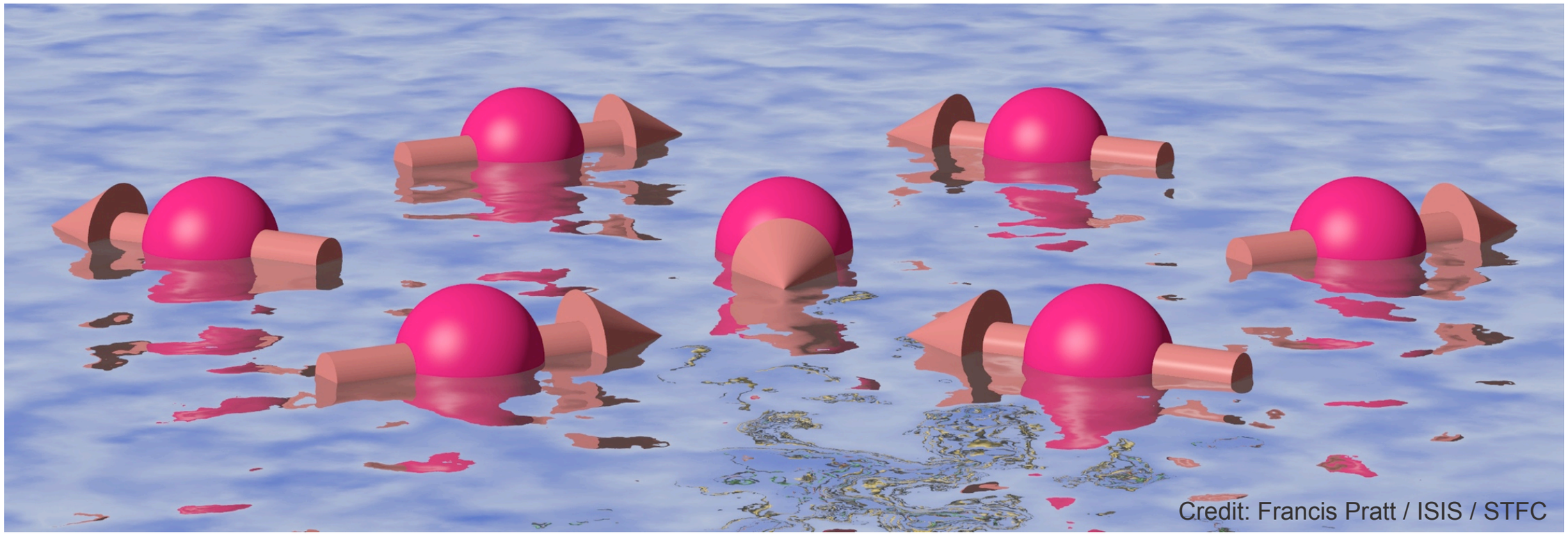
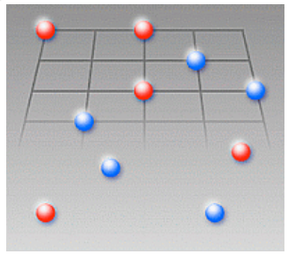
Unusual ordered phases of magnetized frustrated antiferromagnets



Oleg Starykh
University of Utah

Leon Balents and Andrey Chubukov

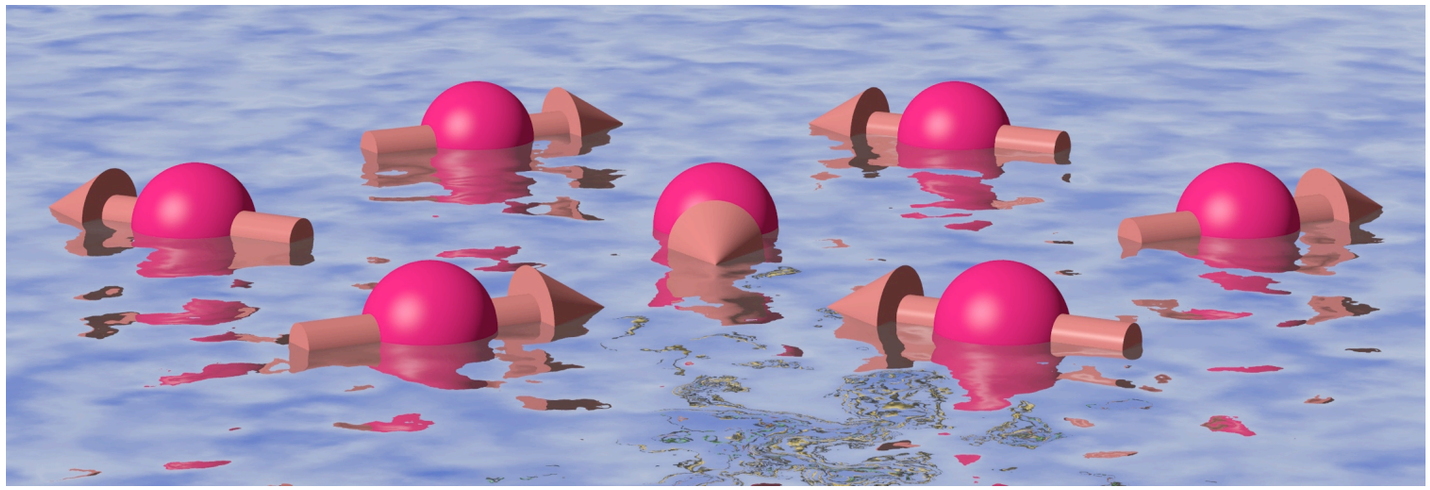


"Novel states in correlated condensed matter – from model systems to real materials", April 8-10, 2014

Outline

- Frustrated magnetism
  - competing orders
  - symmetry breaking vs topological order
- Nematic *vs* SDW in $LiCuVO_4$
  - ✓ spin nematic: “magnon superconductor”
  - ✓ collinear SDW: “magnon charge density wave”
- Instability of the 1/3 plateau: spin-current nematic phase
- Conclusions

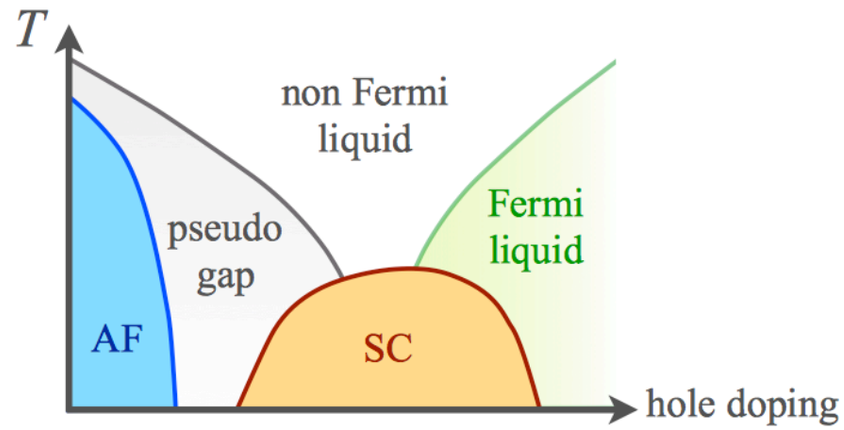
Why to do it?



Immense potential
for practical applications



Different kinds of orders



- **Spontaneous symmetry breaking**
  - ground state has **less** symmetry than Hamiltonian
  - Landau-Ginzburg-Wilson theory
  - **local** order parameter

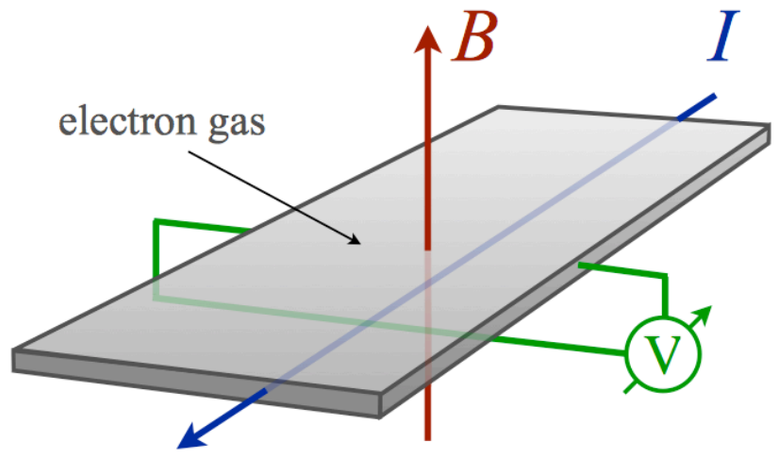


- Fermi liquids
- charge density waves
- superconductors
- spin liquids

- **Topological order**
  - ground state has **more** symmetry than Hamiltonian
  - degenerate ground states
  - **non-local** order parameter



- quantum Hall effect
- topological spin liquids
- topological insulators
- entanglement…



S. Trebst, U Cologne

To break (the symmetry) or not to break?

