Spatially anisotropic triangular antiferromagnet in magnetic field

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EQUAM10, MPIPKS, Dresden, August 27 2010

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Magnetization plateau in triangular lattice antiferromagnets

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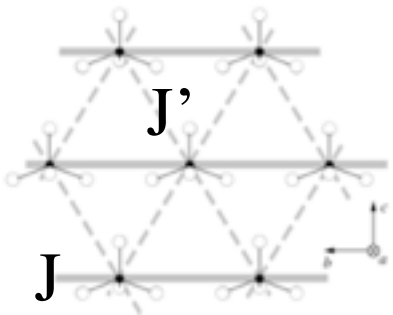
Outline

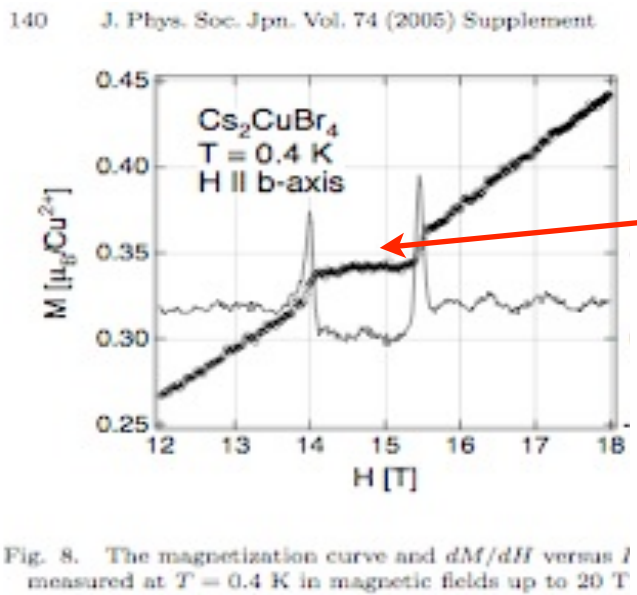
- experimental (Cs_2CuBr_4) and theoretical motivations
- classical antiferromagnet in a field: entropic selection
  - spatial anisotropy - high-T stabilization of the plateau
- Quantum spins: zero-point fluctuations
  - Large-S analysis of interacting spin waves
- Approach from one dimension
  - sequence of plateaux and selection rules
- ★ (attempt at) Unification

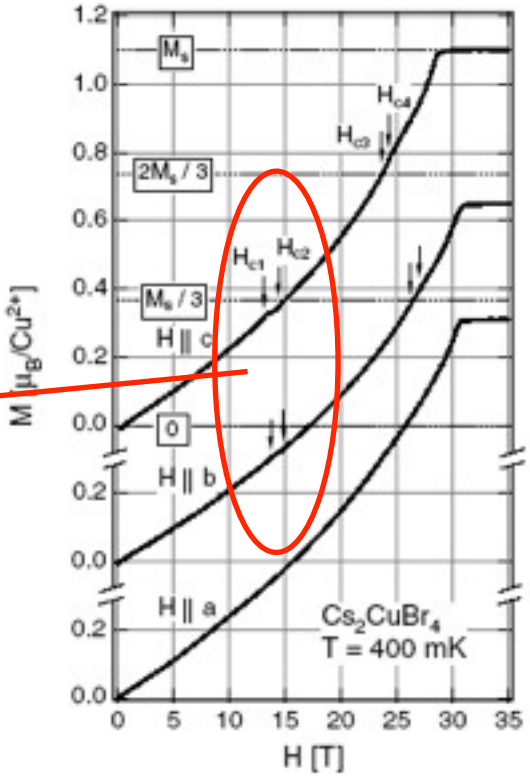
Experiment: M=1/3 magnetization plateau in Cs_2CuBr_4

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.75**
but not Cs_2CuCl_4 [**J'/J = 0.34**]

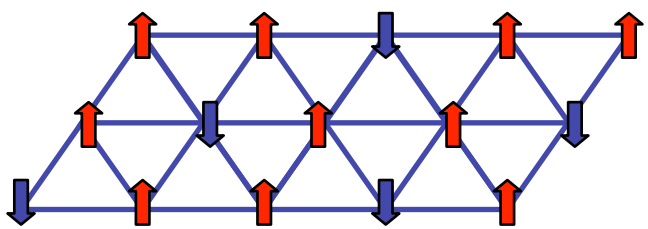
S=1/2





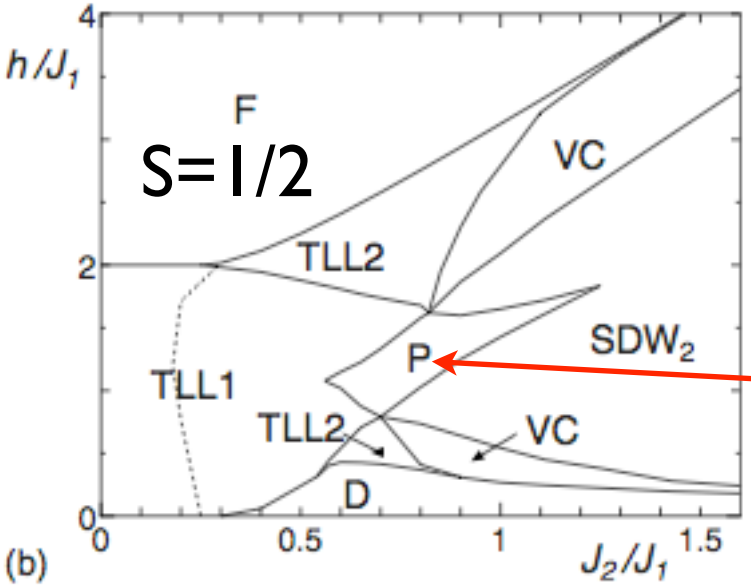


- UUD (up-up-down) structure -- down-spins at the centers of hexagons; commensurate structure -- one down spin per every triangle



★ first observation of "**up-up-down**" state in spin-1/2 triangular lattice antiferromagnet

Progress in one dimensional J_1-J_2 chain (zig-zag ladder)



Okunishi, Tonegawa JPSJ (2003)
Hikihara et al PRB (2010)

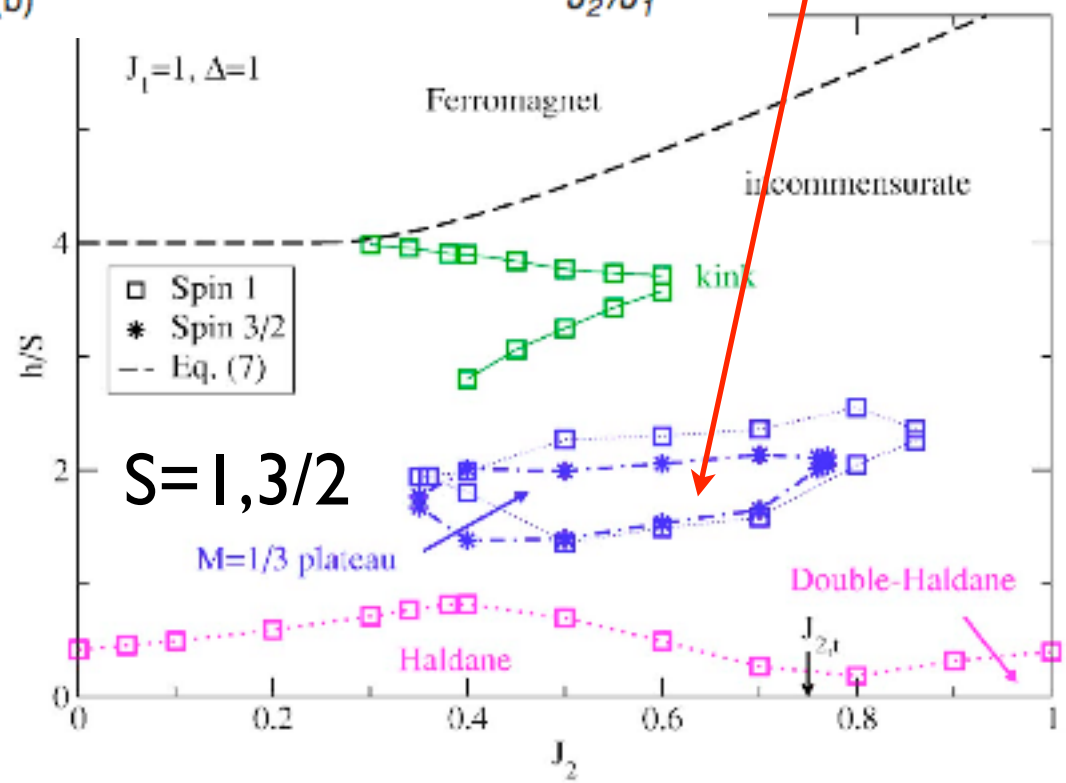
plateau is centered around $J_2 = J_1/2$ point for $S > 1/2$; semi-classical spin wave expansion is possible there (OS 2009)

M=1/3 plateau

agrees with Oshikawa, Yamanaka, Affleck argument (PRL 2007):

p S (1 - M) = integer

p = period, S = spin,
M = magnetization:
M=1/3, p=3
possible for all S



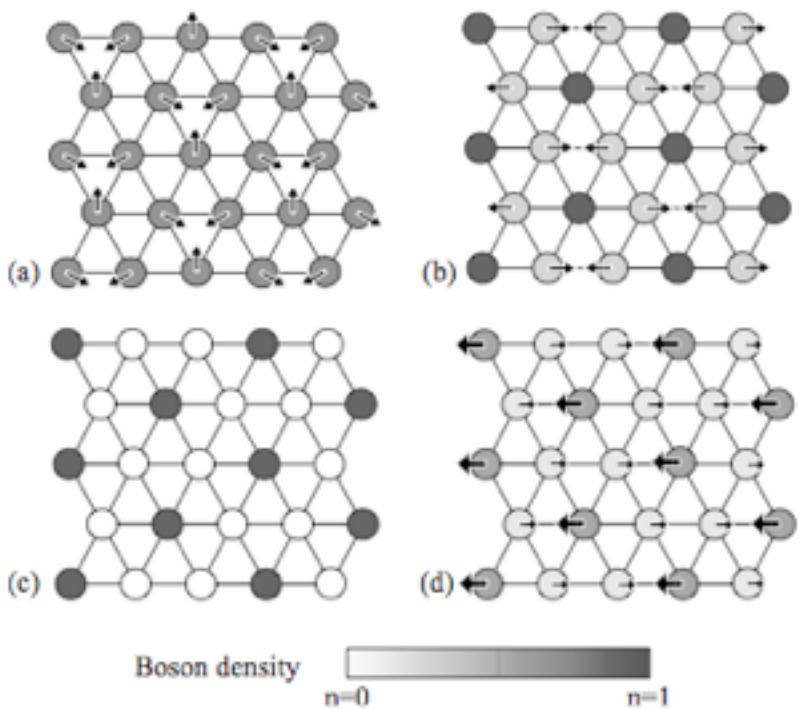
Heirich-Meisner et al PRB (2007)

also seen in frustrated spin ladders, Michaud et al PRB 2010

D = 2

- surprisingly complex phase diagram of spatially anisotropic triangular lattice antiferromagnet
  - no definite conclusions from numerical studies yet...
- connections with interacting boson system
  - Superfluids
  - Mott insulators
  - Supersolids

Nikuni, Shiba 1995
Heidarian, Damle 2005
Wang et al 2009
Jiang et al 2009
Tay, Motrunich 2010

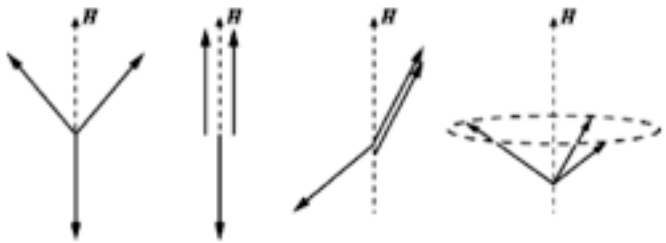


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- experimental (Cs_2CuBr_4) and theoretical motivations
- classical antiferromagnet in a field: entropic selection
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★ (attempt at) Unification

