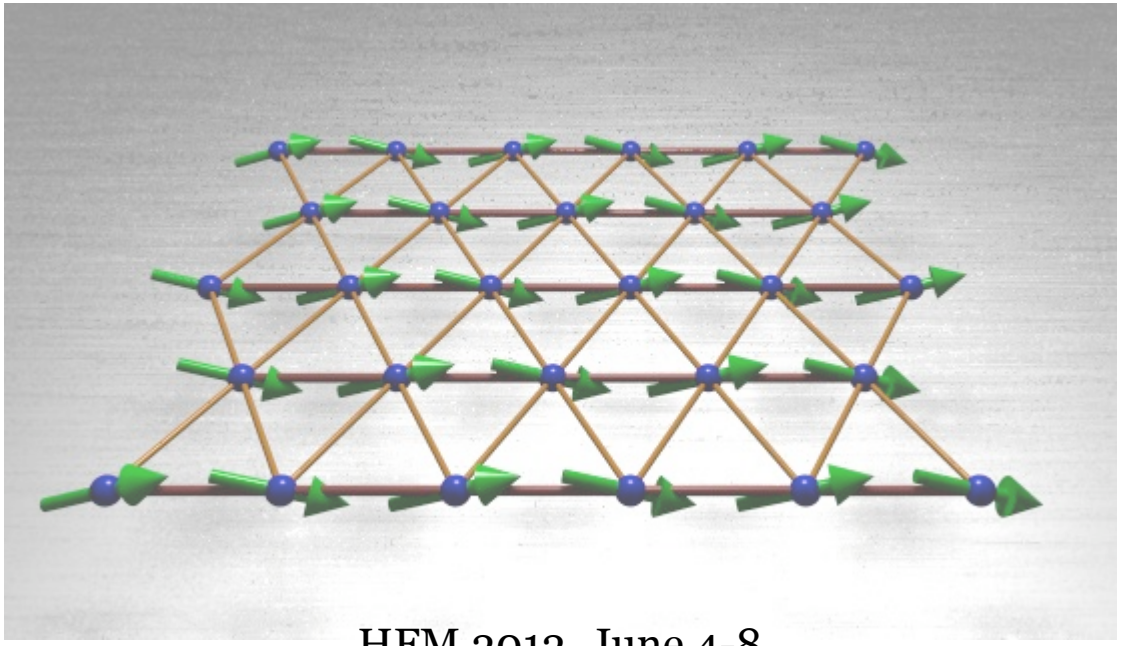
Fate of 1/3-magnetization plateau in spatially anisotropic triangular lattice antiferromagnets

Oleg Starykh, University of Utah

collaborators:
Leon Balents, Ru Chen, Hyejin Ju, Hong-Chen Jiang (KITP and UCSB)
Jason Alicea (UCI), Andrey Chubukov (U Wisconsin)



HFM 2012, June 4-8

frustration

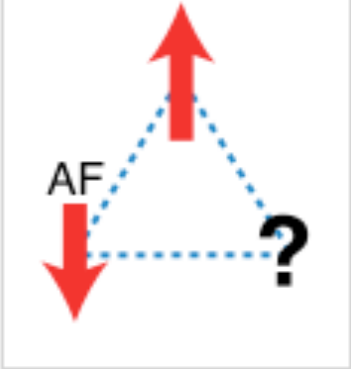


Figure 1: Antiferromagnetically interacting spins in a triangular arrangement

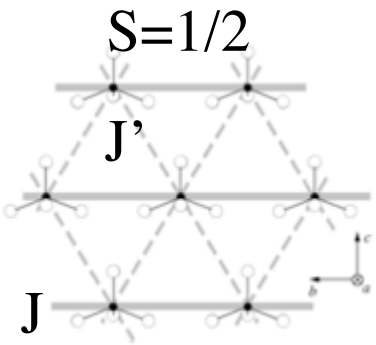
life example:

	Session T4 - Catherine Kallin, McMaster U, Presiding	
15:15 -- 15:45	**Oleg Starykh** Fate of 1/3 magnetization plateau in spatially anisotropic triangular lattice antiferromagnets	U of Utah, USA
15:30 -- 15:45	**Jeffrey Quilliam** - Spin liquid ground state and field-induced gap in S=1/2 triangular lattice system Ba3CuSb2O9	U de Paris Sud, France

Outline

- motivation: Cs_2CuBr_4 , Cs_2CuCs_4 and other materials
- classical antiferromagnet in a field: entropic selection
  - spatial anisotropy - high-T stabilization of the plateau
- Quantum model in magnetic field
  - DMRG (3-leg ladder)
  - Various analytical limits
    - large-S analysis of interacting spin waves
    - weakly coupled spin chains
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- Summary

Cs_2CuBr_4, Cs_2CuCl_4

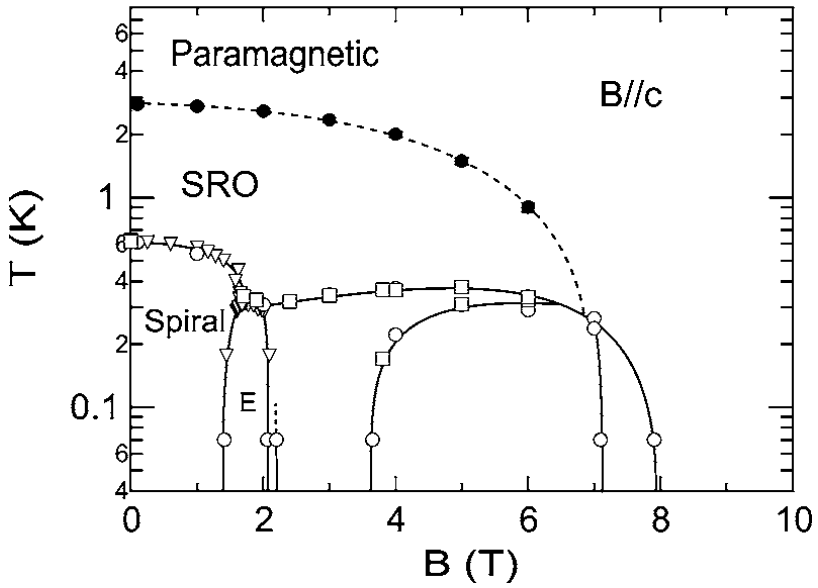


- S=1/2 quantum triangular antiferromagnets with small exchange
  - complex evolution in field!

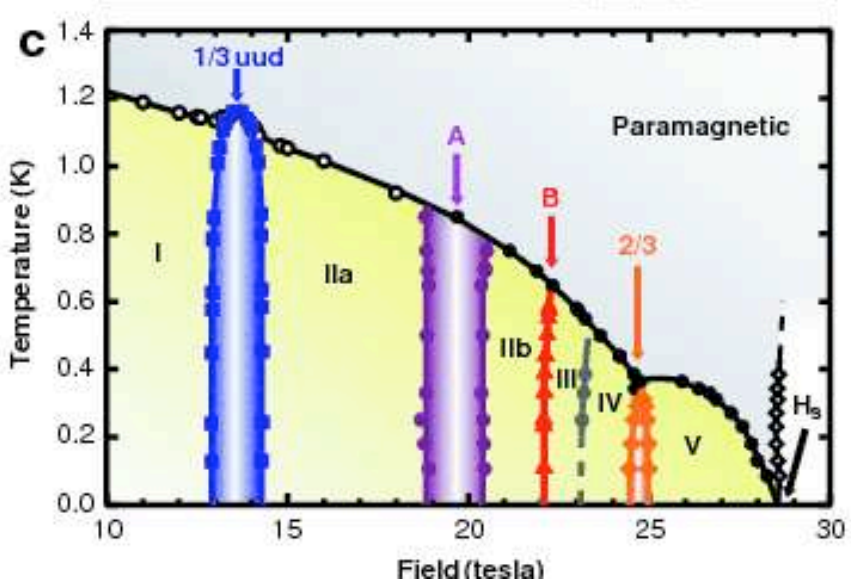
★10 **phases** instead of 3 expected!?