(symmetry → order parameter)

(top. order)

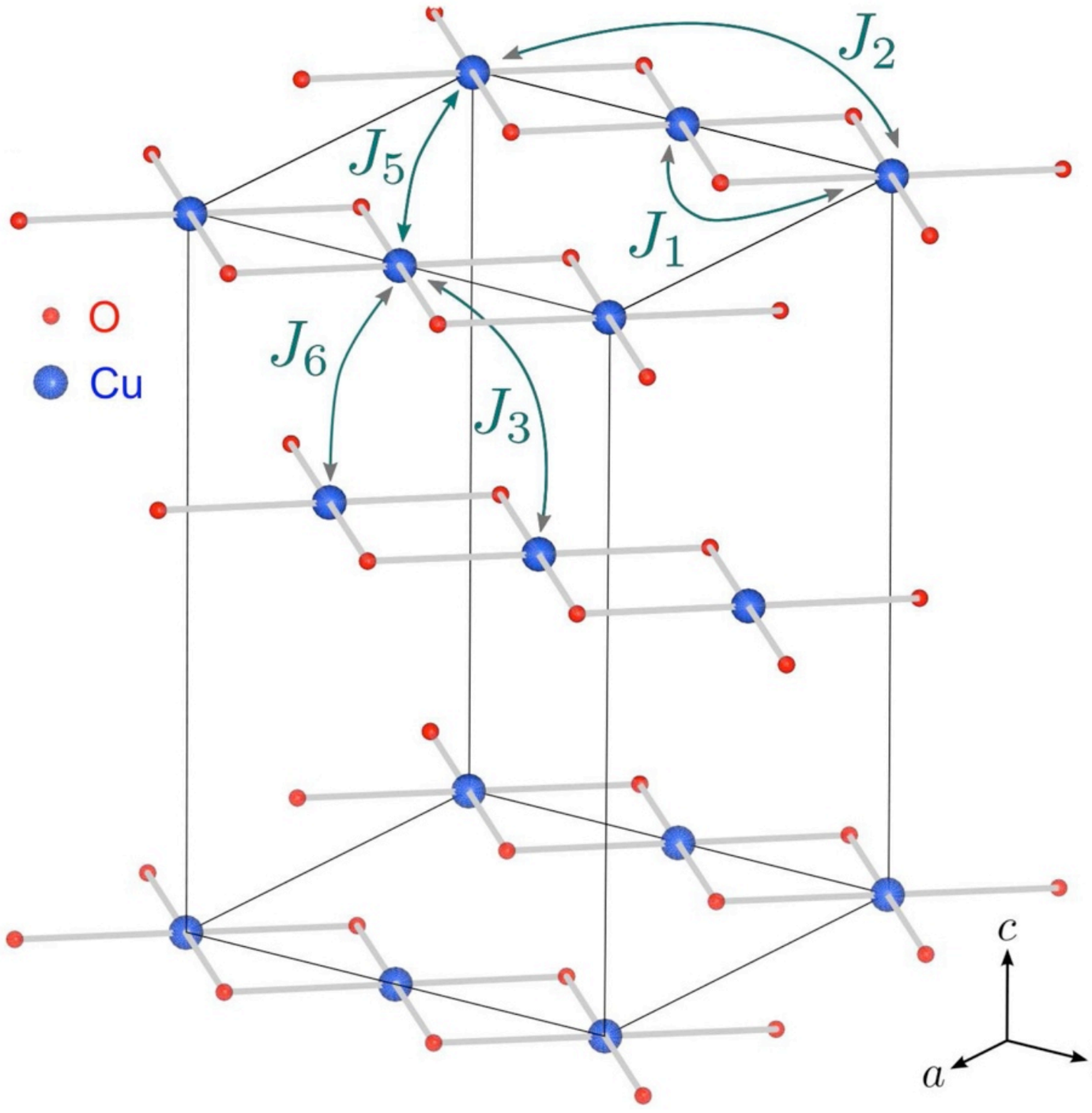
★ This talk: *Break the symmetry in an interesting way*

- spin nematic: “magnon superconductor”
- collinear SDW: “magnon charge density wave”
- spontaneous generation of spin currents

Outline

- Frustrated magnetism
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  - symmetry breaking vs topological order
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$LiCuVO_4$: magnon superconductor?



estimates:

J_1 = - 1.6 meV

J_2 = 3.9 meV

J_5 = -0.4 meV

High-field analysis: condensate of magnon pairs

$$\langle S^+ \rangle = 0 \qquad\qquad \langle S^+ S^+ \rangle \neq 0$$

Ferromagnetic $J_1 < 0$ produces attraction in real space

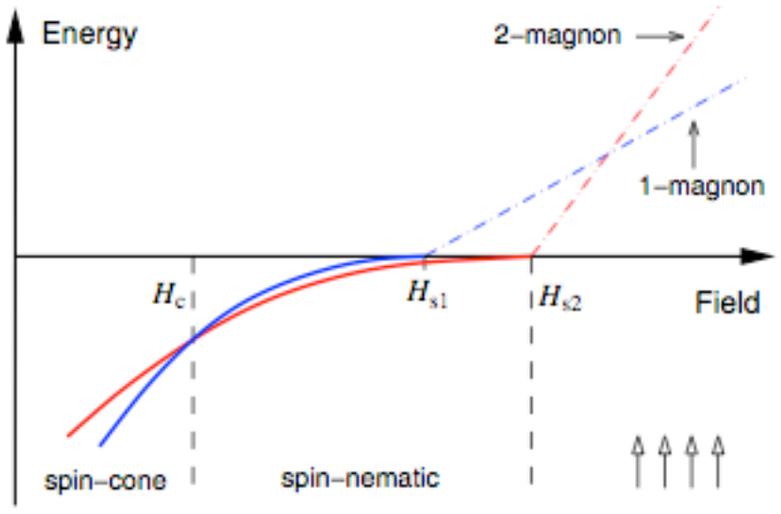


Fig. 1: (Color online) Energy-field diagram for a frustrated quantum magnet close to the saturation field. Dot-dashed lines show lowest one- and two-magnon states. Solid lines represent the ground-state energy for the one-magnon (spin-cone) and the two-magnon (spin-nematic) condensate.

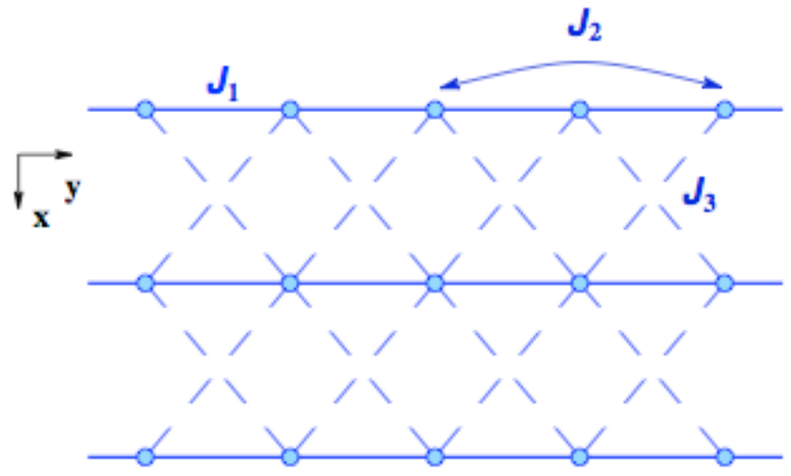


Fig. 2: (Color online) Two-dimensional array of copper ions in $LiCuVO_4$ with principal exchange couplings.

