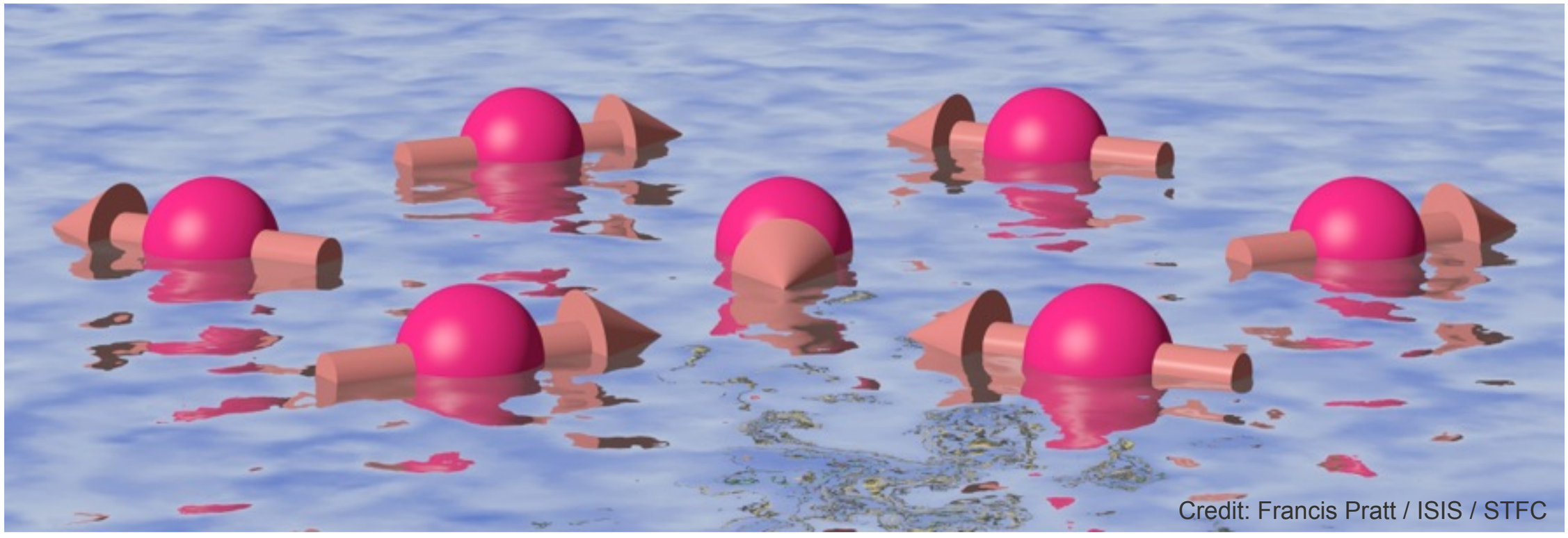
Unusual ordered phases of magnetized frustrated antiferromagnets



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University of Utah

Leon Balents and Andrey Chubukov

Highly Frustrated Magnetism 2014, Cambridge, UK

Outline

- Frustrated magnetism
  - competing orders: SC (SN) *vs* CDW (SDW)
  - ordered state but without low-energy spin waves

- Nematic *vs* SDW in $LiCuVO_4$

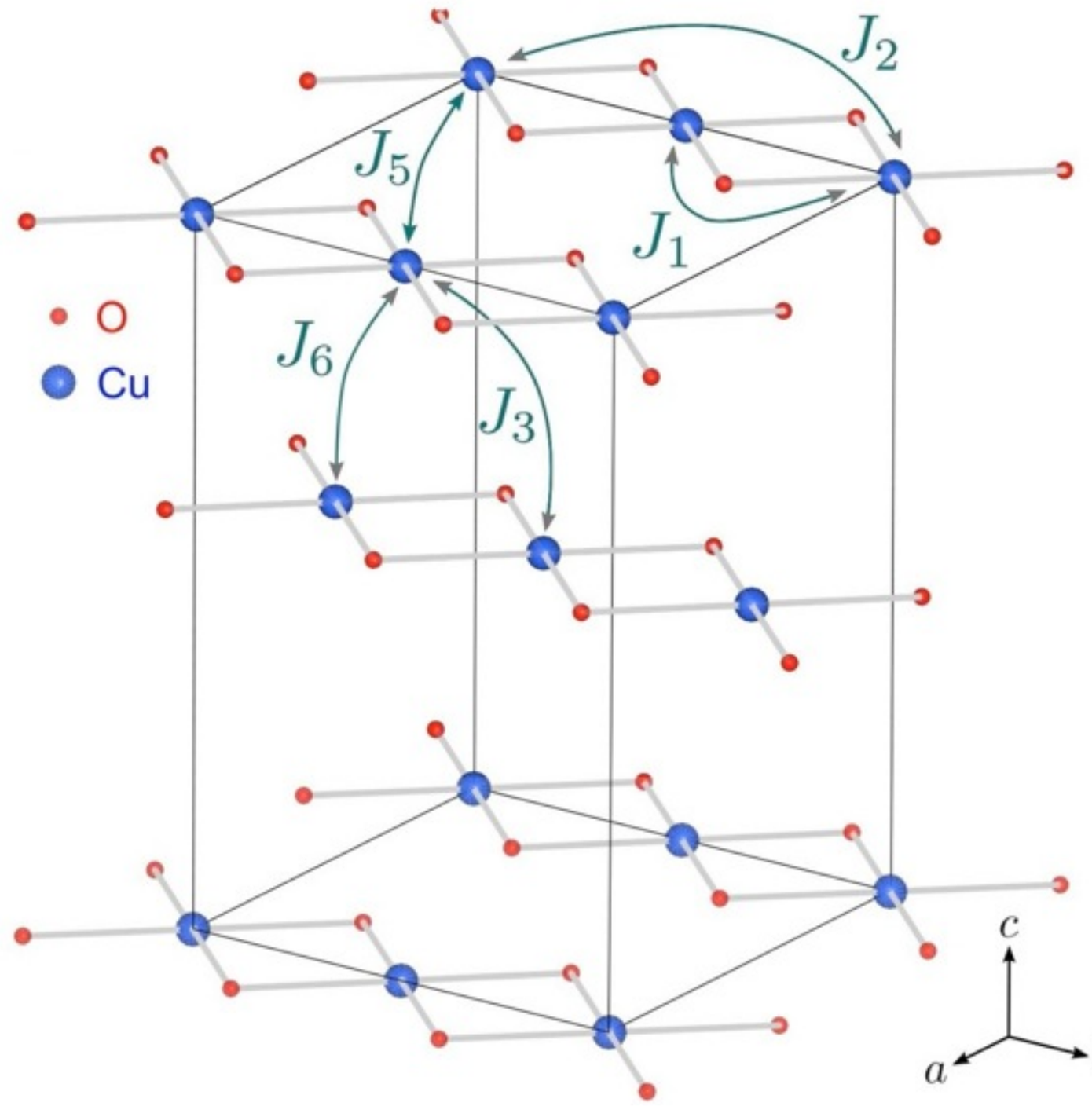
  ✓ spin nematic: “magnon superconductor”

  ✓ collinear SDW: “magnon charge density wave”

- Instability of the 1/3 plateau: spin-current nematic phase -> *open for private discussions!*

- Conclusions

$LiCuVO_4$: magnon superconductor?



estimates:

