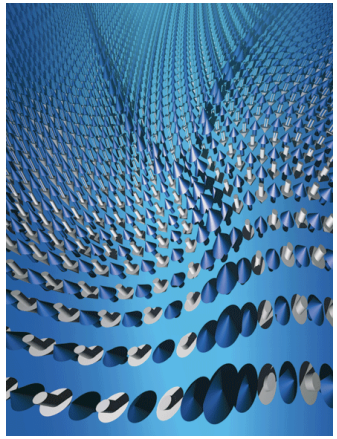
Spinons and triplons in spatially anisotropic triangular antiferromagnet



Oleg Starykh, University of Utah

Leon Balents, KITP, UCSB

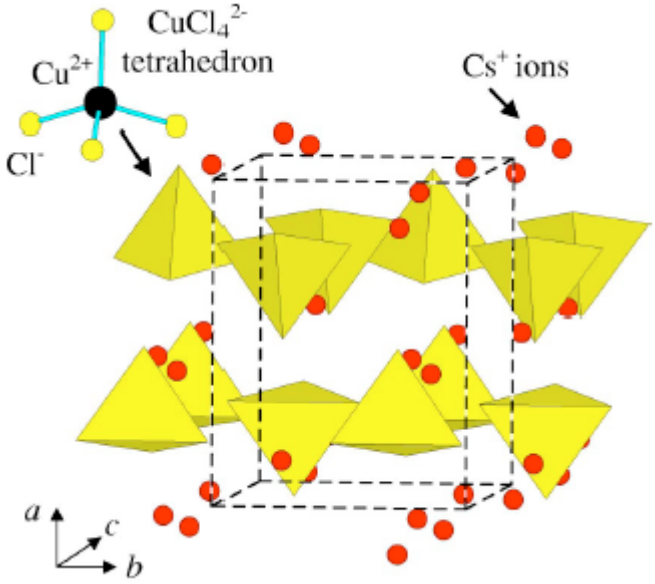
Masanori Kohno, NIMS, Tsukuba

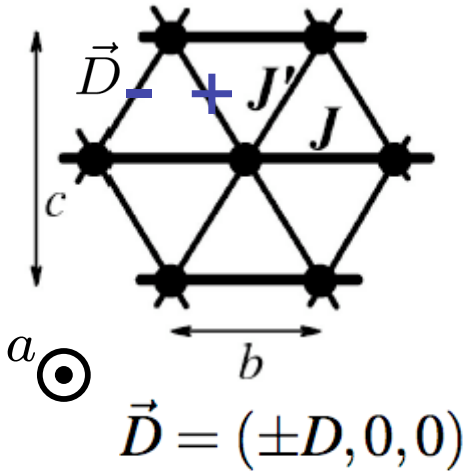
PRL **98**, 077205 (2007); Nature Physics **3**, 790 (2007)

ACS PRF

Dresden, June18, 2008

Anisotropic S=1/2 antiferromagnet Cs_2CuCl_4



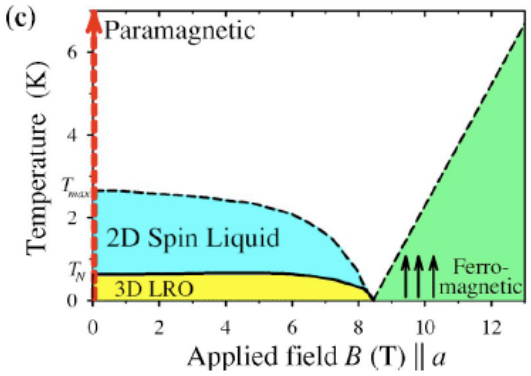


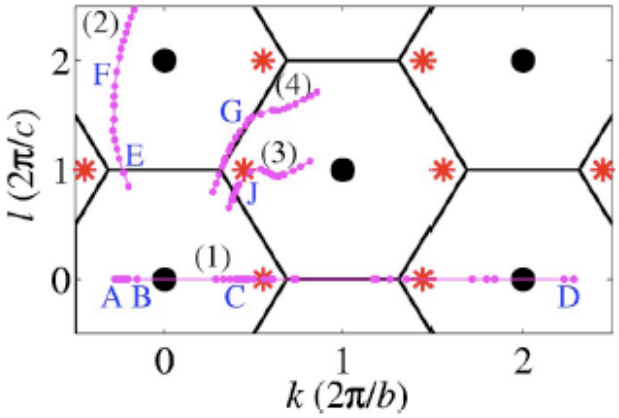
$$\mathcal{H} = \sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{(ij)} \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