Cs_2CuCl_4

Y. Tokiwa et al, 2006



Cs_2CuBr_4



Fortune *et al*, 2009

M=1/3 magnetization plateau in Cs_2CuBr_4

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.75**
but not Cs_2CuCl_4 [**J'/J = 0.34**]

no plateau classically
(at low T)

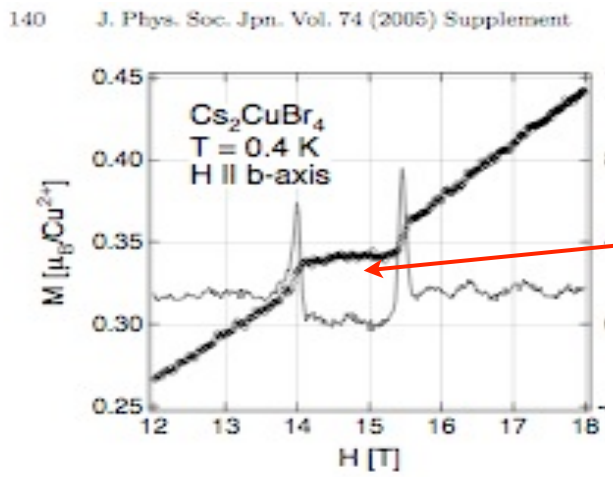
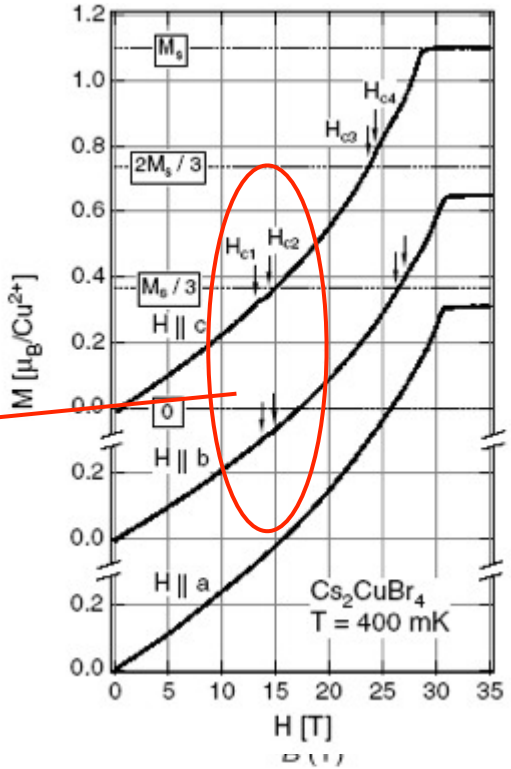


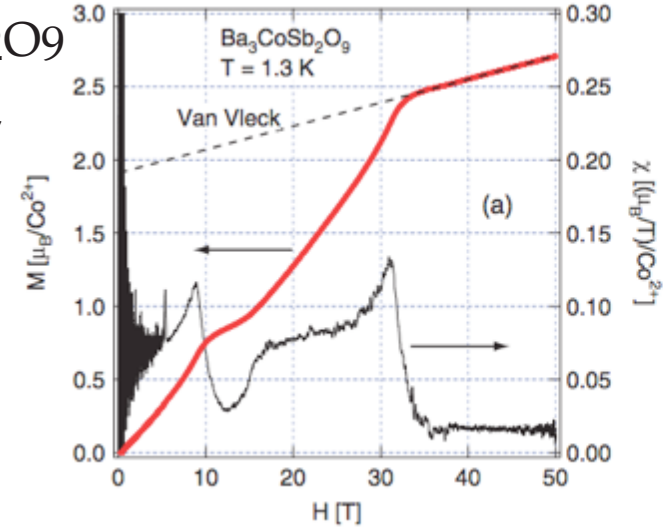
Fig. 8. The magnetization curve and dM/dH versus H measured at $T = 0.4$ K in magnetic fields up to 20 T

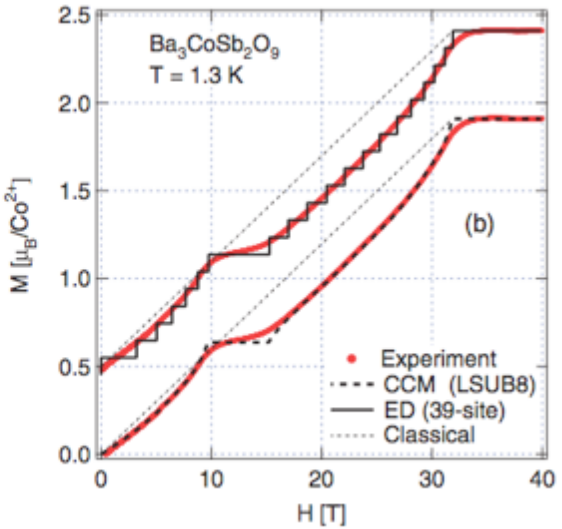


- $Ba_3CoSb_2O_9$

Shirata et al,
PRL 2012

J' = J ?

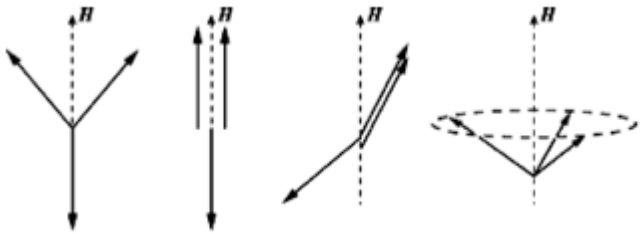




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Classical isotropic Δ AFM in magnetic field T=0



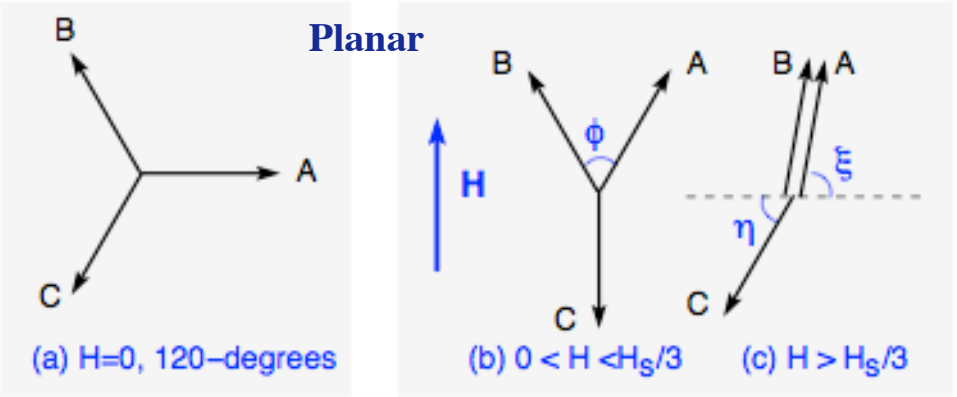
co-planar cone

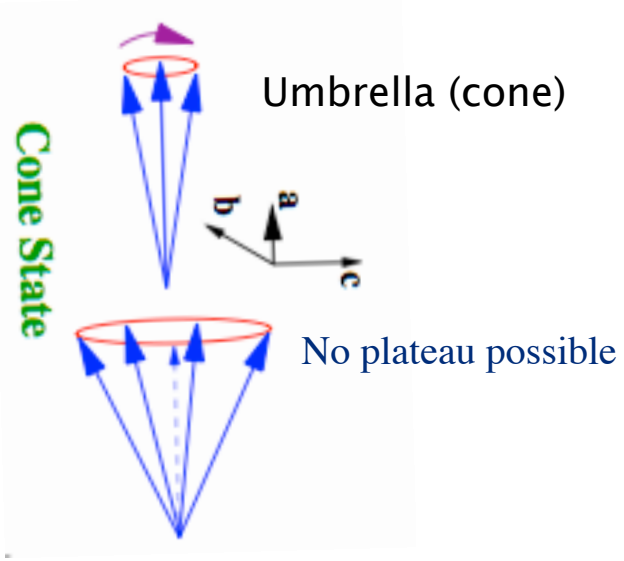
- Zero magnetic field: spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

$$H = J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \sum_i \vec{h}\cdot\vec{S}_i$$

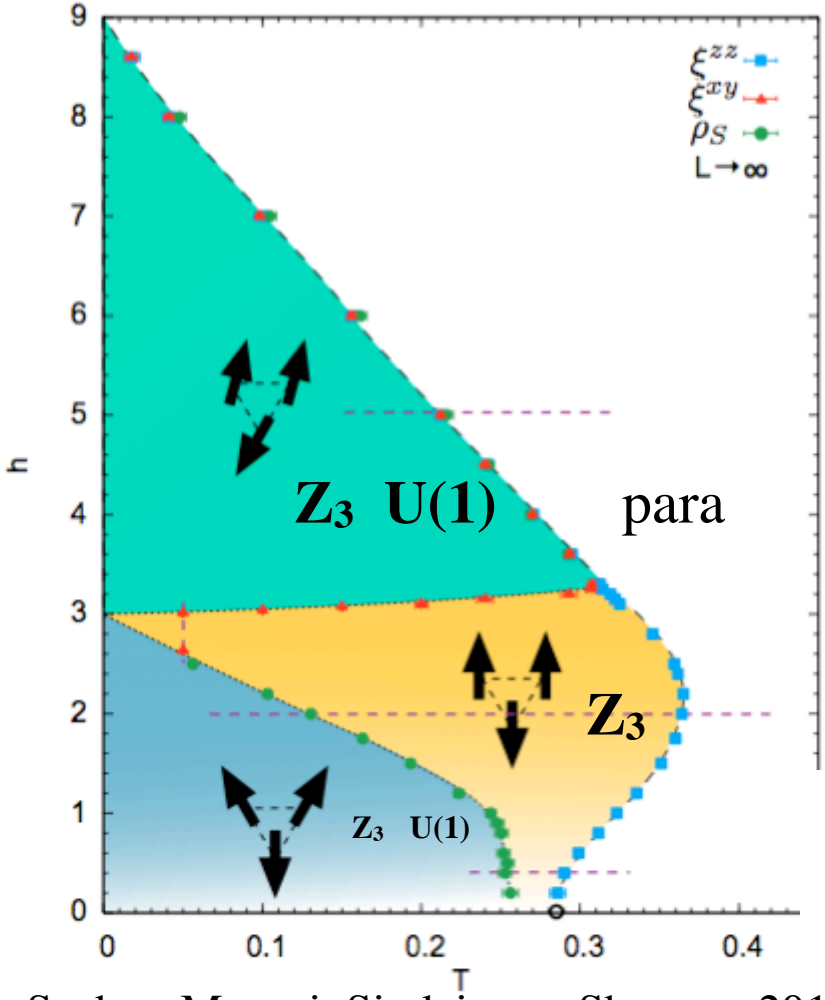
$$H = \frac{1}{2}J\sum_{\triangle}\Big(\sum_{i\in\triangle}\vec{S}_i - \frac{\vec{h}}{3J}\Big)^2$$