Classical isotropic Δ AFM in magnetic field

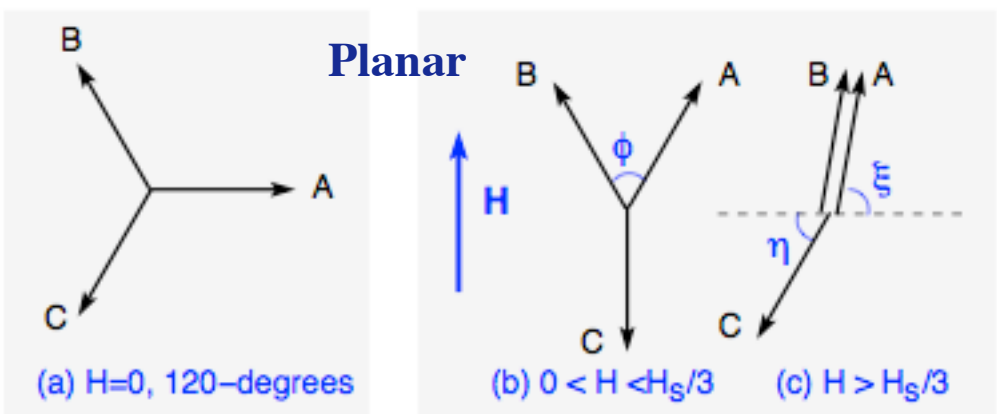


- No field: spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

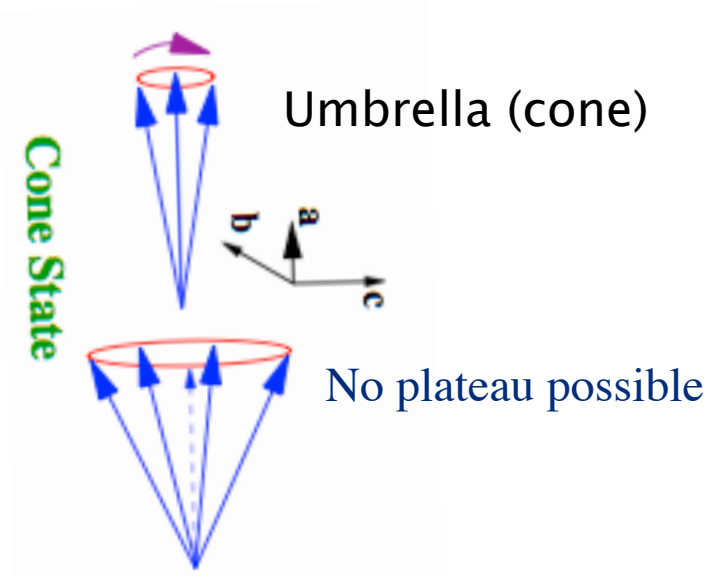
$$H = J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \sum_i \vec{h}\cdot\vec{S}_i$$

$$H = \frac{1}{2}J\sum_{\triangle}\left(\sum_{i\in\triangle}\vec{S}_i - \frac{\vec{h}}{3J}\right)^2$$

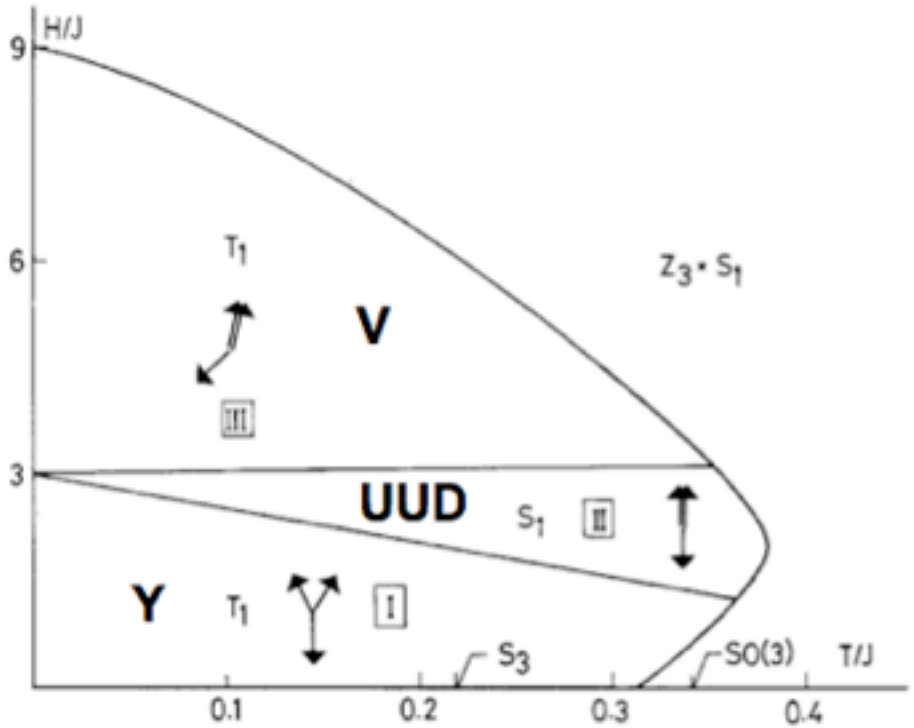
- all states with $\vec{S}_{i1}+\vec{S}_{i2}+\vec{S}_{i3} = \frac{\vec{h}}{3J}$ form the lowest-energy manifold
  - 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about **h**)



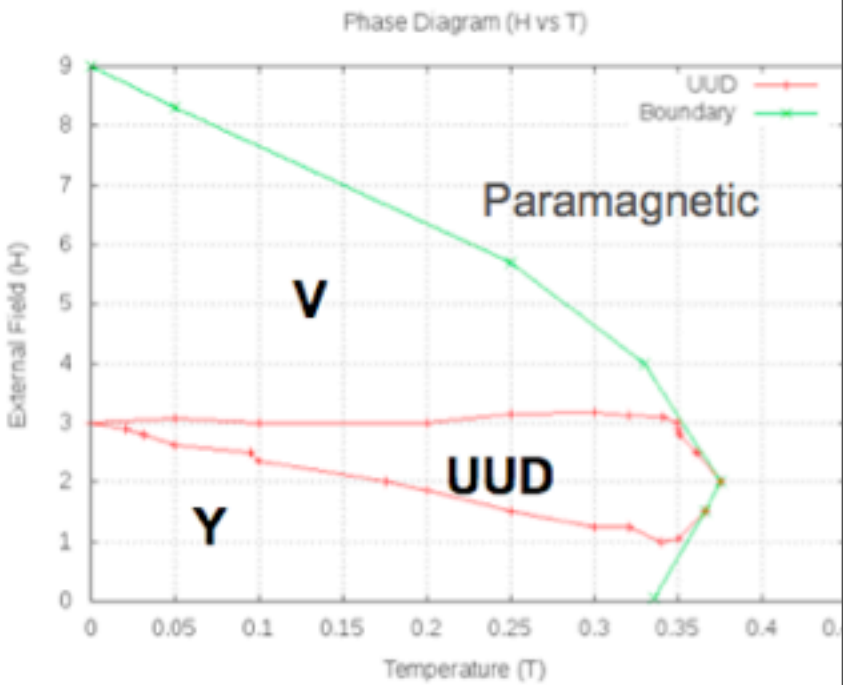
plateau is possible at "collinear" point



Phase diagram



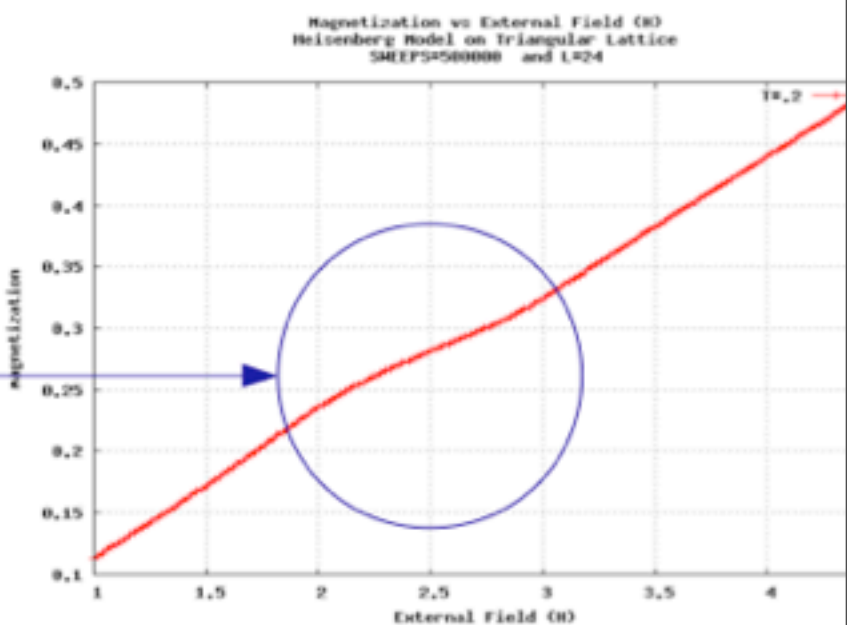
H. Kawamura, S. Miyashita:
JPSJ (1985)



Head, Griset, Alicea, OS 2010

Entropic Selection:

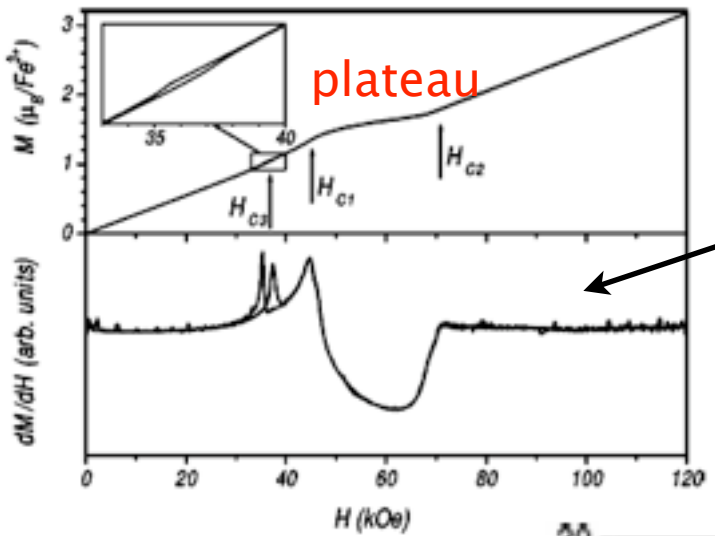
- Planar states favored by thermal fluctuations
- UUD state around m=1/3 resulting in quasi-plateau



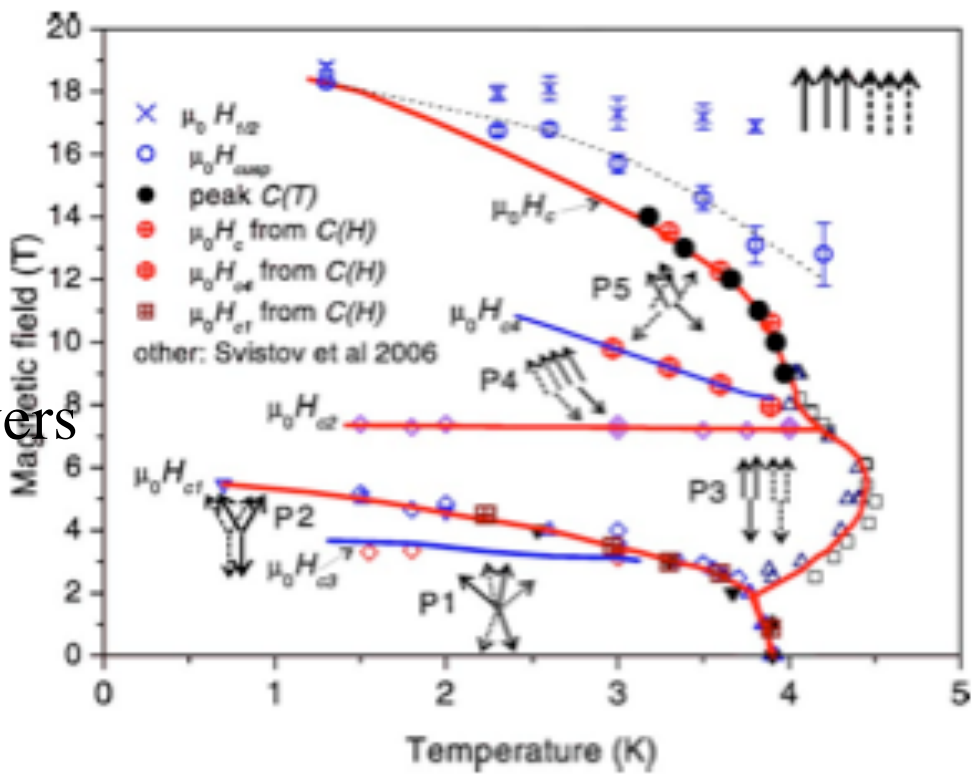
Thermal Order-from-Disorder mechanism

$RbFe(MoO_4)_2$:
$S=5/2$ Fe^{3+}

Svistov et al PRB (2003)
Smirnov et al PRB (2007)