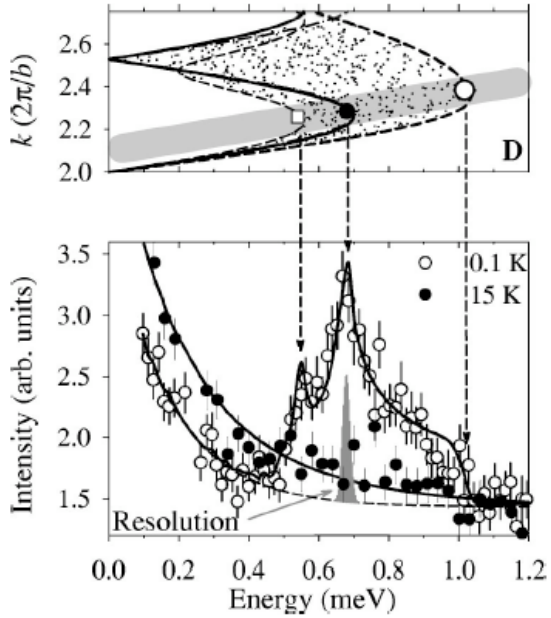
J = 0.37 meV
J' = 0.3 J
D = 0.05 J

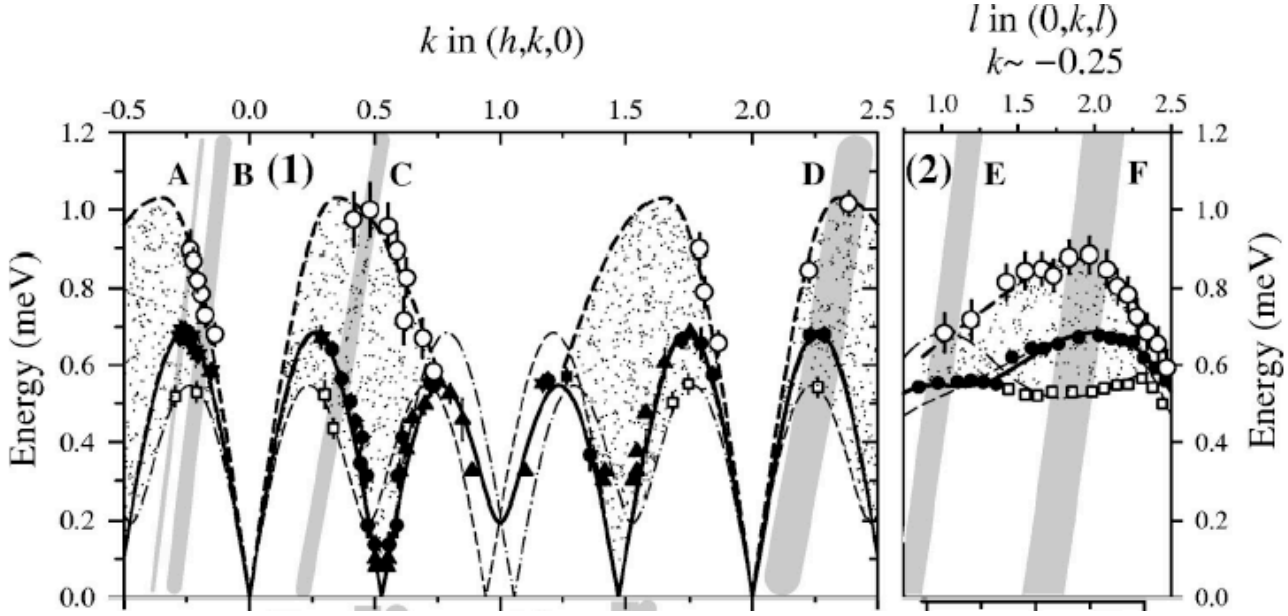
from high-field measurements:
Coldea et al PRL **88** 137203 (2002)