- all states with $\vec{S}_{i1}+\vec{S}_{i2}+\vec{S}_{i3} = \frac{\vec{h}}{3J}$ form the lowest-energy manifold
  - 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about **h**)

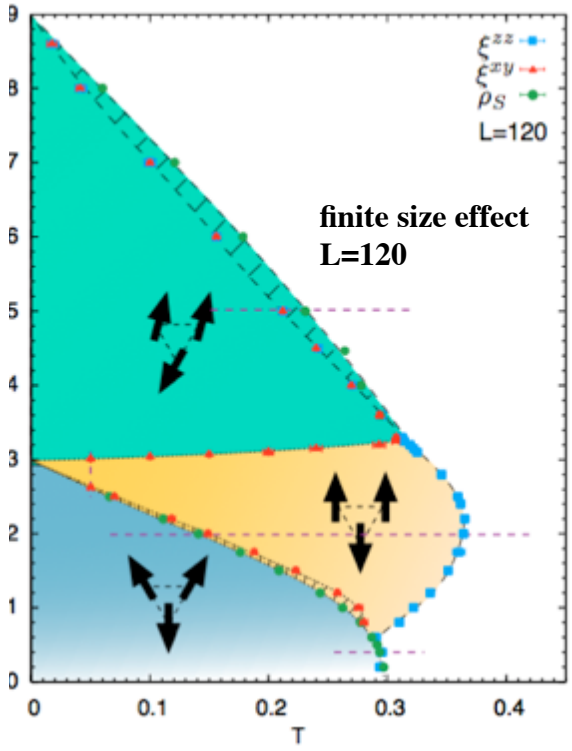


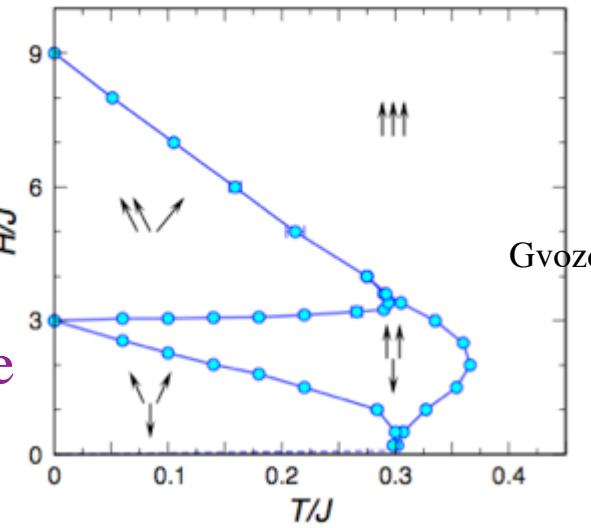


Phase diagram of the classical model: Monte Carlo (Kawamura and Miyashita, 1985)

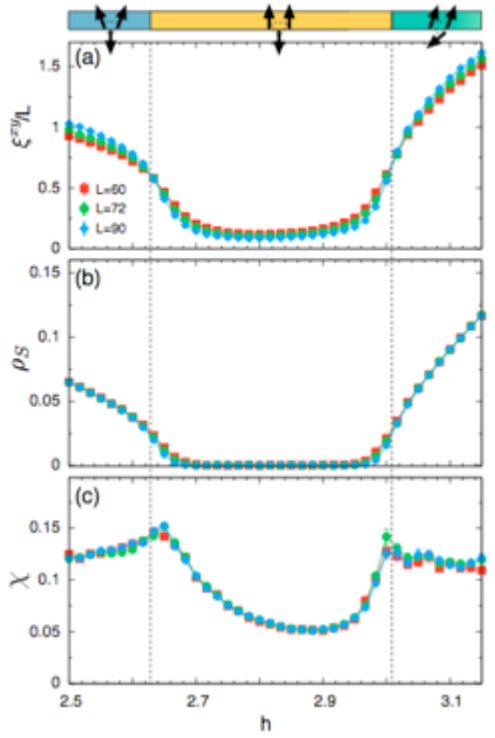


Seabra, Momoi, Sindzingre, Shannon 2011





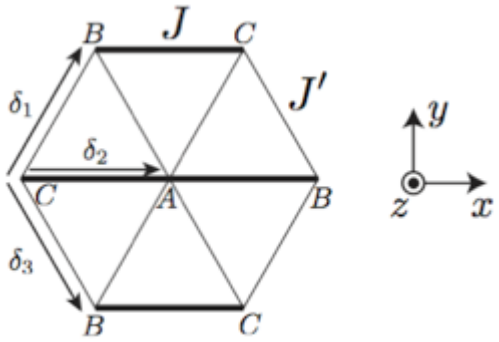
Gvozdikova, Melchy, Zhitomirsky 2010



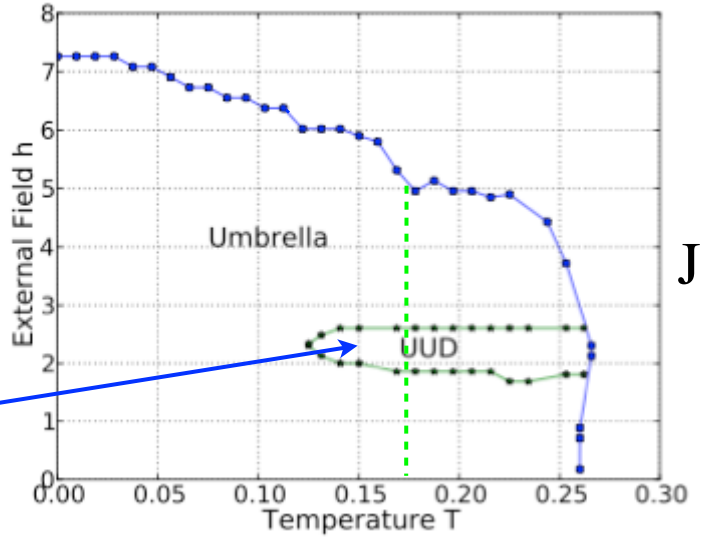
Entropic selection: planar states are favored by thermal fluctuations

Effect of spatial anisotropy J' < J: energy vs entropy

Griset, Head, Alicea, OS 2011

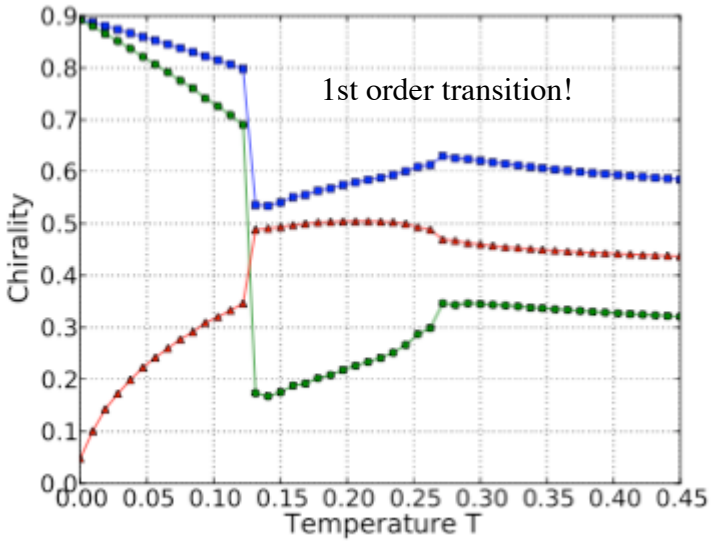


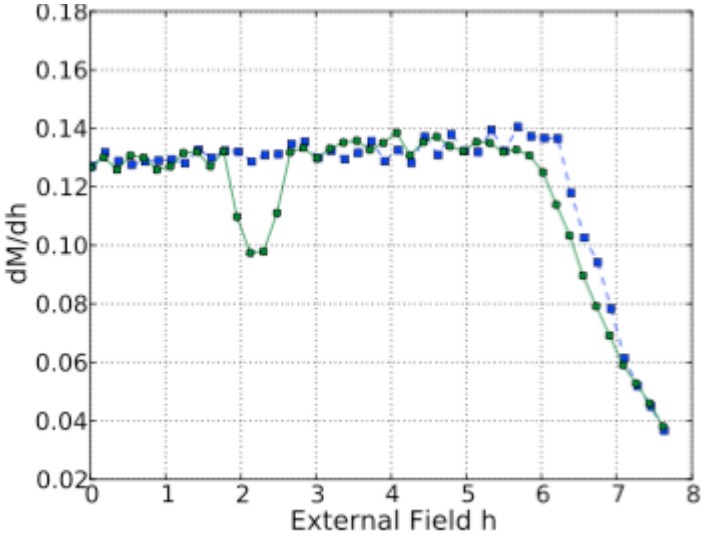
Umbrella state:
favored classically,
energy gain $(J-J')^2/J$