J_1 = - 1.6 meV

J_2 = 3.9 meV

J_5 = -0.4 meV

High-field analysis: condensate of magnon pairs

$$\langle S^+ \rangle = 0 \qquad\qquad \langle S^+ S^+ \rangle \neq 0$$

Ferromagnetic $J_1 < 0$ produces attraction in real space

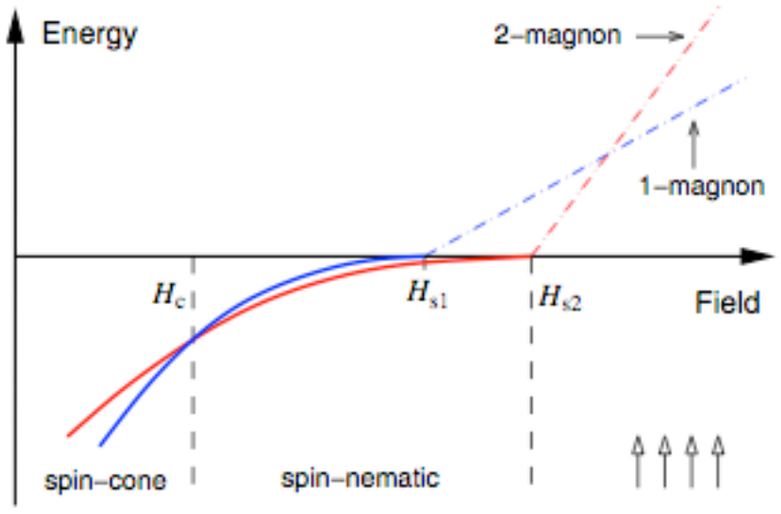


Fig. 1: (Color online) Energy-field diagram for a frustrated quantum magnet close to the saturation field. Dot-dashed lines show lowest one- and two-magnon states. Solid lines represent the ground-state energy for the one-magnon (spin-cone) and the two-magnon (spin-nematic) condensate.

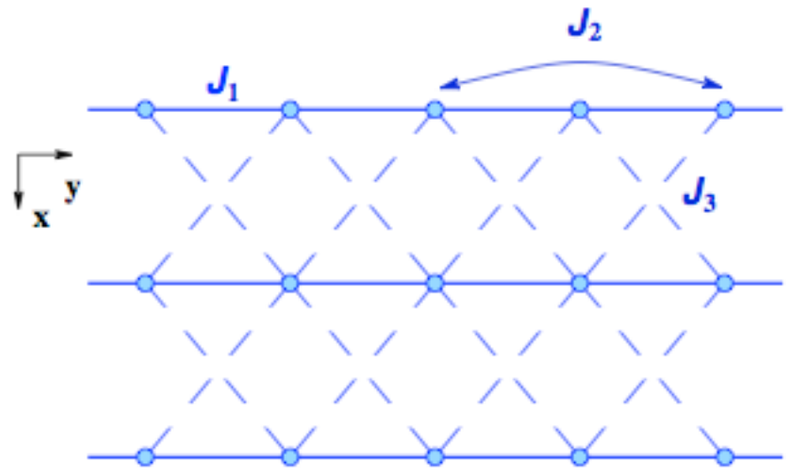


Fig. 2: (Color online) Two-dimensional array of copper ions in $LiCuVO_4$ with principal exchange couplings.

Chubukov, PRB 1991
Zhitomirsky, Tsunetsugu EPL 2010

$LiCuVO_4$: NMR lineshape - collinear SDW along **B**

Hagiwara, Svistov et al, 2011

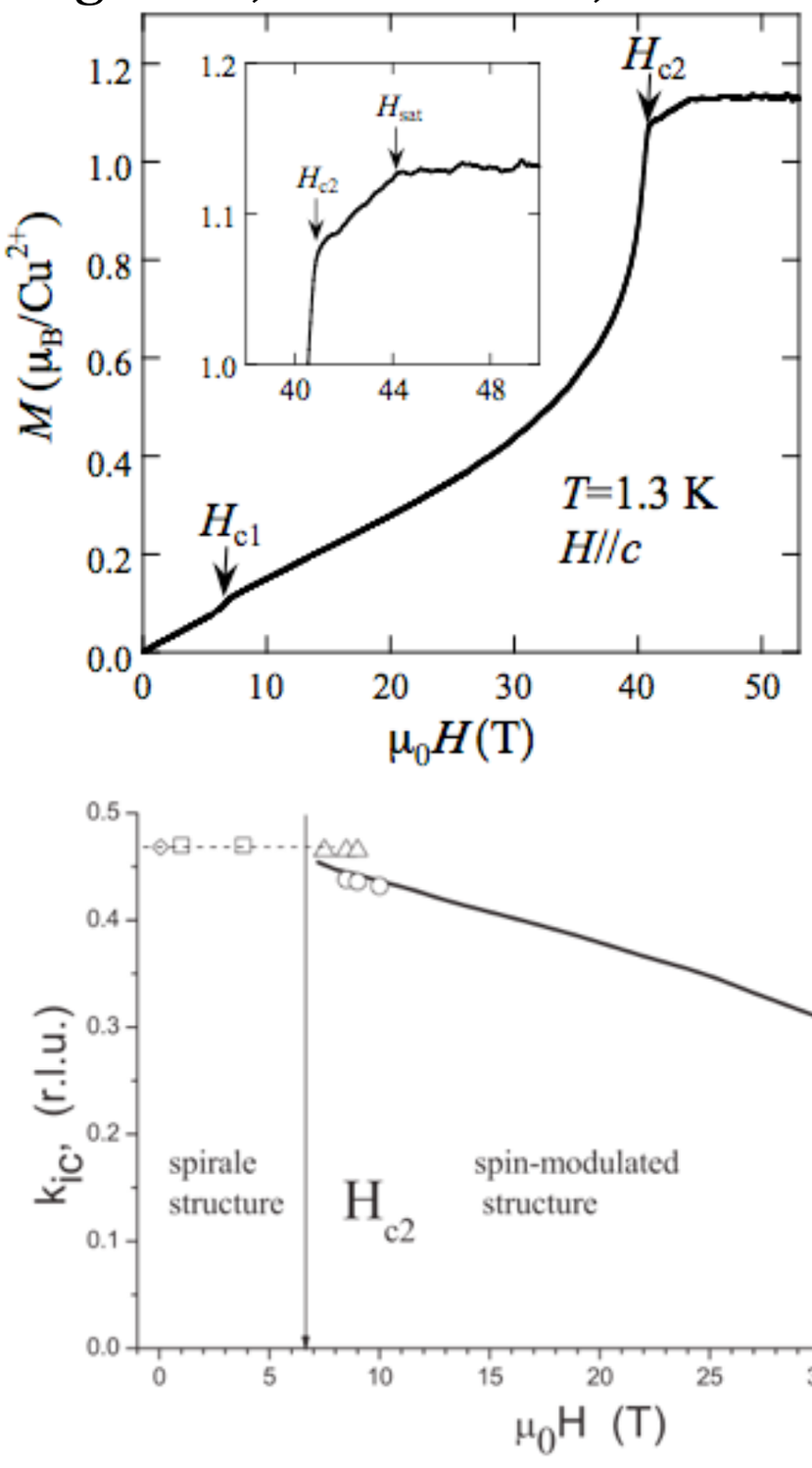
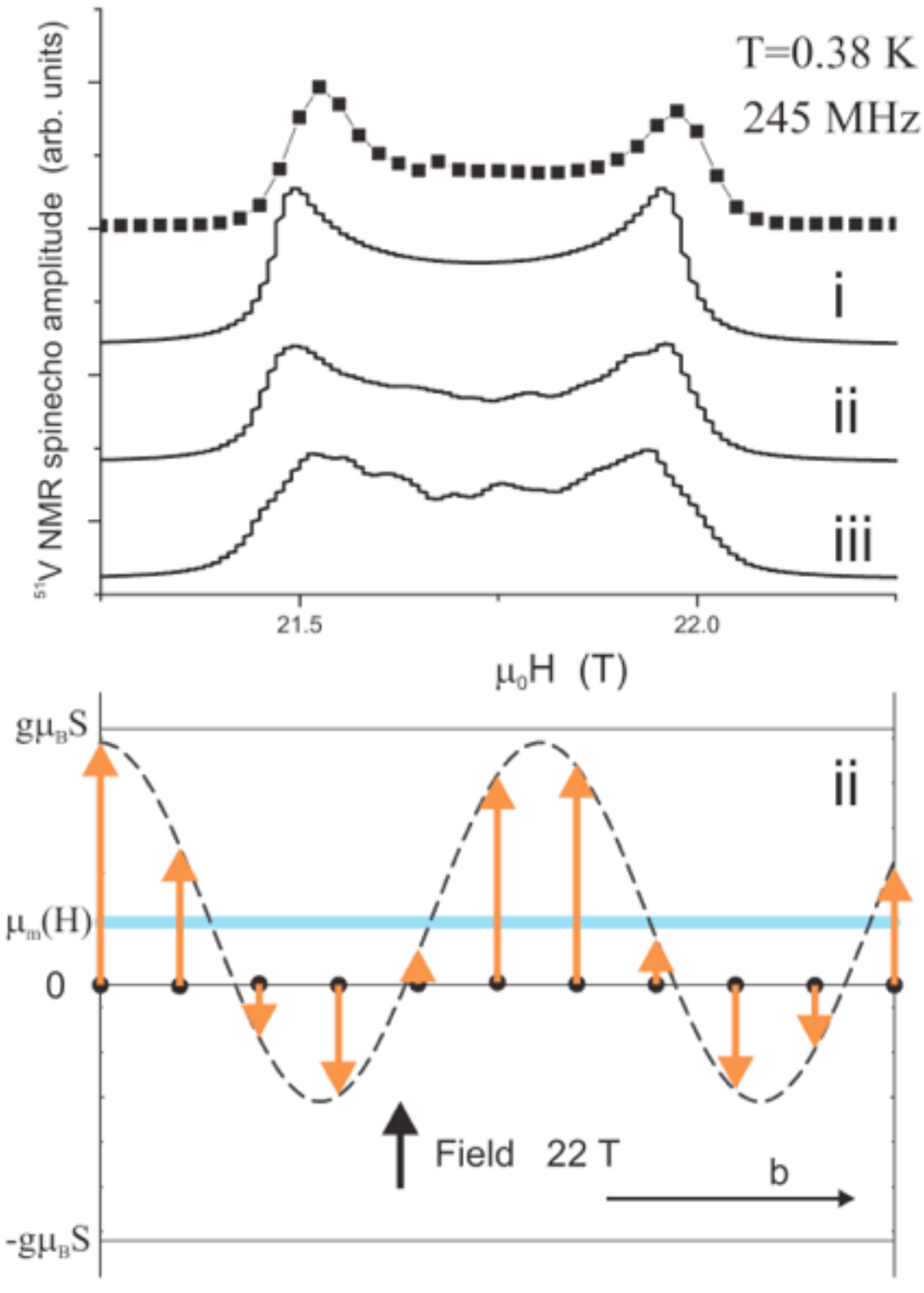


