Spin-orbit-induced spin-density wave in quantum wires and spin chains

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also appears in quasi-1d Kagome antiferromagnet, work with *Andreas Schnyder* (MPI Stuttgart) and *Leon Balents* (KITP)

PRL **98**, 126408; PRL **100**, 156402; PRB **78**, 054436; PRB **78**, 174420
and work in progress

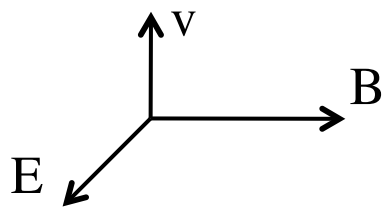
Dahlem Center, Freie Universitat Berlin, Sept. 29, 2010

Motivation:

Why be interested in
weak relativistic interaction
-- spin-orbit?

Spin - Orbital (SO) coupling

—Relativistic effect:



$$\vec{B} = \gamma \frac{\vec{v} \times \vec{E}}{c^2}$$

spin in magnetic field

$$H_{SO} = \frac{\mu_B}{2c^2}(\vec{v} \times \vec{E}) \cdot \vec{\sigma}$$

—Atoms:

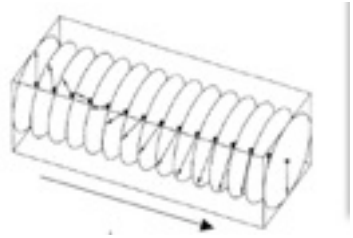
$$H_{SO} = \lambda \vec{L} \cdot \vec{S} \qquad \lambda \sim Z/a^3 \sim (Z^2 e^2/\hbar c)^2 m e^4/\hbar^2$$

—Magnetic materials: Dzyaloshinskii-Moriya interaction *via* exchange + SO (1957)

$$\vec{D}_{rr'} \cdot \vec{S}_r \times \vec{S}_{r'} \rightarrow \vec{M} \cdot \vec{\nabla} \times \vec{M} \qquad D \sim \lambda J$$

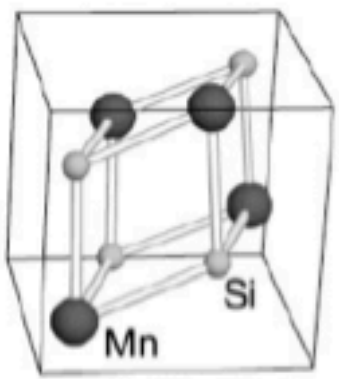
✓ *requires absence of inversion symmetry* **r -> -r**

Textbook (Landau-Lifshits VIII p.286) example: MnSi
pitch ~ 170 A



50 years later: MnSi - quantum phase transition under pressure

Itinerant ferromagnet with long pitch spiral - non-Fermi liquid under pressure



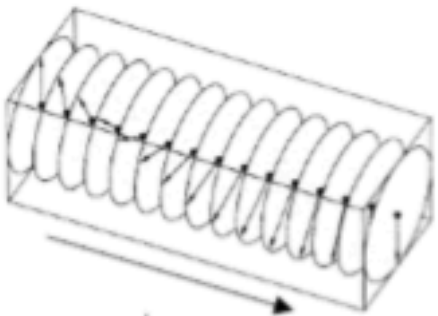
cubic (B20)
no inversion

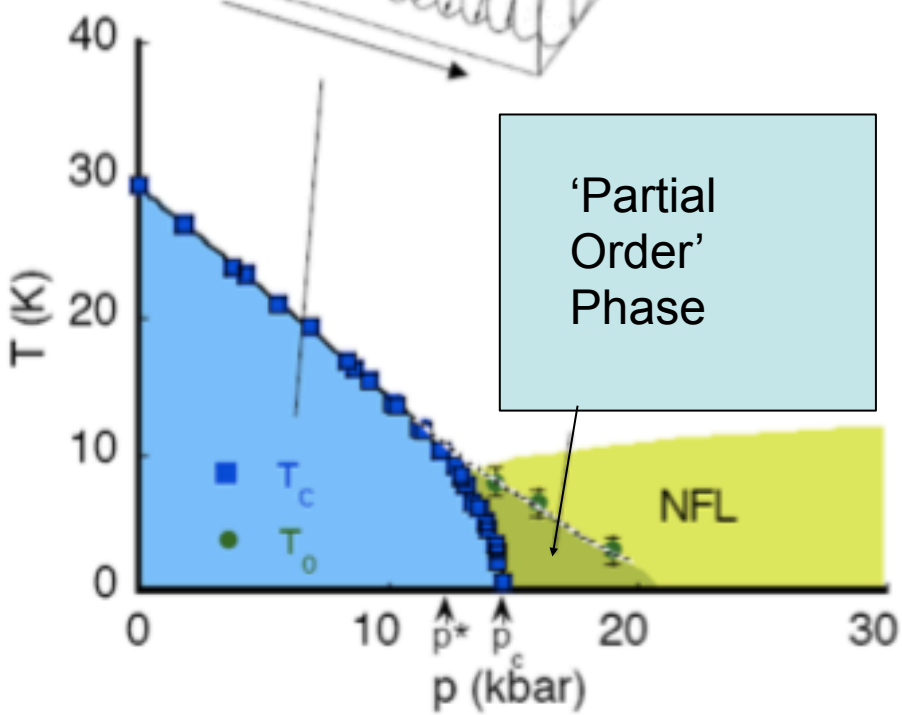
ENERGY

(1) ferromagnetism

(2) spin-orbit coupling:
Dzyaloshinsky-Moriya
rotation-invariant
$\lambda \approx 170$ Å ($a = 4.558$ Å)

(3) crystal field ($P2_13$):
helix locked at <111>



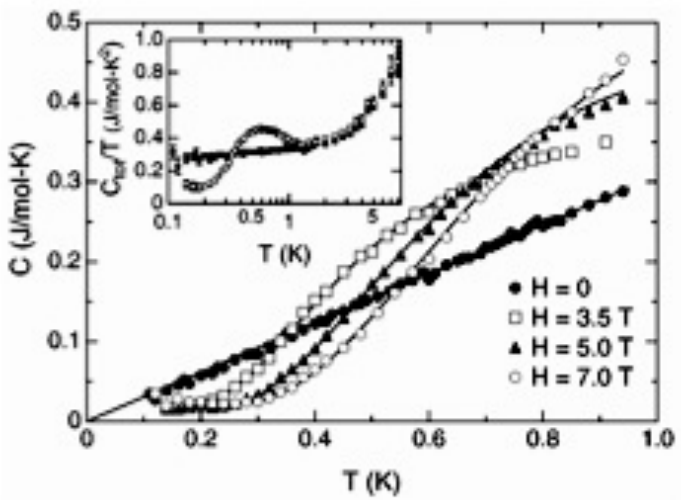


(slide from A. Vishwanath)

Field-induced gap in 1D antiferromagnet

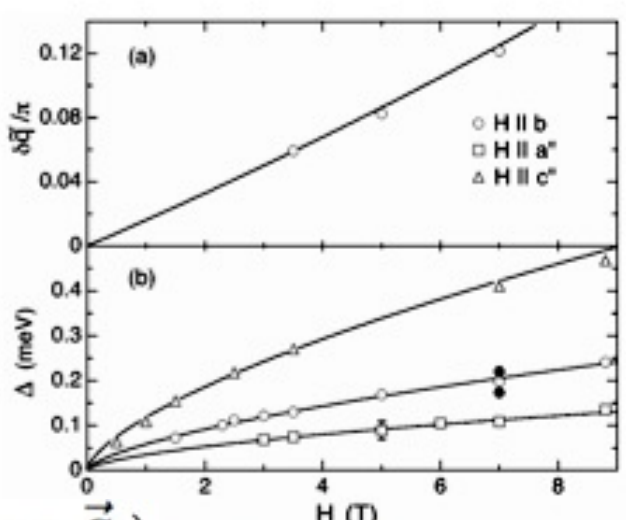
Dender et al PRL 79, 1750 (1997)

•Cu benzoate: specific heat in the magnetic field $C \sim \exp[-\Delta/k_B T]$



$\delta q \sim H$: standard Heisenberg

$\Delta \sim H^{2/3}$: staggered DM



Massive incommensurate S=1 excitations

$$\omega = \sqrt{\Delta^2 + v^2(q-\delta q)^2}$$

$$H_{DM} = \sum_j (-1)^j \vec{D} \cdot (\vec{S}_{j-1} \times \vec{S}_j)$$

Oshikawa, Affleck PRL 79, 2883 (1997)

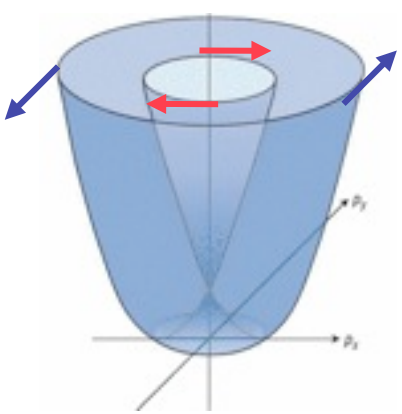
Spintronics

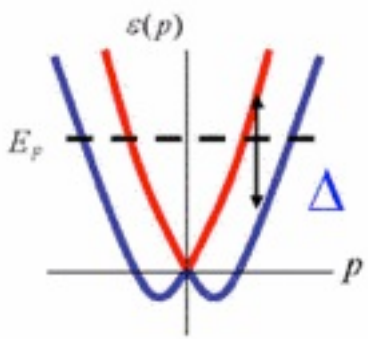
$$\vec{E} = -\vec{\nabla}U(\vec{r}) \rightarrow H_{SO} = \alpha\, \vec{\nabla}U \times \vec{p} \cdot \vec{\sigma}$$

* No inversion symmetry =>
  - 2DEG heterostructures (e.g. GaAs)
  - Surface states (e.g. Au[111])

* Rashba Hamiltonian (1984) $H_R = \alpha_R(p_y\sigma_x - p_x\sigma_y)$ $\alpha_R \sim \vec{\nabla}U(\vec{r}) \sim \frac{dU}{dz} \neq 0$

Free electrons + SO :





Spin-orbit splitting of bands

$$H_{SO} = \alpha(s_x p_y - s_y p_x)$$

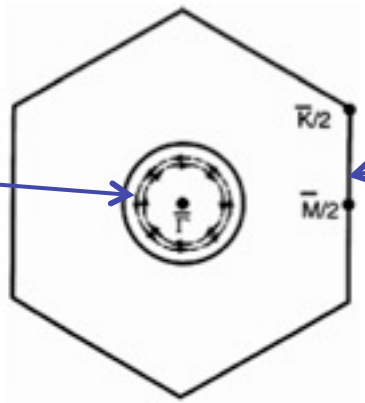
$$\varepsilon(p) = \frac{p^2}{2m} \pm \Delta, \quad \Delta \approx \alpha p_F$$

Spin splitting of an Au(111) surface states: ARPES

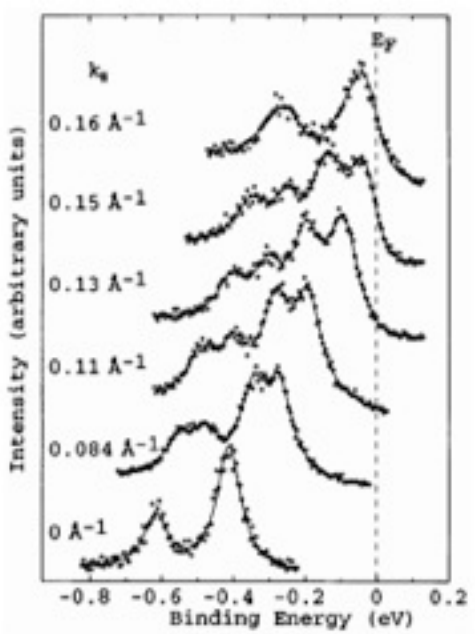
LaShell et al. PRL 77, 3419 (1996)

Surface obtained by cutting along (111) plane $\vec{n} \cdot \vec{k}_{\parallel} = 0$

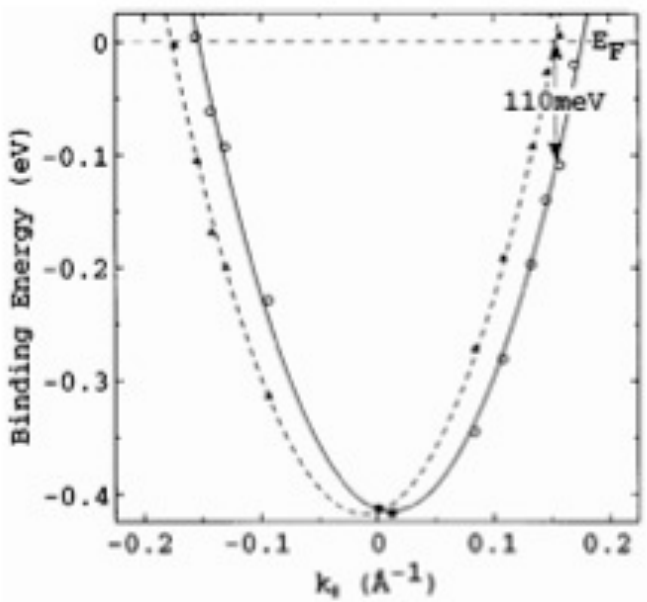
Spin-split Fermi surface



Brillouin zone



ARPES spectra



dispersion fit: $\Delta \sim 55\text{meV} = E_F/8$!