Chubukov, PRB 1991
Zhitomirsky, Tsunetsugu EPL 2010

$LiCuVO_4$: NMR lineshape - collinear SDW along **B**

Hagiwara et a, 2011

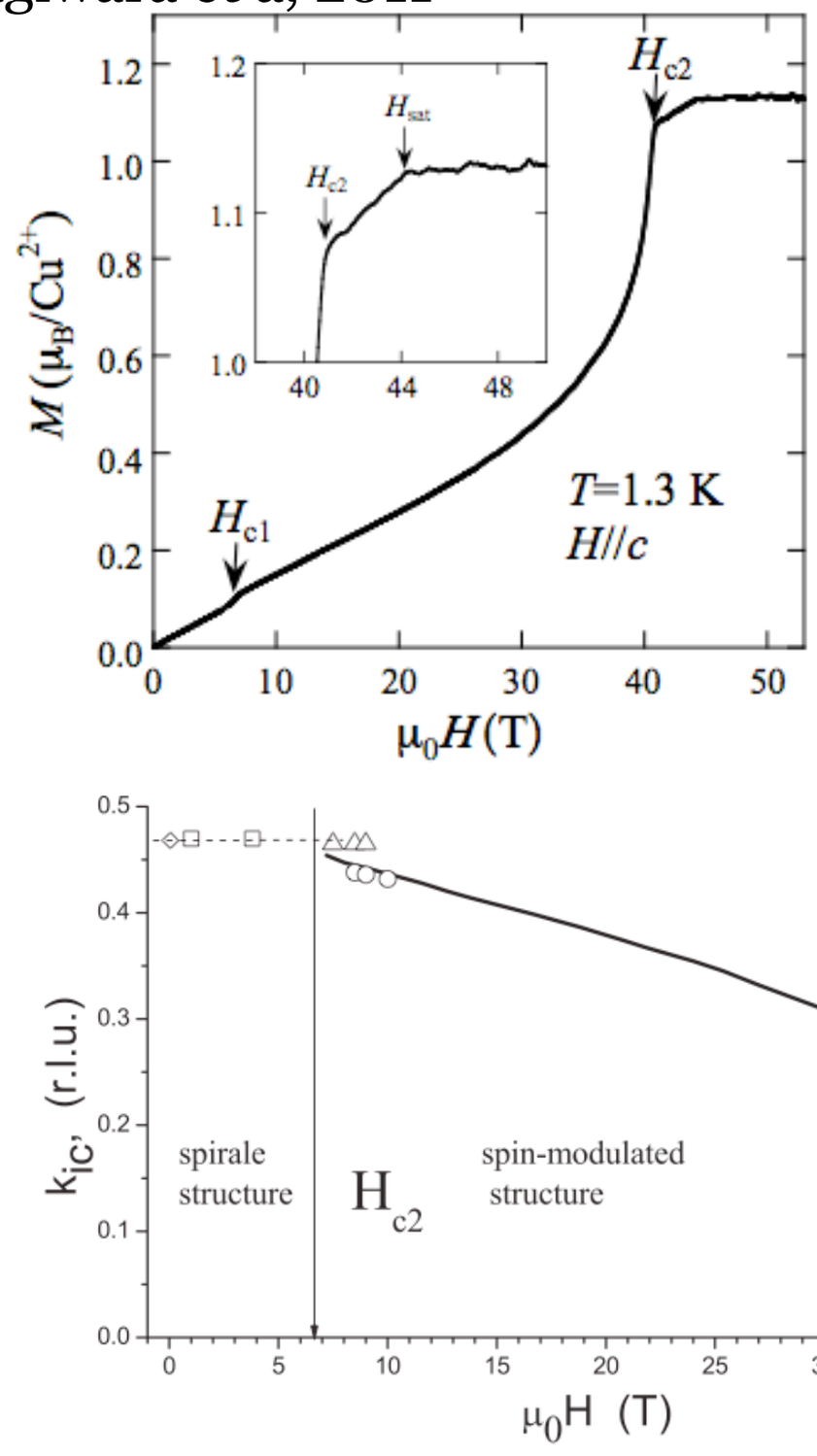
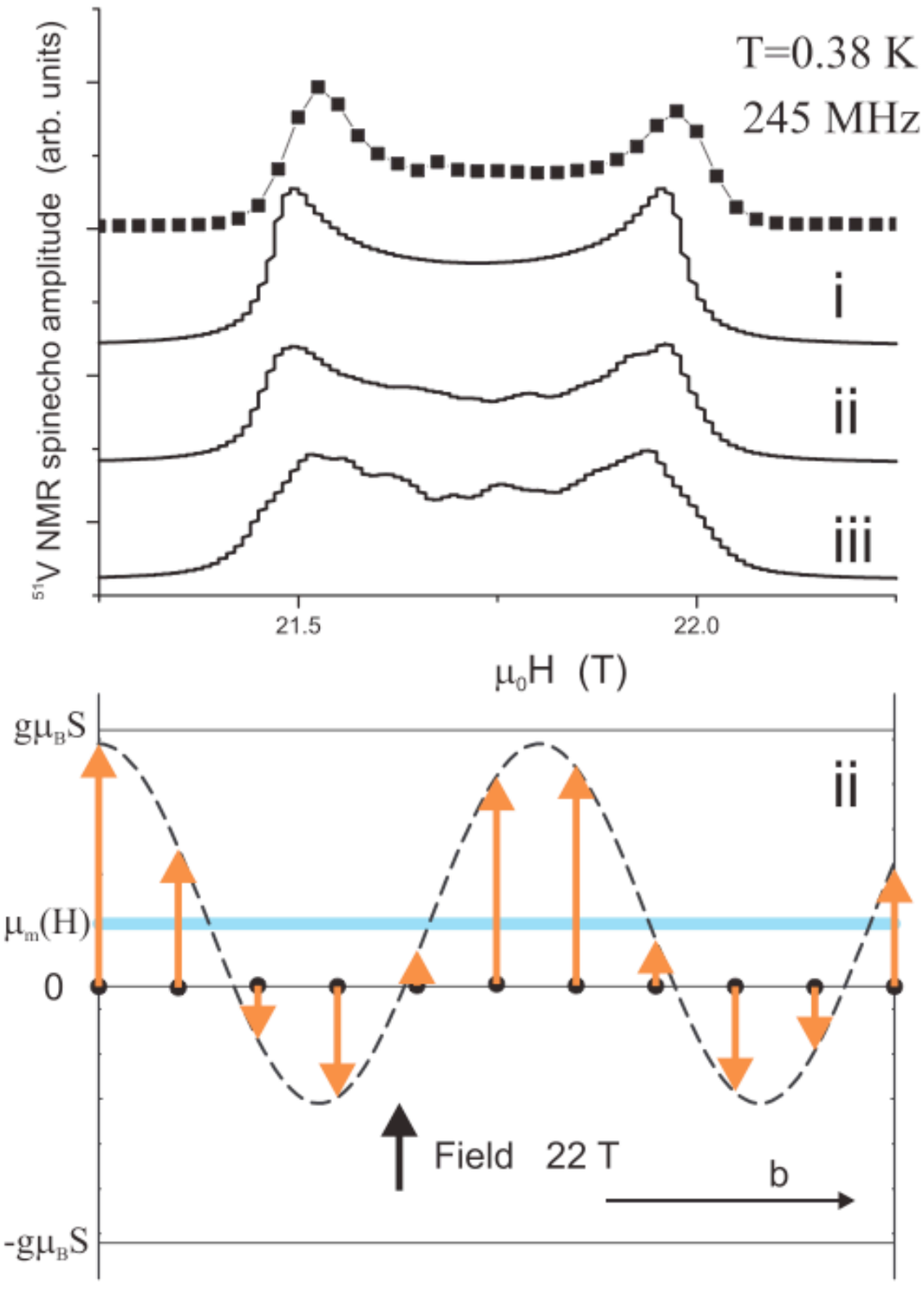


FIG. 2. Field dependence of the incommensurate wave vector k_{ic} for applied magnetic fields **H** ∥ **c** in $LiCuVO_4$. The open symbols

Buttgen et al 2012 PHYSICAL REVIEW B **85**, 214421 (2012)



Evidence of a Bond-Nematic Phase in $LiCuVO_4$

M. Mourigal,[1,2] M. Enderle,[1] B. Fåk,[3] R. K. Kremer,[4] J. M. Law,[4,*] A. Schneidewind,[5] A. Hiess,[1,†] and A. Prokofiev[6,7]

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, ΔS = 1
NSF = no spin flip, ΔS = 0