FIG. 2. Field dependence of the incommensurate wave vector k_{ic} for applied magnetic fields **H** ∥ **c** in $LiCuVO_4$. The open symbols

Buttgen et al 2012 PHYSICAL REVIEW B **85**, 214421 (2012)



Evidence of a Bond-Nematic Phase in $LiCuVO_4$

M. Mourigal,[1,2] M. Enderle,[1] B. Fåk,[3] R. K. Kremer,[4] J. M. Law,[4,*] A. Schneidewind,[5] A. Hiess,[1,†] and A. Prokofiev[6,7]

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, $\Delta S = 1$
NSF = no spin flip, $\Delta S = 0$

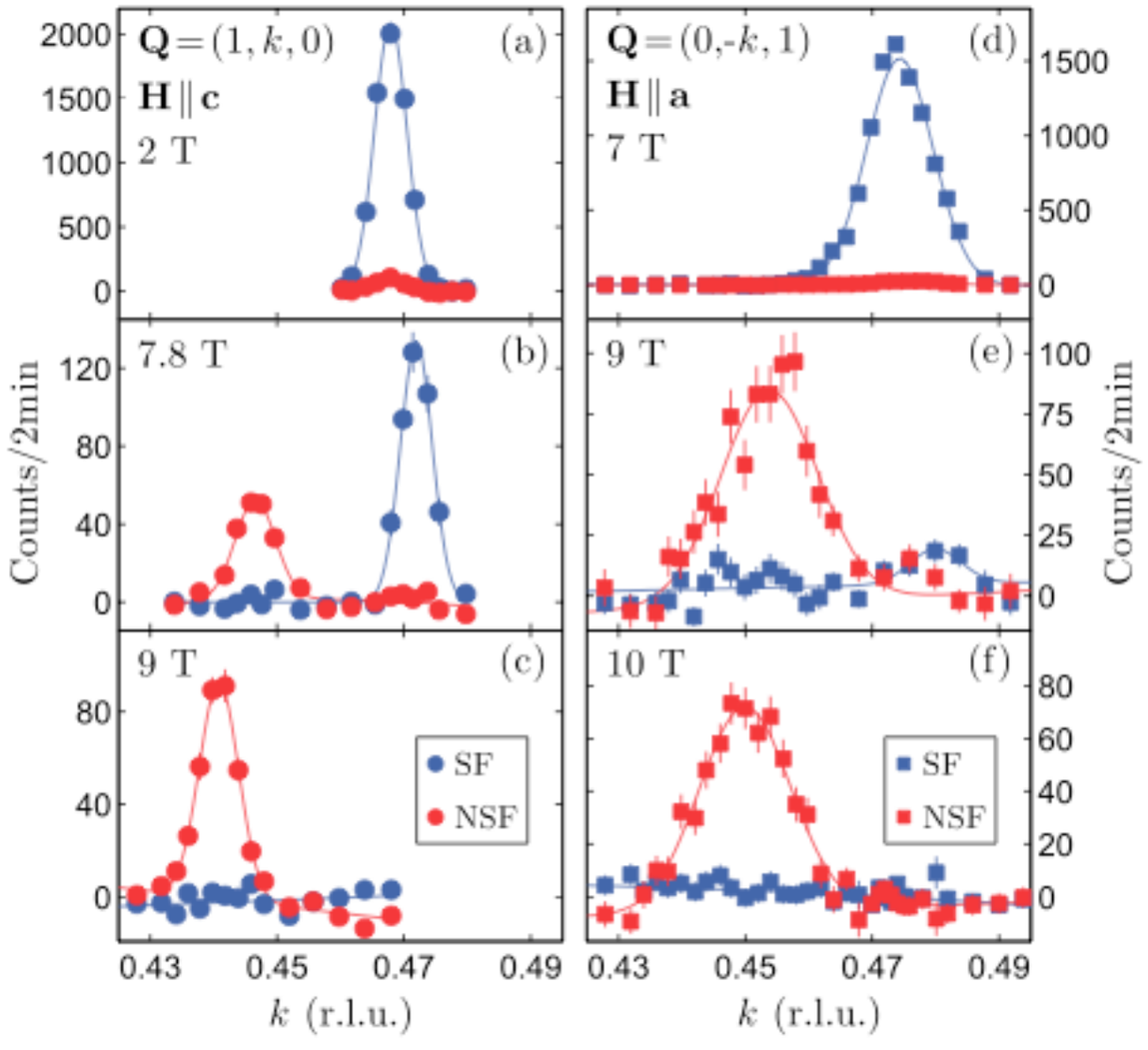
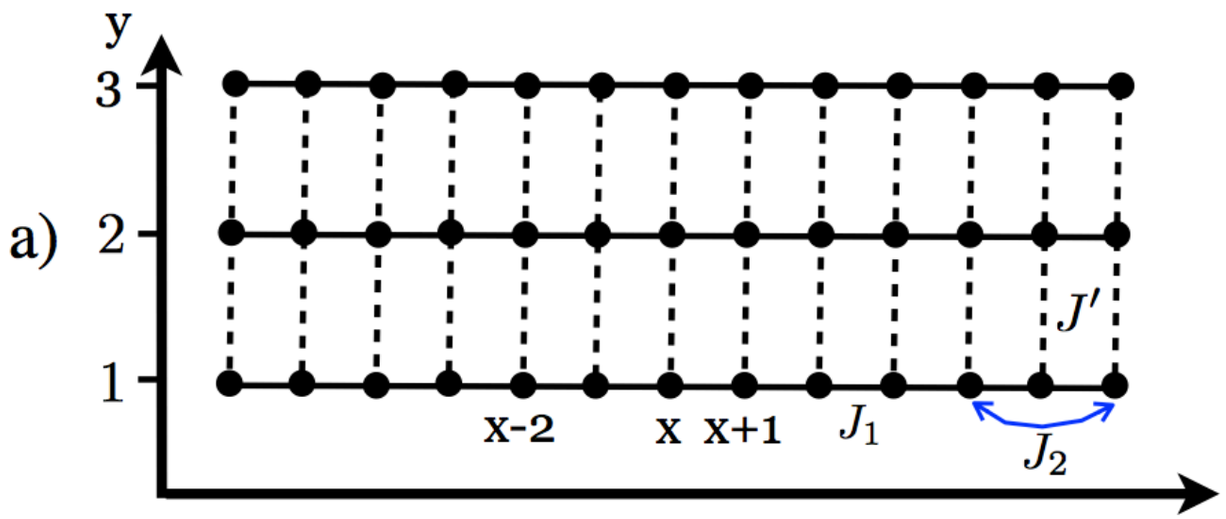
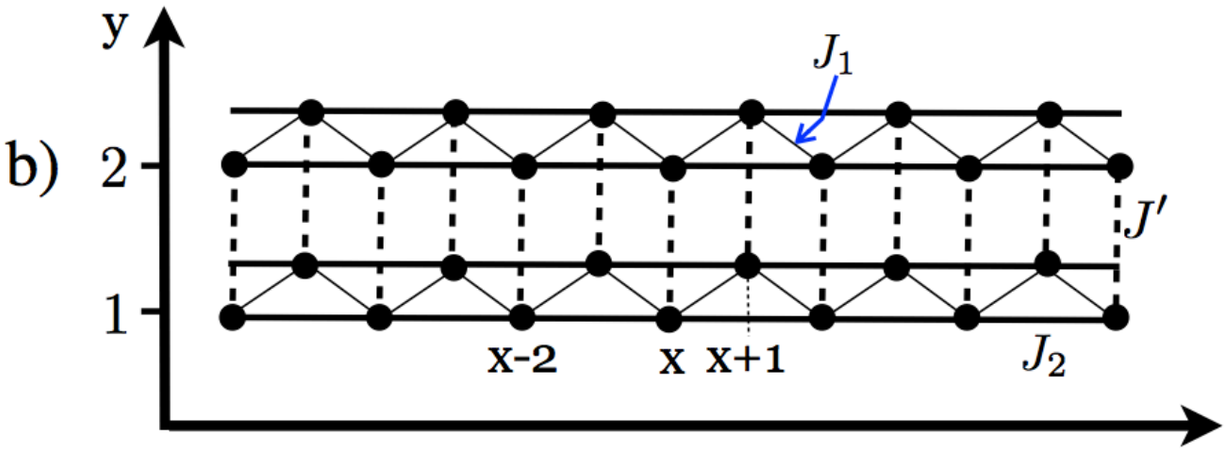


FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $\mathbf{Q} = (1, k_{IC}, 0)$ with $\mathbf{H}\|\mathbf{c}$ [left panels, (a)–(c)] and $\mathbf{Q} = (0, -k_{IC}, 1)$ with $\mathbf{H}\|\mathbf{a}$ [right panels, (d)–(f)].

Geometry (motivated by $LiCuVO_4$)



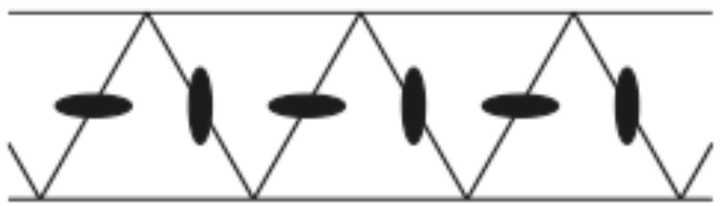


today:

$J_1 < 0$ (ferro)

$J_2 > 0$, $J' > 0$ (afm)

in magnetic field

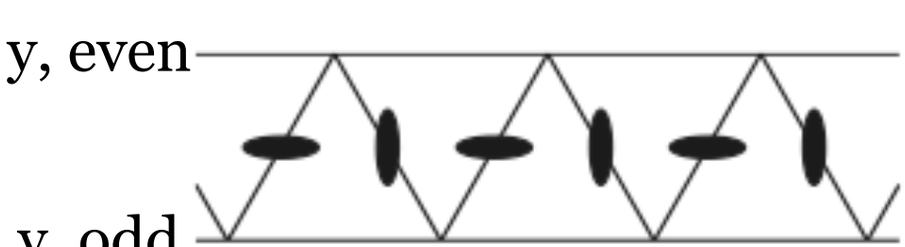


Chubukov 1991;
Kecke et al 2007;
Hikihara et al 2008;
Sudan et al 2009
Zhitomirsky and Tsunetsugu 2010
Sato et al 2013
Starykh and Balents, PRB 2014

Nematic chain

S^z-S^z (SDW) channel: in-chain $J_1 < 0$ gaps out relative mode $\varphi_y^- = (\varphi_{y,\mathrm{odd}} - \varphi_{y,\mathrm{even}})/\sqrt{2}$

$$H_{\mathrm{intra-chain}} = \sum_y \int dx J_1 \sin[\pi M] \cos[\frac{\sqrt{8}\pi}{\beta}\varphi_y^-]$$ local pair formation



J_1-J_2 chain

$$\langle S^+ \rangle = 0$$

$$S_y^+(x) \sim (-1)^x A_3 e^{i\frac{\beta}{\sqrt{2}}\theta_y^+(x)} e^{i(-1)^x \frac{\beta}{\sqrt{2}}\theta_y^-(x)}$$

quantum-disordered,
decays *exponentially:*
S^z = 1 excitations are gapped

$$T_y^+ = S_y^+(x) S_y^+(x+1) \sim e^{i\sqrt{2}\beta\theta_y^+(x)}$$

Standard (in 1d) power-law decay:
critical nematic spin correlations,
but U(1) *is preserved*

$$\langle T^+ \rangle = \langle S^+ S^+ \rangle \to 0$$

Physical picture: 1d magnon "superconductor"

Kolezhuk, Vekua (2005); Hikihara et al. (2008); Sato, Hikihara, Momoi (2013)

Inter-chain interaction $H_{\text{inter-chain}} = \sum_y \int dx\ \vec{S}_y \cdot \vec{S}_{y+1} \sim \sum_y \int dx\ S_y^+ S_{y+1}^- + S_y^z S_{y+1}^z$

Superconducting analogy: single-particle (magnon) tunneling between magnon superconductors is strongly suppressed at low energy (below the single-particle gap)

$$H_{\text{inter}}^{\perp} = \sum_y \int dx\ J' \langle S_y^+(x) S_{y+1}^-(x+1) \rangle_{\text{nematic ground state}} \to 0$$

Superconducting analogy: fluctuations generate two-magnon (Josephson = 4 spin interaction) tunneling between chains. They are generically weak, ~ $J_1(J'/J_1)^2$ << J' , but responsible for a true **two-dimensional nematic order**

$$H_{\text{nem}} \sim (J'^2/J_1) \sum_y \int dx\ [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

At the same time, density-density inter-chain interaction does not experience any suppression. It drives the system toward a **two-dimensional collinear SDW order**.

$$S_y^z = M - 2n_{\text{pair}} = M - \tilde{A}_1 e^{i\frac{\sqrt{2}\pi}{\beta}\varphi_y^+(x)} e^{ik_{\text{sdw}}x}$$

$$H_{\text{inter-chain}}^z = H_{\text{sdw}} \sim J' \sum_y S_y^z S_{y+1}^z \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi_y^+ - \varphi_{y+1}^+)]$$