Topological Insulators,
M. Z. Hasan and C. L. Kane,
arxiv 1002.3895

Strong spin-orbit
+
surface states

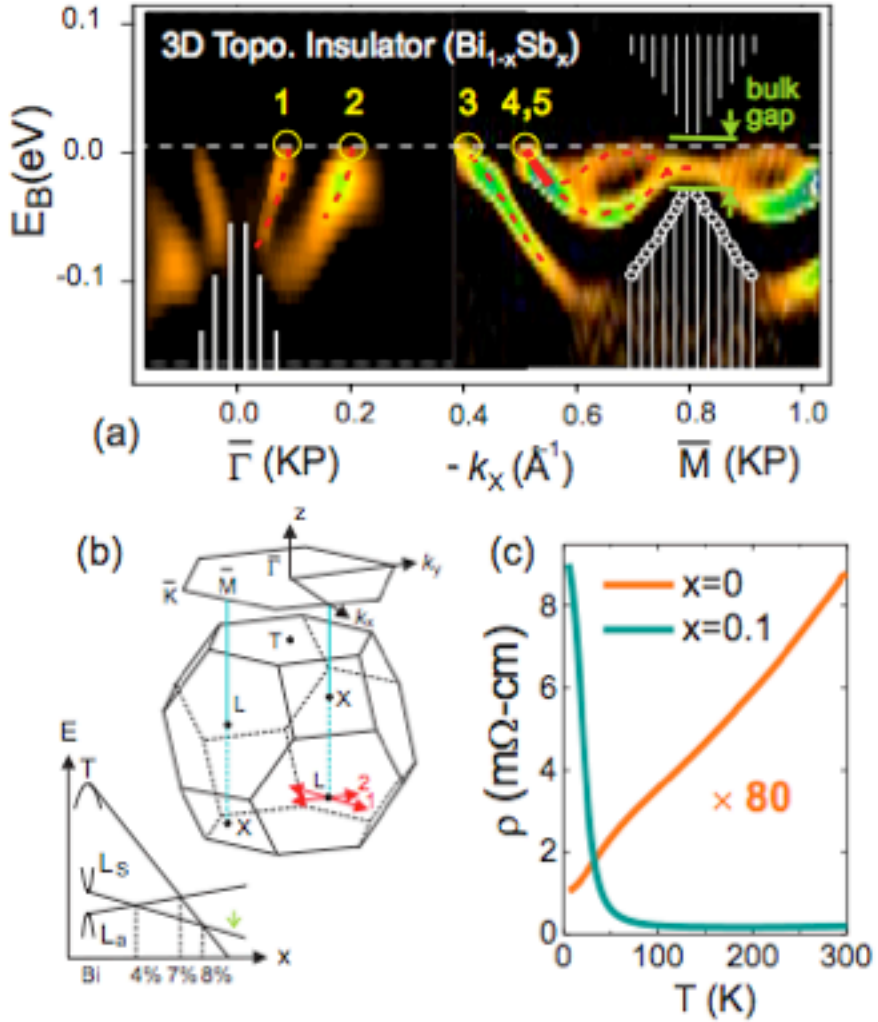


FIG. 9 Topological surface states in $Bi_{1-x}Sb_x$: (a) ARPES data on the 111 surface of $Bi_{0.9}Sb_{0.1}$ which probes the occupied surface states as a function of momentum on the line connecting the time reversal invariant points $\bar{\Gamma}$ and $\bar{M}$ in the surface Brillouin zone. Only the surface bands cross the Fermi energy 5 times. This, along with further detailed ARPES results (Hsieh, *et al.*, 2008) establish that the semiconducting alloy $Bi_{1-x}Sb_x$ is a strong topological insulator in the (1; 111) class. (b) shows a schematic of the 3D Brillouin zone and its (111) surface projection. (c) contrasts the resistivity of semimetallic pure Bi with the semiconducting alloy.

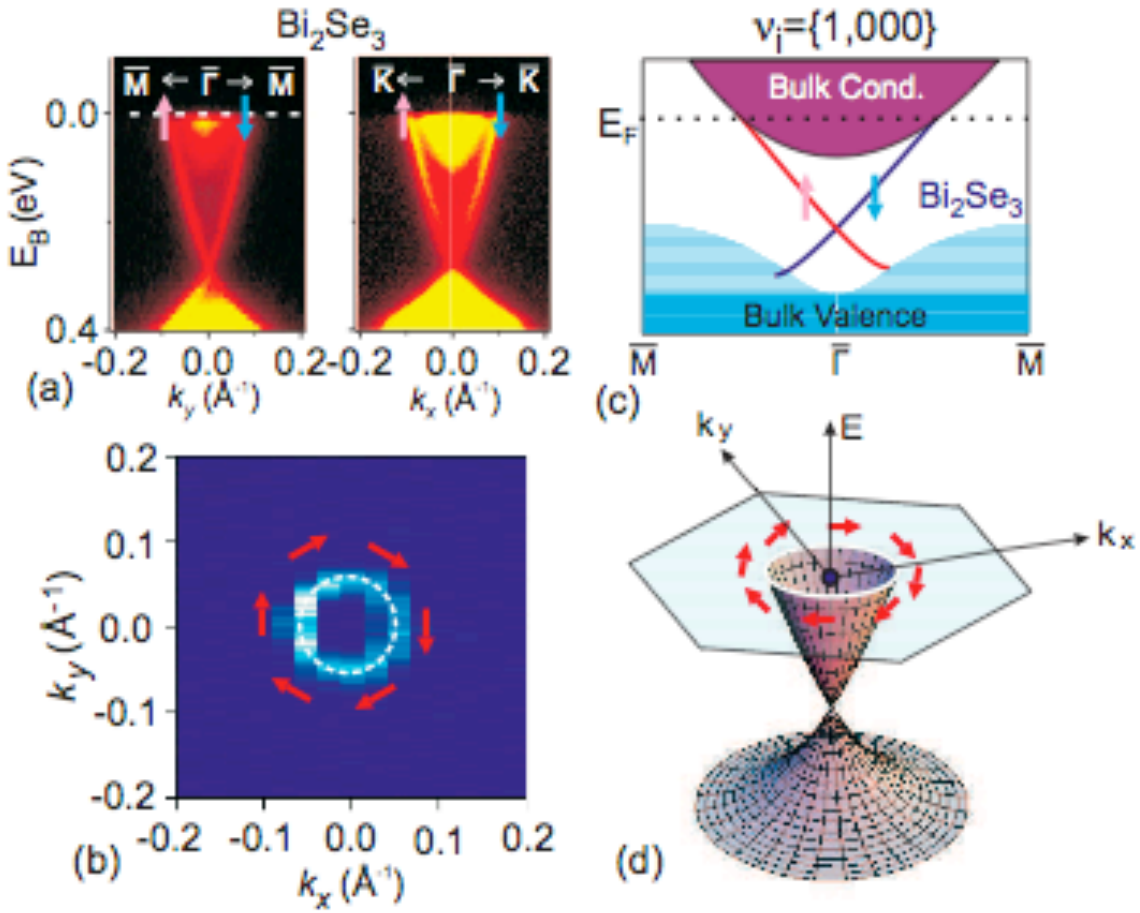
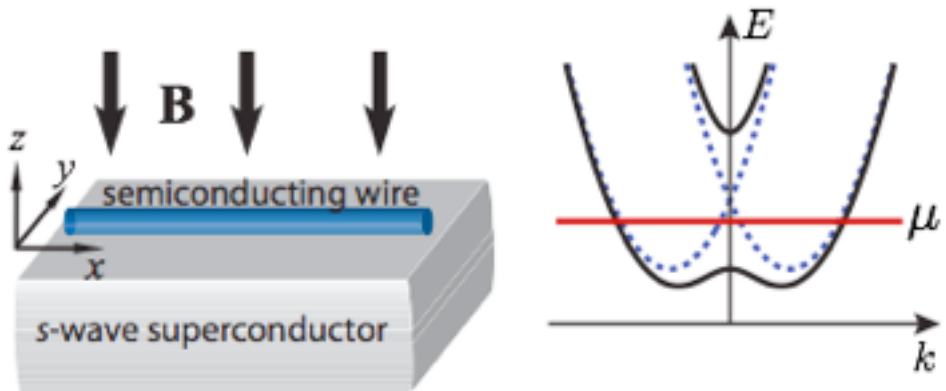


FIG. 12 Helical fermions: Spin-momentum locked helical surface Dirac fermions are hallmark signatures of topological insulators. (a) Surface dispersion relation of electronic states reveal a spin-polarized Dirac cone. (b) Surface Fermi surface exhibits chiral left-handed spin textures. Schematic presentations of (c) surface and bulk band topology and (d) a single spin-polarized Dirac fermion on the surface of Bi_2X_3 (1; 000) topological insulators (Hsieh, *et al.*, 2009b; Xia, *et al.*, 2009a).

1D setting: magnetized wires with SOI and in proximity with s-wave superconductors



Helical liquids and Majorana bound states in quantum wires

Yuval Oreg,[1] Gil Refael,[2] and Felix von Oppen[3]

Non-Abelian statistics and topological quantum information processing in 1D wire networks

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[2]*Department of Physics and Astronomy, University of California, Irvine, California 92697*
[3]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[4]*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, 14195 Berlin, Germany*
(Dated: September 21, 2010)

arxiv 1003.1145 and 1006.4395

Thus

- Spin-orbit interactions show up in different physical situations
  - Dresselhaus, Rashba, Dzyaloshinskii-Moriya...

- Result in interesting symmetry reductions
  - Momentum dependent magnetic field
  - Symmetry reduction SU(2) => U(1)

- Are not *that* [$(v/c)^2$] small : can be (and, *are*) observed currently!

Interplay of e-e interactions *and* spin-orbit is very interesting

Outline

- *Warm-up*: van der Waals like coupling between spins in quantum dots

- Brief intro to quantum wires
  - Key scattering processes in a single-subband wire
  - Effect of magnetic field: Zeeman splitting

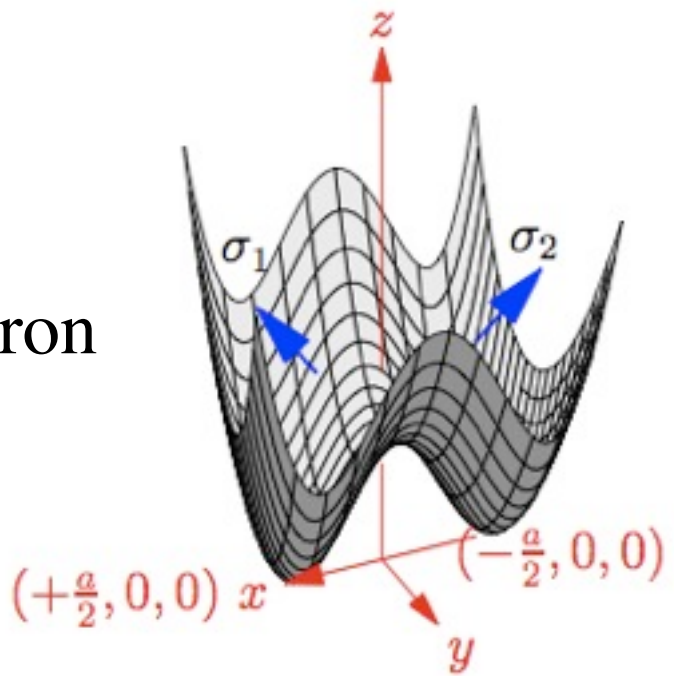
- Spin-orbital effects
  - Cooper channel
  - Spin-density wave formation (orthogonal magnetic field)
  - Transport: suppressed backscattering
  - Connection with spin chain physics: *uniform DM* interaction
  - Unusual implications for ESR experiments

- Conclusions

Warm-up exercise: spin-orbit mediated coupling of spins in the absence of exchange (no tunneling!)