J' = 0.765 J

similar with Pomeranchuk effect (He3):
crystal-like UUD is stabilized at high temperature





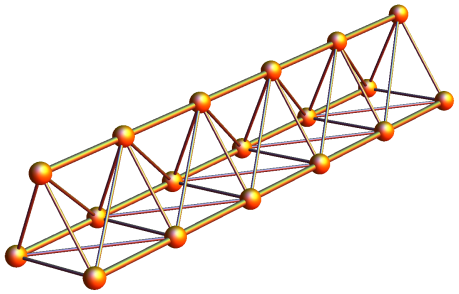
$$\kappa = \frac{2}{3\sqrt{3}}\frac{1}{N}\sum_{\mathbf{r}}\left(\mathbf{S}_{\mathbf{r}}\times\mathbf{S}_{\mathbf{r}+\delta_1}+\right.$$
$$\left.+\mathbf{S}_{\mathbf{r}+\delta_1}\times\mathbf{S}_{\mathbf{r}+\delta_2}+\mathbf{S}_{\mathbf{r}+\delta_2}\times\mathbf{S}_{\mathbf{r}}\right)$$

Low T: energetically preferred umbrella
High T: entropically preferred UUD
Y and V are less stable.

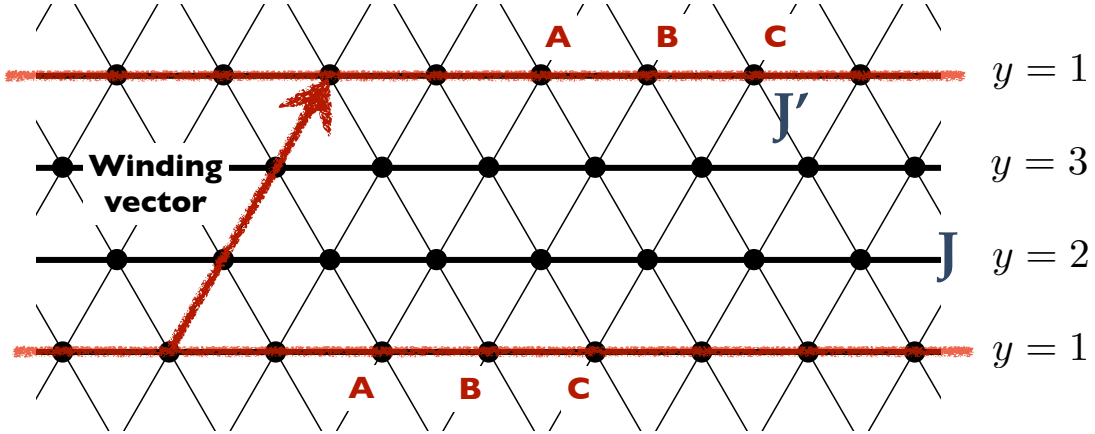
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Model







$y = 1$

$y = 3$

$y = 2$

$y = 1$

Periodic boundary conditions along y in numerical studies

- Hamiltonian

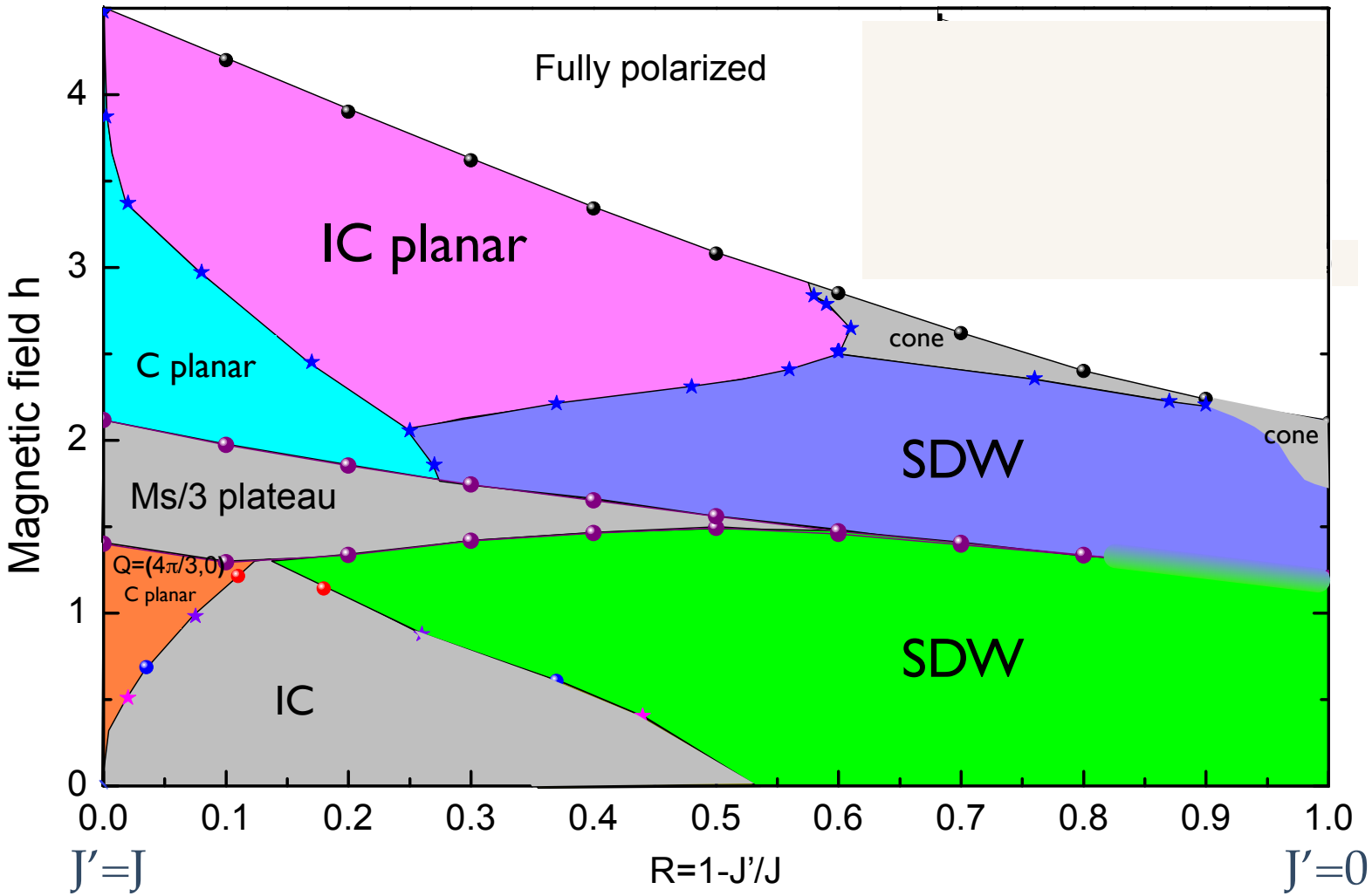
$$H = \sum_{x,y} J S_{x,y} \cdot S_{x+1,y} + J' \left(S_{x,y} \cdot S_{x,y+1} + S_{x,y} \cdot S_{x-1,y+1} \right)$$