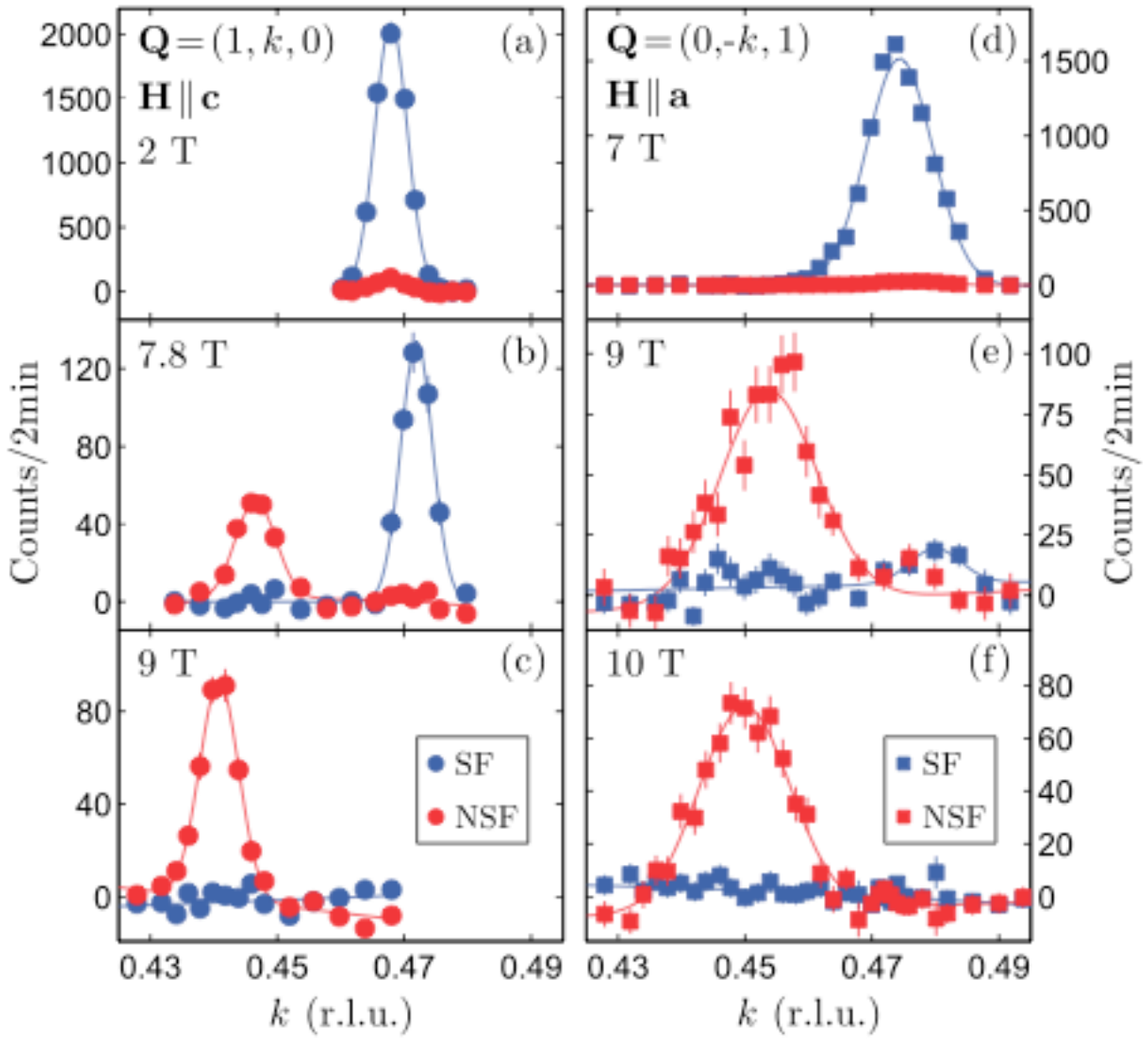
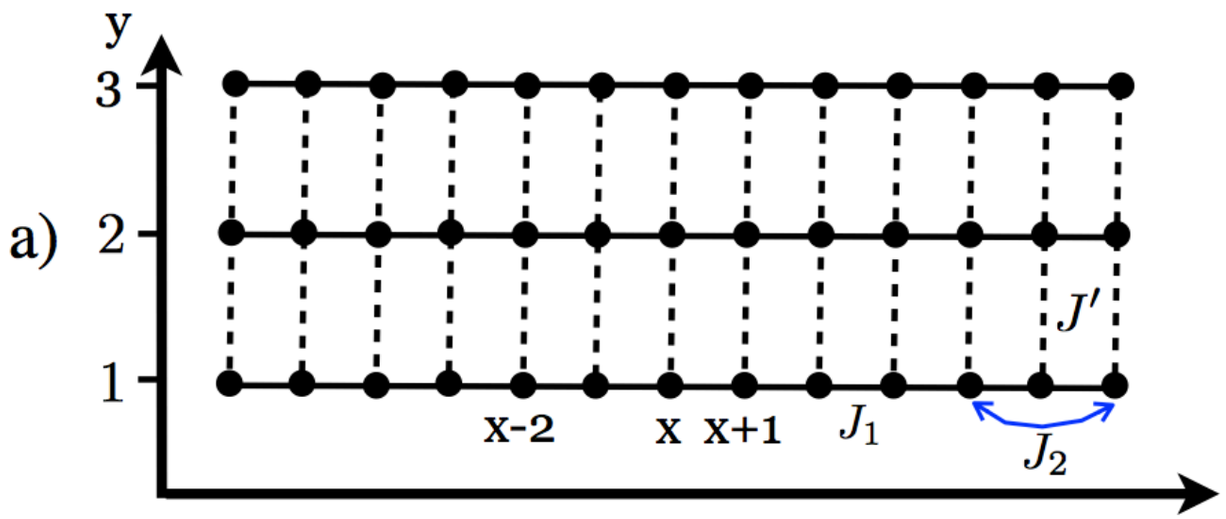
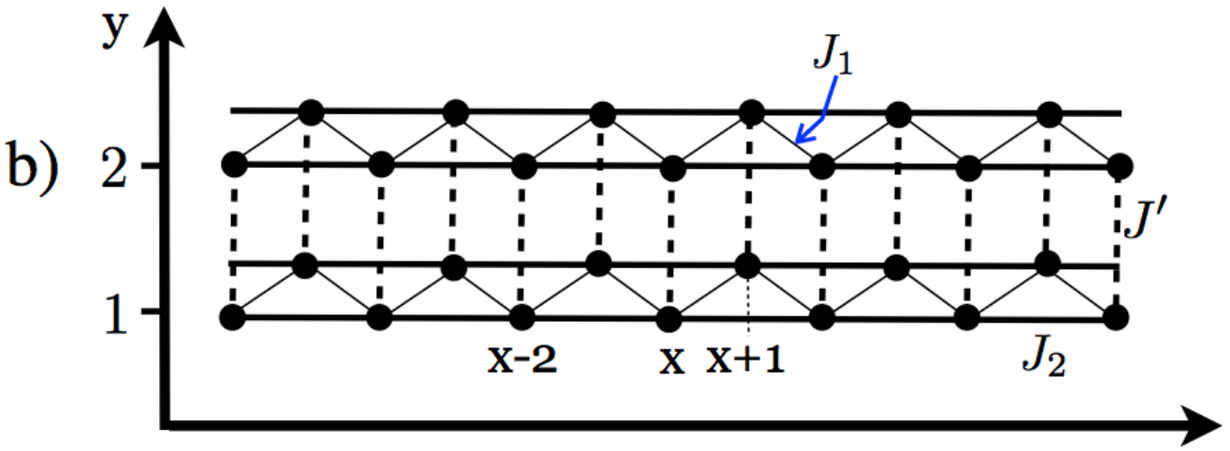
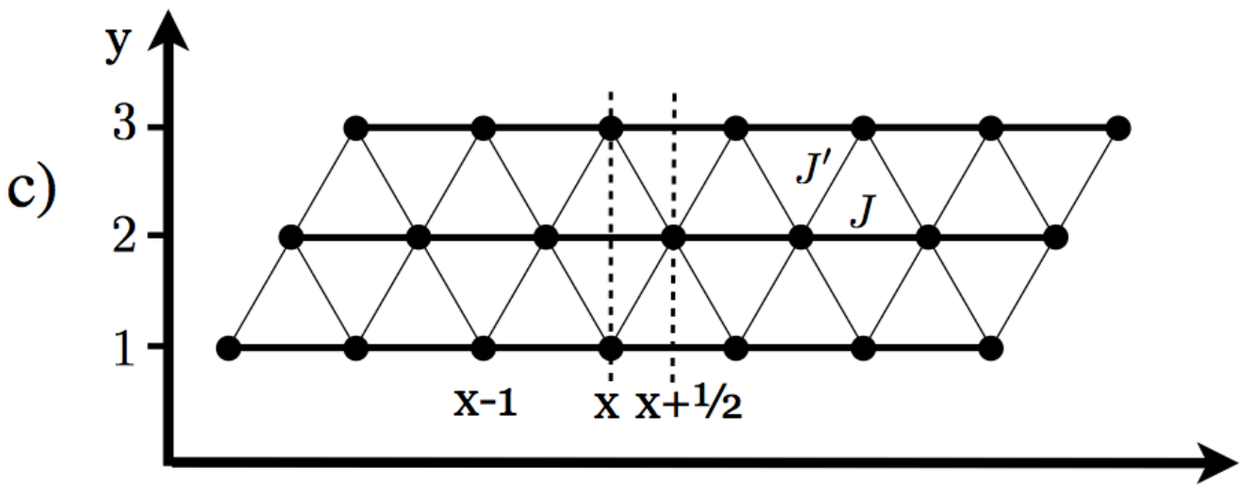


FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $\mathbf{Q} = (1, k_{IC}, 0)$ with $\mathbf{H}\|\mathbf{c}$ [left panels, (a)–(c)] and $\mathbf{Q} = (0, -k_{IC}, 1)$ with $\mathbf{H}\|\mathbf{a}$ [right panels, (d)–(f)].

Geometry (motivated by $LiCuVO_4$)





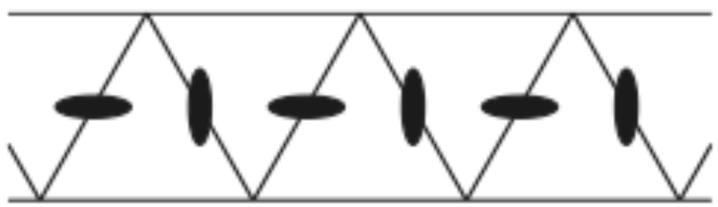


today:

$J_1 < 0$ (ferro)

$J_2 > 0$, $J' > 0$ (afm)

in magnetic field

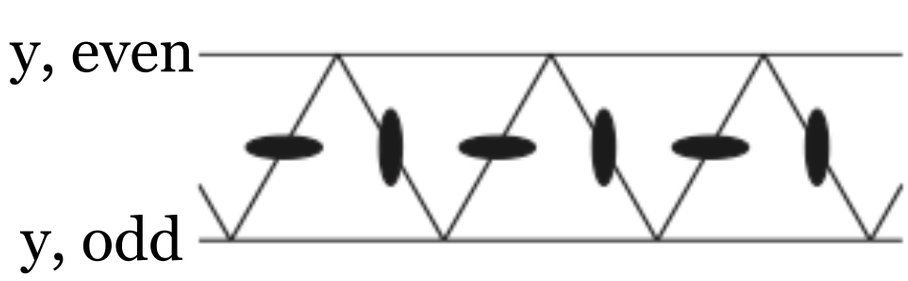


Chubukov 1991;
Kecke et al 2007;
Hikihara et al 2008;
Zhitomirsky and Tsunetsugu 2010
Sato et al 2013
Starykh and Balents 2014

Nematic chain

S^z-S^z (SDW) channel: in-chain $J_1 < 0$ gaps out relative mode $\varphi_y^- = (\varphi_{y,\mathrm{odd}} - \varphi_{y,\mathrm{even}})/\sqrt{2}$

$$H_{\mathrm{intra-chain}} = \sum_y \int dx J_1 \sin[\pi M] \cos[\frac{\sqrt{8}\pi}{\beta}\varphi_y^-]$$



J_1-J_2 chain

$$\langle S^+ \rangle = 0$$

$$S_y^+(x) \sim (-1)^x A_3 e^{i\frac{\beta}{\sqrt{2}}\theta_y^+(x)} e^{i(-1)^x \frac{\beta}{\sqrt{2}}\theta_y^-(x)}$$

quantum-disordered,
decays *exponentially:*
S^z = 1 excitations are gapped

$$T_y^+ = S_y^+(x) S_y^+(x+1) \sim e^{i\sqrt{2}\beta\theta_y^+(x)}$$

Standard (in 1d)
power-law decay:
critical nematic spin correlations

$$\langle T^+ \rangle = \langle S^+ S^+ \rangle \neq 0$$

Physical picture: 1d magnon “superconductor”

Kolezhuk, Vekua (2005); Hikihara et al. (2008); Sato, Hikihara, Momoi (2013)

Inter-chain interaction $H_{\text{inter-chain}} = \sum_y \int dx\ \vec{S}_y \cdot \vec{S}_{y+1} \sim \sum_y \int dx\ S_y^+ S_{y+1}^- + S_y^z S_{y+1}^z$

Superconducting analogy: single-particle (magnon) tunneling between magnon superconductors is strongly suppressed at low energy (below the single-particle gap)

$$H_{\text{inter}}^{\perp} = \sum_y \int dx\ J' \langle S_y^+(x) S_{y+1}^-(x+1) \rangle_{\text{nematic ground state}} \to 0$$

Superconducting analogy: fluctuations generate two-magnon (Josephson) tunneling between chains. They are generically weak, ~ $J_1(J'/J_1)^2 << J_1$, but responsible for a true **two-dimensional nematic order**.

$$H_{\text{nem}} \sim (J'^2/J_1) \sum_y \int dx\ [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