Idea: Spin-Orbit correlates spin and orbital motion, while Coulomb correlates orbital motion of electrons

Coupled single-electron quantum dots



Unitary transformation to "remove" Spin-Orbit

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2) + \alpha_R \vec{p} \times \vec{\sigma} \cdot \hat{z}$$

Unitary rotation:

$$U = \exp[im\alpha_R \, \hat{z} \cdot \vec{r} \times \vec{\sigma}]$$

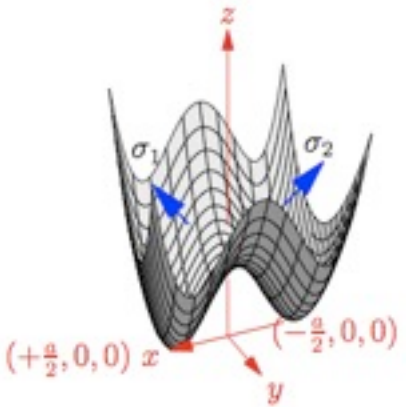
Transforms SOI into: $-m\alpha_R^2(xp_y - yp_x)\sigma^z + O(\alpha_R^3)$

- Spin-orbit form *indeed* $\boxed{-m\alpha_R^2 \, L_j^z \, \sigma_j^z}$

- Assumes that (SO length) << (confining length)

Shahbazyan, Raikh (1994)
Aleiner, Fal'ko (2001)

Two single-electron dots coupled by Coulomb interaction



$$\frac{e^2}{|\vec{r}_1 - \vec{r}_2 + a\hat{x}|} \rightarrow \frac{e^2}{2a^3}\left(2(x_1 - x_2)^2 - (y_1 - y_2)^2\right)$$

- Four harmonic oscillators: along X (Y), symmetric (anti-symmetric)
- Perturbation: spin-orbit $\alpha_R^2(L_1^z\sigma_1^z + L_2^z\sigma_2^z)$
- 2nd order energy correction $\Delta E = -\sum_{i,j=S,A} \frac{|\langle 0|H_{so}|1x_i\ 1y_j\rangle|^2}{\omega_{ix} + \omega_{jy}}$
- van der Waals-like **spin-spin** interaction

$$H_{\mathrm{vdW}} = -\frac{\alpha_R^4 e^4}{4\omega^5\, a^6}\,\sigma_1^z\sigma_2^z$$

Generalizations

- vdW interaction is absent in strict d=1 limit, when $\omega_y \longrightarrow \infty$

  but external magnetic field will again result in two non-commuting perturbations!

- Effect of magnetic field: appearance of dipolar coupling

$$\Delta E_Z = -(g\mu_B B\alpha_R)^2 \frac{e^2}{a^3}\left(2\frac{\sigma_1^x\sigma_2^x}{\omega_x^4} - \frac{\sigma_1^y\sigma_2^y}{\omega_y^4}\right)$$

for $g\mu_B B \gg \sqrt{m\omega}\alpha_R$ Flindt et al 2006, Trif et al 2007

- Implications for exchange interactions: expect symmetry breaking of DM form only in α_R^4 order.

  - Hidden SU(2) symmetry (Shekhtman et al 1992, Koshibae et al 1994)

$$H_{ex}/J = b\vec{S}_1\cdot\vec{S}_2 + D\hat{d}\cdot\vec{S}_1\times\vec{S}_2 + \Gamma(\hat{d}\cdot\vec{S}_1)(\hat{d}\cdot\vec{S}_2)$$

Serious consequences for Wigner crystals

- Electron lattice with *exponentially small* exchange
  - competing multi-spin exchanges
  - extensive spin degeneracy (e.g. Pomeranchuk effect)

- Spin vdW coupling: ferromagnetic Ising interaction
  - Non-exchange type (no overlap of wave functions)
  - No frustration
  - lifts degeneracy

- Ferromagnetic ground state (GaAs r_s~100; InAs r_s~20)

SOI + Coulomb does lead to interesting new physics

Sun, Gangadharaiah, OS, PRL 100, 156402 (2008)

Outline

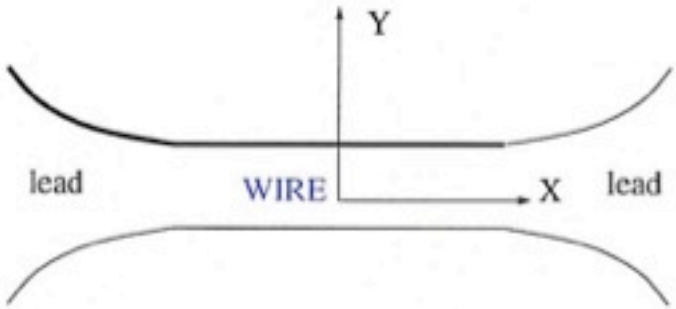
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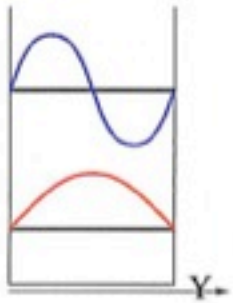
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Quantum wire



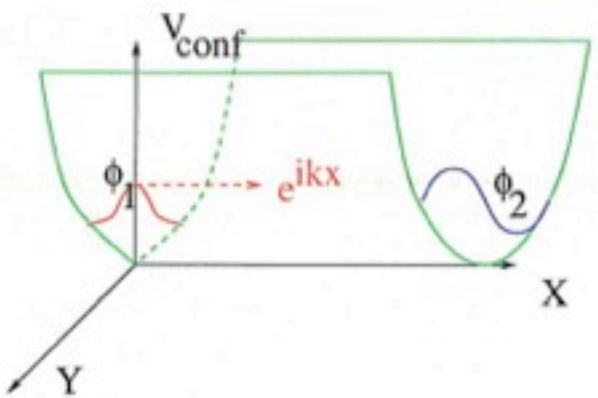


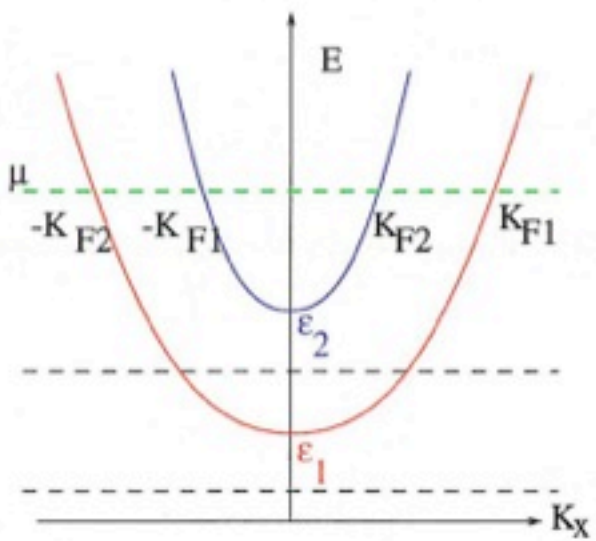
width $\propto \lambda_F \approx 300 - 400$ Å,
length $\sim 1 - 10\mu m \ll L_{elastic}, L_{inelastic}$

• electron is confined in Y, Z directions but free to move in X.

$$H_0(x,y) = -\frac{\nabla_x^2}{2m} - \mu - \frac{\nabla_y^2}{2m} + V_{conf}(y)$$

$$\Psi(x,y) = \sum_n \varphi_n(y)\ e^{\pm ik_n x} \Rightarrow E = \frac{k_n^2}{2m} - \mu + \epsilon_n$$

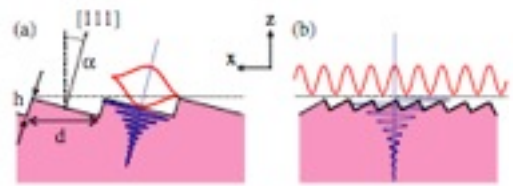




Vicinal Au(111) surface states: one-dimensional electrons on terraces

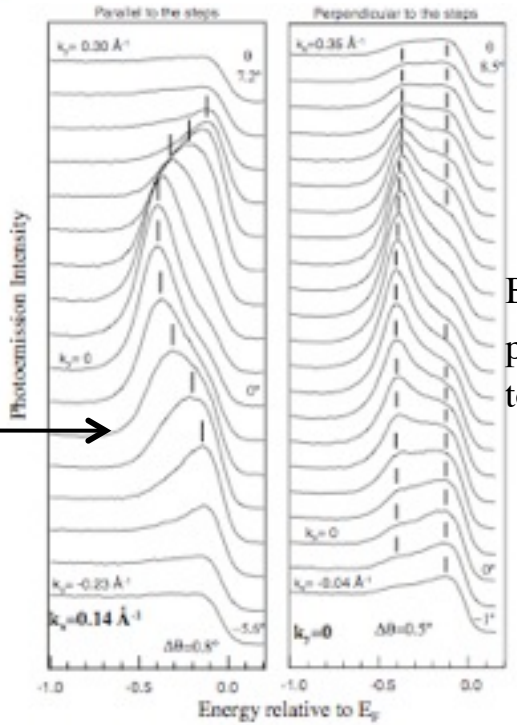
•Cut at small miscut angle $\alpha \sim 3.5^0$: surface composed of {111} steps (terraces)

d ~ 38 A



•Terrace: one-dimensional states ($d \sim \lambda_F$)

Disperses along the terrace



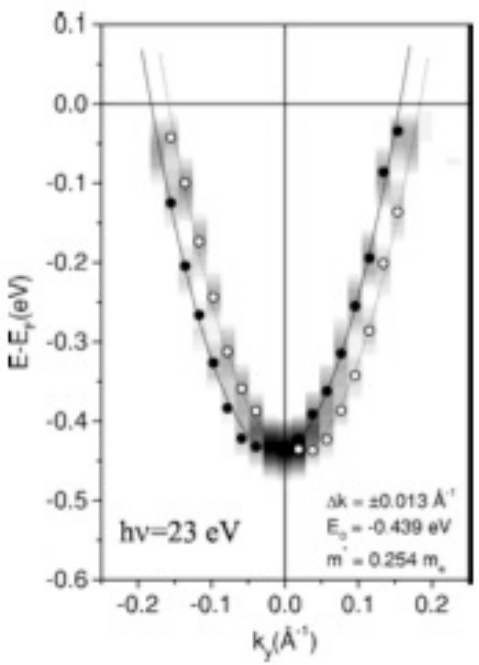
But **not** perpendicular to it!

•Dispersing states are *spin-split* :

$k_{F1} = 0.157\ A^{-1}$

$k_{F2} = 0.184\ A^{-1}$

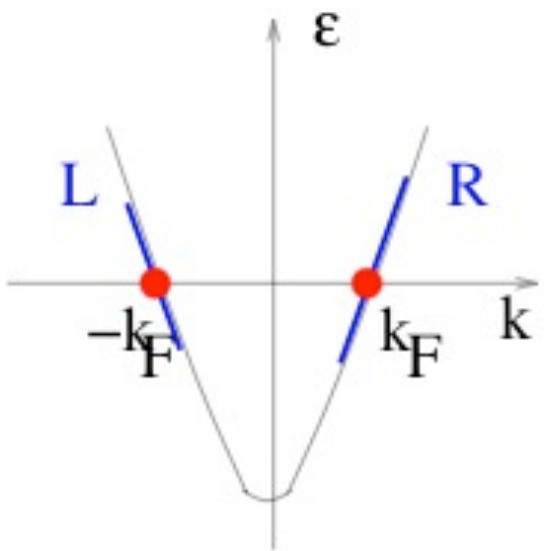
no magnetic field here



Mugarza et al
PRL 87, 107601 (2001)
PRB 66, 245419 (2002)