$$-h \sum_{x,y} S^z_{x,y}$$

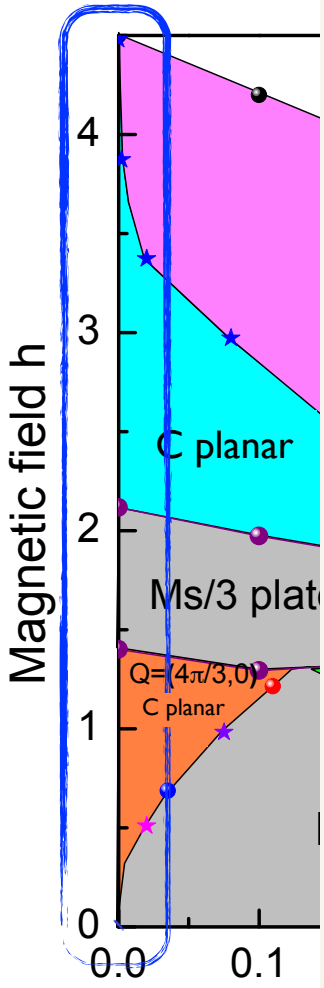
Cs_2CuCl_4: $J'/J = 0.34$

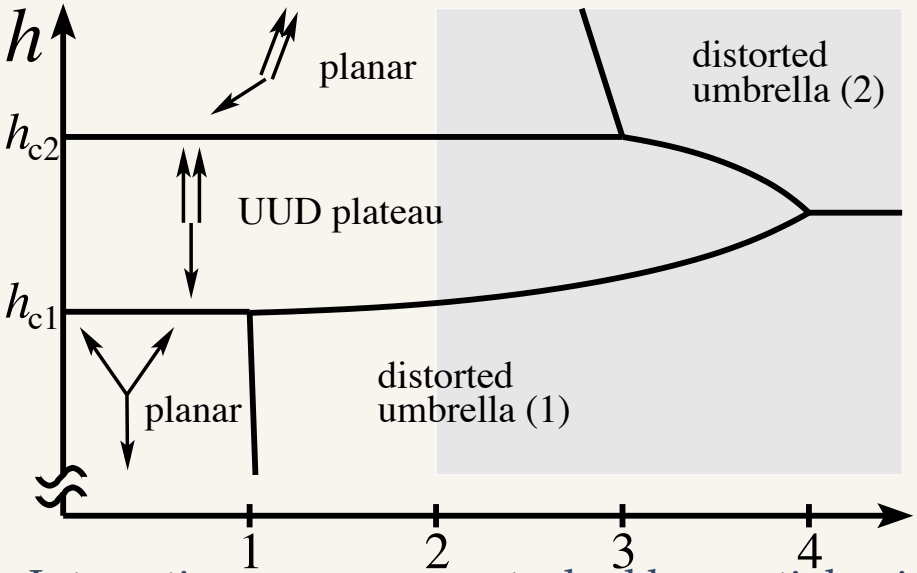
Cs_2CuBr_4: $J'/J = 0.5\text{-}0.7$

Phase diagram (3 leg ladder)



Isotropic case: quantum fluctuations select co-planar states





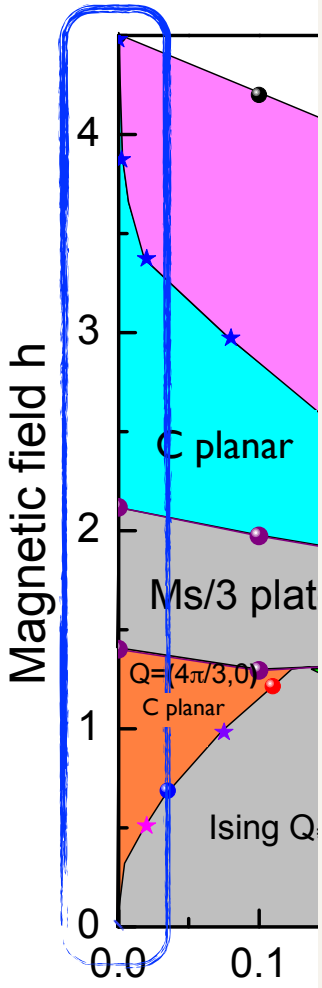
$$\delta \sim S(1 - J'/J)^2$$

Interacting magnons perturbed by spatial anisotropy

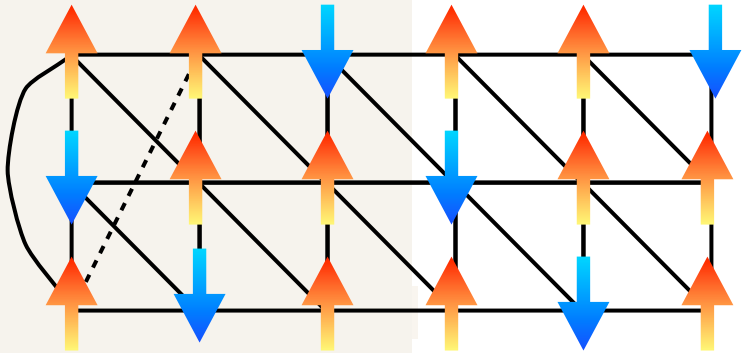
isotropic case: Chubukov+Golosov, 1991; Alicea, Chubukov, Starykh, 2008

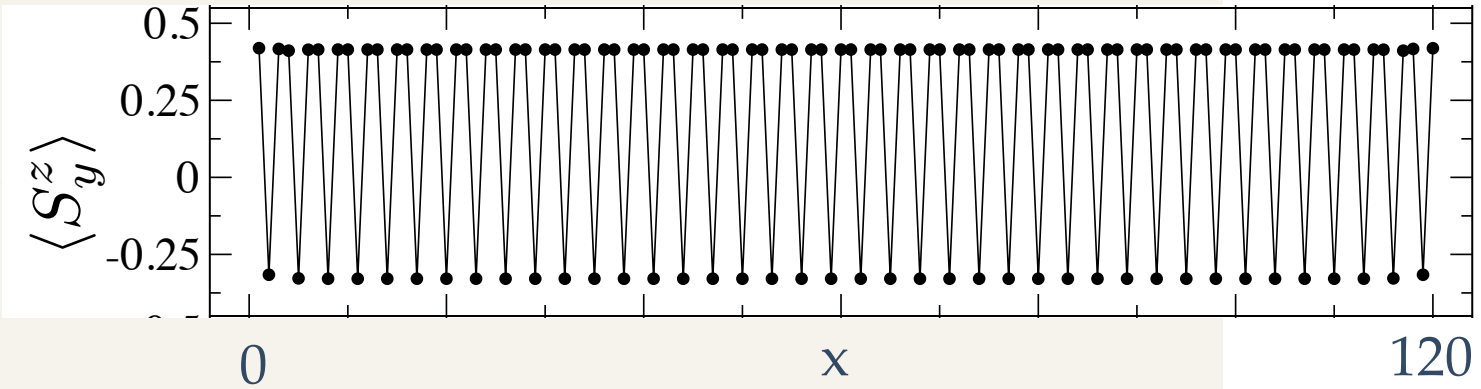
UUD preserves U(1) symmetry. Gapped spin waves