plateau is signaled by depression in dM/dH

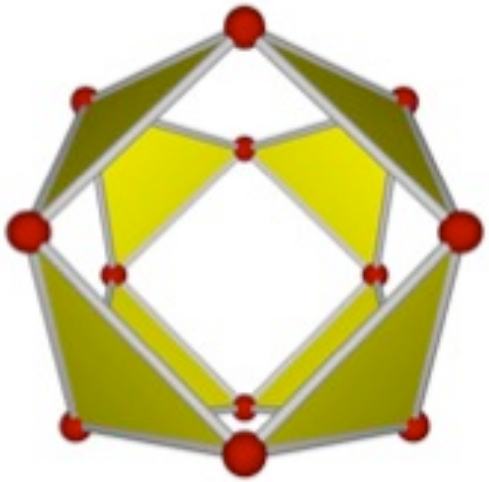


Two antiferromagnetically coupled layers

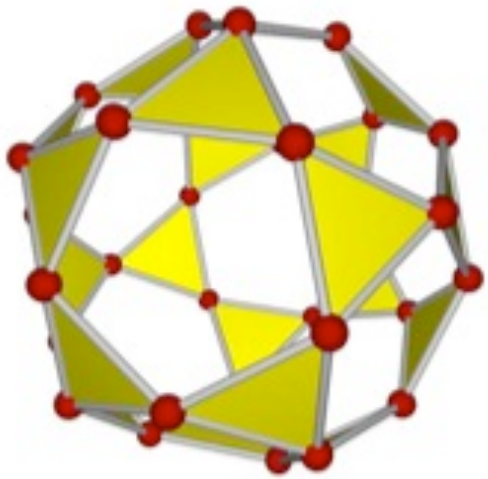
Plateau width increases with T

Persistence of plateau in triangular structures

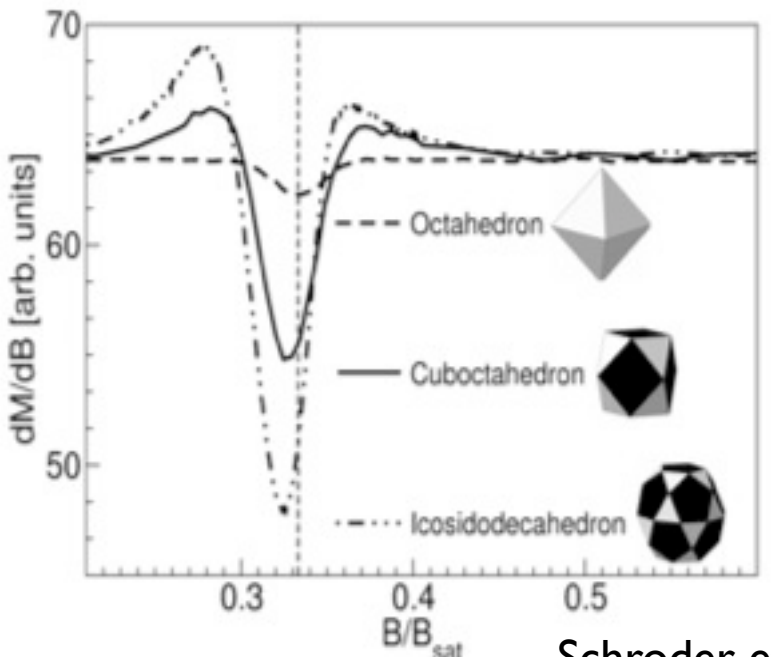
cuboctahedron

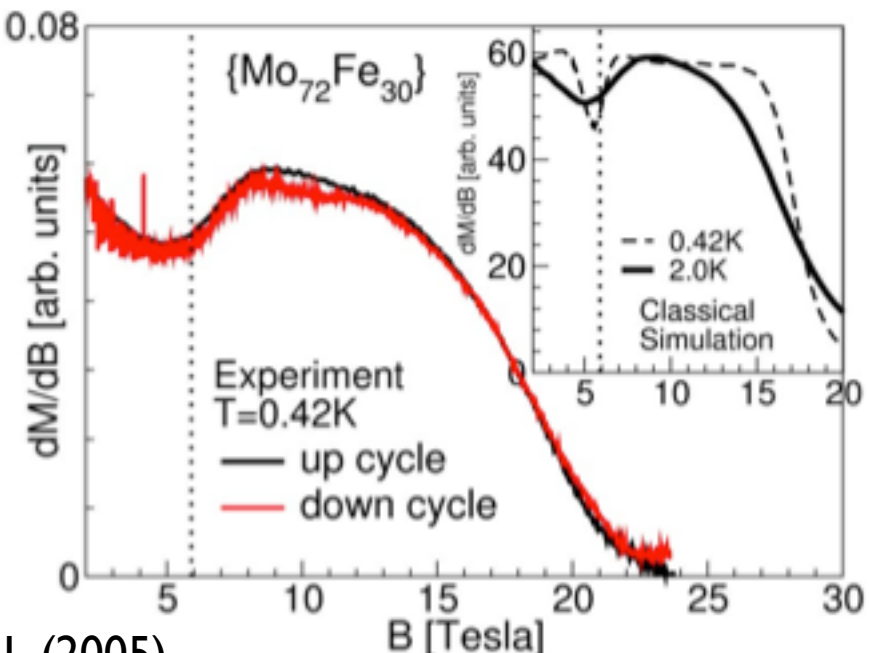


icosidodecahedron



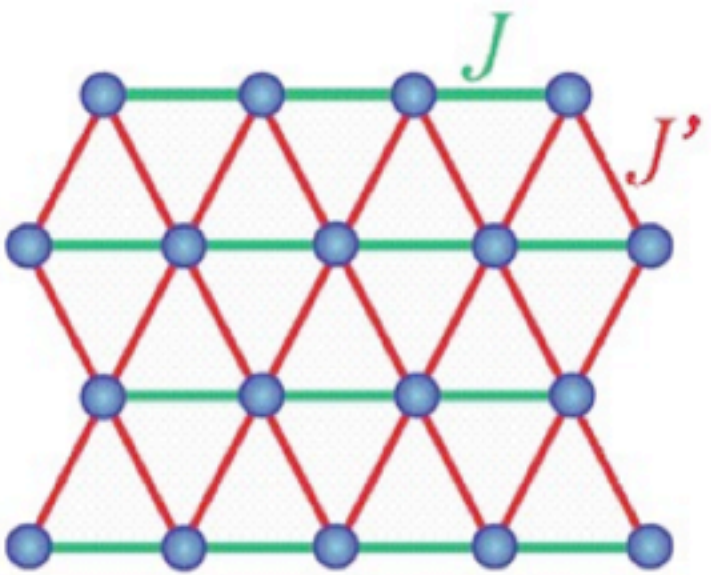
$Mo_{72}Fe_{30}$
S=5/2 Heisenberg
(kagome on a sphere)





Schroder et al, PRL (2005)
Rousochatzakis et al, PRB(2008)

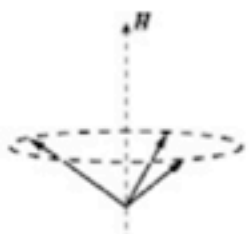
Spatial Anisotropy (J'/J = 0.765)



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

J' ≠ J removes degeneracy at T=0

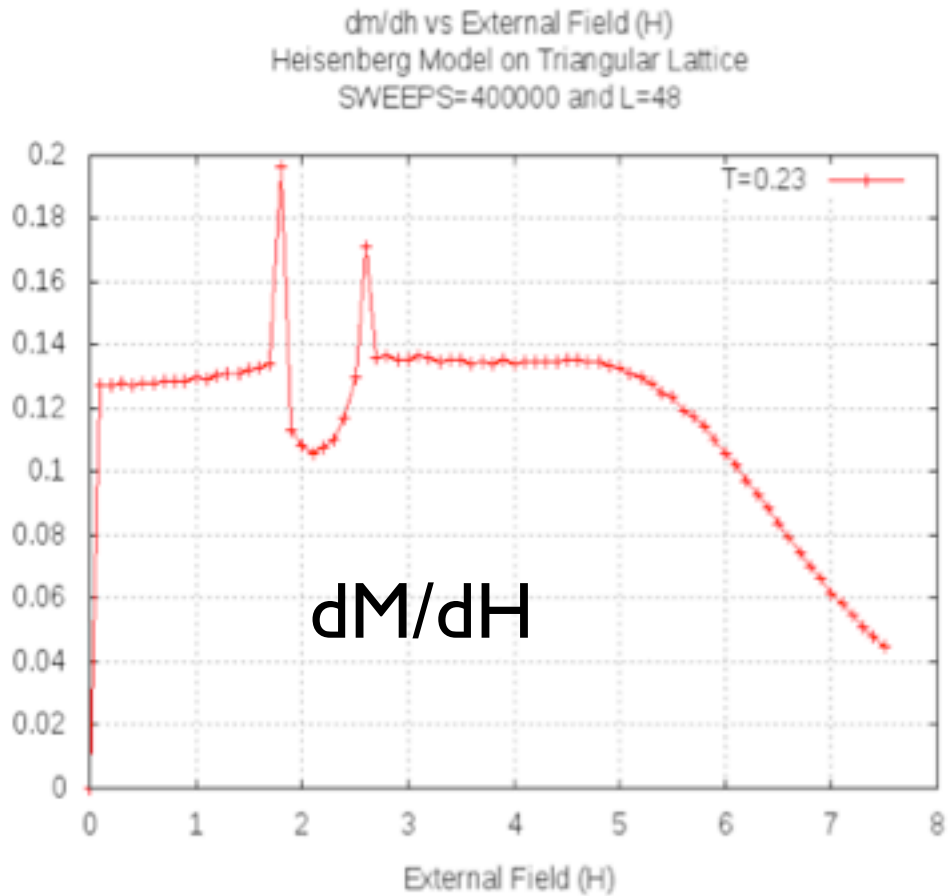
Non-planar ground state at T=0
(Umbrella)

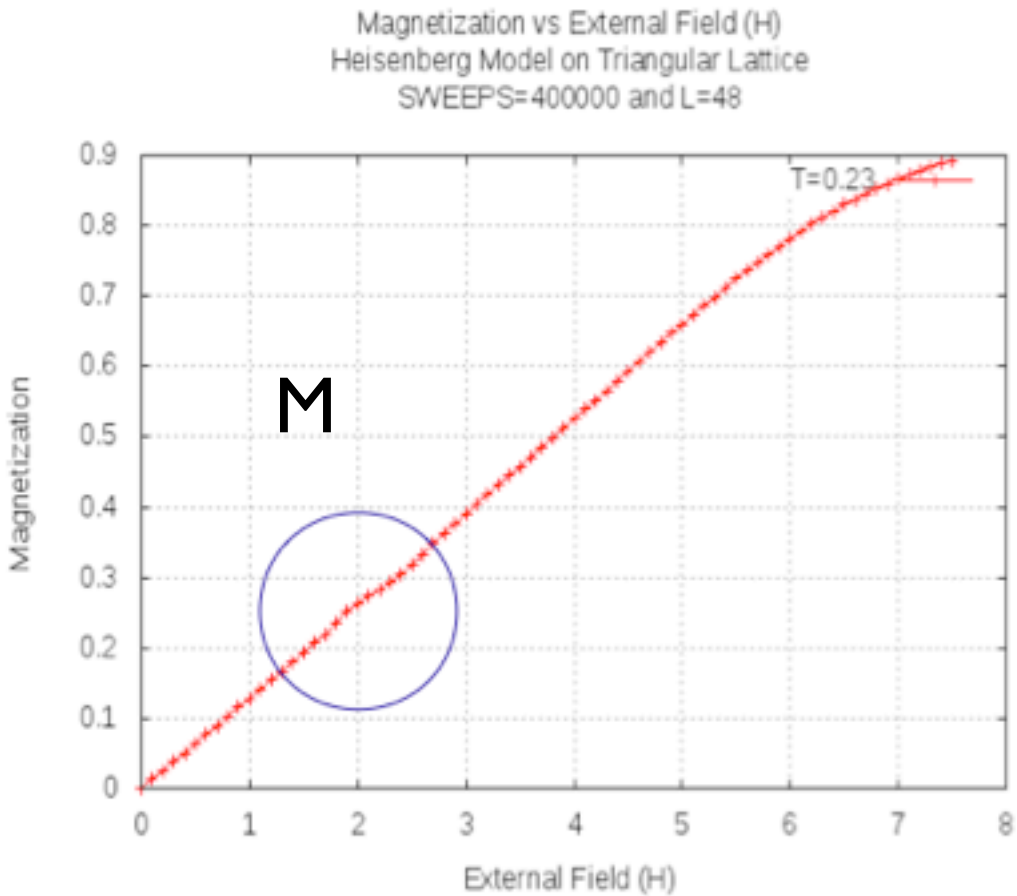


If (J – J')/J is sufficiently small:

Expect thermal fluctuations to stabilize planar states above some finite temperature

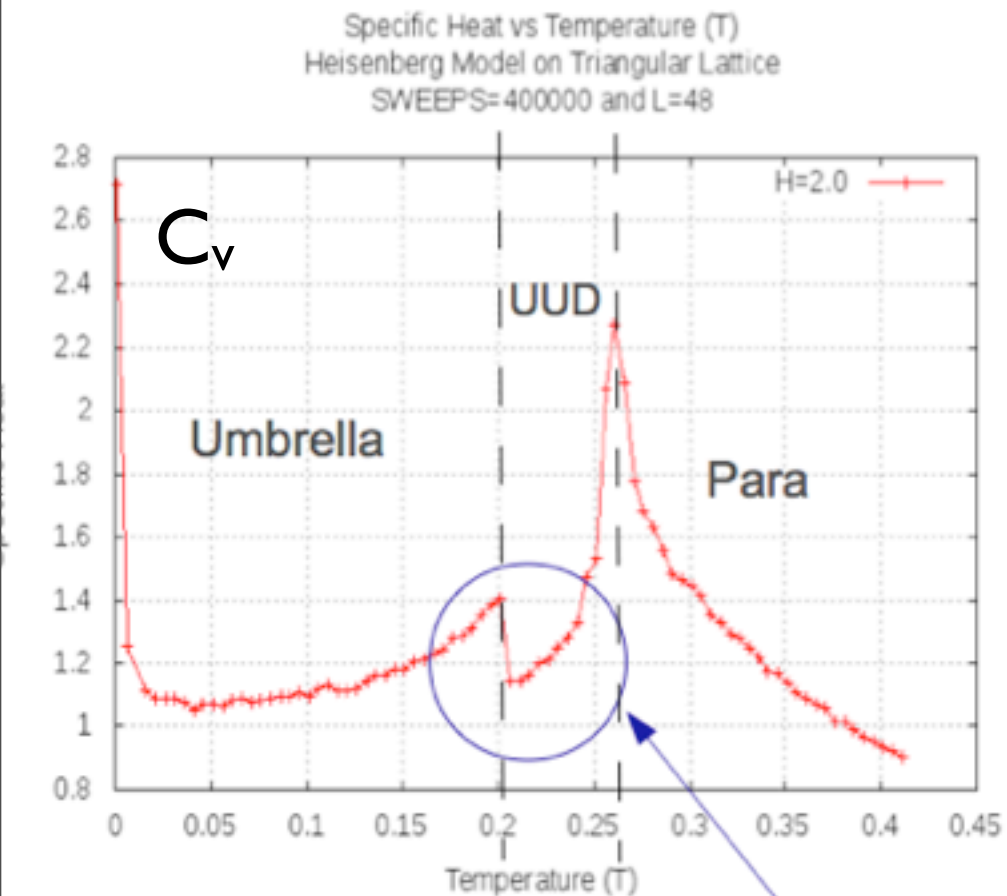
Monte Carlo Data (J'/J = 0.765)



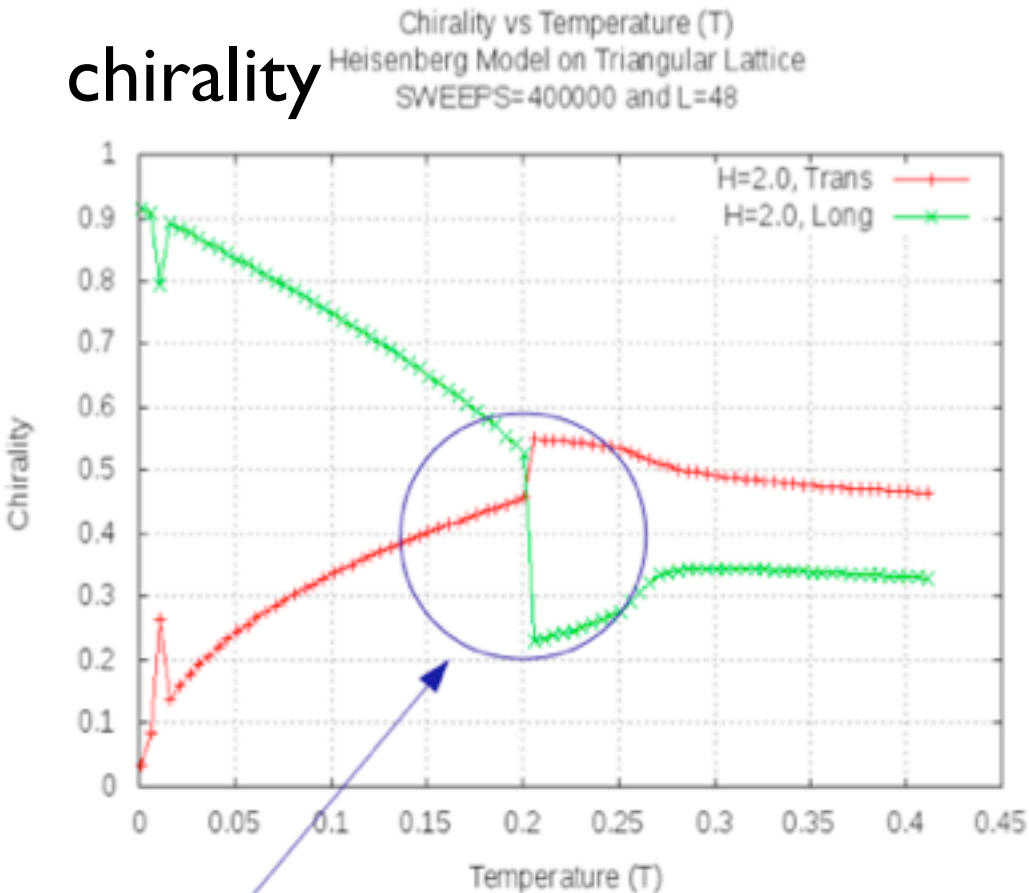


crossing UUD state at finite T

First order transition



chirality



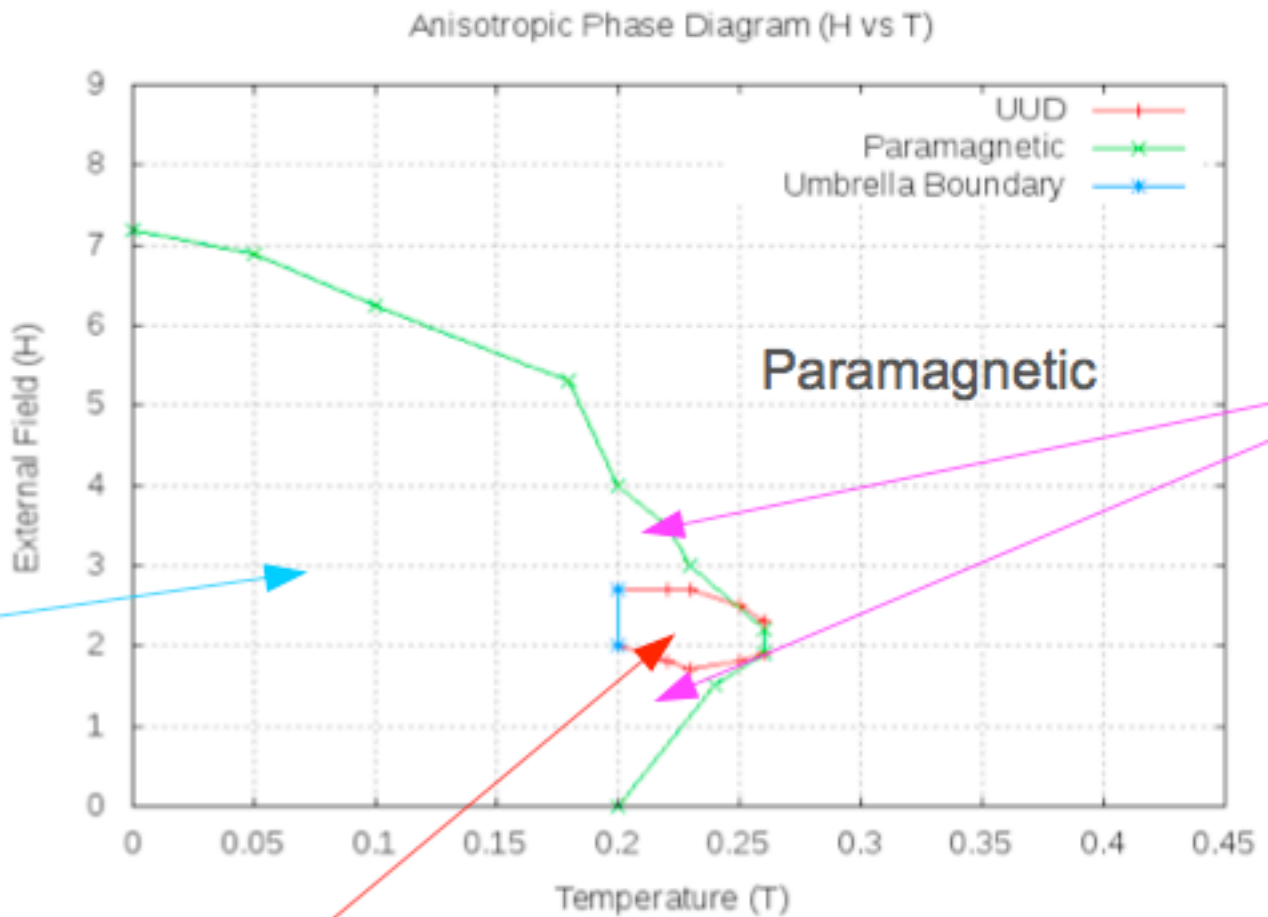
First order transition

Fixed H scan
Phase transition from umbrella to UUD

Chirality: $\kappa = \frac{2}{3\sqrt{3}}(\vec{S}_1 \times \vec{S}_2 + \vec{S}_2 \times \vec{S}_3 + \vec{S}_3 \times \vec{S}_1)$

Phase diagram of the classical anisotropic model

Griset, Head, Alicea, OS 2010



Dual role of thermal fluctuations: promotes high-entropy states; destabilizes order

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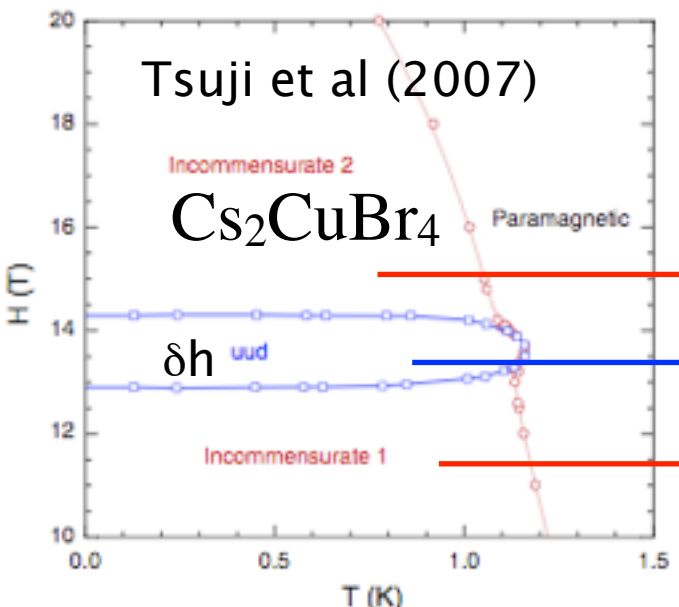
★ (attempt at) Unification