Quantum wire

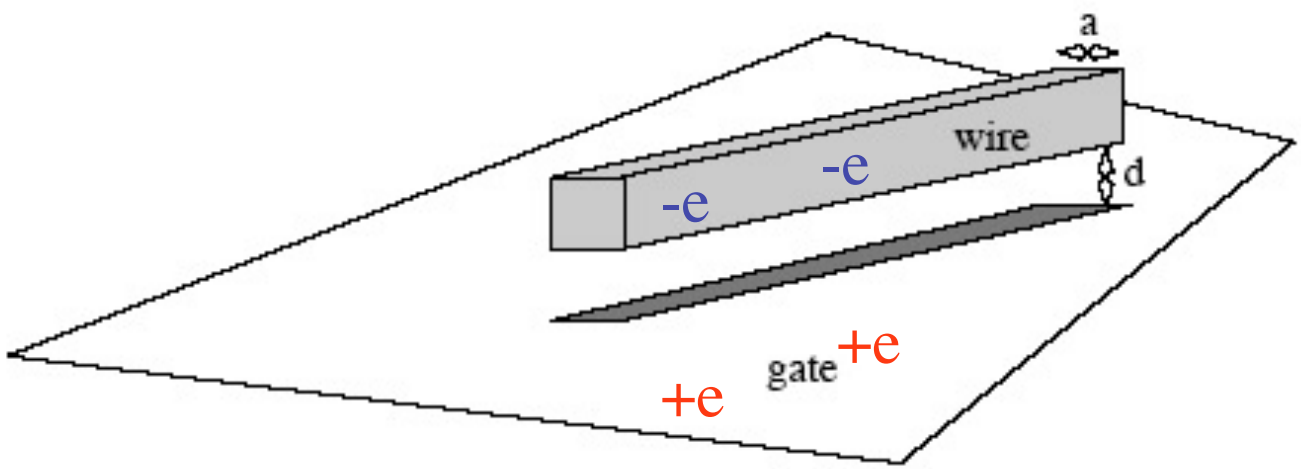
$$H_{\text{wire}} = \sum_s \int_x \Psi_s^+(x)\left(-\frac{\partial_x^2}{2m} - \mu\right)\Psi_s(x) + \frac{1}{2}\sum_{s,s'}\int_{x,x'} U(x-x')\Psi_s^+(x)\Psi_{s'}^+(x')\Psi_{s'}(x')\Psi_s(x)$$



$$\Psi_s(x) \to e^{ik_F x}R_s(x) + e^{-ik_F x}L_s(x)$$

Slow modes: **right** and **left** movers

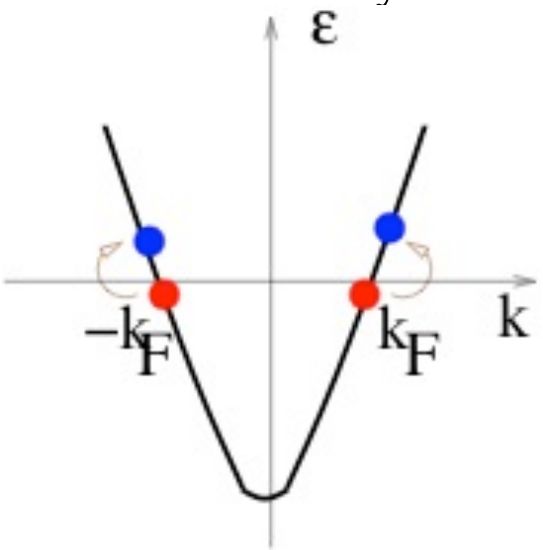
Coulomb interaction is screened by the gate => short-ranged U(x)



a/d=0.1-1

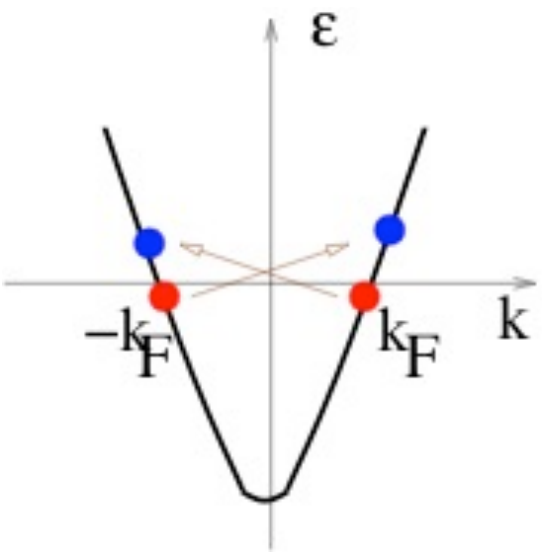
Interaction leads to two-particle scattering

- Must conserve momentum (at T=0)
- characterized by momentum transfer q



Forward q~ 0 (mostly controls charge)

$$U(0) = \int_x U(x)$$



Backscattering q~ $2k_F$ (mostly spin)

$$U(2k_F) = \int_x U(x) e^{i2k_F x}$$

Long-range interaction: U(0) >> $U(2k_F)$

Screened interaction: U(0) ~ $U(2k_F)$

Hydrodynamic description: bosonization

- All excitations are density waves



- Two independent liquids: charge and spin are decoupled

Charge density $\rho_c \sim \partial_x \phi_c$ ϕ_c = "coordinate"

Charge current $j_c \sim \partial_x \theta_c$ j_c = "momentum"

Dual pair ϕ and θ

$$[\phi_c(x), \partial_y \theta_c(y)] = i\hbar\delta(x-y)$$

$$H_{\text{wire}} = \frac{1}{2} \sum_{i=\text{charge, spin}} \int_x \frac{v_i}{K_i} (\partial_x \phi_i)^2 + v_i K_i (\partial_x \theta_i)^2$$

PE KE

charge $$K_c \approx 1 - \frac{2U(0) - U(2k_F)}{2\pi v_F} < 1 \ , \ v_c = v_F / K_c > v_F$$

spin $$K_s = 1 + \frac{U(2k_F)}{2\pi v_F} \approx 1$$

controlled by spin-rotational [SU(2)] symmetry

Correlation functions are determined by interaction-dependent K_c & K_s

- Charge correlations

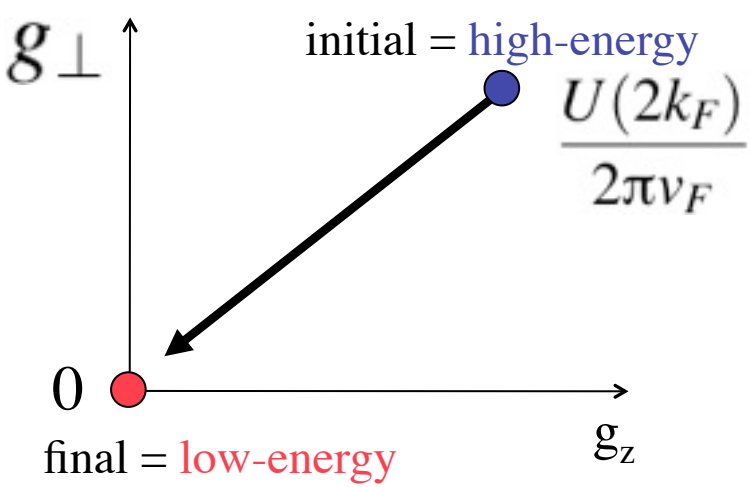
$$\langle \rho(x)\rho(0)\rangle = \frac{1}{x^2} + \frac{\cos[2k_F x]}{x^{K_c+K_s}}$$

- Spin correlations

$$\langle S^z(x)S^z(0)\rangle = \frac{1}{x^2} + \frac{\cos[2k_F x]}{x^{K_c+K_s}}$$

$$\langle S^+(x)S^-(0)\rangle = \frac{1}{x^2} + \frac{\cos[2k_F x]}{x^{K_c+1/K_s}}$$

(zz) and (xx, yy) are equivalent only if $K_s = 1/K_s$ => $K_s = 1$ (SU(2) fixed point)



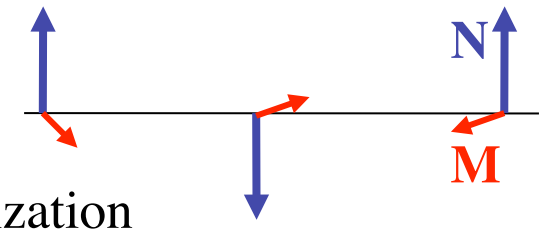
This happens via BKT renormalization:
spin backscattering is marginally irrelevant
Thus initially $g_\perp = g_z = \frac{U(2k_F)}{2\pi v_F}$

But $g_\perp \cos[\sqrt{8\pi K_s}\phi_s] \to 0$

at the "end" $K_s = 1 + g_z \to 1$

Spin backscattering *is* noticeable: NMR in Sr_2CuO_3

Spin decomposition: $\vec{S} = \vec{M}_R + \vec{M}_L + (-1)^x \vec{N}$



uniform and staggered magnetization

free part, H_0

$$H_{\text{chain}} = \frac{2\pi v}{3}\left(\vec{M}_R\vec{M}_R + \vec{M}_L\vec{M}_L + g_{\text{bs}}\vec{M}_R\vec{M}_L\right)$$

Spin correlations

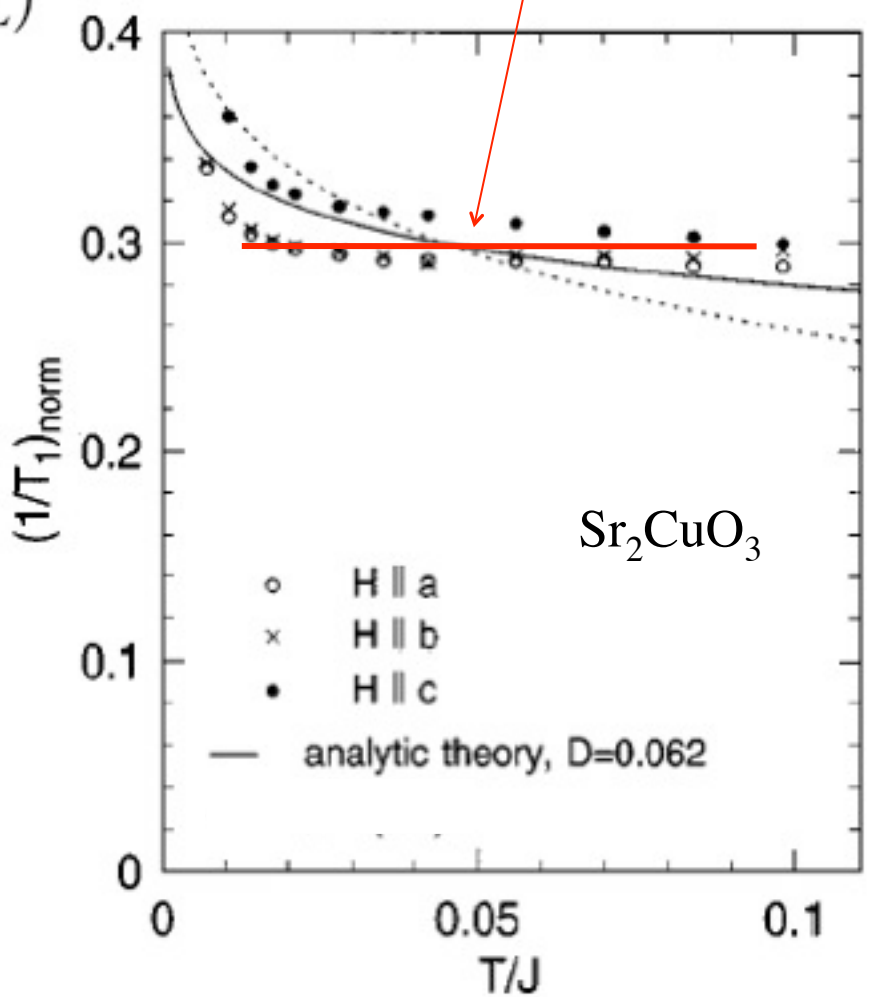
$$\langle \vec{S}(x)\vec{S}(0)\rangle = \frac{1}{x^2} + \frac{e^{i\pi x}}{x}\sqrt{\ln x}$$

NMR relaxation rate

$$\frac{1}{T_1} \sim T\int dk \frac{\text{Im}\chi(k,\omega)}{\omega} \rightarrow \sqrt{\ln(\Lambda/T)}$$

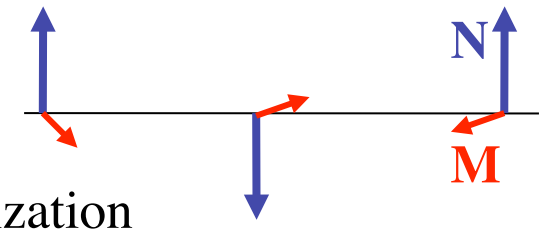
(OS, Singh, Sandvik;
Takigawa,OS,Sandvik,Singh 1997)