Plateau phase



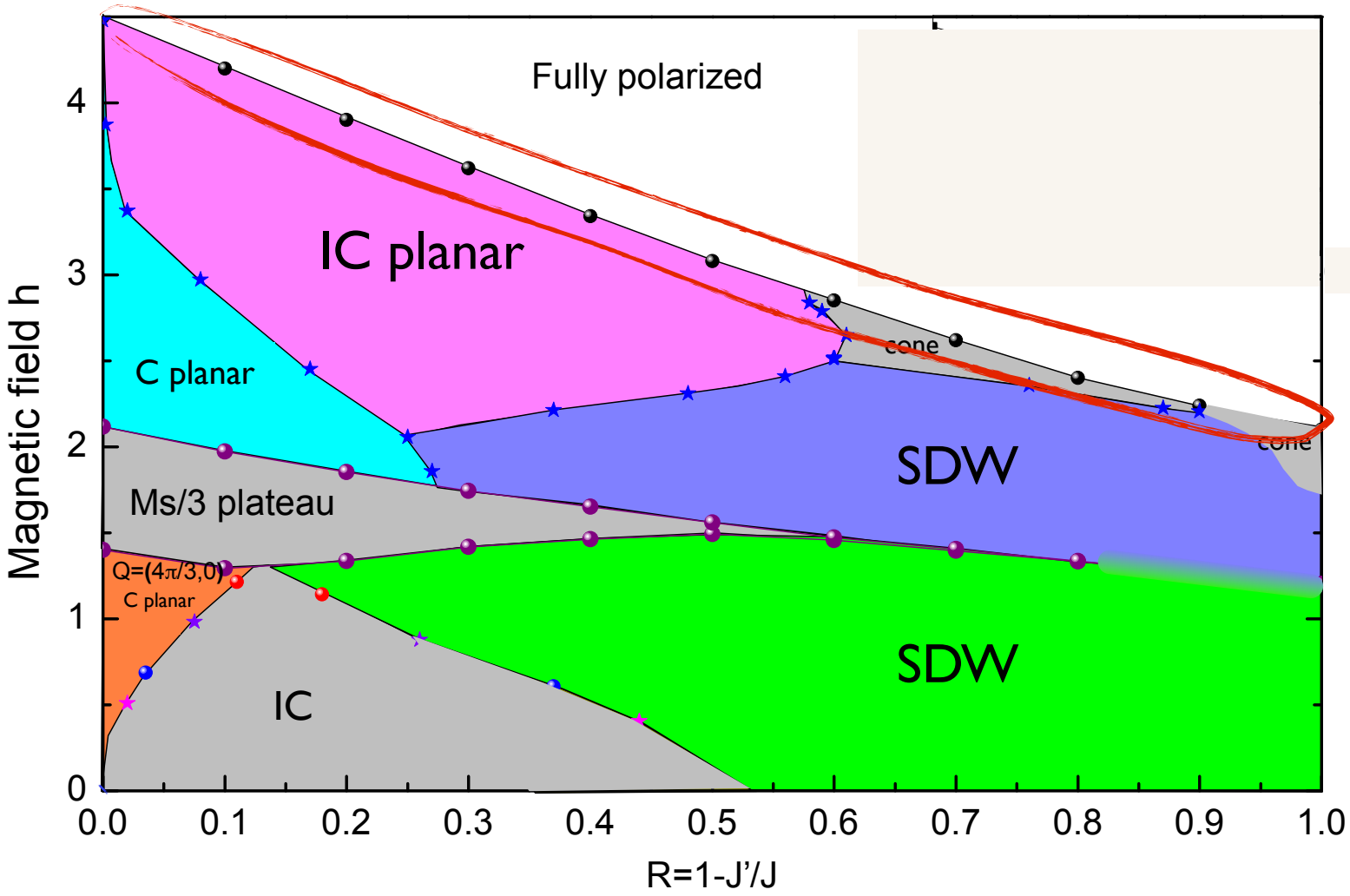
M/M$_s$=1/3 plateau:
uud state



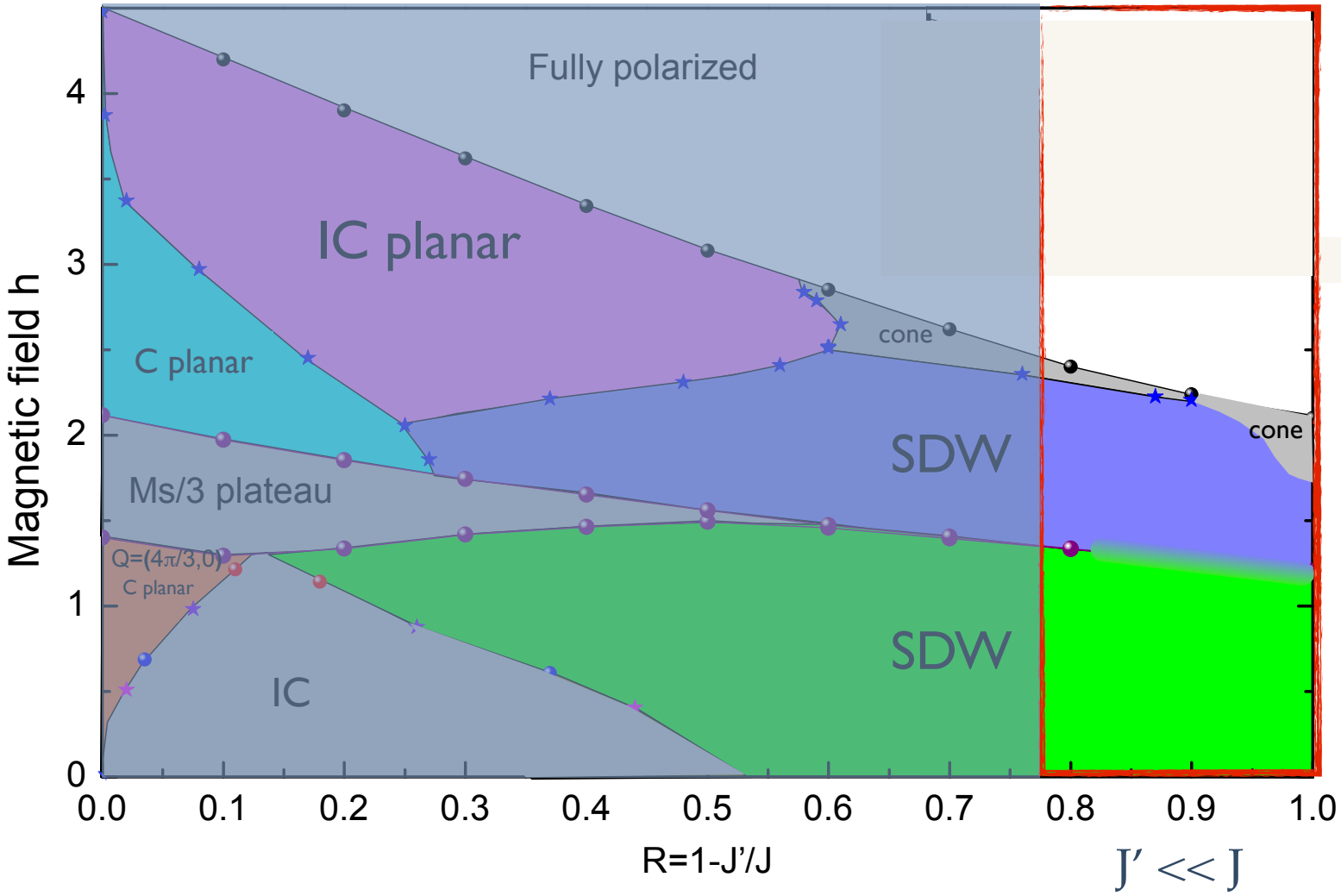


$R = 0.2$

High field: condensation of spin flips - see poster by Ru Chen, Thursday June 7



Weakly coupled chains

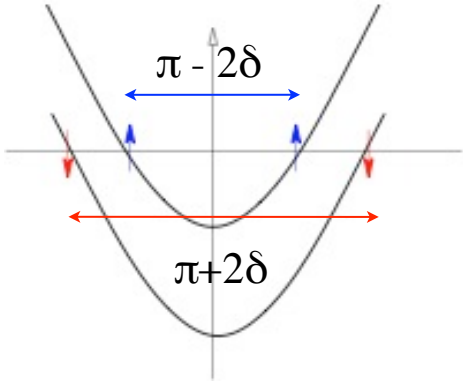


weakly coupled chains

J' << J

S=1/2 AFM Chain in a Field

$$\mathcal{H} = J\sum_x \vec{S}(x)\cdot\vec{S}(x+1) - h\sum_x S^z(x)$$



- Field-split Fermi momenta:
  - ✓ Uniform magnetization
  - ✓ Half-filled condition

$$k_{F\uparrow} = \frac{\pi}{2} - \delta\ ,\ k_{F\downarrow} = \frac{\pi}{2} + \delta$$

$$M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$$

$$n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$$

- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$ scaling dimension **increases** $1/4\pi R^2$

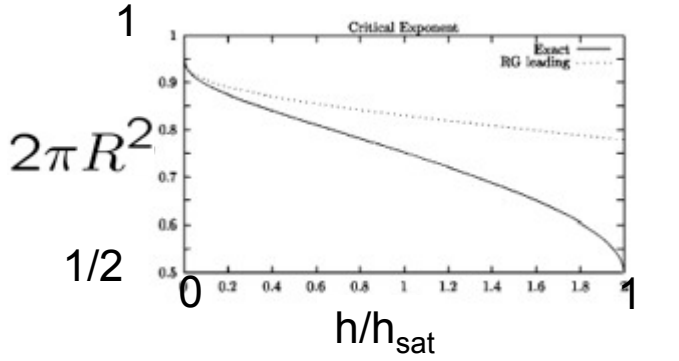
$S^z_{\pi\pm 2\delta}$

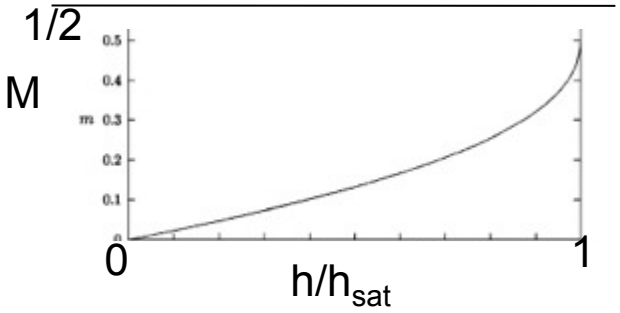
- $S^{x,y}$ components ($\Delta S=1$) remain at π scaling dimension **decreases** πR^2

$S^{\pm}_{\pi}$

- Derived for free electrons but correct always - *Luttinger Theorem*

Affleck and Oshikawa, 1999





$h_{sat}=2J$

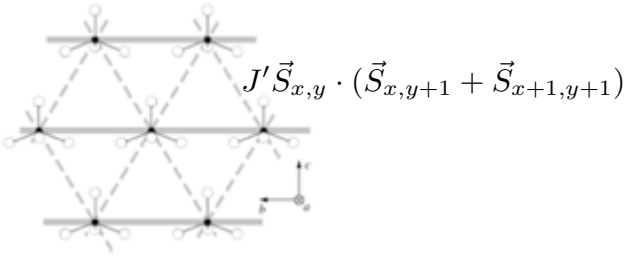
$2k_F$ spin density fluctuations

- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

Ideal J-J' model in magnetic field

OS, Balents 2007

- Two important couplings for h>0
- Quantum phase transition between SDW and Cone states



Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2

"collinear" SDW ordering momentum scales with magnetic field

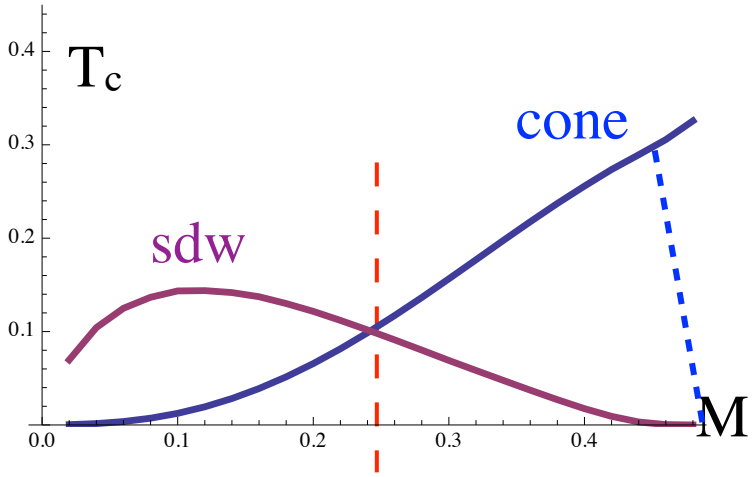
$$k_{F\downarrow} - k_{F\uparrow} = 2\delta = 2\pi M$$

