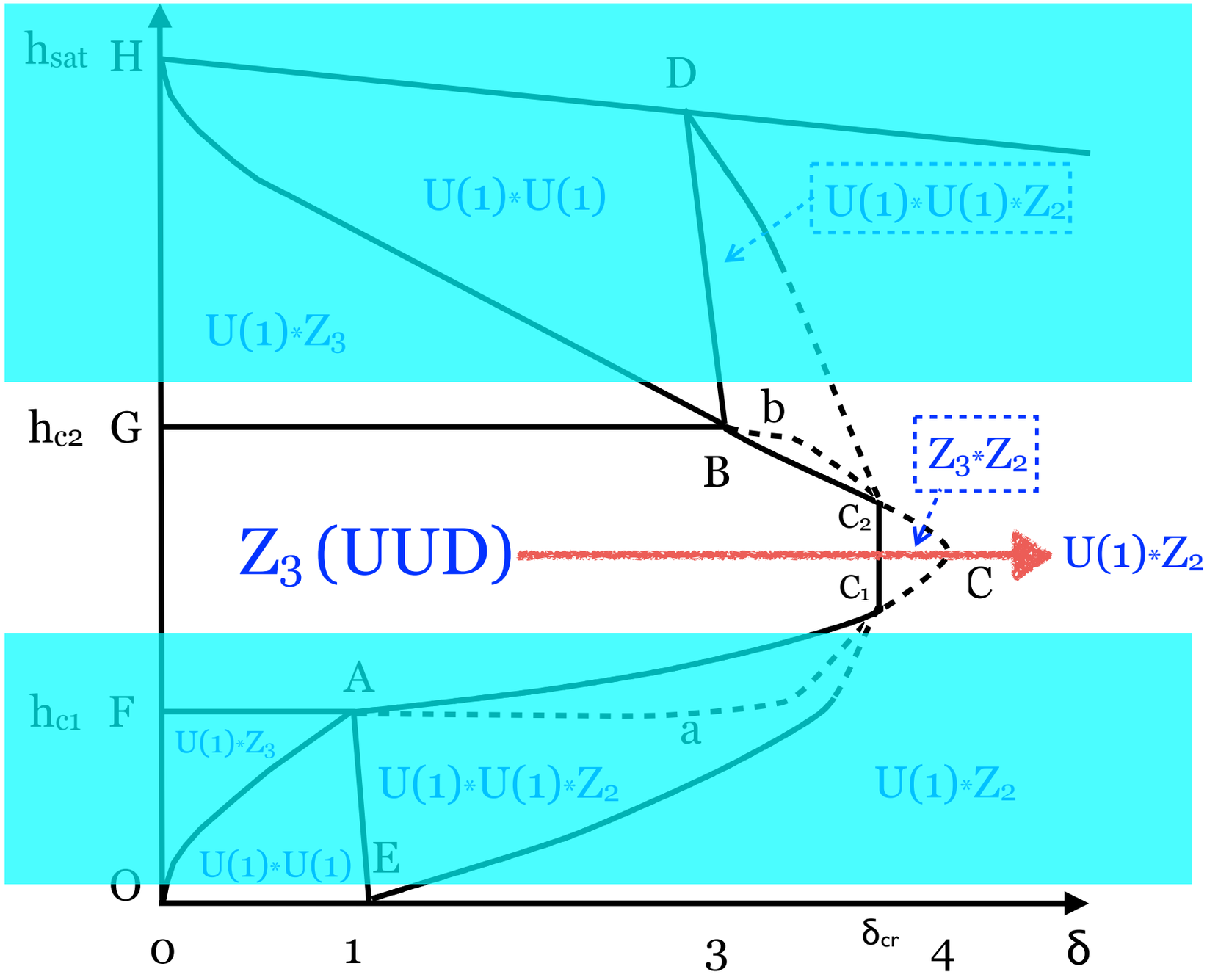
Continuous transition: plateau —> spin-current —> cone !

$$Z_3 \to Z_3 \times Z_2 \to U(1) \times Z_2$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan,[1] Alexander O. Gogolin,[2] and Fabian H. L. Eßler[3]

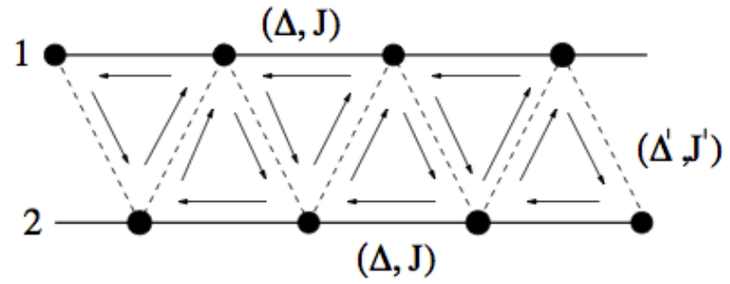


FIG. 3. Structure of the spin currents in the spin nematic phase.

PHYSICAL REVIEW B **87**, 174501 (2013)

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar,[1] Tapan Mishra,[2] Maheswar Maji,[3] R. V. Pai,[4] Subroto Mukerjee,[3,5] and Arun Paramekanti[2,3,6,7]

Spin-current phase = chiral Mott insulator

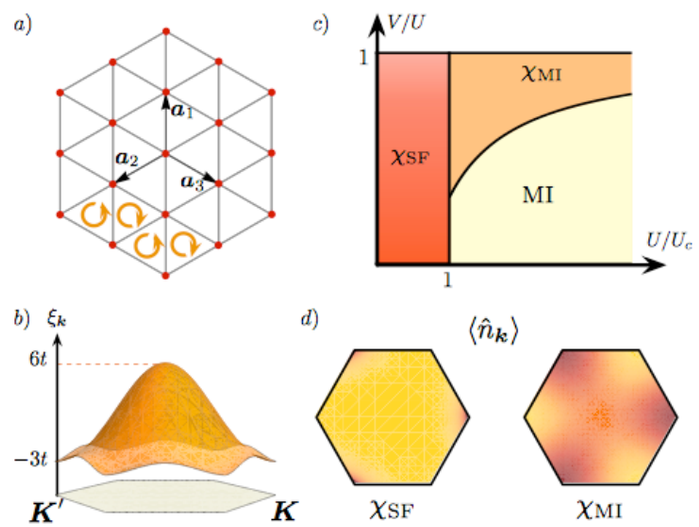


FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion $\xi_{\boldsymbol{k}}$, with minima at the $\boldsymbol{K}, \boldsymbol{K}'$ points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution $\langle \hat{n}_{\boldsymbol{k}} \rangle$ for the chiral phases.

PHYSICAL REVIEW B **89**, 155142 (2014)

Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel,[1] S. A. Parameswaran,[1,2] Andreas Rüegg,[1,3] and Ehud Altman[1,4]

gapped single particles;
but
spontaneously broken time-reversal
= spontaneous circulating
currents

Conclusions

Magnon pairing is a fascinating problem

Route to multipolar orders of frustrated ferromagnets

extention to d=2 problems?

Spin-current/Chiral Mott insulators