noninteracting spinons



Spin backscattering *is* noticeable: NMR in Sr_2CuO_3

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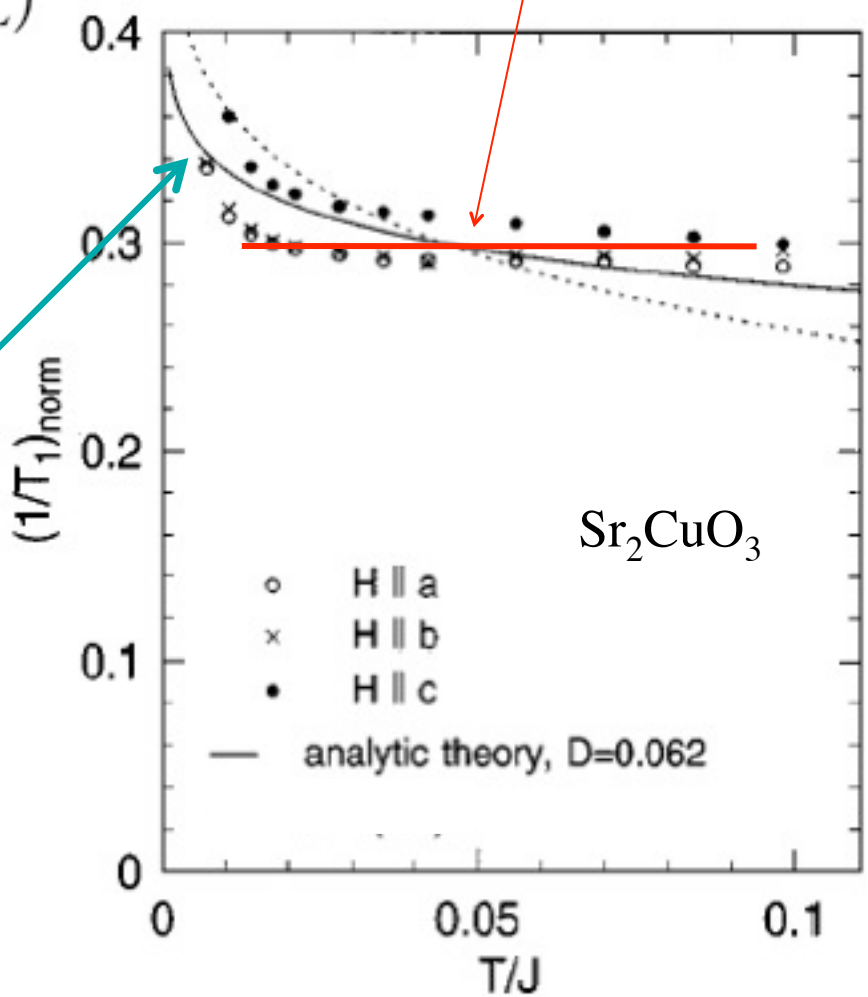
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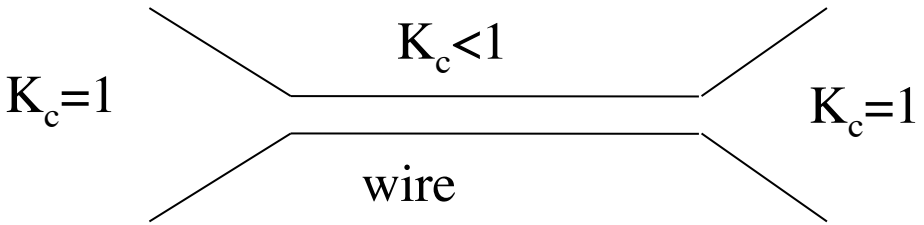
$$\frac{1}{T_1} \sim T\int dk \frac{\text{Im}\chi(k,\omega)}{\omega} \rightarrow \sqrt{\ln(\Lambda/T)}$$

noninteracting spinons



(OS, Singh, Sandvik;
Takigawa,OS,Sandvik,Singh 1997)

Transport: Ballistic conductance G=I/V



$$G = 2\frac{e^2}{h}N$$

Number of subbands

spin degeneracy

perfect transmission
due to multiple scattering
of plasmon waves

- Very fragile: single impurity "cuts" the wire

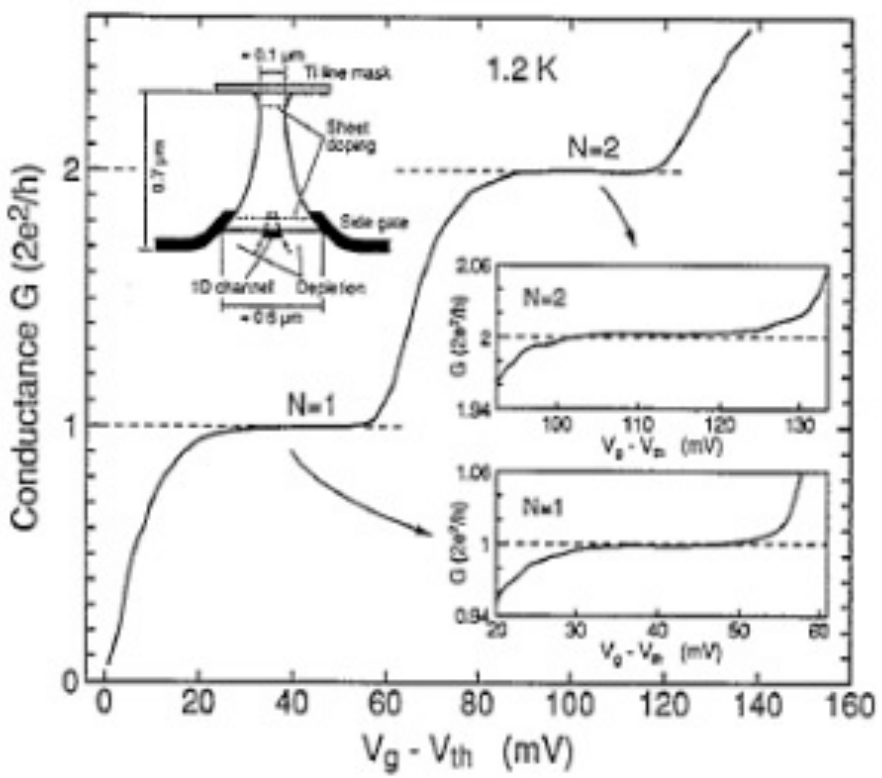
$$\delta G = -V_0^2\, T^{K_c+K_s-2}$$

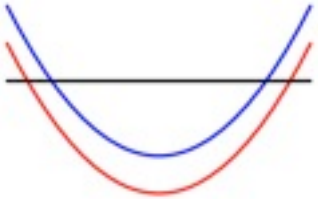
[Kane,Fisher 1992]

- spins play no role !

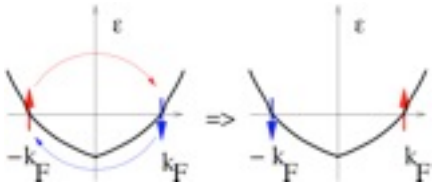
[Solid State Communications, 94, 413 (1995)]

S. Tarucha, T. Honda, and T. Saku





Quantum wire in magnetic field

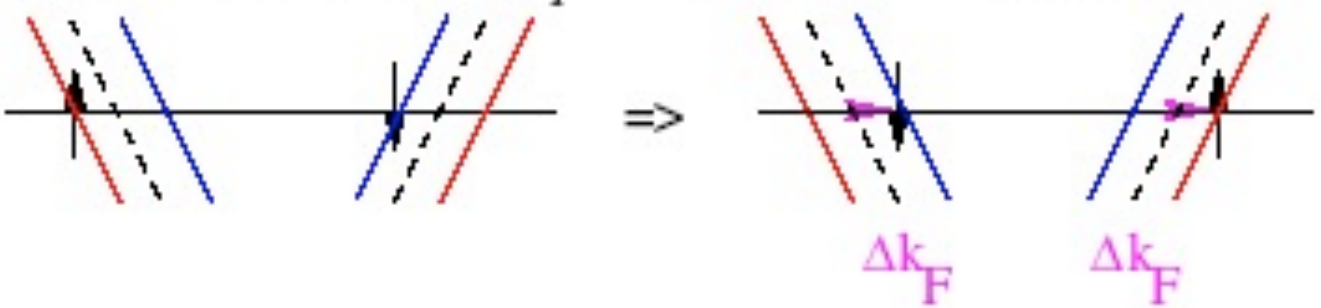


without the field

- Magnetic field $H_B = -\frac{1}{2} g \mu_B B \int dx (\rho_\uparrow - \rho_\downarrow)$

  splits Fermi-momenta of $s = \uparrow = +1$ and $s = \downarrow = -1$ spins:

$$k_{F,s} = k_F + s \cdot \frac{g\mu_B B}{2v_F} \Rightarrow \Delta k_F = g\mu_B B / v_F$$



$g_\perp$: BS with spin-flip

g_z : BS without spin-flip

- Spin backscattering *does not* conserve momentum

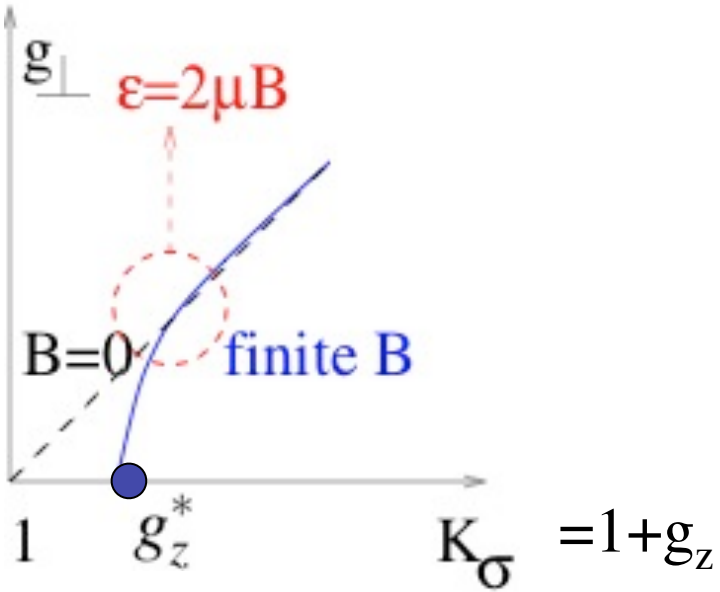
$$V_{\text{bs}} = g_\perp \sum_s \int_x e^{-i2s\Delta k_F x} R_s^+(x) L_s(x) L_{-s}^+(x) R_{-s}(x)$$

$$\sim g_\perp \int_x \cos[\sqrt{8\pi}\varphi_\sigma + 2\Delta k_F x]$$

marginal+oscillating = irrelevant

$\Rightarrow$ Irrelevant for any $\Delta k_F \neq 0$

Renormalization Group: BKT flow in magnetic field



- Initial values of BS constants: $g_\perp = g_z = \frac{U(2k_F)}{2\pi v_F}$

  The fixed point $g_z^* = \frac{1}{2\log[E_F/g\mu B]}$ but $g_\perp^* = 0$

- The meaning: spin-flip scattering is frozen.

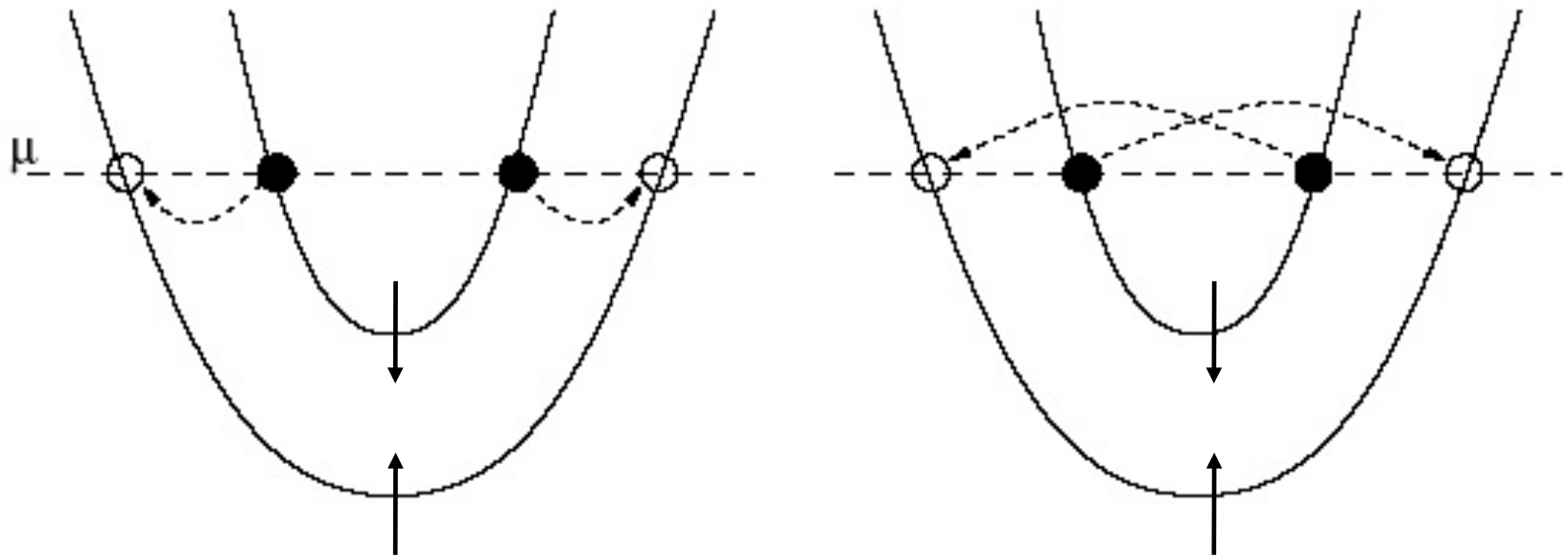
- Note $\langle S^+S^-\rangle_{2k_F} > \langle S^zS^z\rangle_{2k_F}$ since $K_s > 1/K_s$

SU(2) → U(1) : spins are in the plane perpendicular to B

$\mathbf{K_s^* > 1}$

Hint of a new scattering channel: ***Cooper scattering***

$$k_{\text{initial}} = 0 [= -k_{F\downarrow} + k_{F\downarrow}] => k_{\text{final}} = 0 [= -k_{F\uparrow} + k_{F\uparrow}]$$



- But $\mathbf{S^z}$ is conserved $\langle \downarrow | \uparrow \rangle = 0$
- Consequence of U(1) symmetry - need to break it!