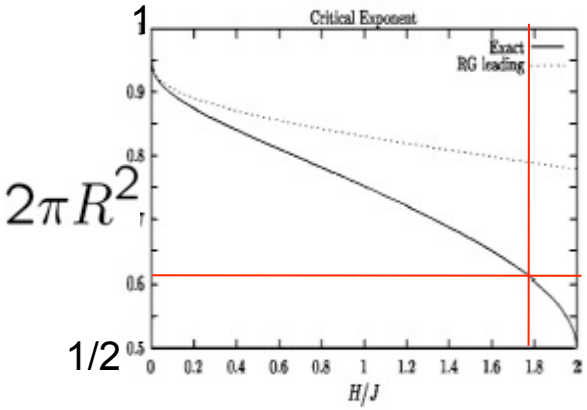
dim $1+2\pi R^2$: 2 -> 3/2

spiral "cone" state

- "Critical point": $1+2\pi R^2 = 1/2\pi R^2$ gives at M = 0.3

$$2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$$





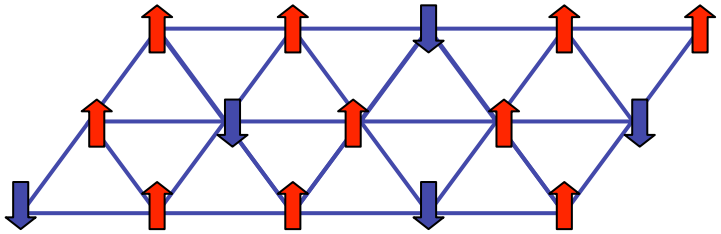
also: Kolezhuk, Vekua 2005

h/h_{sat}

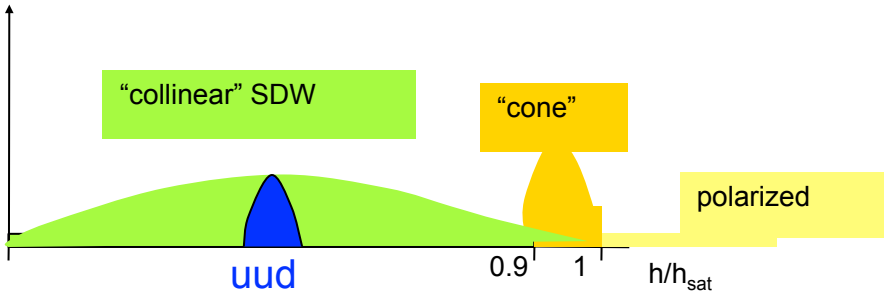
Plateau from SDW

- "Collinear" SDW state *locks* to the lattice
  - -"irrelevant" (1d) umklapp terms become relevant once SDW order is present (when *commensurate*): multiparticle umklapp scattering

  - -strongest locking is at M=1/3 M_{sat}



$$\delta = \pi M$$



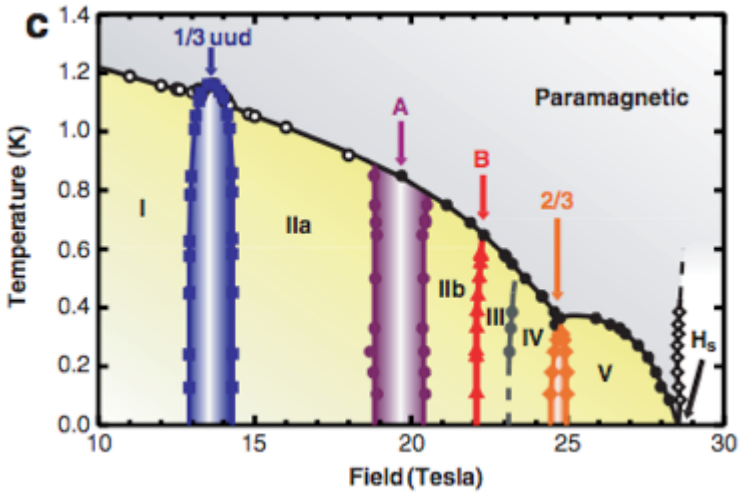
$$\left(\Psi_R^\dagger \Psi_L\right)^n \rightarrow (\pi - 2\delta)n = 2\pi m \rightarrow 2M = 1 - 2m/n$$

n	3	4	5	5	6
m	1	1	1	2	1
2M	1/3	1/2	3/5	1/5	2/3

naively thinking

Cs_2CuBr_4 Fortune et al 2009

1/3 2/3



Plateau more carefully

OS, Katsura, Balents PRB 2010

$$M^{(n,m)} = \frac{1}{2}\left(1 - \frac{2m}{n}\right)$$

- Umklapp must respect triangular lattice symmetries
  - translation along chain direction $\phi_y(x) \to \phi_y(x+1) - R(\pi - 2\delta)$
  - translation along diagonal $\phi_y(x) \to \phi_{y+1}(x+1/2) - R(\pi - 2\delta)/2$
  - spatial inversion $\phi_y(x) \to \pi R - \phi_y(-x)$

$$H^{(n)}_{umk} = \sum_y \int dx\, t_n \cos[\frac{n}{R}\phi_y] \quad \textbf{and} \quad n = m \ (mod\ 2)$$

same parity condition

- n-th plateau width (in field) $width \sim \left(J'/J\right)^{n^2/(4(4\pi R^2-1))}$

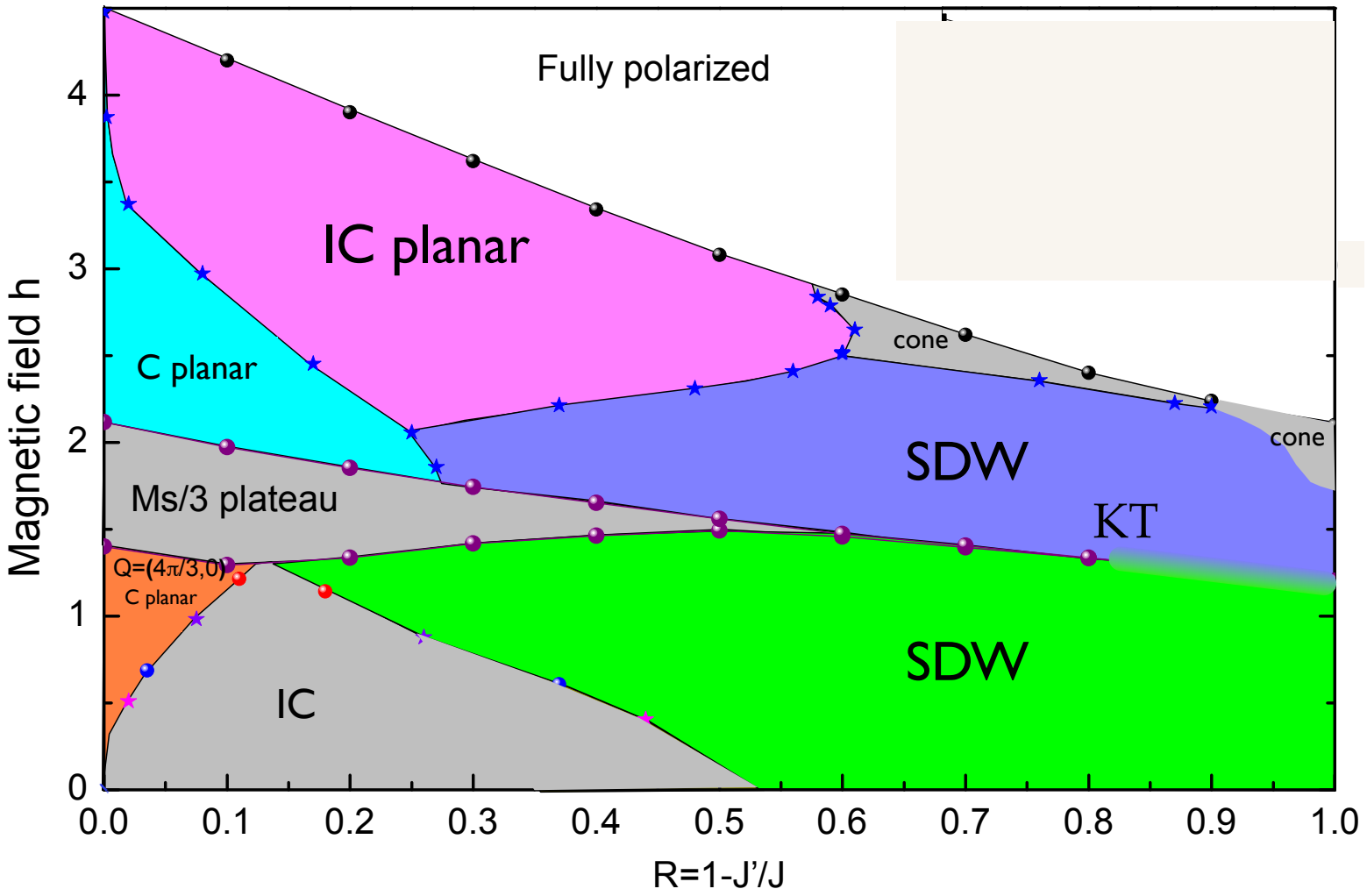
n	3	8	5	10	12
m	1	2	1	4	2
2M	1/3	1/2	3/5	1/5	2/3