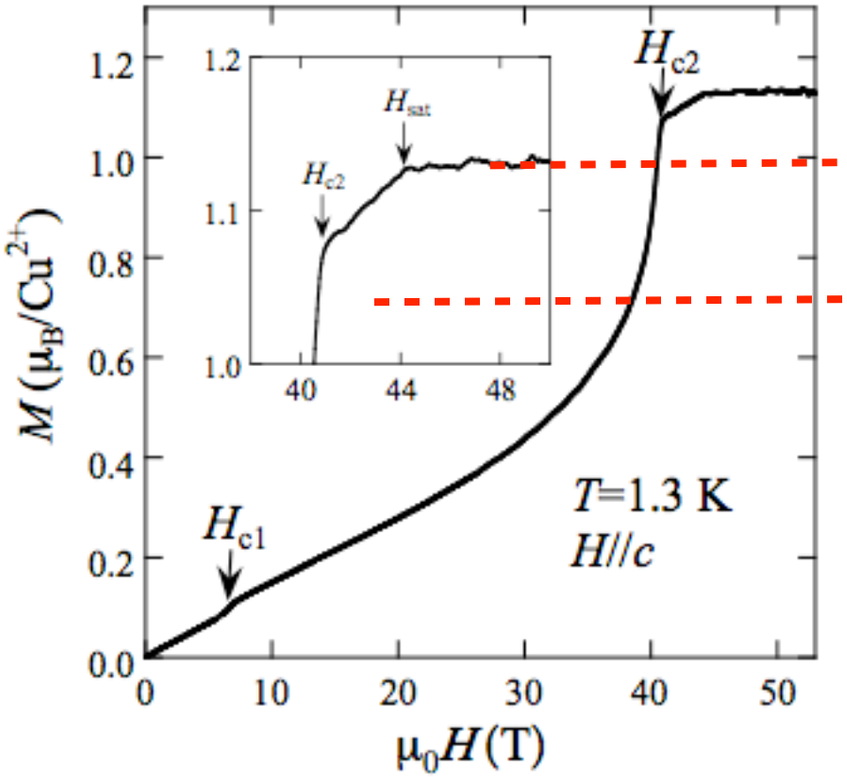
At the same time, density-density inter-chain interaction does not experience any suppression. It drives the system toward a **two-dimensional collinear SDW order**.

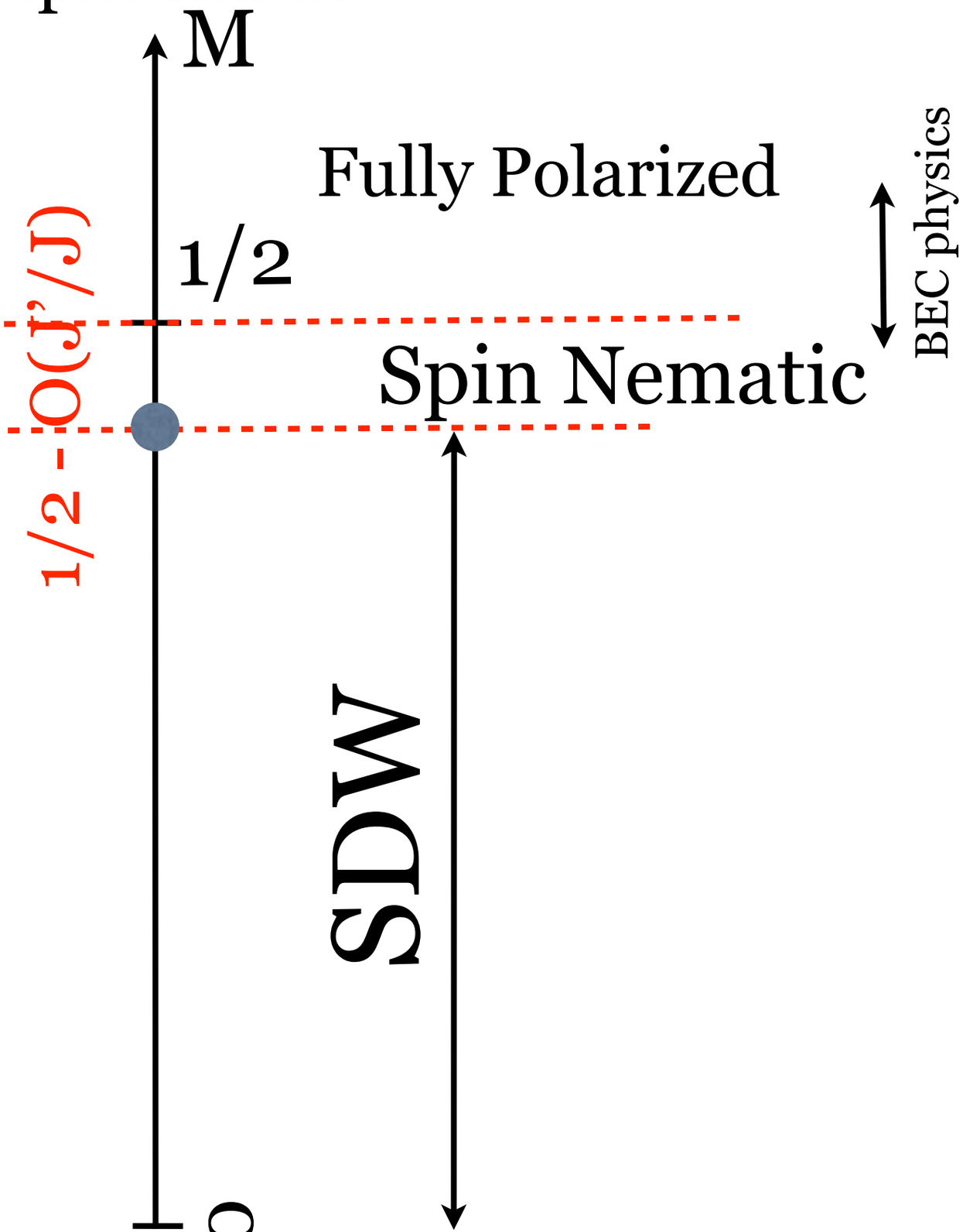
$$S_y^z \sim M - 2n_{\text{pair}} = M - \tilde{A}_1 e^{\frac{\sqrt{2}\pi}{\beta}\varphi_y^+(x)}$$

$$H_{\text{inter-chain}}^z = H_{\text{sdw}} \sim J' \sum_y S_y^z S_{y+1}^z \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi_y^+ - \varphi_{y+1}^+)]$$

Away from the saturation, **SDW** is more relevant [and stronger, via $J_1 >> J_1(J'/J_1)^2$] than the **nematic interaction**: **coupled** 1d **nematic chains order in a** 2d **SDW state.**

T=0 schematic phase diagram of weakly coupled **nematic** spin chains



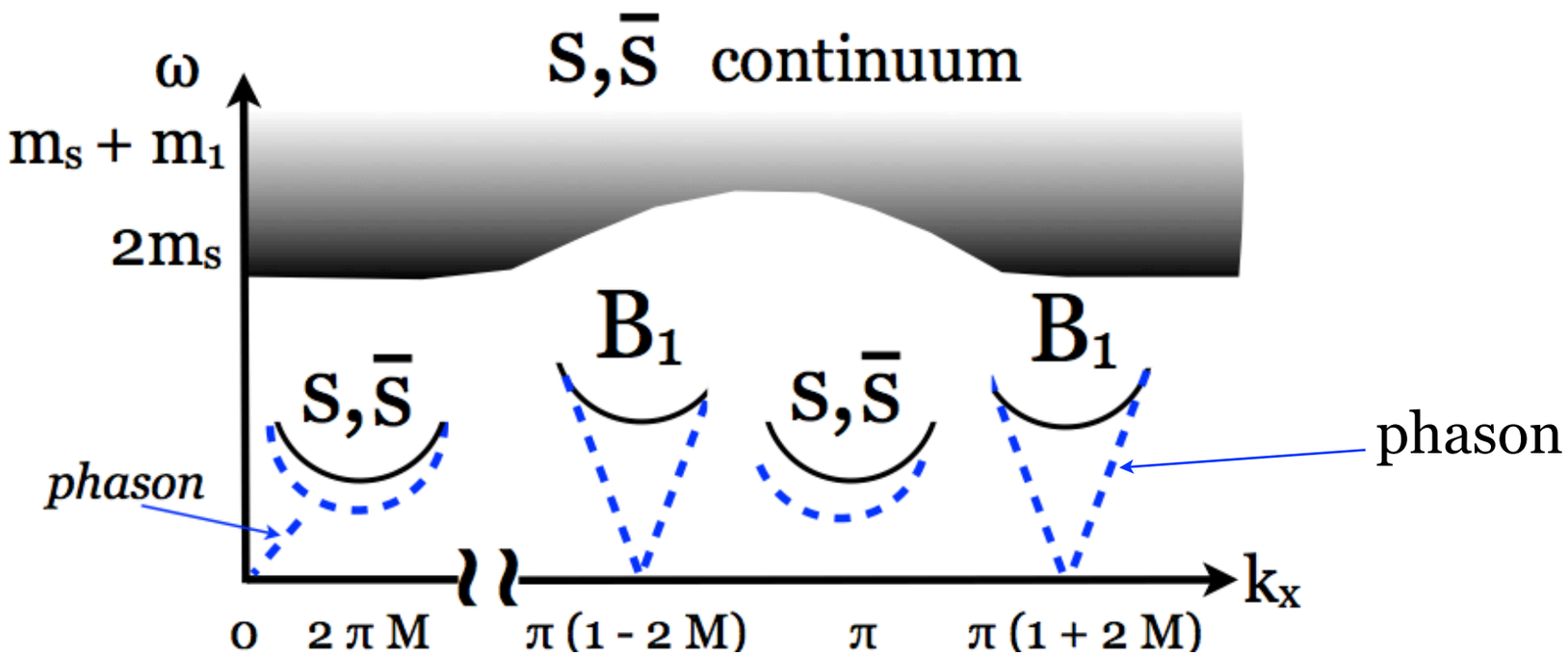


Excitations (via spin-spin correlation functions)

- 2d SDW $\langle S^z(\mathbf{r})\rangle = M + \mathrm{Re}\left(\Phi e^{i\mathbf{k}_{\mathrm{sdw}}\cdot\mathbf{r}}\right)$

  1. preserves U(1) [with respect to magnetic field] -> no transverse spin waves

  2. breaks translational symmetry - longitudinal phason mode at $k_{sdw} = \pi(1\text{-}2M)$ and k=0