S^z conservation forbids Cooper scattering

Outline

- *Warm-up*: van der Waals like coupling between spins in quantum dots

- Brief intro to quantum wires

  - Key scattering processes in a single-subband wire
  - Effect of magnetic field: Zeeman splitting

- Spin-orbital effects

  - Cooper channel
  - Spin-density wave formation (orthogonal magnetic field)
  - Transport: suppressed backscattering
  - Connection with spin chain physics: *uniform DM* interaction
  - Unusual implications for ESR experiments

- Conclusions

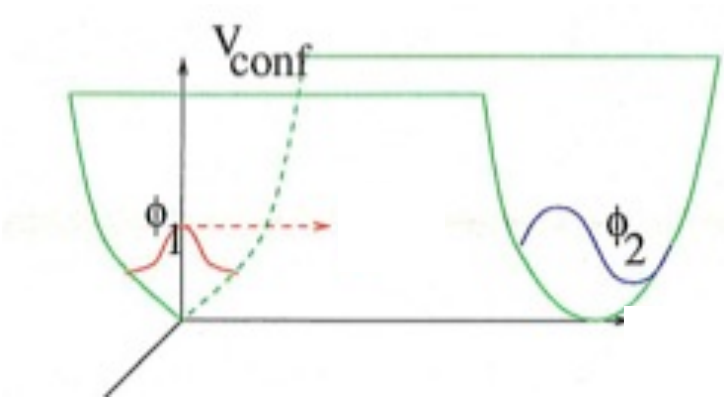
Spin - orbit interaction

• **Two** dimensions: Rashba Hamiltonian $$H_R = \alpha(p_y\hat{\sigma}_x - p_x\hat{\sigma}_y)$$

Confining potential $V_{conf}(x) = m\omega\, x^2/2$

Transverse momentum is quantized $\langle p_x \rangle = 0$

(standing wave)



• **One** dimension: $$H_R = \alpha p_y\hat{\sigma}_x$$

SOI = momentum-dependent magnetic field

Preferred axis - σ_x : spin-rotational symmetry is reduced to U(1)

Single particle problem $\vec{B} \perp \vec{SO}$

$$H_{B+SO} = \frac{p_y^2}{2m} + \alpha \hat{\sigma}_x p_y - \frac{1}{2} g \mu_B B \hat{\sigma}_z$$

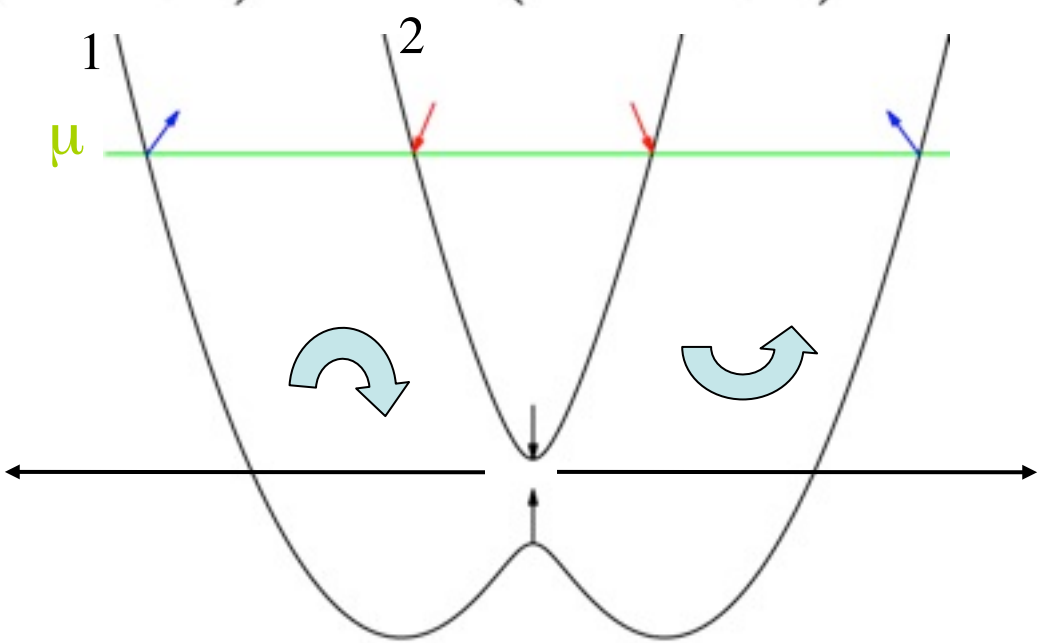
- Eigenvalues $$E_\pm = \frac{p^2}{2m} \pm \sqrt{(\alpha p)^2 + (g\mu B/2)^2}$$

- Eigenstates $$\Psi_\pm = e^{iky}\, \chi_\pm(k)$$

χ_+ and χ_- : orthogonal at the same **k** but not at the same **energy**

spinors $$\chi_+ = \begin{pmatrix} \sin \delta_k/2 \\ \cos \delta_k/2 \end{pmatrix} \quad \chi_- = \begin{pmatrix} \cos \delta_k/2 \\ -\sin \delta_k/2 \end{pmatrix} \qquad \delta_k = \arctan\left[\frac{2\alpha k}{g\mu_B B}\right]$$

k < 0:
clock-wise rotation of spins



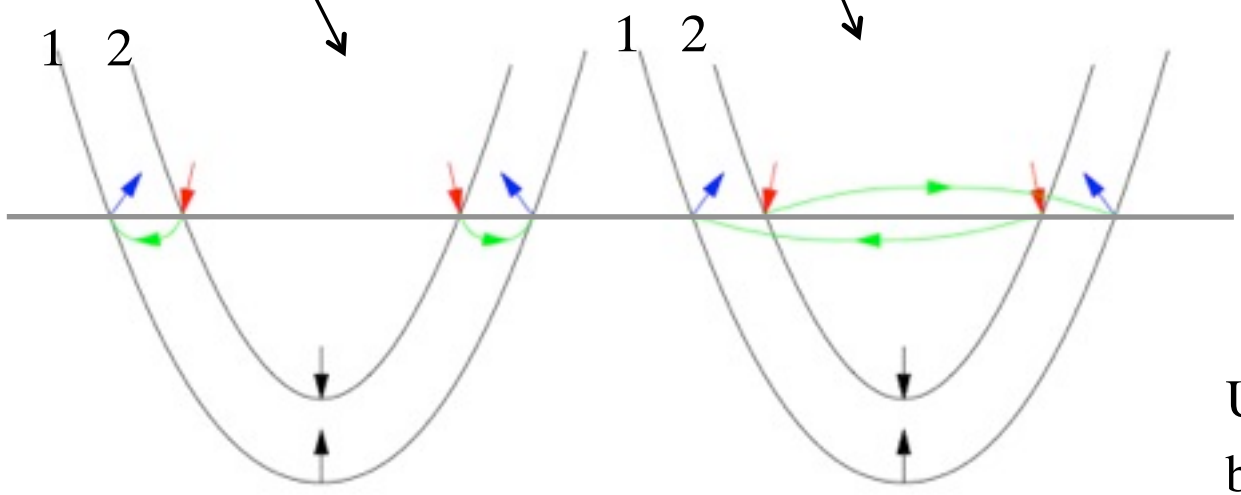
k > 0:
counterclock-wise rotation of spins

N.B: different precession frequencies at $\mathbf{k}_1$ and $\mathbf{k}_2$

Cooper scattering $R^{\dagger}_{1\downarrow} L^{\dagger}_{1\downarrow} R_{2\uparrow} L_{2\uparrow} + \text{h.c.}$

- **Cooper channel**: spin non-conserving inter-subband pair tunneling possible due to **Spin-Orbit** only

(almost) always: $U(g\mu B/v_F) \ll U(2k_F)(v_F k_F/g\mu B)^2$



$U(k_1 - k_2)$ but small overlap

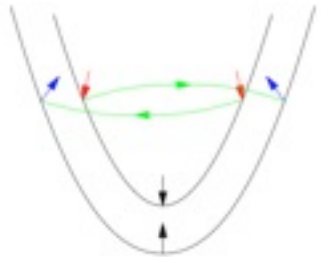
$\propto \sin[\alpha(k_1 - k_2)/g\mu B]$

$U(k_1 + k_2)$ but bigger overlap

$\propto \sin[\alpha(k_1 + k_2)/g\mu B]$

[relative **minus** sign]

SDW instability



- Easy limit: $E_F >> g\mu B >> \alpha k_F$

Free charge: $H_c = \frac{v_c}{2K_c}(\partial_x \phi_c)^2 + \frac{1}{2} v_c K_c (\partial_x \theta_c)^2$ **$K_c < 1$**

Interacting spin: $H_s = \frac{v_s}{2K_s}(\partial_x \phi_s)^2 + \frac{1}{2} v_s K_s (\partial_x \theta_s)^2$ **+ Cooper process**

$$H_{\text{Cooper}} = U(2k_F)\left(\frac{\alpha k_F}{g\mu B}\right)^2 \cos[\sqrt{8\pi}\theta_s]$$ **$K_s > 1$** relevant!

- Strong-coupling limit: minimal energy @ $\sqrt{2\pi}\theta_s = (2M+1) \times \frac{\pi}{2}$

Thus θ_s is frozen, hence ϕ_s fluctuates wildly.

- **$2k_F$** component of spin operators:

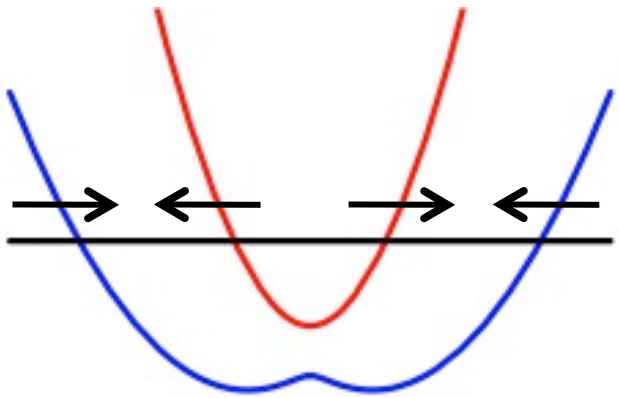
$$S^z_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c] \sin[\sqrt{2\pi}\phi_s] \to 0$$
$$S^y_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c] \cos[\sqrt{2\pi}\theta_s] \to 0$$

but $S^x_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c] \sin[\sqrt{2\pi}\theta_s] \to \cos[\sqrt{2\pi}\phi_c]$

Power-law decay is controlled by charge sector: $\langle S^x(x) S^x(0) \rangle \sim \frac{\cos[2k_F x]}{x^{K_c}}$

quasi Long Range Order

SDW: transport properties



S^x ordered component

- Density: suppressed Friedel oscillations

  at $\mathbf{2k_1}$ and $\mathbf{2k_2}$ $\quad \rho_{2k_1/2k_2} \sim \cos[\sqrt{2\pi}\phi_c]\cos[\sqrt{2\pi}\phi_s] \to 0$

  at $\mathbf{2k_F=k_1+k_2}$ $\quad \rho_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c]\cos[\sqrt{2\pi}\theta_s] \to 0$

- Should we expect better conductance?
  Impurity = potential scatterer => preserves spin

$$V(x)\sum_{s=\uparrow\downarrow} R_s^\dagger L_s + \text{h.c.} \to V[\rho_{2k_1} + \rho_{2k_2} + \frac{\alpha k_F}{g\mu B}\rho_{2k_F}] \to 0$$

N.B. magnetic impurity will scatter strongly

No single particle scattering off the potential impurity in SDW phase!