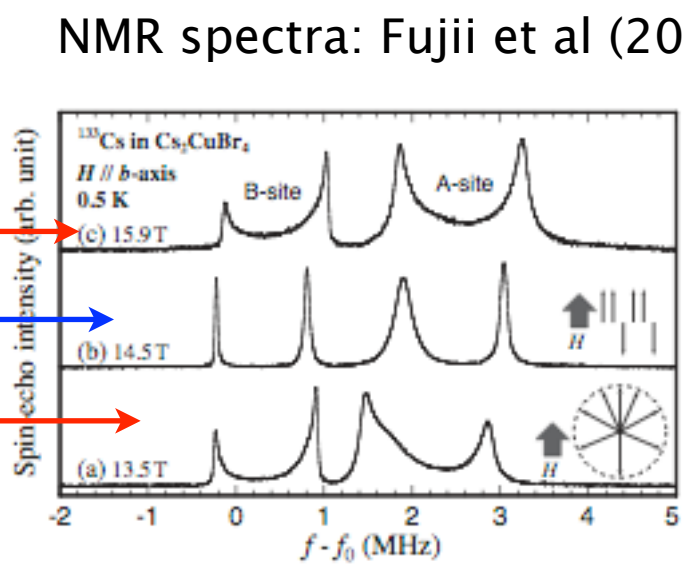
Order-from-Disorder via Quantum fluctuations

finite S effect

- fluctuation spectra of *different* spin structures are *different*: $E = E_{class} + \Delta E^{sw}$
- quantum fluctuations prefer planar arrangement $\Delta E^{sw}_{planar} = \frac{S}{2}\sum_k \omega_{planar}(k) < \Delta E^{sw}_{umbrella} = \frac{S}{2}\sum_k \omega_{umbrella}(k)$
- prefer collinear configuration even more, when possible: state with maximum number of soft modes wins.
- plateau is a quantum effect, width $\delta h = 1.8\ J/(2S)$ ($h_{saturation} = 9\ J$)

  Chubukov, Golosov (1991)
- plateau is the effect of interactions (hence, width ~ 1/S) between spin waves





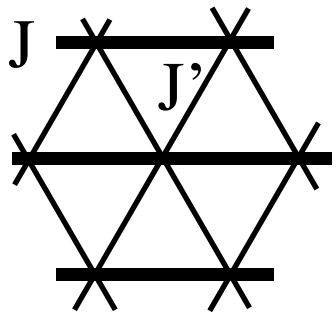
T-independent width

The problem: spatial anisotropy stabilizes umbrella

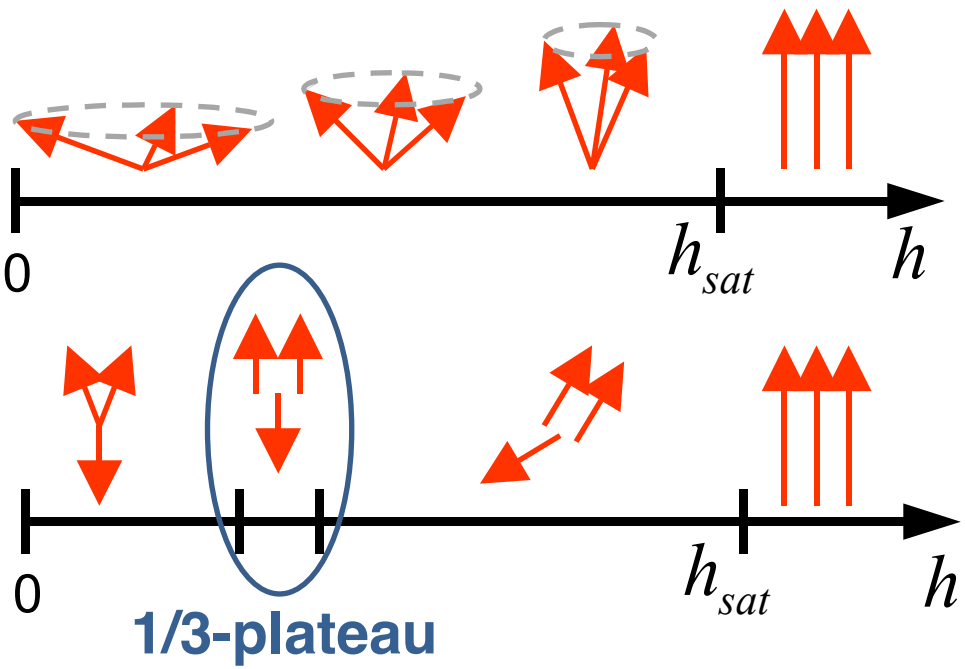


Spatially anisotropic model: classical analysis *fails*

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$J' \neq J$



Umbrella state:
favored classically;
energy gain $(J\text{-}J')^2/J$

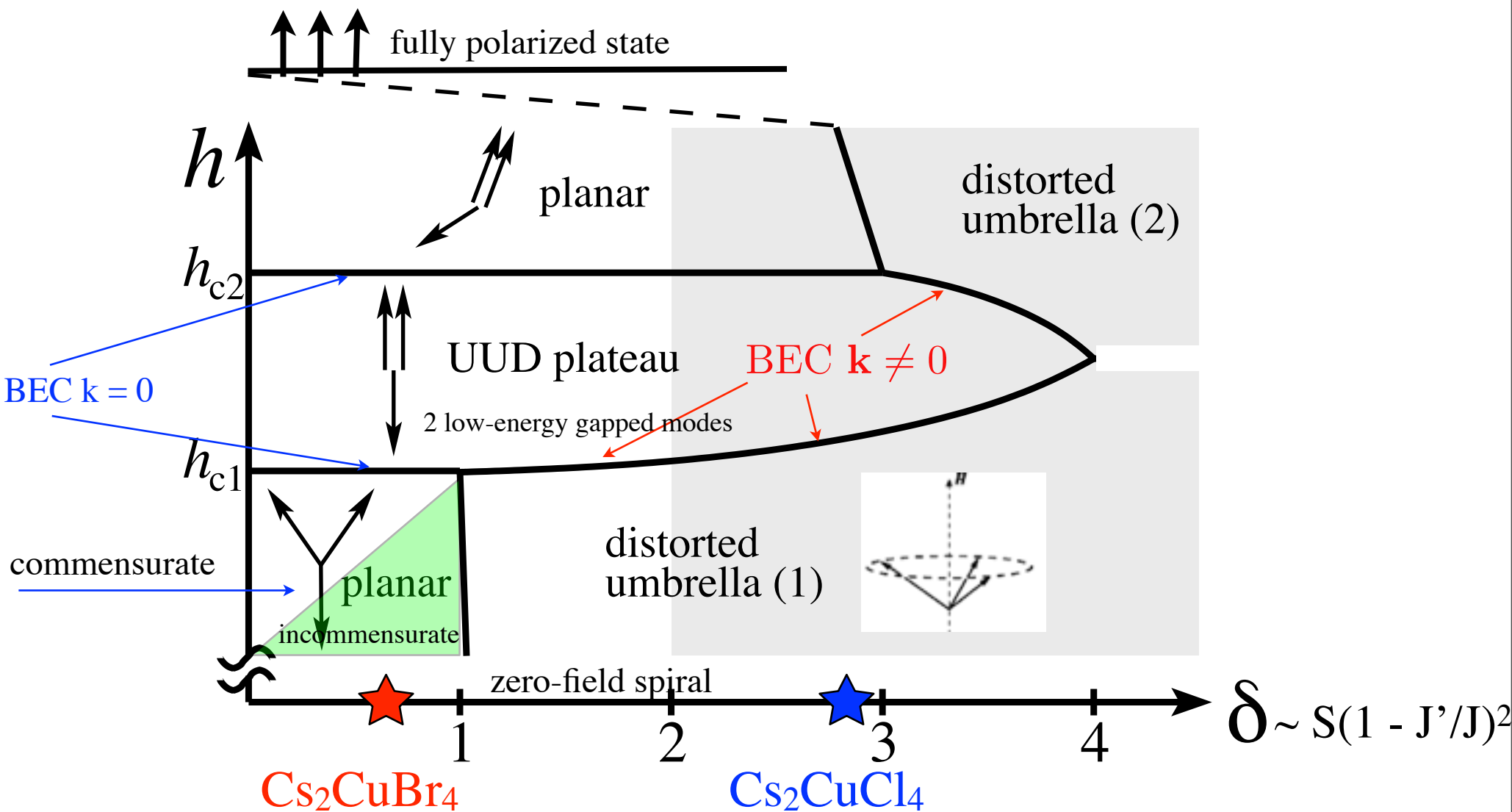
Planar states: favored by
quantum fluctuations;
energy gain J/S

The competition is controlled by dimensionless parameter $\delta = S(J - J')^2 / J^2$

- Technical formulation: spatial anisotropy J-J' causes softening of interacting (including 1/S correction) spin waves

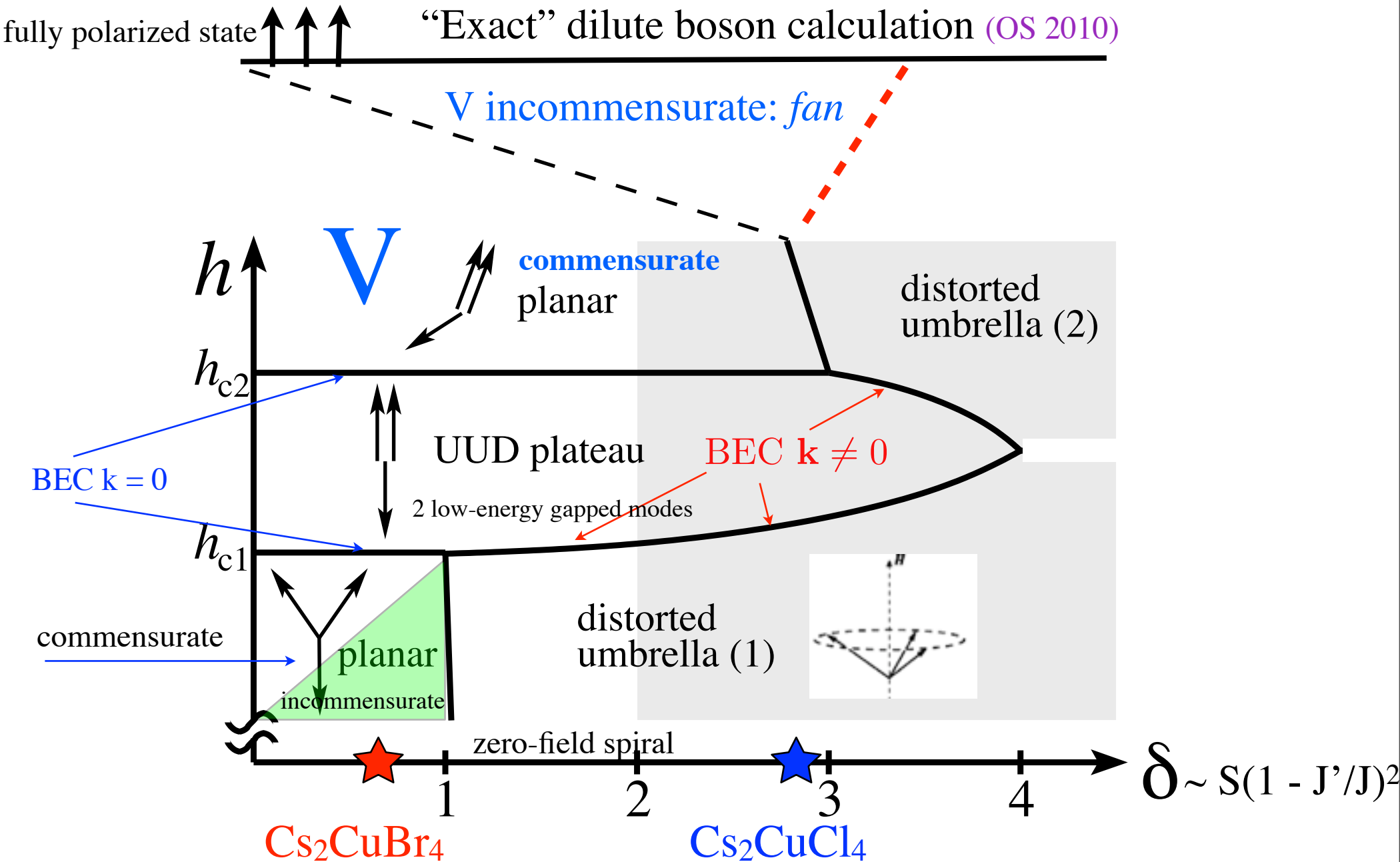
Our *semiclassical* approach: treat spatial anisotropy (J–J') as a perturbation to **interacting** spin waves

- single dimensionless parameter $\delta = S(1 - J'/J)^2$:

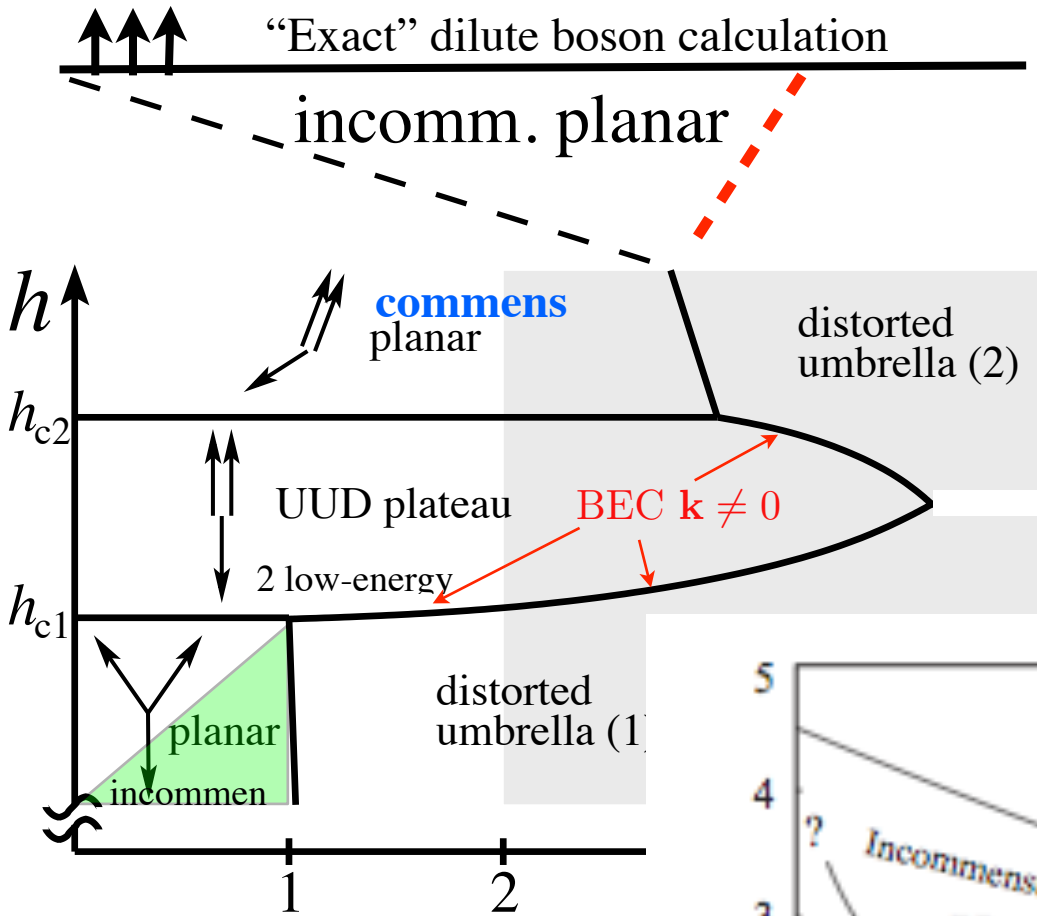


Alicea, Chubukov, Starykh PRL 102, 137201 (2009)

More detailed phase diagram

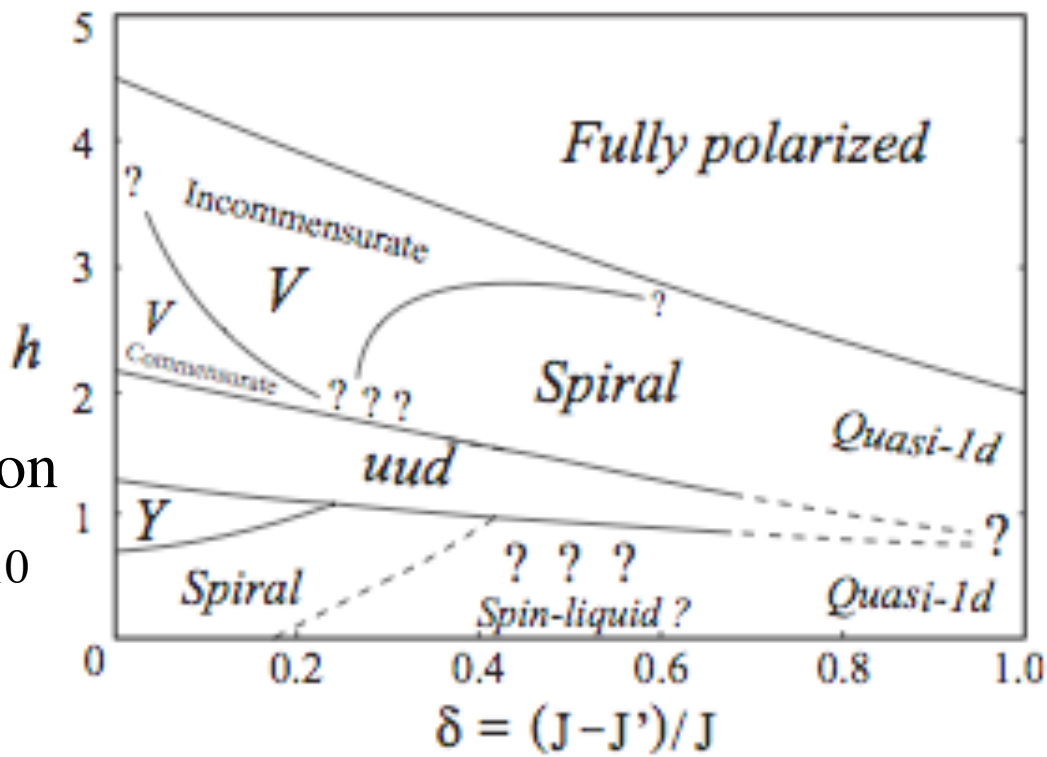


Alicea, Chubukov, Starykh PRL 102, 137201 (2009)



Variational wave function calculation

Tay, Motrunich PRB 2010



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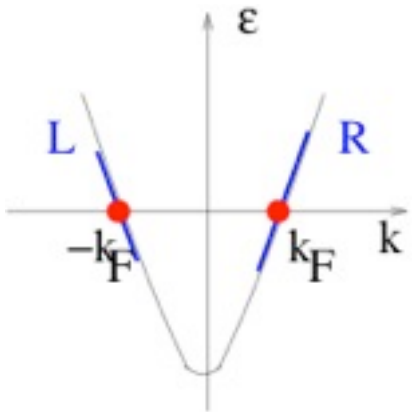
Heisenberg spin chain via free Dirac fermions

- Spin-1/2 AFM chain = half-filled (1 electron per site, $k_F=\pi/2a$) fermion chain



- Spin-charge separation

$$H_{\text{dirac}} = iv \int dx \sum_{s=\uparrow,\downarrow} (\Psi^+_{L,s}\partial_x\Psi_{L,s} - \Psi^+_{R,s}\partial_x\Psi_{R,s})$$



- *q=0 fluctuations: right- and left- spin currents*

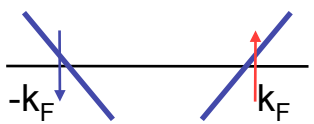
$$\vec{J}_R = \Psi^+_{Rs}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Rs'}\ ,\ \vec{J}_L = \Psi^+_{Ls}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Ls'}$$

- *$2k_F$ (= π/a) fluctuations: charge density wave ε , spin density wave N*

Staggered Magnetization N

$$\begin{cases} N^+ \sim \Psi^+_{R\uparrow}\Psi_{L\downarrow} + \text{h.c.} \\ N^z \sim \Psi^+_{R\uparrow}\Psi_{L\uparrow} - \Psi^+_{R\downarrow}\Psi_{L\downarrow} + \text{h.c.} \end{cases}$$

Spin flip ΔS=1



Susceptibility

1/q $\quad \chi_{1d}(q)$

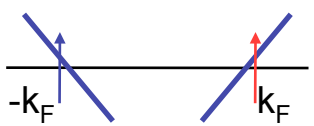
1/q

Staggered Dimerization

$\varepsilon = (-1)^x S_x S_{x+a}$

$$\varepsilon \sim i\left(\Psi^\dagger_{R\uparrow}\Psi_{L\uparrow} + \Psi^\dagger_{R\downarrow}\Psi_{L\downarrow} - \text{h.c.}\right)$$

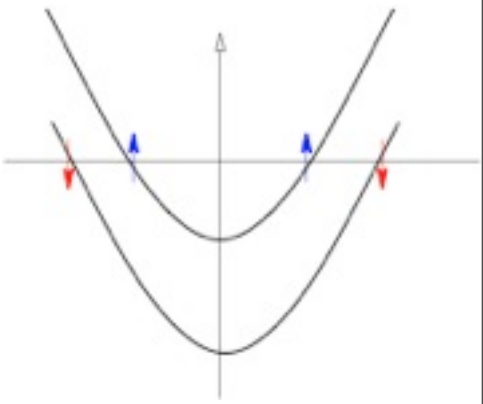
ΔS=0



1/q

- Must be careful: **often** spin-charge separation must be enforced by hand

S=1/2 AFM Chain in a Field



$$\mathcal{H} = J\sum_x \vec{S}(x)\cdot\vec{S}(x+1) - h\sum_x S^z(x)$$

- Field-split Fermi momenta: $k_{F\uparrow} = \frac{\pi}{2} - \delta\ ,\ k_{F\downarrow} = \frac{\pi}{2} + \delta$
  - ✓ Uniform magnetization $M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$
  - ✓ Half-filled condition $n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$
- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$ scaling dimension $1/4\pi R^2$ **increases**

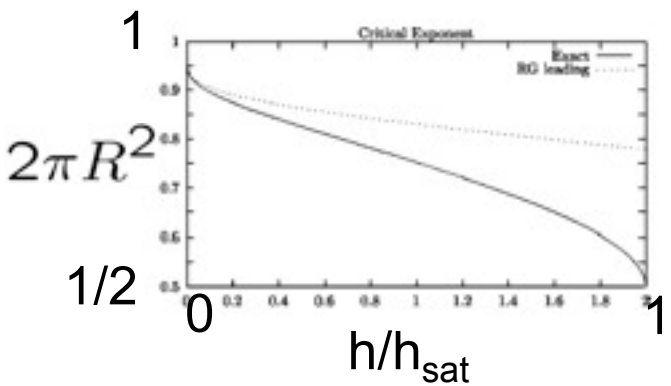
$S^z_{\pi\pm 2\delta}$

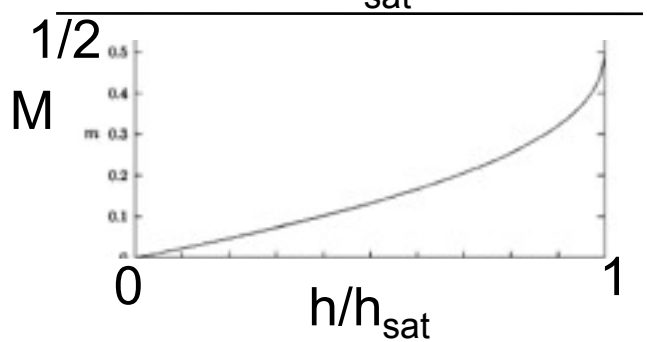
- $S^{x,y}$ components ($\Delta S=1$) remain at π scaling dimension πR^2 **decreases**

$S^{\pm}_{\pi}$

- Derived for free electrons but correct always - *Luttinger Theorem*

Affleck and Oshikawa, 1999





h$_{sat}$=2J

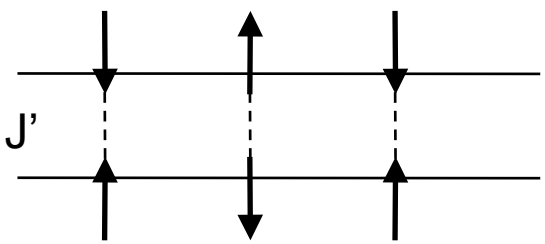
- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

Weakly coupled Heisenberg chains in magnetic field

- non - frustrated inter-chain coupling $\vec{S}_r \cdot \vec{S}_{r'} \rightarrow N_r^x N_{r'}^x + N_r^y N_{r'}^y + N_r^z N_{r'}^z$

most relevant

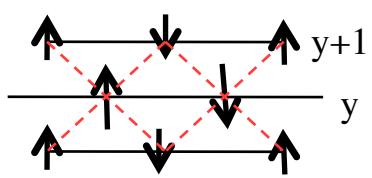
less relevant



$2\pi R^2 < 1/(2\pi R^2)$

spins order in the plane perpendicular to the direction of magnetic field (z): **umbrella / cone / spin-flop states**

- frustrated inter-chain coupling



$\vec{S}_{x,y} \cdot (\vec{S}_{x,y+1} + \vec{S}_{x+1,y+1}) \rightarrow N_y^x \partial_x N_{y+1}^x + N_y^y \partial_x N_{y+1}^y + \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1)$

less relevant

most relevant (small to intermediate fields)

$1+2\pi R^2 > 1/(2\pi R^2)$

★ frustration promotes collinear **SDW** order

Ideal J-J' model in magnetic field

OS, Balents 2007

- Two important couplings for h>0
- Quantum phase transition between SDW and Cone states

Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2

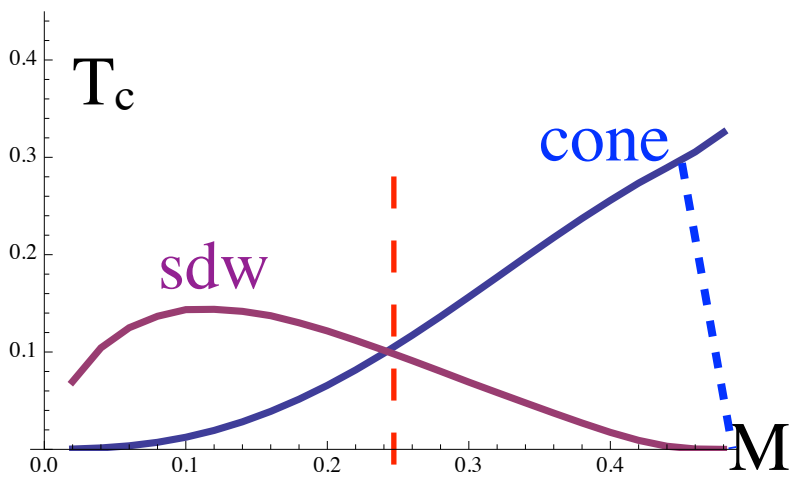
"collinear" SDW

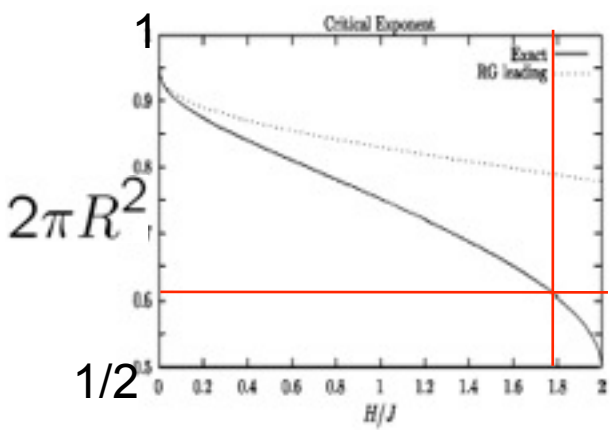
dim $1+2\pi R^2$: 2 -> 3/2

spiral "cone" state

$$k_{F\downarrow} - k_{F\uparrow} = 2\delta = 2\pi M$$

- "Critical point": $1+2\pi R^2 = 1/2\pi R^2$ gives $2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$ at M = 0.3



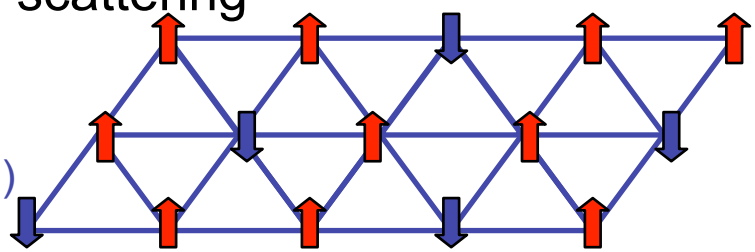


also: Kolezhuk, Vekua 2005

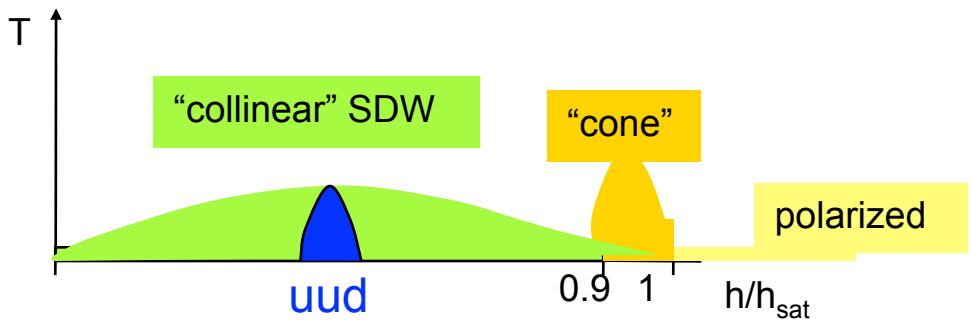
h/h$_{sat}$

J-J' model: magnetization plateaux via commensurate locking of **SDW**

- "Collinear" SDW state *locks* to the lattice at low-T
  - -"irrelevant" (1d) umklapp terms become relevant once SDW order is present (when *commensurate*): multiparticle umklapp scattering



  - -strongest locking is at M=1/3 M_{sat}
  - ✓Observed in **Cs_2CuBr_4** (Ono 2004, Tsuji 07, Fortune 09)
    - ***down***-spins at the centers of hexagons



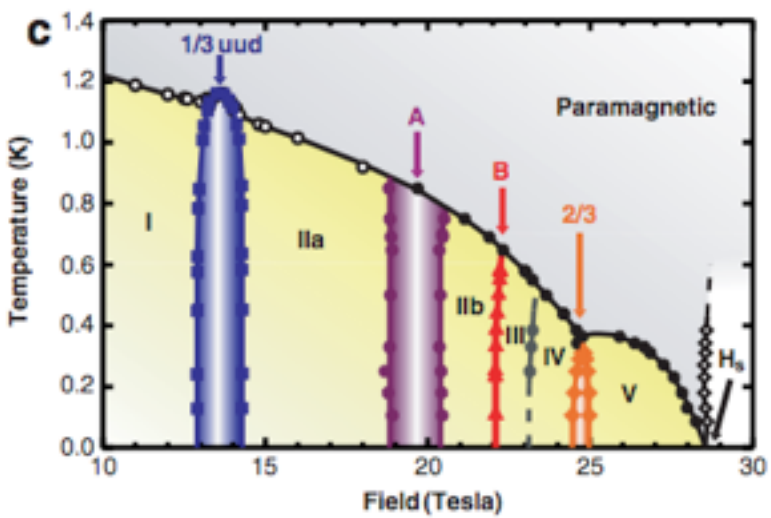
$$\left(\Psi_R^\dagger \Psi_L\right)^n \rightarrow (\pi - 2\delta)n = 2\pi m \rightarrow \boxed{2M = 1 - 2m/n}$$

n	3	4	5	5	6
m	1	1	1	2	1
2M	1/3	1/2	3/5	1/5	2/3

naively thinking

Cs_2CuBr_4 Fortune et al 2009

1/3 2/3



Plateau more carefully

OS, Katsura, Balents PRB 2010

$$M^{(n,m)} = \frac{1}{2}\left(1 - \frac{2m}{n}\right)$$

- Umklapp *must* respect triangular lattice symmetries
  - translation along chain direction $\phi_y(x) \to \phi_y(x+1) - R(\pi - 2\delta)$
  - translation along diagonal $\phi_y(x) \to \phi_{y+1}(x+1/2) - R(\pi - 2\delta)/2$
  - spatial inversion $\phi_y(x) \to \pi R - \phi_y(-x)$

$$H^{(n)}_{umk} = \sum_y \int dx\, t_n \cos[\frac{n}{R}\phi_y] \quad \textbf{and} \quad n = m \ (mod\ 2)$$

same parity condition

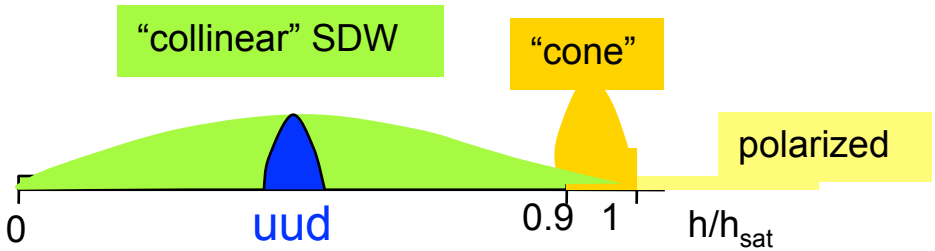
- **n**-th plateau width (in field) $width \sim \left(J'/J\right)^{n^2/(4(4\pi R^2 - 1))}$

n	**3**	**8**	**5**	**10**	**12**
m	**1**	**2**	**1**	**4**	**2**
2M	**1/3**	1/2	**3/5**	1/5	2/3

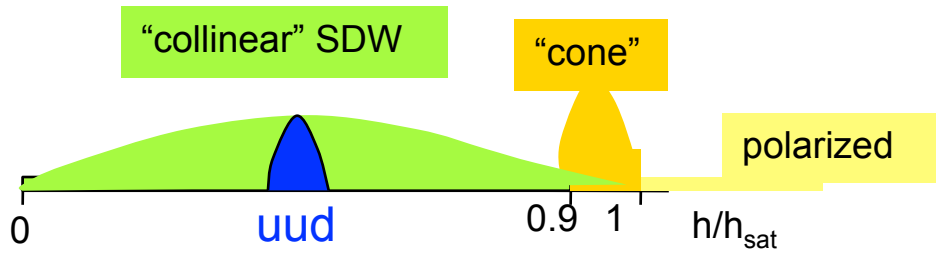
large **n** leads to exponential suppression

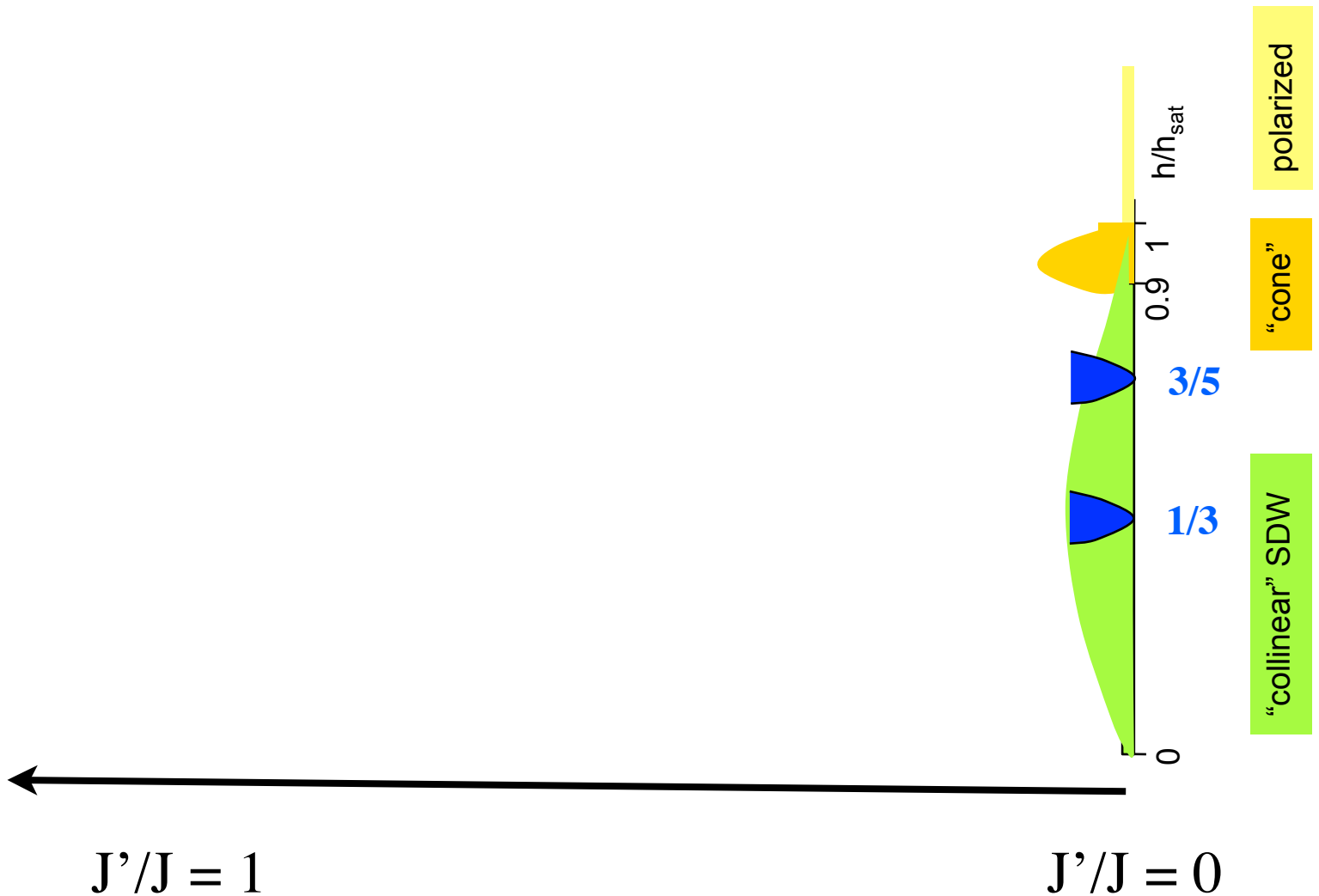
- 1/3-plateau is most prominent, 3/5 is possible (if falls within the SDW region). Exponentially weak 1/2- and 2/3- plateaux, if any !