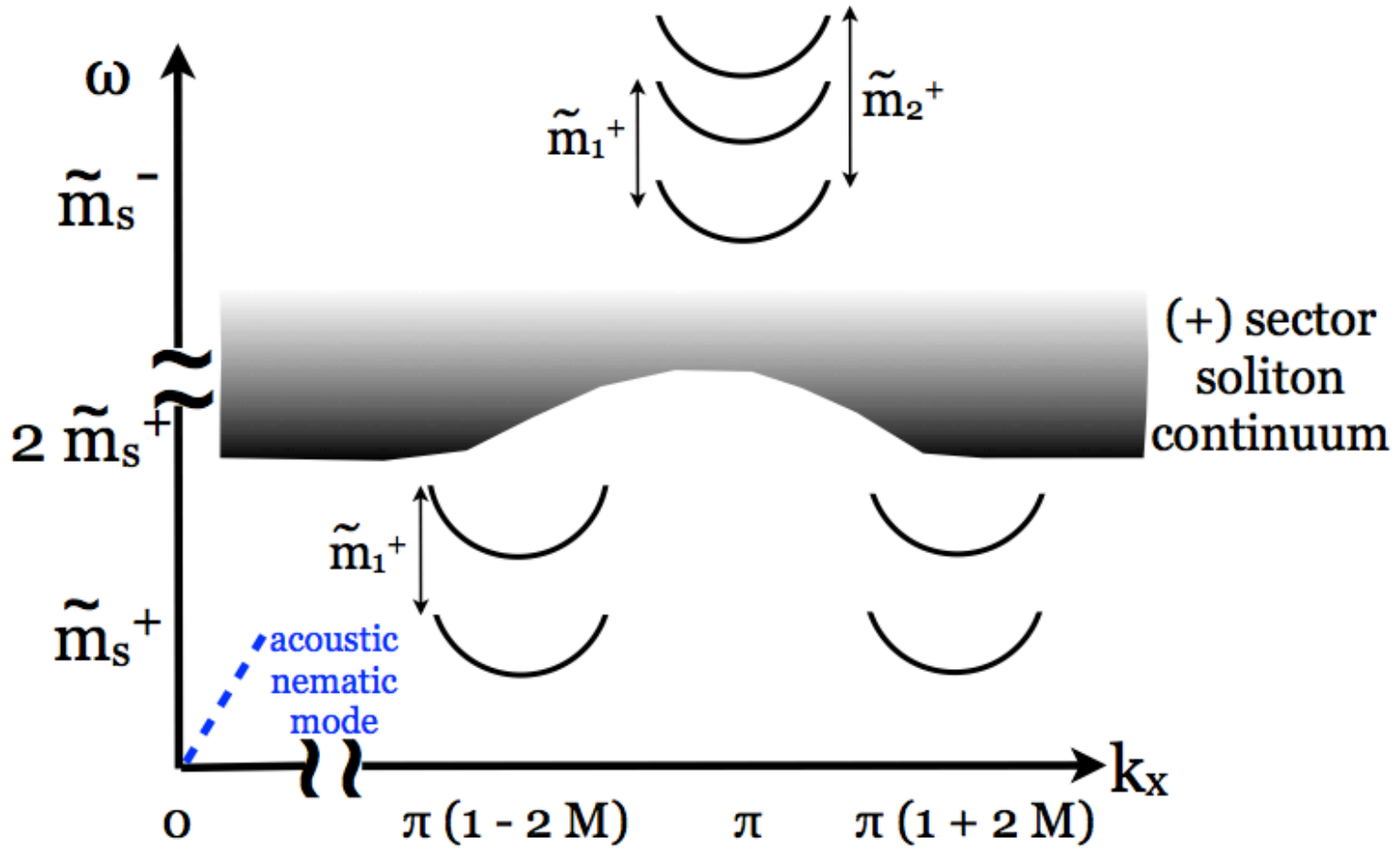
OS, Balents PRB 2014

Excitations (via spin-spin correlation functions)

- 2d Spin Nematic $\langle S^+(\mathbf{r})S^+(\mathbf{r}')\rangle = \Psi e^{i\mathbf{k}_{\rm nm}\cdot\mathbf{r}_{\rm cm}}$

  1. breaks U(1) but ΔS=1 excitations are gapped (magnon superconductor) $\langle S^+(\mathbf{r})\rangle = 0$

  2. gapless density fluctuations at k=0



OS, Balents PRB 2014

Conclusions I

- Interesting magnetically ordered states: SDW and Spin Nematic
  - Gapped $\Delta S=1$ excitations
  - Linearly-dispersing ***phason*** mode with $\Delta S=0$ in SDW
  - Linearly-dispersing ***magnon density*** waves in SN
  - useful analogy with superconductor/charge density wave competition

Outline

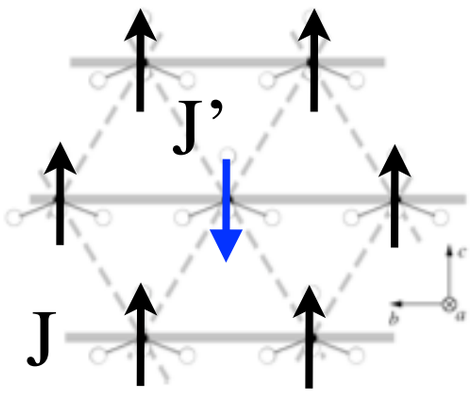
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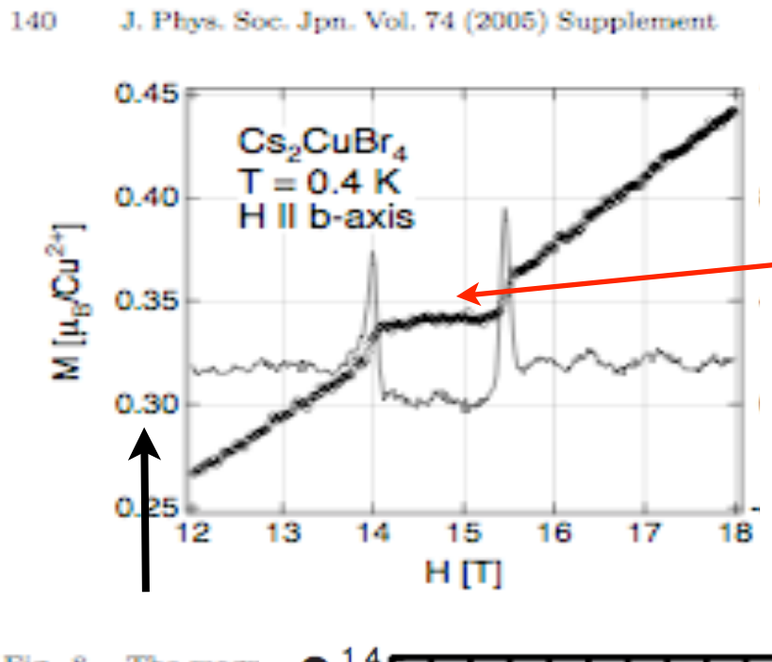
Exp: M=1/3 magnetization plateau in Cs_2CuBr_4

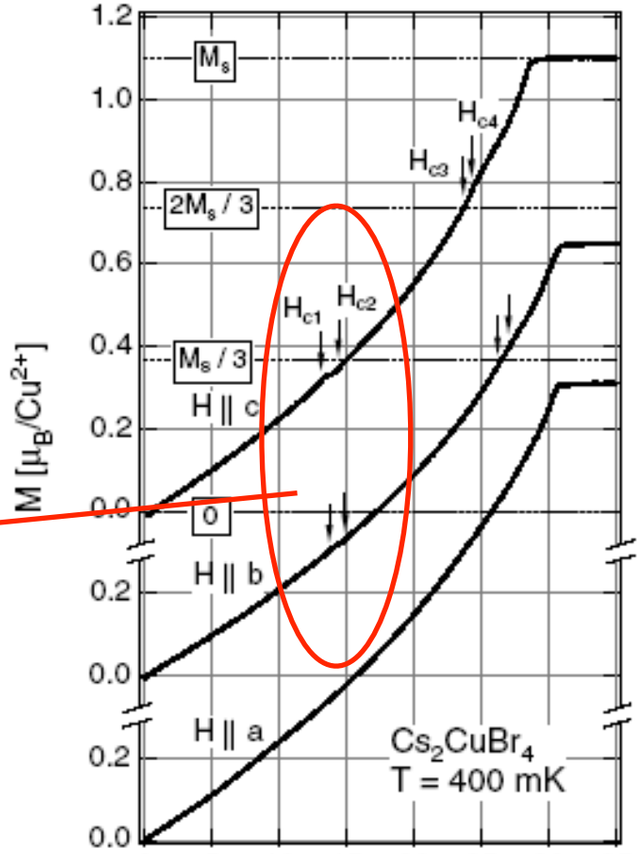
★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007)

$$J'/J \approx 0.75|_{\text{year } 2007} \rightarrow 0.4|_{\text{year } 2014}$$

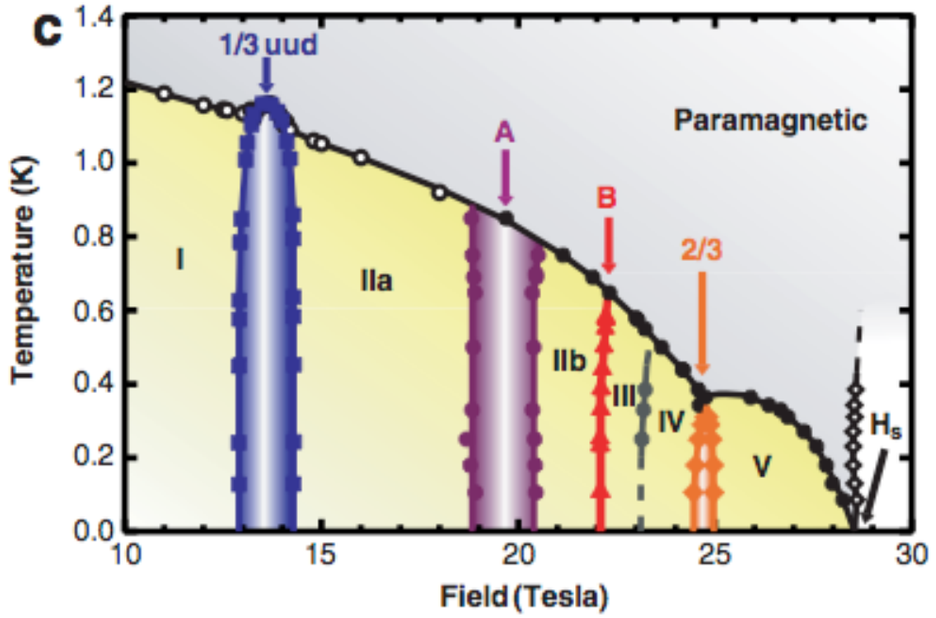
S=1/2







★ "**up-up-down**" state is **commensurate** collinear **SDW**



Important: the lattice is strongly anisotropic

Quantum fluctuations, S >> 1, T=0.