- But two-particle backscattering off the impurity does get generated $\quad \propto \frac{V^2}{\Delta_{SDW}}\cos[\sqrt{8\pi}\phi_c]$

  Correction to conductance $\quad G = \frac{2e^2}{h} - (V^2/\Delta_{SDW})^2 T^{4K_c-2}$

  Relevant (divergent) for strong e-e interaction: $K_c < 1/2$

- The physics: $\mathbf{k_{1/2}}$ **=>** $\mathbf{-k_{1/2}}$ backscattering suppressed due to opposite ordering of $\mathbf{S_x}$
  Inter-subband backscattering $\mathbf{k_{1/2}}$ **=>** $\mathbf{-k_{2/1}}$ suppressed by *destructive interference*

Close parallels with helical liquids and topological insulators

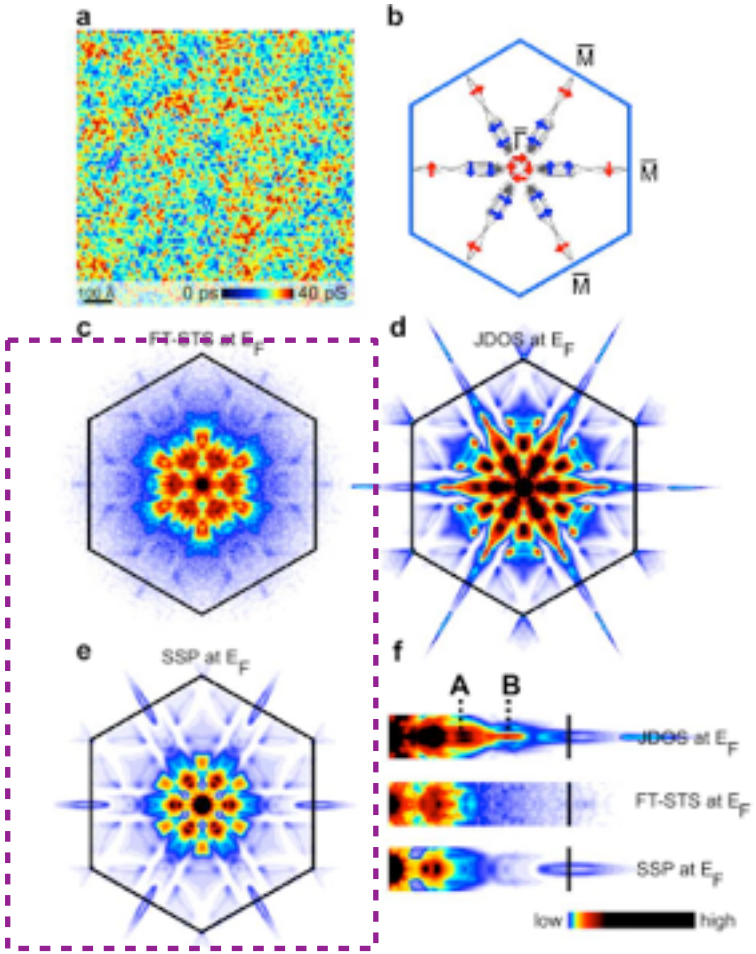


FIG. 11 Absence of backscattering: Quasiparticle interference observed at the surface of $Bi_{0.92}Sb_{0.08}$ exhibits and absence of elastic backscattering: (a) Spatially resolved conductance maps of the (111) surface obtained at 0 mV over a 1000Å×1000Å. (b) ARPES intensity map of the surface state measured at the Fermi level. The spin textures from spin-ARPES measurements are shown with arrows. (c) Fourier transform scanning tunneling spectroscopy (FT-STS) at E_F. (d) The joint density of states (JDOS) at E_F. (e) The spin-dependent scattering probability(SSP) at E_F. (f) Close-up of the JDOS, FT-STS and SSP at E_F, along the Γ-M direction (Roushan, *et al.*, 2009).

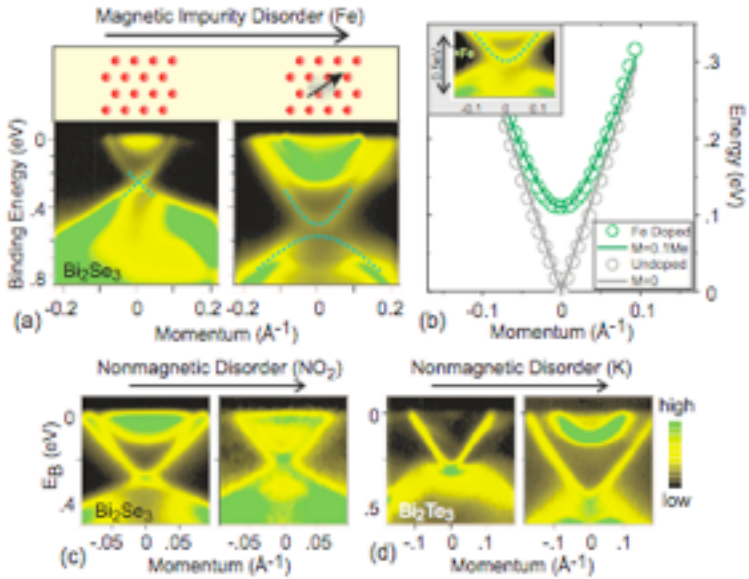


FIG. 15 Protection by time-reversal symmetry: Topological surface states are robust in the presence of strong non-magnetic disorder but open a gap in the presence of time-reversal breaking magnetic impurities and disorder. (a) Magnetic impurity such as Fe on the surface of Bi_2Se_3 opens a gap at the Dirac point. The magnitude of the gap is set by the interaction of Fe ions with the Se surface and the time-reversal breaking disorder potential introduced on the surface. (b) A comparison of surface band dispersion with and without Fe doping. (c) Non-magnetic disorder created via molecular absorbent NO_2 or alkali atom adsorption (K or Na) on the surface leaves the Dirac node intact in both Bi_2Se_3 and Bi_2Te_3 (Hsieh, *et al.*, 2009b; Xia, *et al.*, 2009b; Wray, *et al.*, 2010).

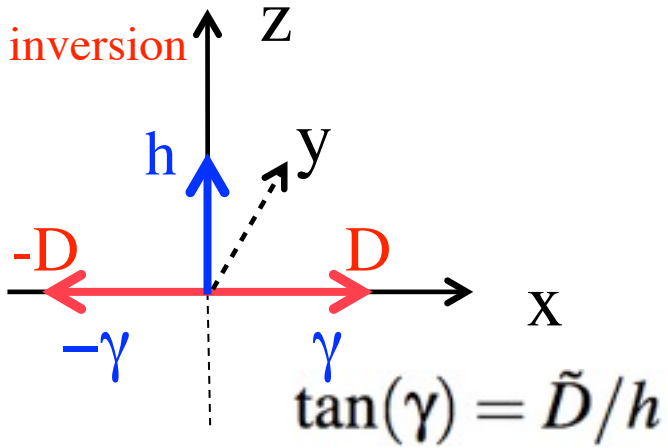
Topological Insulators,
M. Z. Hasan and C. L. Kane,
arxiv 1002.3895

Spin chain with uniform DM term via non-abelian rotations

$$\sum_j D\hat{x} \cdot \vec{S}_j \times \vec{S}_{j+1} \to \tilde{D} \int_x (J_R^x - J_L^x)$$

odd under inversion

$$-h \sum_j S_j^z \to -h \int_x (J_R^z + J_L^z)$$



- Rotate right (left) current by γ (−γ) about ***y*** axis

$$\tilde{D}(J_R^z - J_L^z) - h(J_R^z + J_L^z) \to -\sqrt{\tilde{D}^2 + h^2}(M_R^z + M_L^z)$$

- This rotation leaves invariant, thanks to emergent $SU(2)_R$ x $SU(2)_L$ symmetry

$$H_0 = \frac{2\pi v}{3} \int_x \vec{J}_R^2 + \vec{J}_L^2 \to \frac{2\pi v}{3} \int_x \vec{M}_R^2 + \vec{M}_L^2$$

- Backscattering interaction of spin currents is modified

$$H_{\text{bs}} = -g\vec{J}_R \cdot \vec{J}_L \to$$

$$= -gM_R^y M_L^y - g\cos[2\gamma](M_R^x M_L^x + M_R^z M_L^z) - g\sin[2\gamma](M_R^x M_L^z - M_R^z M_L^x)$$

Gangadharaiah, Sun, OS, PRB **78**, 054436; and Schnyder, OS, Balents, PRB **78**, 174420

- Magnetic field can now be absorbed

$$-\sqrt{\tilde{D}^2+h^2}(M_R^z+M_L^z)\rightarrow -t\partial_x\varphi$$

- Transverse to total field ***t*** components $M^{x,y}$ oscillate with ***x***

$$M_R^{\pm}\rightarrow M_R^{\pm}e^{\mp itx}\,;\; M_L^{\pm}\rightarrow M_L^{\pm}e^{\pm itx}$$

So that $\quad M_R^{+}M_L^{-}\rightarrow M_R^{+}M_L^{-}e^{i2tx}\,;\; M_R^{+}M_L^{+}\rightarrow M_R^{+}M_L^{+}$

- The final (momentum-conserving) Hamiltonian

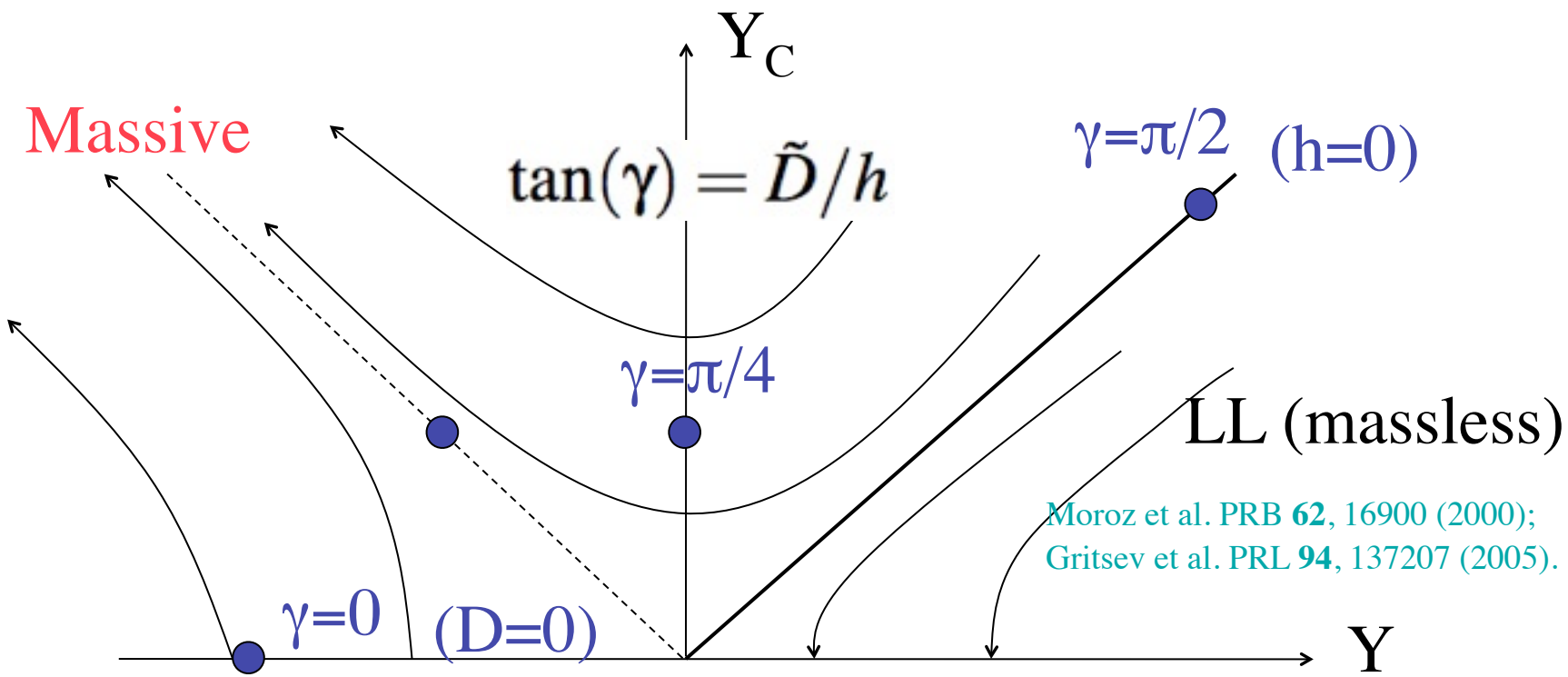
$$H_{\rm bs}=-gM_R^yM_L^y-g\cos[2\gamma](M_R^xM_L^x+M_R^zM_L^z)-g\sin[2\gamma](M_R^xM_L^z-M_R^zM_L^x)$$

$$\rightarrow -g\cos[2\gamma]\underbrace{M_R^zM_L^z}+g\sin^2[\gamma]\underbrace{(M_R^+M_L^++M_R^-M_L^-)}$$

$$(\partial_x\varphi)^2-(\partial_x\theta)^2 \qquad \cos[\sqrt{8\pi}\,\theta]$$

Cooper term

BKT phase diagram: *always* in strong coupling phase for ***h*** perp. ***D***



$$Y = -g\cos[2\gamma]\ ,\ Y_C = g\sin^2[\gamma]$$

$$H = \frac{v}{2}\int_x (1+Y)(\partial_x\varphi)^2 + (1-Y)(\partial_x\theta)^2 + Y_C\cos[\sqrt{8\pi}\,\theta]$$

- SDW for arbitrary ratio of D/h = S.O. coupling/Zeeman

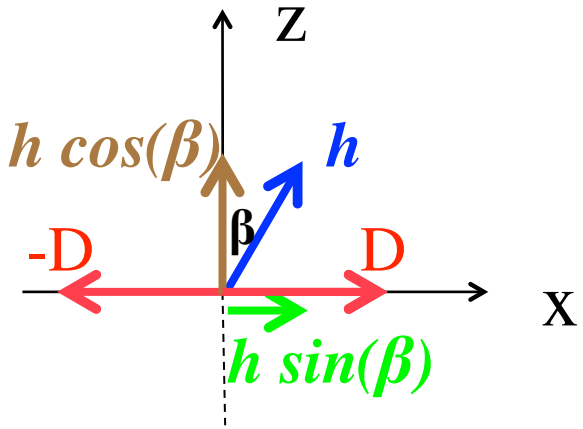
Arbitrary angle between SO axis and magnetic field

Field experienced by **right**-moving electrons

$$(D + h\sin[\beta])J_R^x + hJ_R^z$$

Field experienced by **left**-moving electrons

$$(-D + h\sin[\beta])J_L^x + hJ_L^z$$

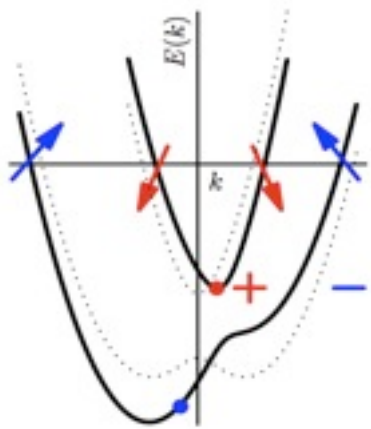


Chiral rotation angles for **right/left** currents are **different**:
linear shifts in both $\partial_x\varphi_\sigma$ and $\partial_x\theta_\sigma$ are required.