J' << J limit

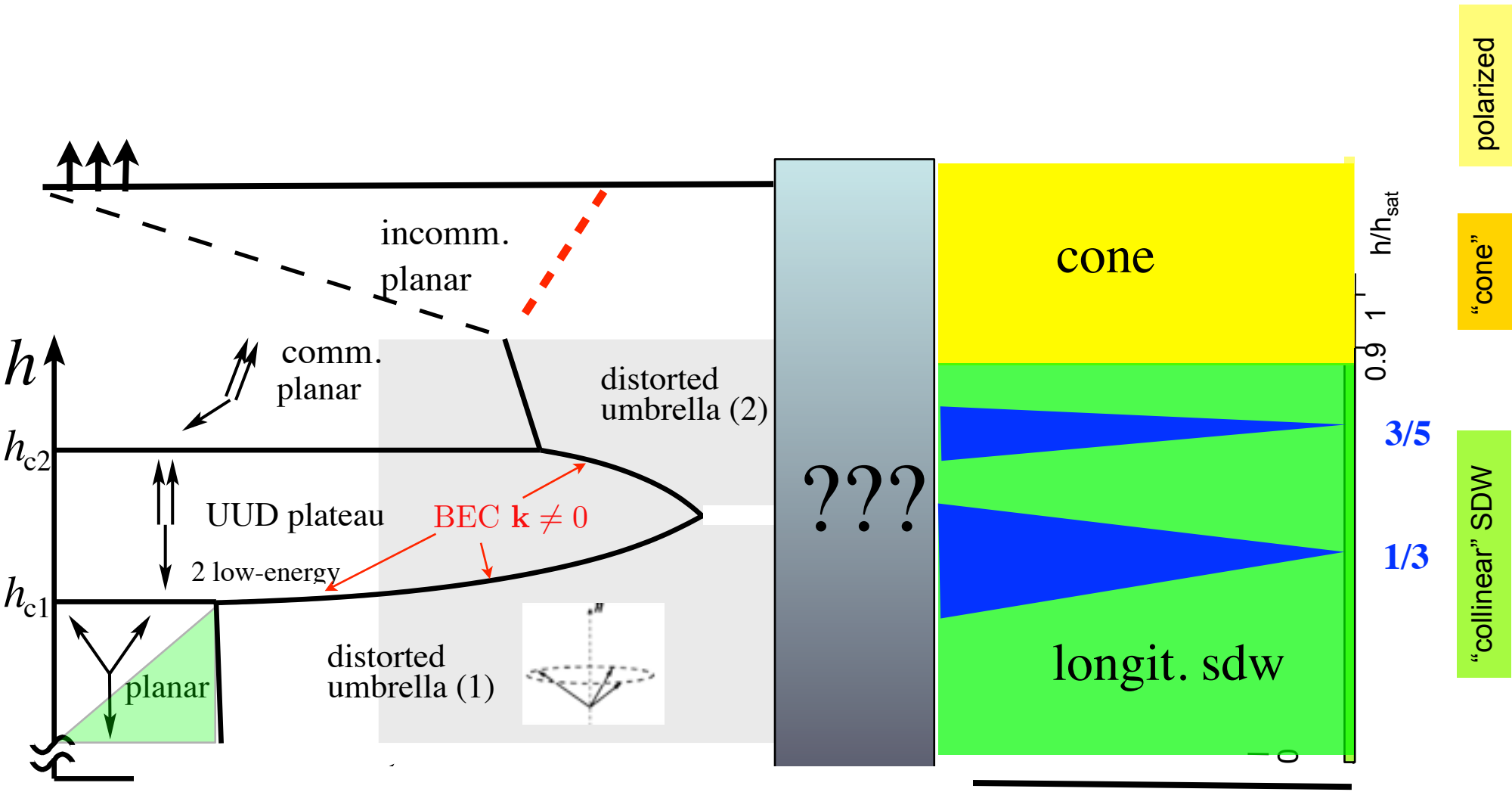


J' << J limit


"collinear" SDW
"cone"
polarized
0
uud
0.9
1
h/h_sat


h/h_sat
polarized
"cone"
"collinear" SDW
0
0.9
1
3/5
1/3
J'/J = 1
J'/J = 0

J' << J limit to J' = J point...

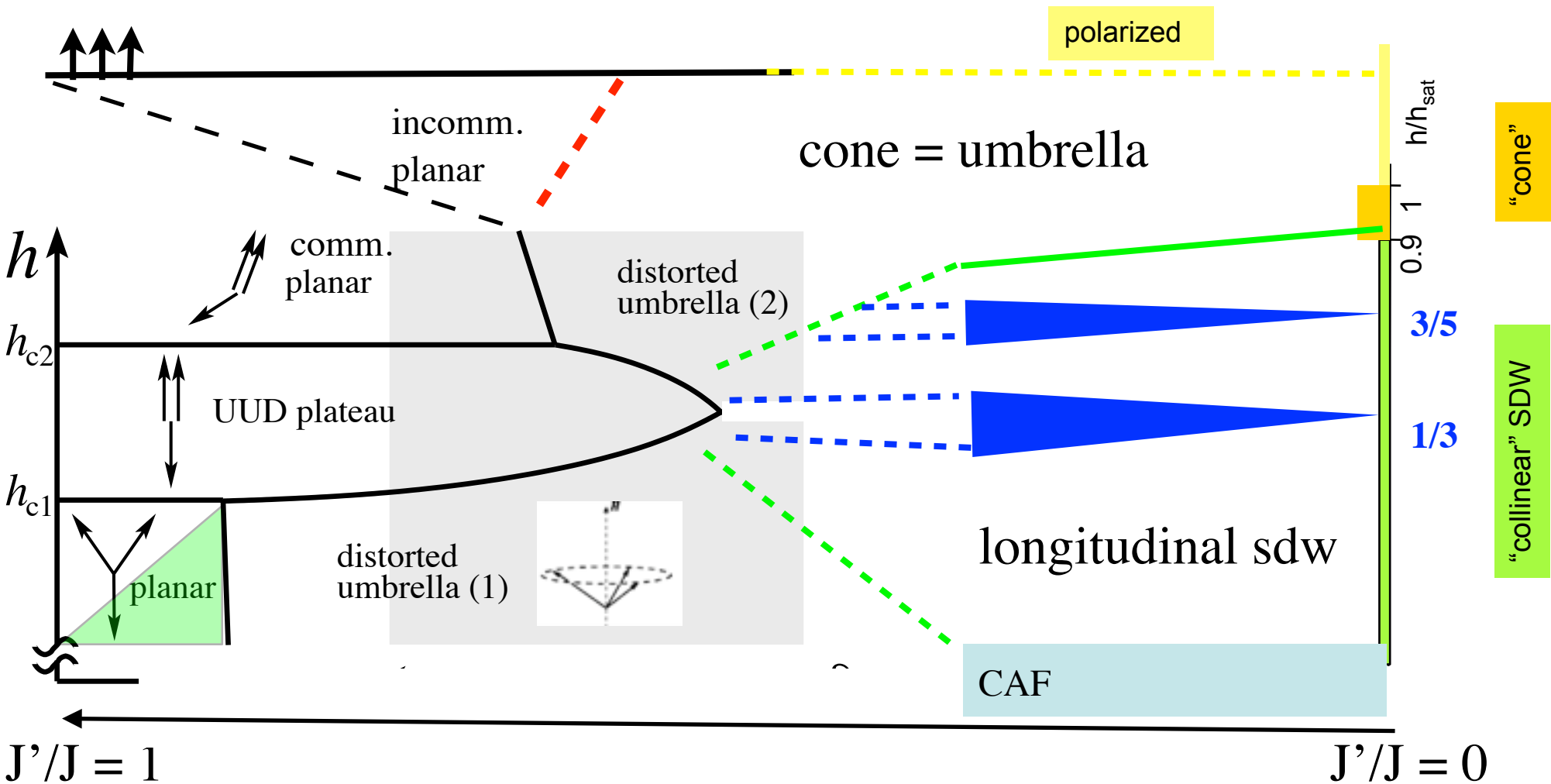


J'/J = 1

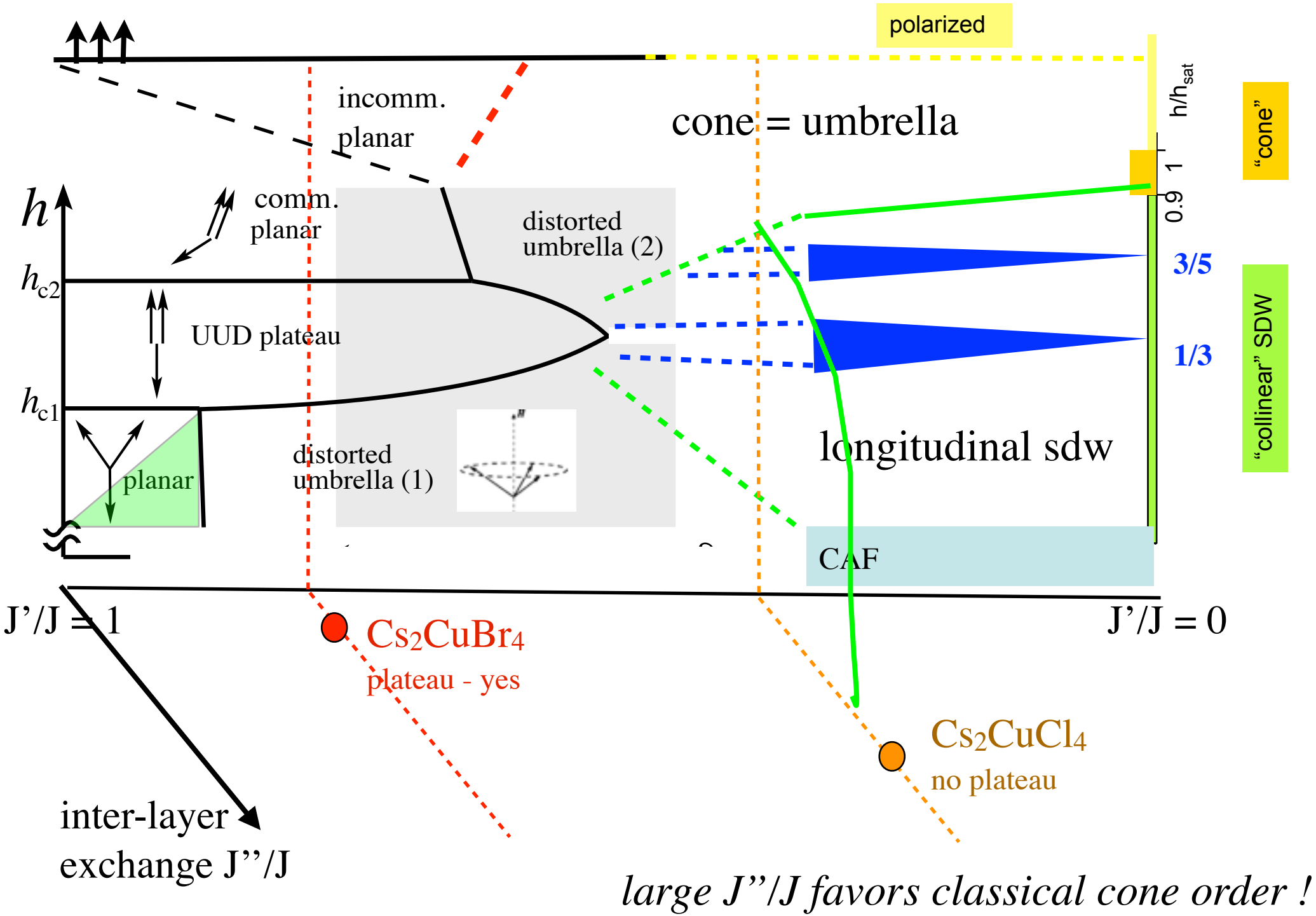
J'/J = 0

Global phase diagram

Hypothesis: 1/3 plateau extends for all $0 < J'/J < 1$;
other magnetization plateaux terminate above some critical J'/J ratio.



Experimental relevance



large J''/J favors classical cone order !

Conclusions

- Magnetization plateau persists for all J'/J
  - semiclassical interacting spin waves near J - J' << J
  - 1d scaling + symmetry arguments near J' << J
- Many other interesting phases
  - Longitudinal SDW
  - commensurate-incommensurate transitions
- Many open questions, excellent problem for numerical studies