J' = **J**: Quantum fluctuations select co-planar and collinear phases

J. Phys.: Condens. Matter 3 (1991) 69–82. Printed in the UK

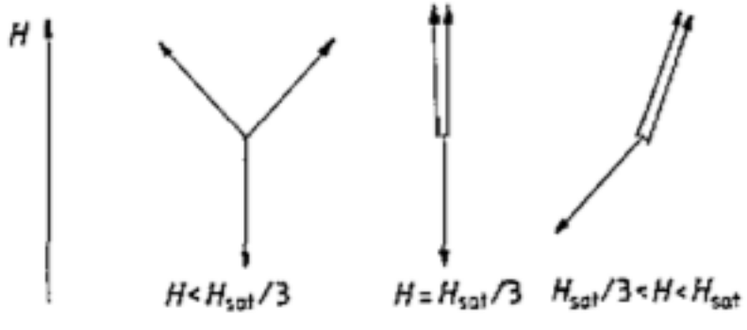
UUD plateau is due to interactions between spin waves

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

A V Chubukov and D I Golosov
Institute for Physical Problems, USSR Academy of Sciences, 117334 ul. Kosygina 2, Moscow, USSR

Received 9 February 1990

Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and *XY* quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.



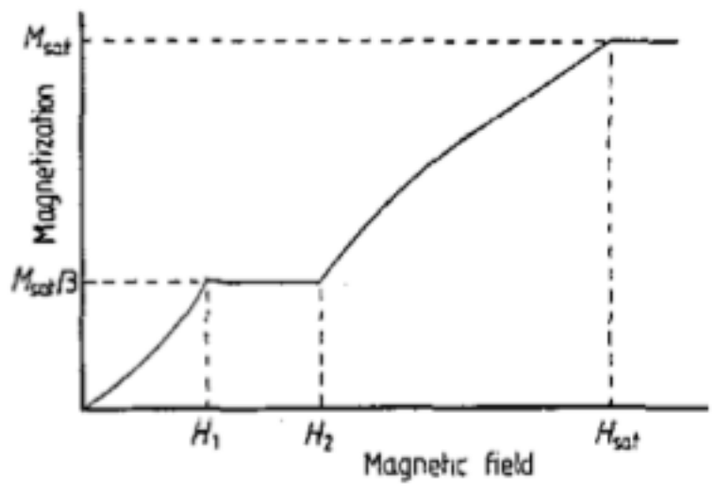


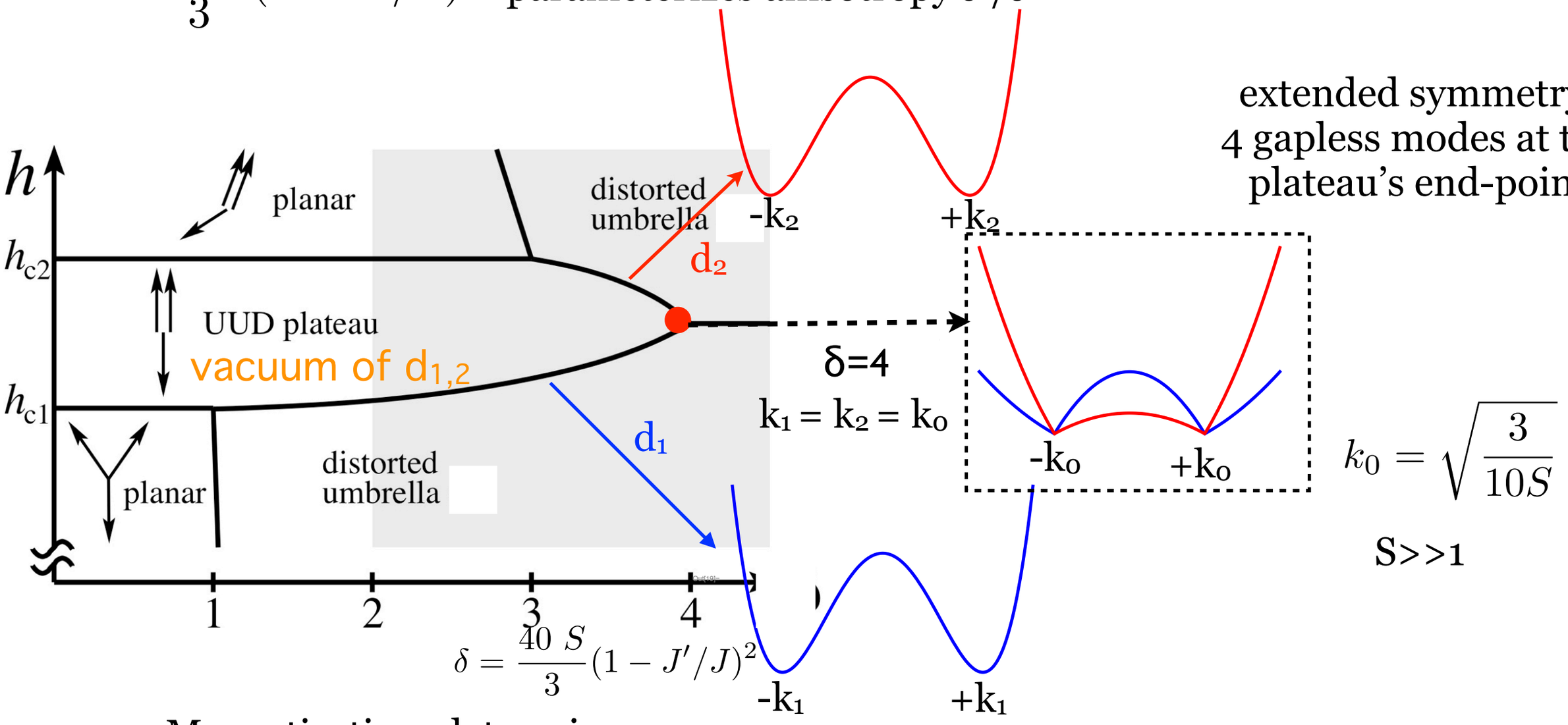
Figure 3. The anticipated behaviour of longitudinal magnetization in 2D Heisenberg AFM on a triangular lattice. The plateau on the magnetization curve results from the stabilization of the collinear phase in the finite region of magnetic fields due to zero-point motion.

$$h_{c2} - h_{c1} = (0.6/2S)\, h_{sat}$$

Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_2$ in the vicinity of $H_{sat}/3$.

Low-energy excitation spectra near the plateau's end-point

$\delta = \frac{40\ S}{3}(1 - J'/J)^2$ parameterizes anisotropy J'/J



extended symmetry:
4 gapless modes at the plateau's end-point

$k_0 = \sqrt{\frac{3}{10S}}$

S>>1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3. Hence, ***very stable***.

Alicea, Chubukov, OS PRL 2009

Bosonization of 2d interacting magnons

$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\Big(d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}\Big) + \text{h.c.}$$

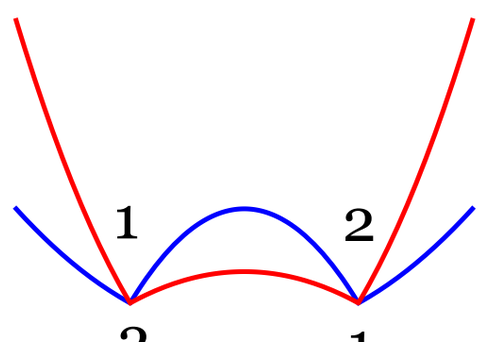
$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

singular magnon interaction

$$\Psi^\dagger_{1,p}\Psi_{2,q} \qquad \Psi^\dagger_{1,p}\Psi^\dagger_{2,q}$$

magnon **pair operators**

$$\begin{cases}\Psi_{1,p} = d_{1,k_0+p}d_{2,-k_0-p}\\ \Psi_{2,p} = d_{1,-k_0+p}d_{2,k_0-p}\end{cases}$$



Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}},\Psi_{2,\mathbf{q}}] = \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}\Big(1 + d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d_{1,\mathbf{k}_0+\mathbf{p}} + d^\dagger_{2,\mathbf{k}_0+\mathbf{p}}d_{2,\mathbf{k}_0+\mathbf{p}}\Big) \to \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state $\quad \langle d^\dagger_1 d_1\rangle_{\text{uud}} = \langle d^\dagger_2 d_2\rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of $\mathbf{d}_{1,2}$ bosons = non-interacting Hamiltonian in terms of $\Psi_{1,2}$ magnon pairs

Chubukov, OS PRL 2013

Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for Ψ - Hamiltonian

$$\langle \Psi^\dagger_{1,\mathbf{p}} - \Psi_{1,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{2,\mathbf{q}} - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi^\dagger_{2,\mathbf{p}} - \Psi_{2,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{1,\mathbf{q}} - \Psi_{1,\mathbf{q}} \rangle$$

`Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4$ - $O(1/S^2)$

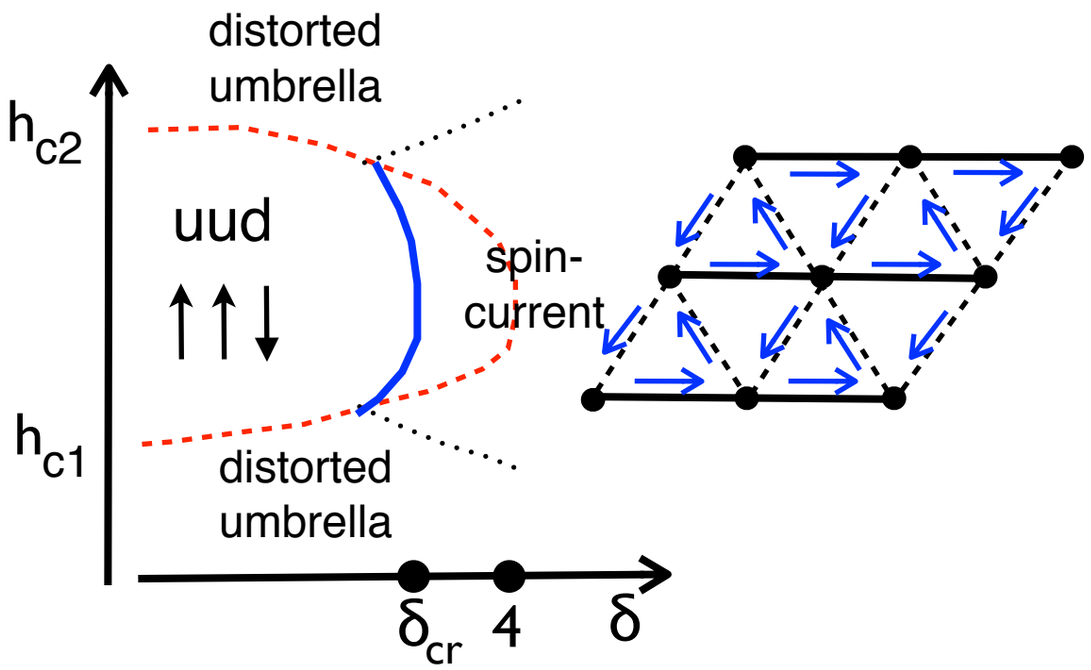
$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1-\delta/4)k_0^2}}$$

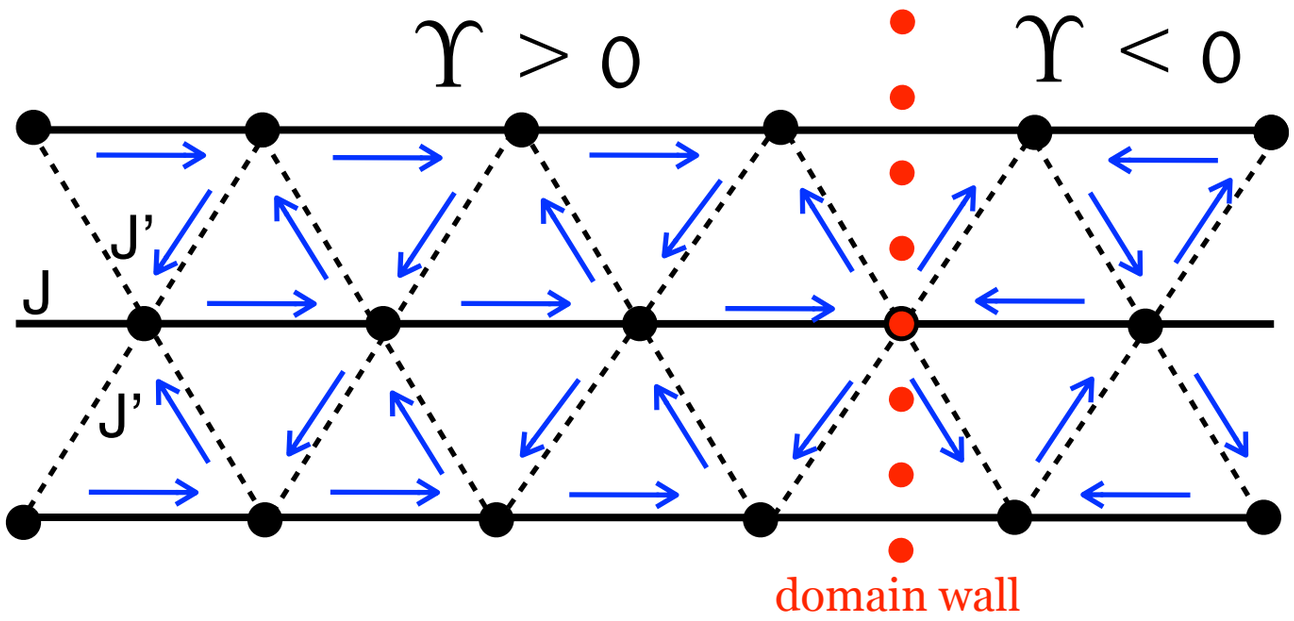
no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

Chubukov, OS PRL 2013

Two-magnon condensate = Spin-current nematic state





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

Finite scalar (and vector) chiralities. Sign of Υ determines sense of spin-current circulation

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

Spontaneously broken Z_2 -- spatial inversion [in addition to broken Z_3 inherited from the UUD state]

Leads to **spontaneous generation of Dzyaloshisnkii-Moriya interaction**

Chubukov, OS PRL 2013

Conclusions

- Nematic *vs* SDW in *real material* $LiCuVO_4$

  ✓ spin nematic: “magnon superconductor”

  ✓ collinear SDW: “magnon charge density wave”

- Two-magnon instability of the 1/3 plateau: spin-current nematic phase

  ✓ breaks spatial inversion (Z_2)

  ✓ spontaneous generation of DM interaction

- ***Ordered states can be quite interesting!***

Chiral Bosonic Mott Insulator on the Frustrated Triangular Lattice

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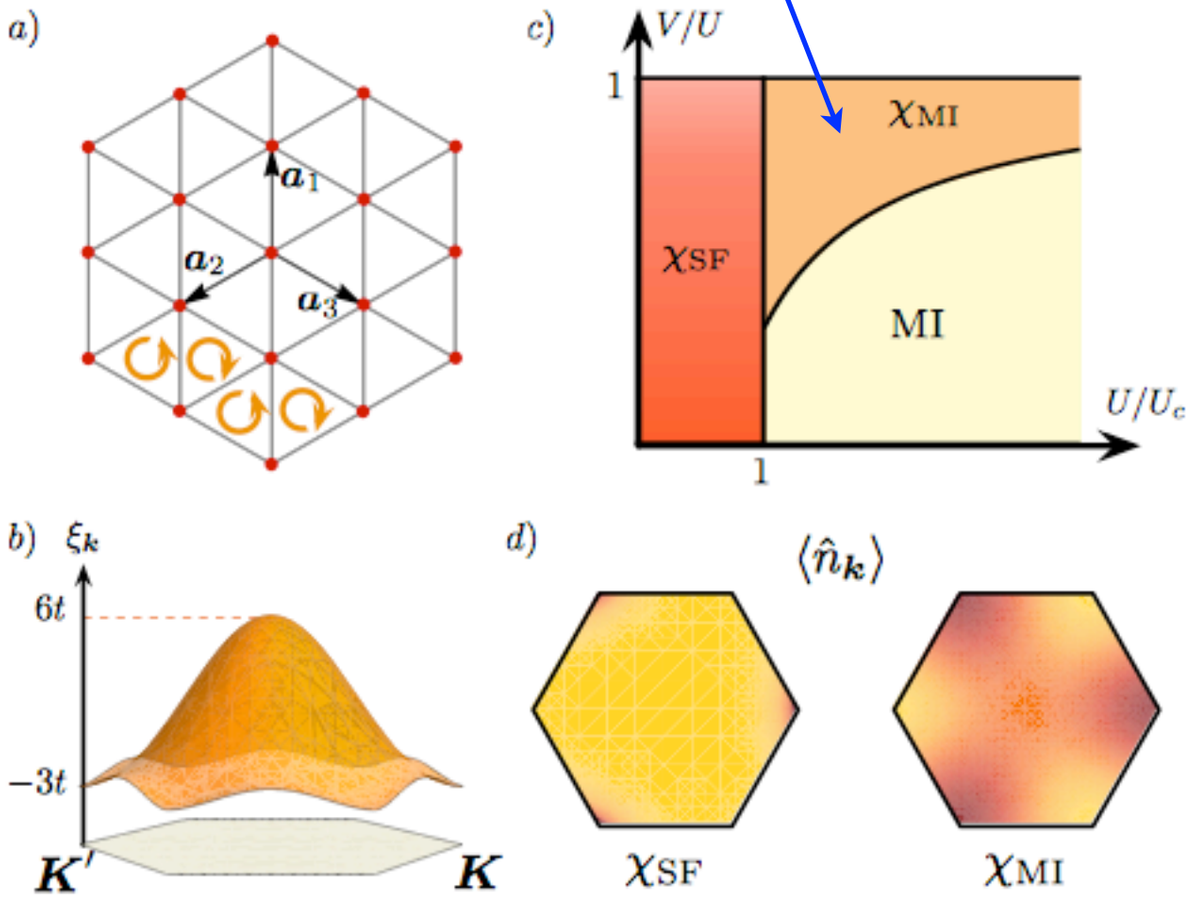


FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion ξ_k, with minima at the K, K' points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution $\langle \hat{n}_k \rangle$ for the chiral phases.

ar lattice with an inverted
rticle dispersion has mul-
)-wavevector minimum of
eveloping additonal chiral
te at nonzero wavevector.
the chiral Mott insulator,
onal estimates, as well as
r permit a fairly complete
ptical lattice experiments,

gapped single particles; spontaneous circulating currents

Motivation: cold atoms in optical lattices