Cooper process does not conserve momentum anymore.

Backscattering is reduced to purely marginal term:

$$H_{bs} \rightarrow -g\cos[\gamma_R - \gamma_L]\, M_R^z M_L^z$$



$$k_1^{\text{initial}} + k_2^{\text{initial}} \neq k_1^{\text{final}} + k_2^{\text{final}}$$

★ End result: critical Luttinger state with slightly renormalized exponents

Detailed phase diagram via numerical solution of coupled RG equations:
Garate and Affleck, PRB **81**, 144419 (2010)

Implications for ESR experiments

Measures absorption of linearly polarized, and perpendicular to external magnetic field, radiation $I_{esr}(\omega) \propto E_r^2\, \omega\, \chi''_{xx}(q=0,\omega)$

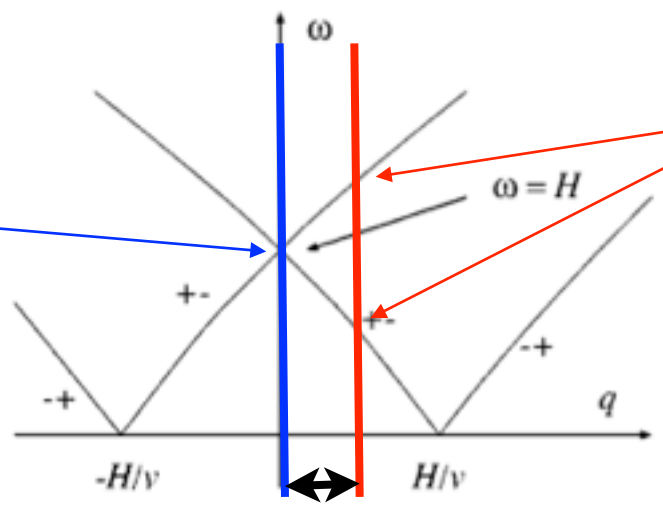
SU(2) symmetric system of spins: $\chi''_{xx}(q=0,\omega) \propto \delta(\omega - g\mu_B H)$

Oshikawa, Affleck PRB 65, 134410 (2002)

Spin chain with ***uniform DM*** (quantum wire with SO interaction):
right and ***left*** movers absorb at ***different*** frequencies !

$$\chi''_{xx}(q=0,\omega) \propto \delta\left(\omega - \sqrt{(D - h\sin\beta)^2 + (h\cos\beta)^2}\right) + \delta\left(\omega - \sqrt{(D + h\sin\beta)^2 + (h\cos\beta)^2}\right)$$

ideal Heisenberg chain



Chain with uniform DM

carbon nanotubes:
A. De Martino et al, PRL (2002);
generation of DC currents
in quantum wires:
Ar. Abanov et al, arxiv 1008.1225

shift due to momentum boost ~ D/J

Conclusions

• Interplay of magnetic field, spin-orbit and interactions: novel and interesting many-body physics

• SDW driven by electron *pair* tunneling between Zeeman-split subbands

✓ Possible due to SU(2) breaking by the ***spin-orbit*** interaction

• Spin-density wave instability affects (charge) conductance

• Spin chains with uniform DM interaction

✓ Chiral rotations of right- and left- spin currents

❖ ESR experiments as a ***chiral*** probe of 1d excitations

★ *Consequences for Majorana fermions?!*

ESR study of Cs_2CuCl_4

Schrama et al, Physica B 256-258, 637 (1998)

Single peak at T = 4.2 K
evolves into ***two*** peaks at
T < 1.1 K

This spin-1/2 quasi-1d material
is known to possess uniform
DM couplings,

OS, Katsura, Balents PRB 82, 014421 (2010)

Experiments in Institute
for Physical Problems, Moscow:
K. Povarov, A.I. Smirnov et al
(unpublished) confirm orientation
dependent ESR doublets

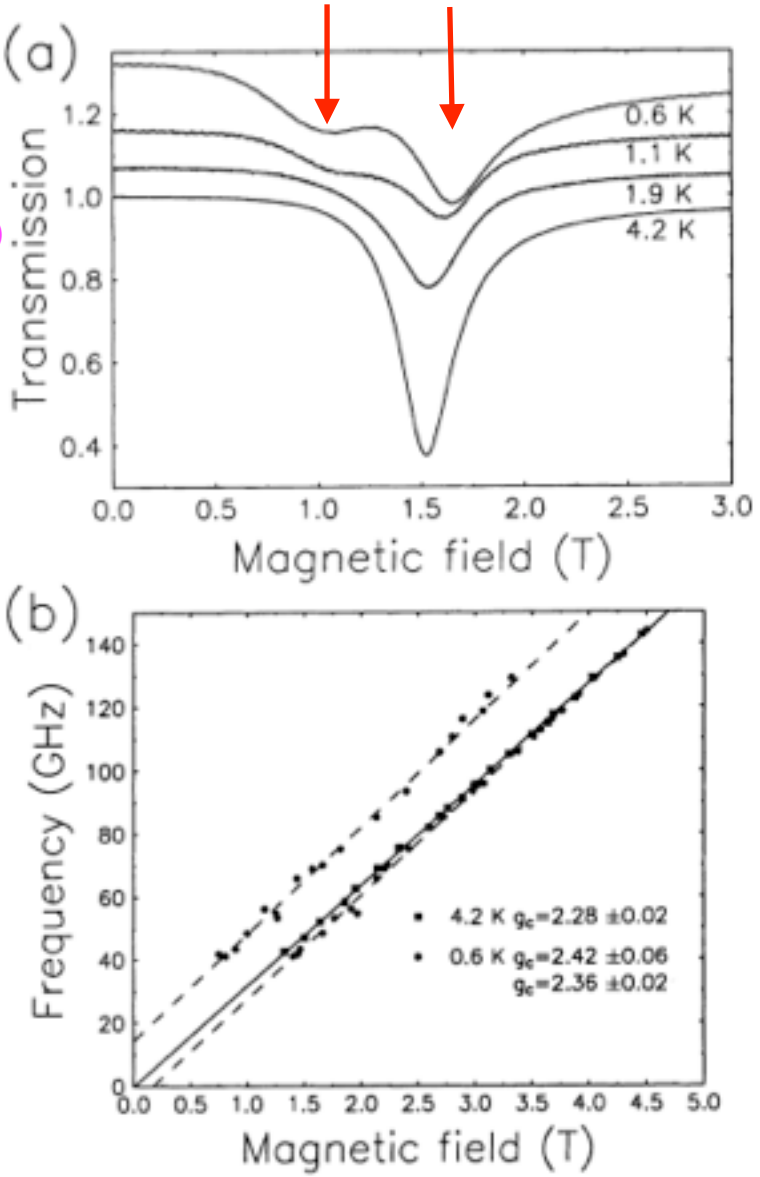


Fig. 1. (a) The temperature dependence of the transmission of a cavity containing a single crystal of Cs_2CuCl_4. The steady magnetic field is aligned along the *c*-axis. The traces are normalised and offset for clarity. The measurement frequency is 48.5 GHz. (b) Resonance positions as a function of frequency when the steady magnetic field is parallel to the crystalline *c*-axis at 4.2 and 0.6 K. Note that there are two resonances at 0.6 K. The lines are fits to the data. The corresponding *g*-factors, calculated from the gradients of the straight line fits, are indicated on the plot.

Can we *really* get there?

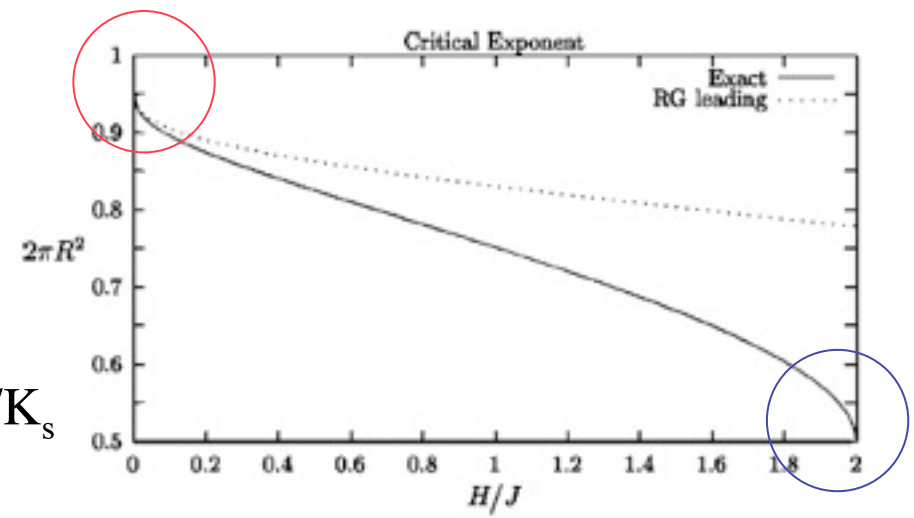
- So far: assumed fully developed SDW state
- With impurities present, what happens first:
  SDW instability or strong-impurity limit - detailed RG required.

Naively: impurity is washed away if $\mathbf{V_0 < \Delta_{SDW}}$

$$\Delta_{SDW} \propto v_F \left[\frac{U(2k_F)}{v_F}\left(\frac{\alpha k_F}{g\mu B}\right)^2\right]^{\frac{K_s}{2(K_s-1)}} \qquad 1 < \frac{K_s}{2(K_s-1)} < \ln\left[\frac{E_F}{g\mu B}\right]$$

Weak field: $K_s = 1 + 1/[2 \ln(E_F/g\mu B)]$

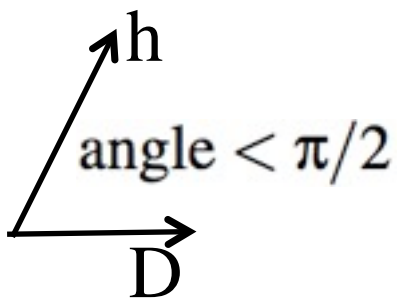
Strong field: $K_s = 2$

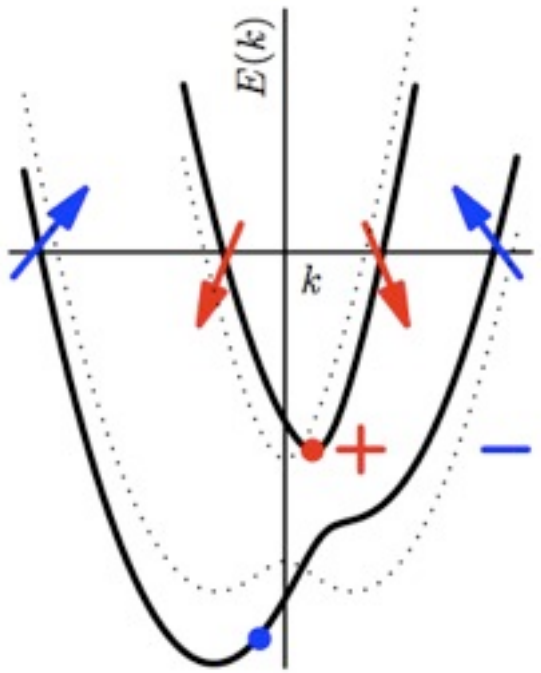


Magnetic field

Affleck, Oshikawa PRB **60**, 1038 (1999)

Tilted magnetic field: pair momentum is NOT conserved





$$k_1^{\text{initial}} + k_2^{\text{initial}} \neq k_1^{\text{final}} + k_2^{\text{final}}$$

- SDW stable when SO axis and magnetic field are orthogonal. Narrow (but finite) angular stability.

Monolayer Graphene on Ni (111)

Dedkov et al. PRL 2008