Away from the saturation, **SDW** is more relevant [and stronger, via J' >> $(J')^2/J_1$] than the **nematic interaction**: **coupled** 1d **nematic chains order in a** 2d **SDW state.**

Simple scaling

$$H_{\rm nem} \sim (J'^2/J_1) \sum_y \int dx \; [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

- describes kinetic energy of magnon pairs, linear in magnon pair density $n_{\rm pair}$

$$H^z_{\rm inter-chain} = H_{\rm sdw} \sim J' \sum_y S^z_y S^z_{y+1} \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi^+_y - \varphi^+_{y+1})]$$

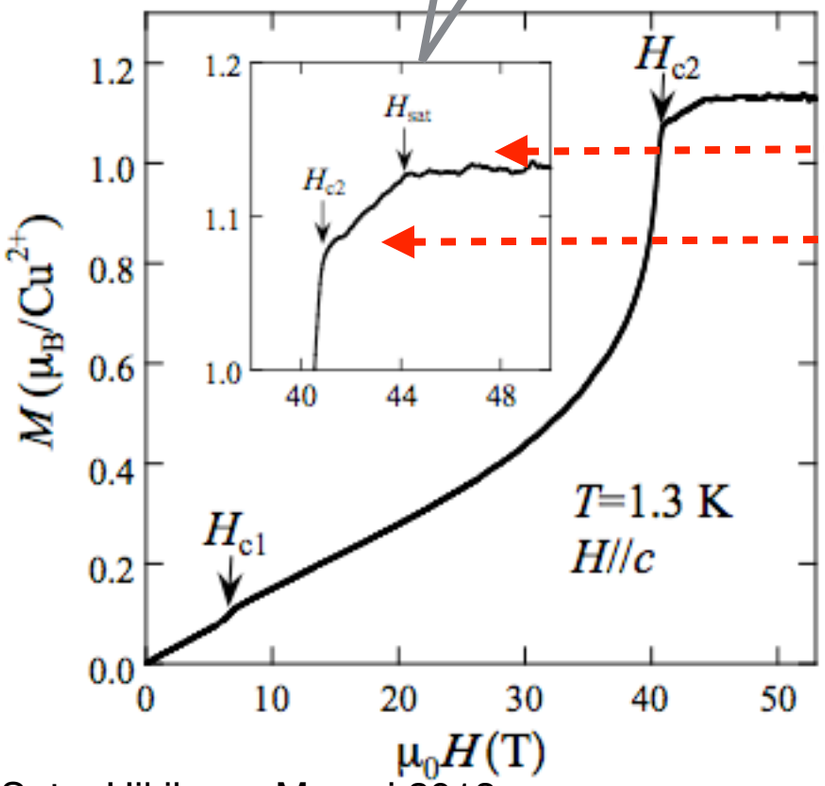
- describes potential energy of interaction between magnon pairs on neighboring chains, quadratic in magnon pair density $n_{\rm pair}$

- Competition $\frac{(J')^2}{J_1} n_{\rm pair} \sim J' n^2_{\rm pair}, \text{ hence } n_{\rm pair,c} \sim J'/J_1$

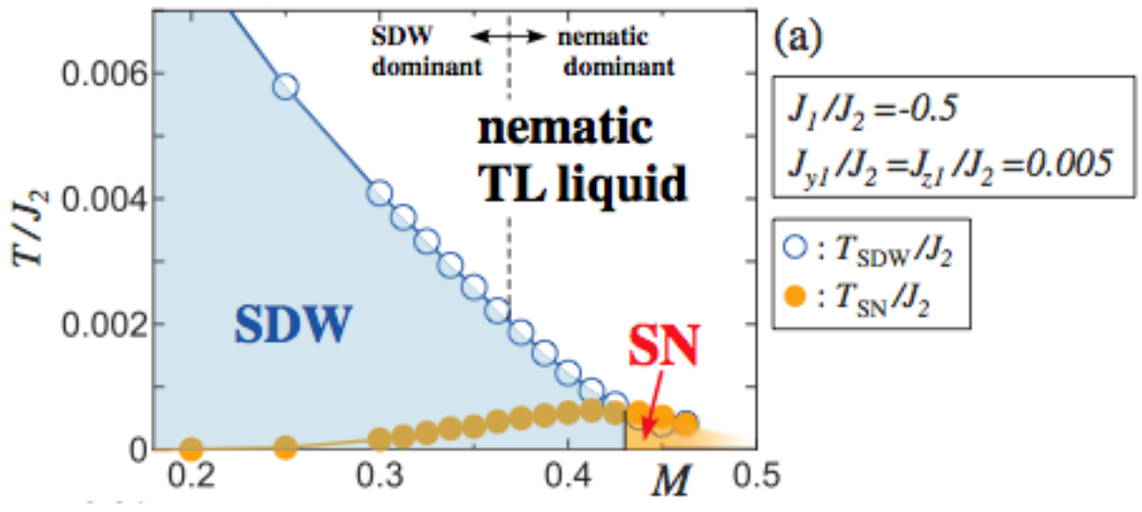
- Spin Nematic *near saturation*, for $n_{pair} < n_{pair,c}$
- SDW for $n_{pair} > n_{pair,c}$

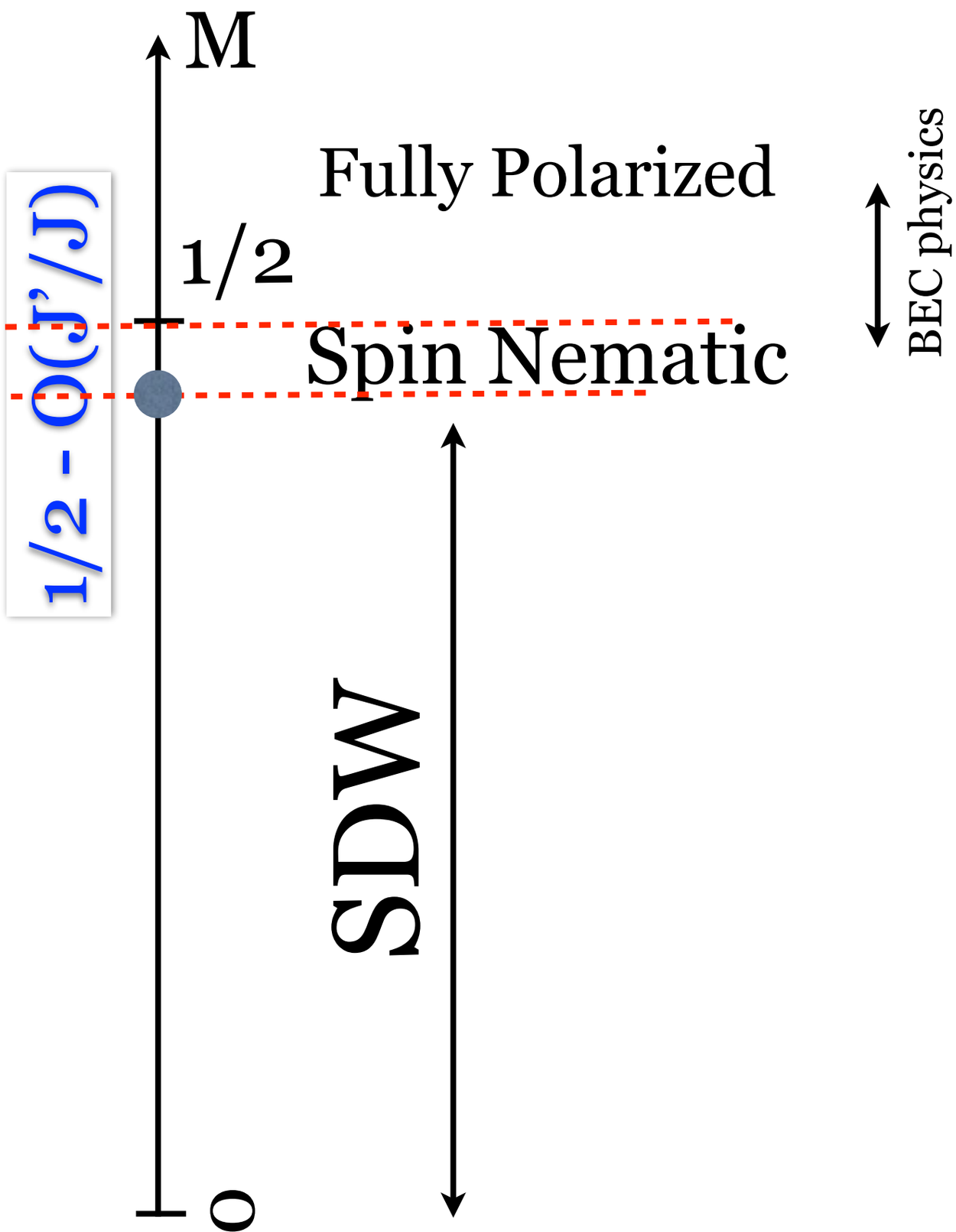
T=0 schematic phase diagram of weakly coupled **nematic** spin chains