large **n** leads to exponential suppression

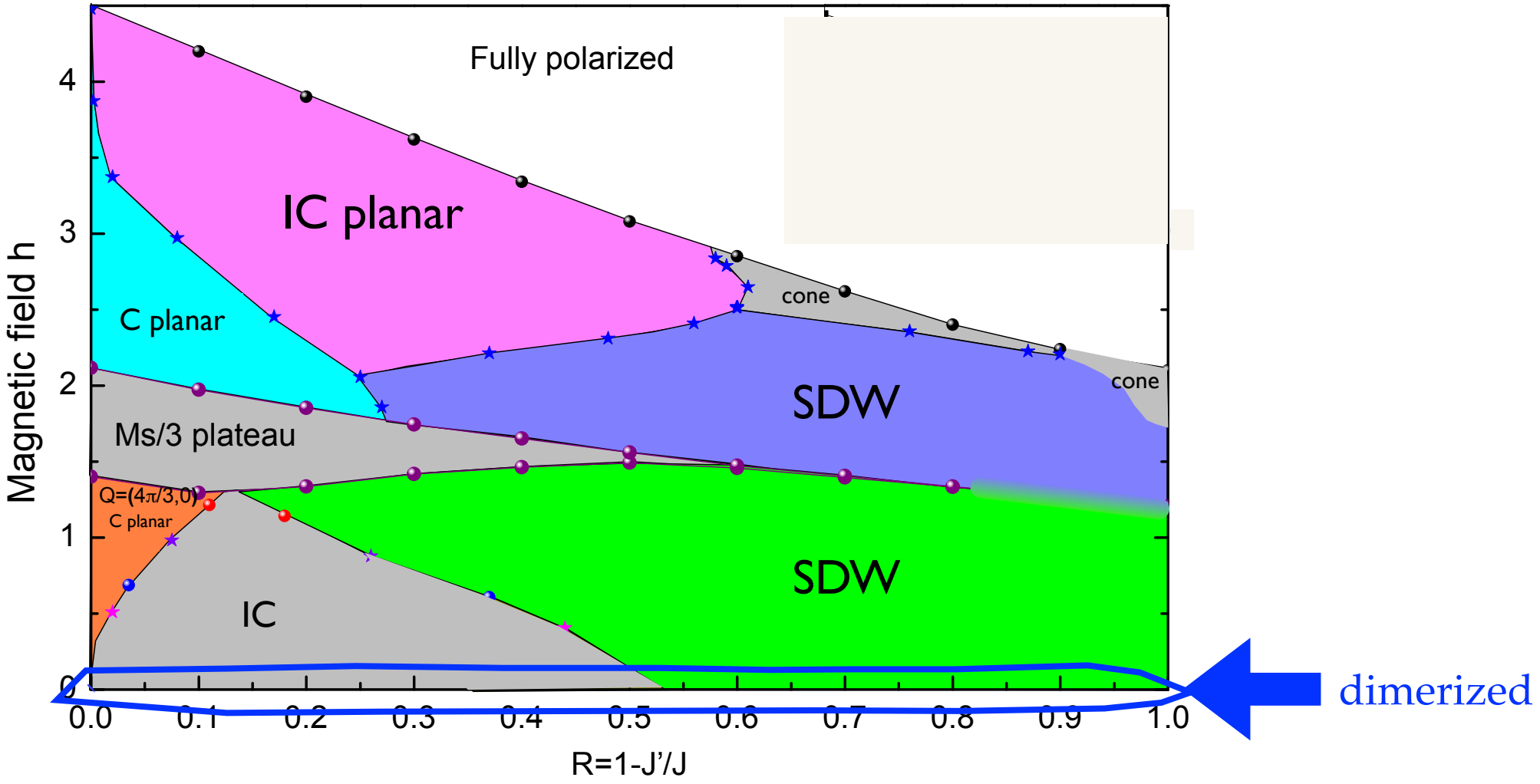
- Ladder: Kosterlitz-Thouless transition to the plateau state @ R=0.7±0.1 (J'/J = 0.3) because umklapp operator becomes irrelevant for J'/J -> 0

Plateau endpoint (ladder)

- Ladder: Kosterlitz-Thouless transition to the plateau state @ R=0.7±0.1 (J'/J = 0.3) because umklapp operator becomes irrelevant for J'/J -> 0

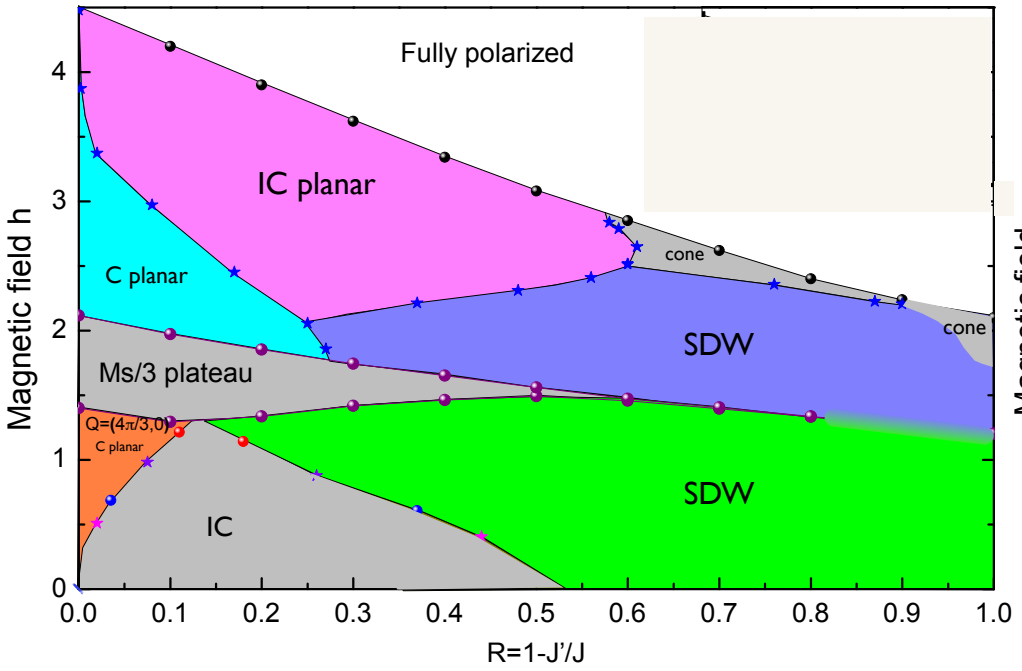


Zero field (ladder)

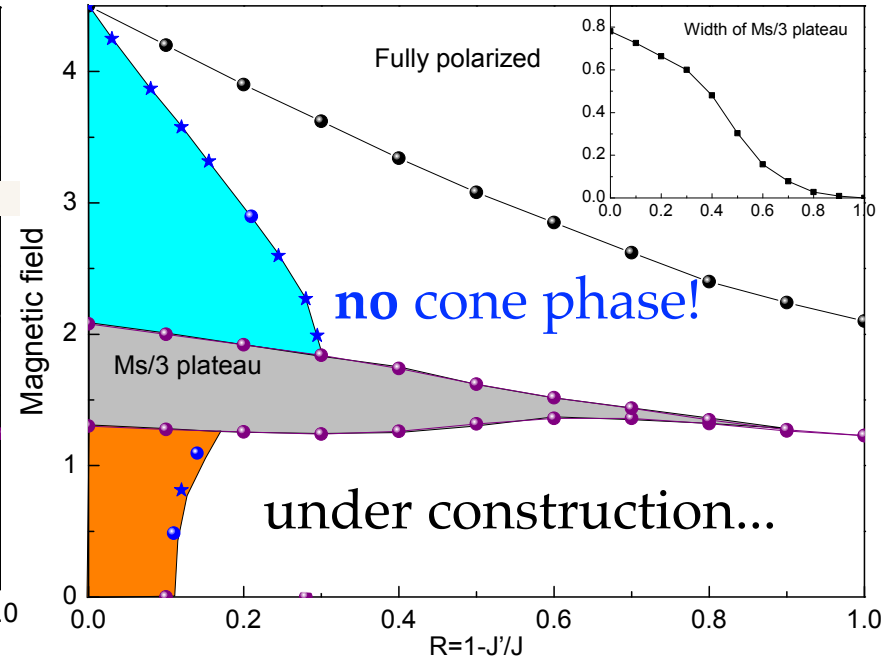


Summary

ladder



2d DMRG and theory



- plateau and co-planar phases are surprisingly stable

✓ 7 out of 8 phases are of quantum origin