Cautionary remark:
maybe impurity effect



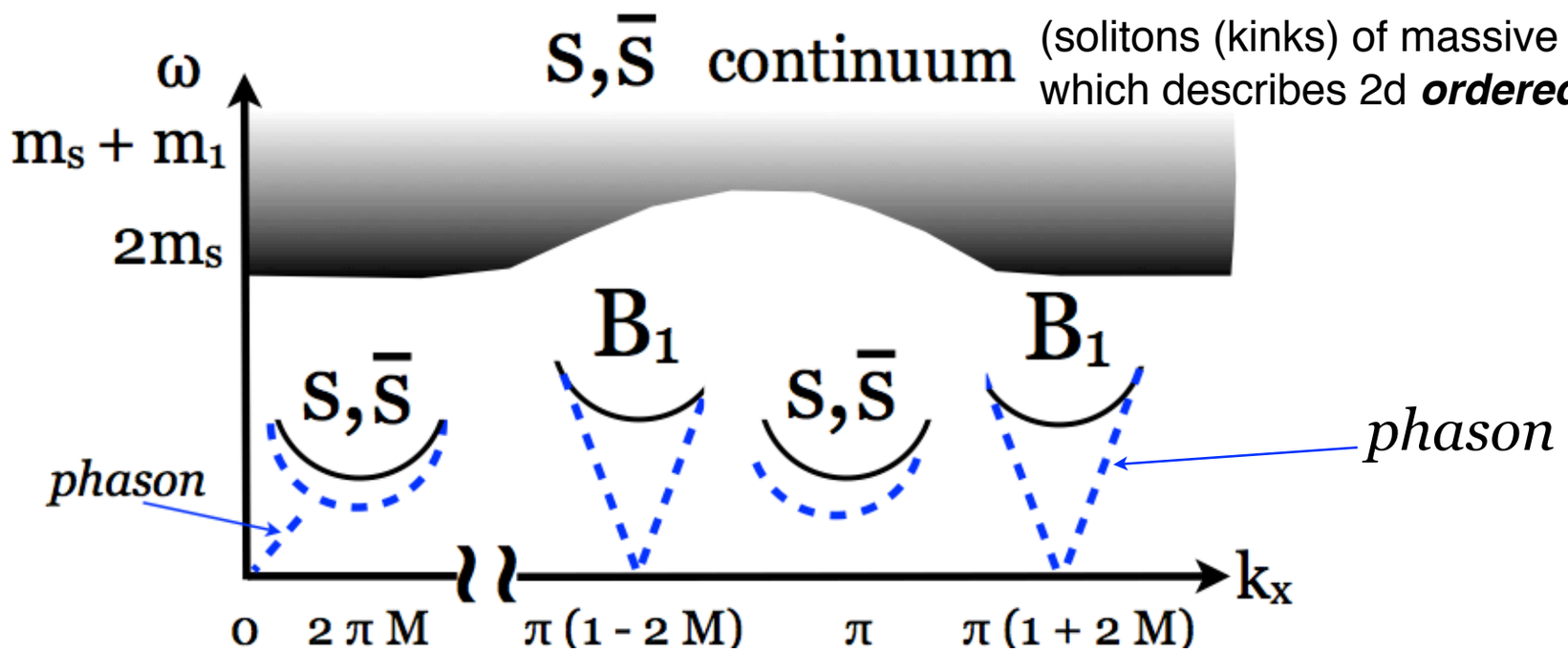
cf: Sato, Hikihara, Momoi 2013





Excitations (via spin-spin correlation functions)

- 2d SDW $\langle S^z(\mathbf{r})\rangle = M + \mathrm{Re}\left(\Phi e^{i\mathbf{k}_{\mathrm{sdw}}\cdot\mathbf{r}}\right)$
- preserves U(1) [with respect to magnetic field] -> hence NO transverse spin waves
- breaks translational symmetry -> longitudinal phason mode at $k_{sdw} = \pi(1\text{-}2M)$ and k=0

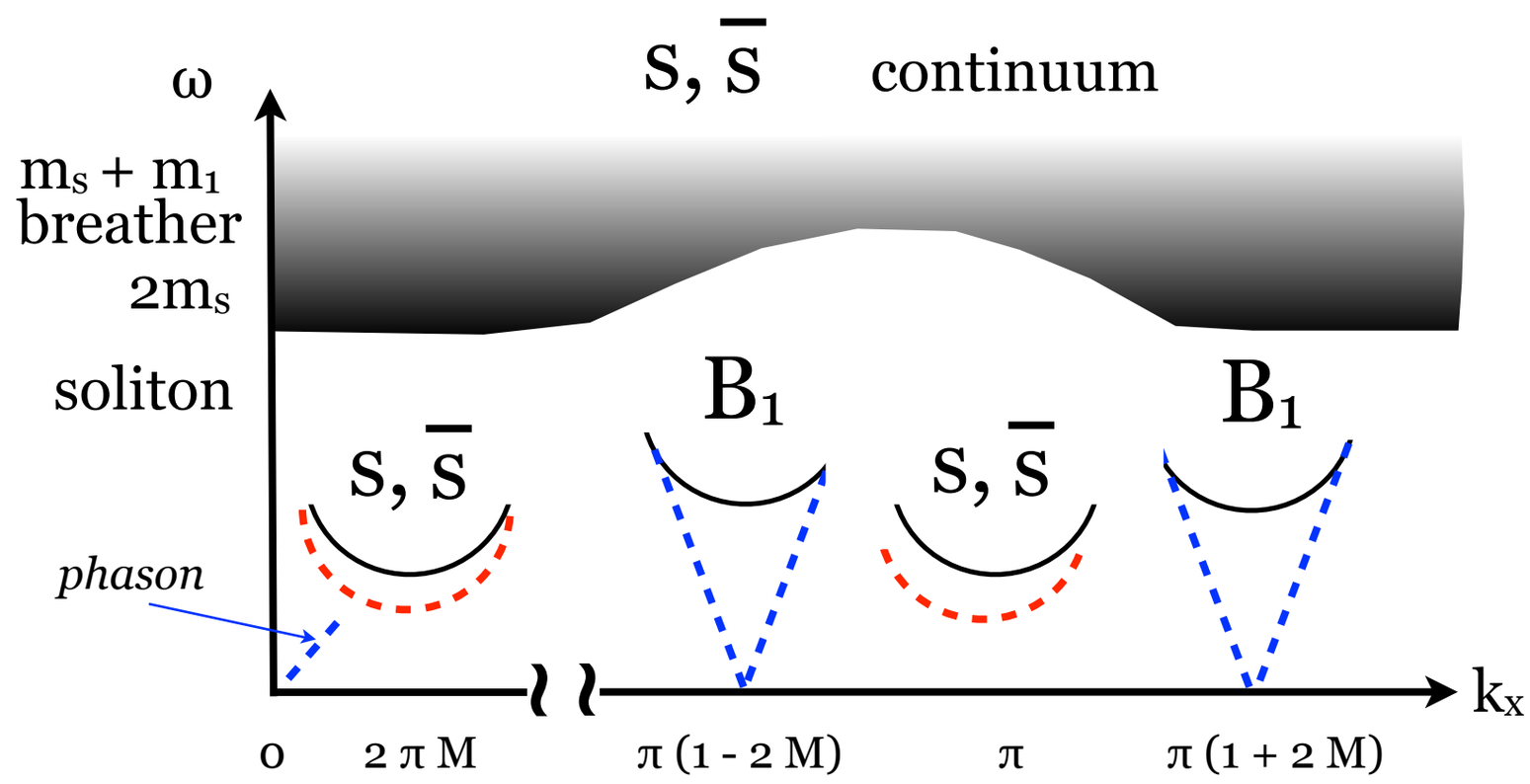


(solitons (kinks) of massive sine-Gordon model which describes 2d ***ordered*** state)

OS, Balents PRB 2014

bosonization, sine-Gordon + inter-chain RPA/LG analysis

2d SDW



gapped transverse excitations

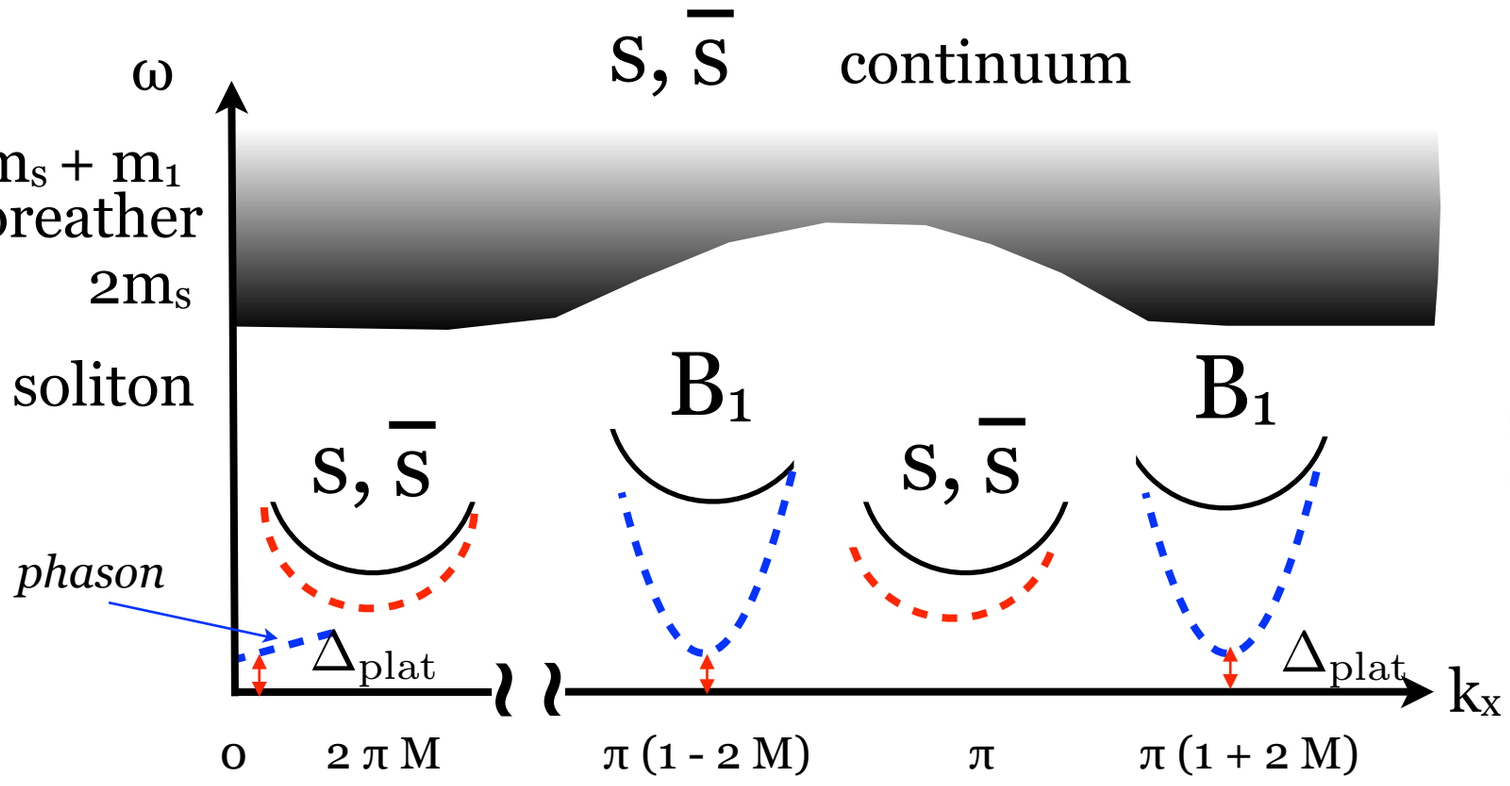
vanishing spectral weight as k_x -> 0 (anisotropic)

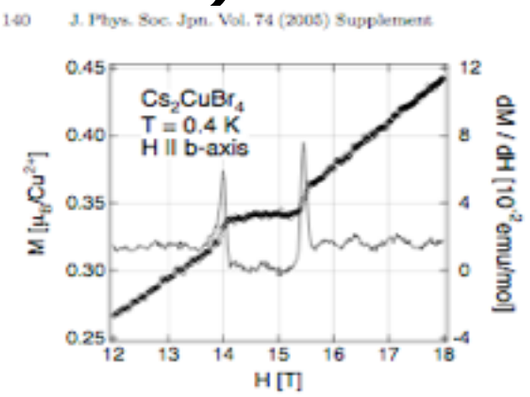
$$\chi_{2d}^{zz}(k_x,k_y,\omega) = \frac{vk_x^2/\beta^2}{v^2k_x^2 + v_\perp^2 k_y^2 - \omega^2},$$

phason (longitudinal)

$$\chi_{2d}^{zz}(q,\pi+q_y,\omega) \sim \frac{Z_{zz;2d}}{\left(v^2q^2 + v_\perp^2 q_y^2\right) - \omega^2}$$

2d commensurate SDW (such as 1/3 magnetization plateau)





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Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel b$ measured at $T = 0.4$ K in magnetic fields up to 20 T.

gapped transverse excitations

gapped phason

$$\chi^{zz}_{2d;plat}(q, \pi + q_y, \omega) \sim \frac{Z_{zz;2d}}{\Delta^2_{plat} + \left(v^2 q^2 + v^2_\perp q^2_y\right) - \omega^2}$$

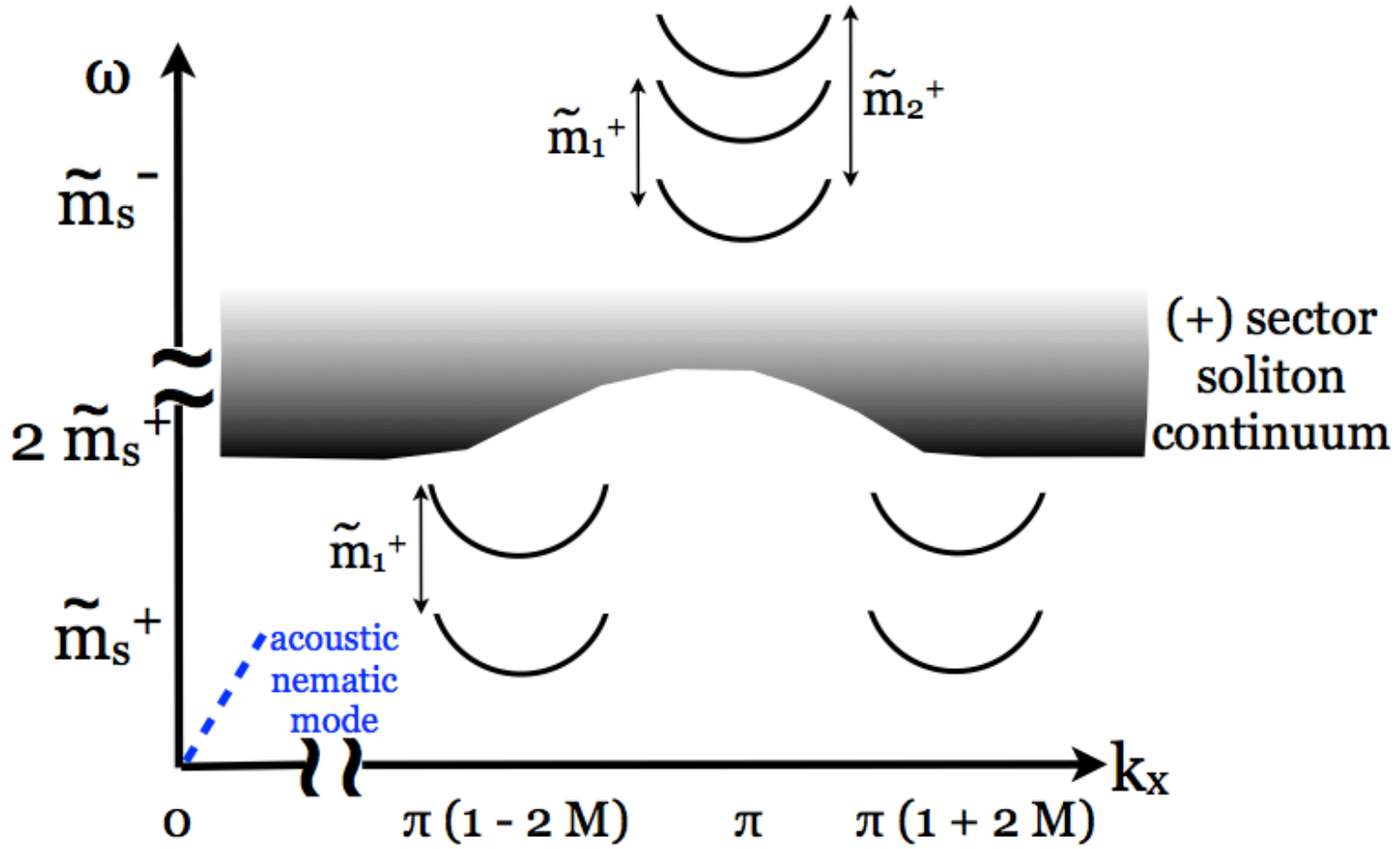
Excitations (via spin-spin correlation functions)

- 2d Spin Nematic $\langle S^+(r)S^+(r')\rangle \sim \Psi \neq 0$

- breaks U(1) but ΔS=1 excitations are gapped (magnon superconductor) $\langle S^+(\mathbf{r})\rangle = 0$

- gapless density fluctuations at k=0

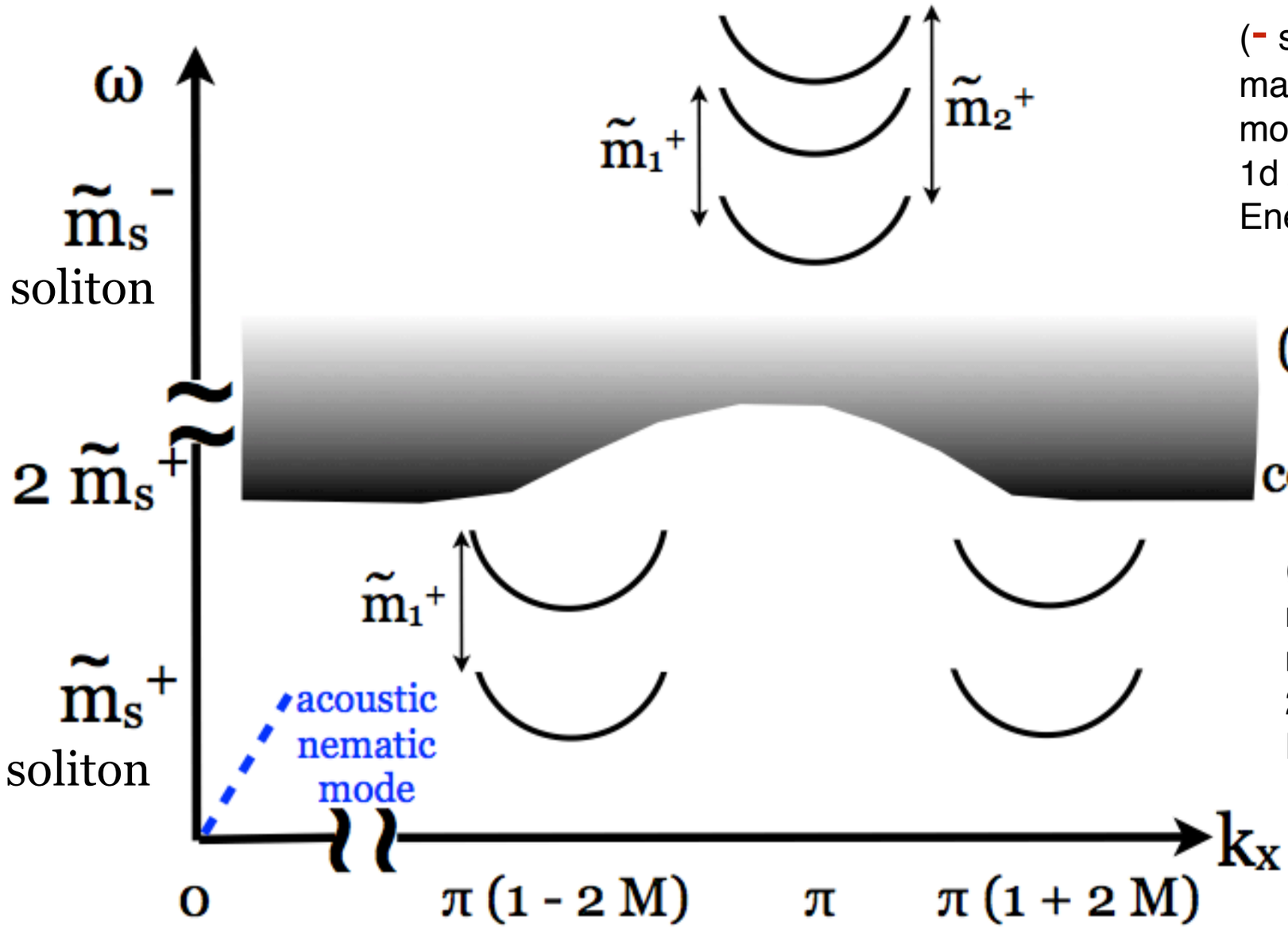
(- sector: solitons of massive sine-Gordon model describing 1d ***zig-zag chain.***)
Energy scale J_1



(+ sector: solitons of massive sine-Gordon model which describes 2d ***ordered*** state.)
Energy scale $(J')^2/J_1$

OS, Balents PRB 2014

2d Spin Nematic



(- sector: solitons of massive sine-Gordon model describing 1d ***zig-zag chain.***)
Energy scale J_1

(+ sector: solitons of massive sine-Gordon model which describes 2d ***ordered*** state.)
Energy scale $(J')^2/J_1$

$$\mathrm{Im}\chi^{zz}_{2d}(k,\omega) \sim \frac{\rho_0 \epsilon_k}{\omega_B(k)} \delta[\omega - \omega_B(k)]$$

vanishing spectral weight as **k** -> 0 (isotropic)

Conclusions

- Interesting magnetically ordered states: SDW and Spin Nematic
  - Gapped $\Delta S=1$ excitations (No usual spin waves)
  - Linearly-dispersing ***phason*** mode with $\Delta S=0$ in **2d** SDW
  - SDW naturally sensitive to structural disorder
  - Linearly-dispersing ***magnon density*** waves in **2d** Spin Nematic
  - analogy with superconductor/charge density wave competition