Very well characterized material









Coldea et al, PRB 2003

VOLUME 86, NUMBER 7 PHYSICAL REVIEW LETTERS 12 FEBRUARY 2001

Experimental Realization of a 2D Fractional Quantum Spin Liquid

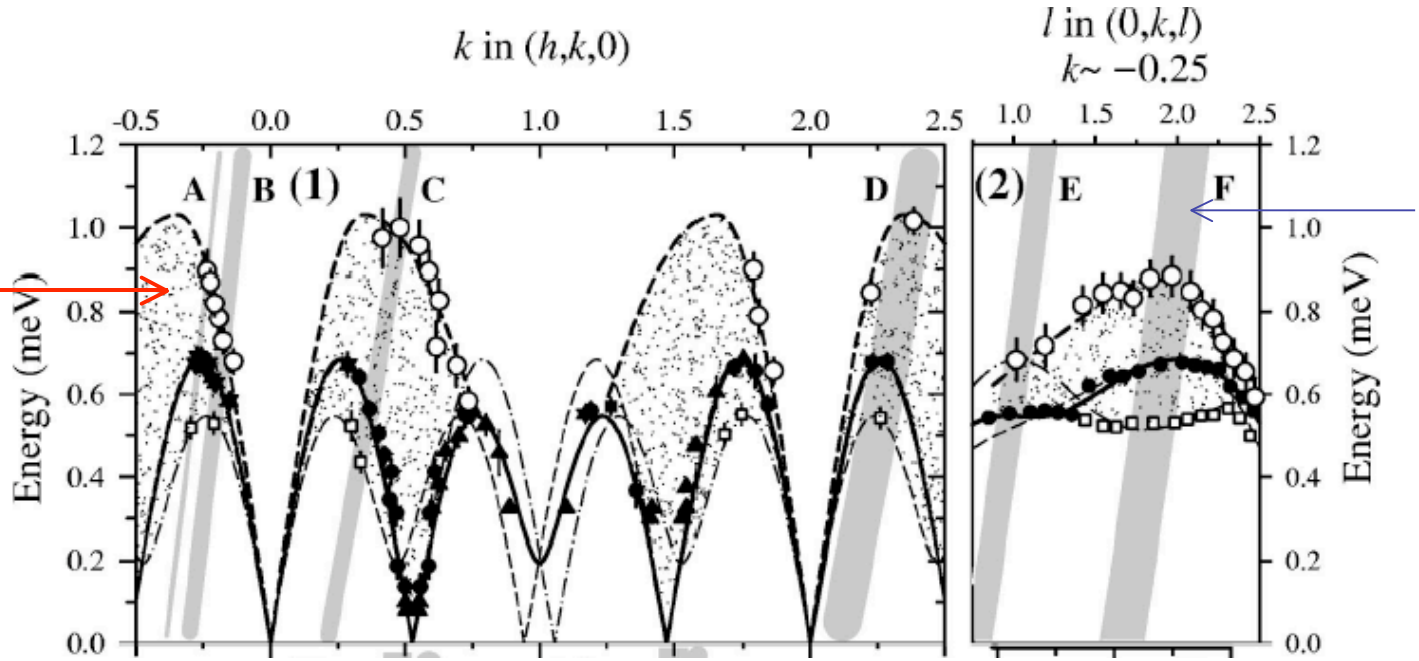
R. Coldea,[1,2] D. A. Tennant,[2,3] A. M. Tsvelik,[4] and Z. Tylczynski[5]

PHYSICAL REVIEW B 68, 134424 (2003)

Extended scattering continua characteristic of spin fractionalization in the two-dimensional frustrated quantum magnet Cs_2CuCl_4 observed by neutron scattering

R. Coldea,[1,2,3] D. A. Tennant,[1,3] and Z. Tylczynski[4]

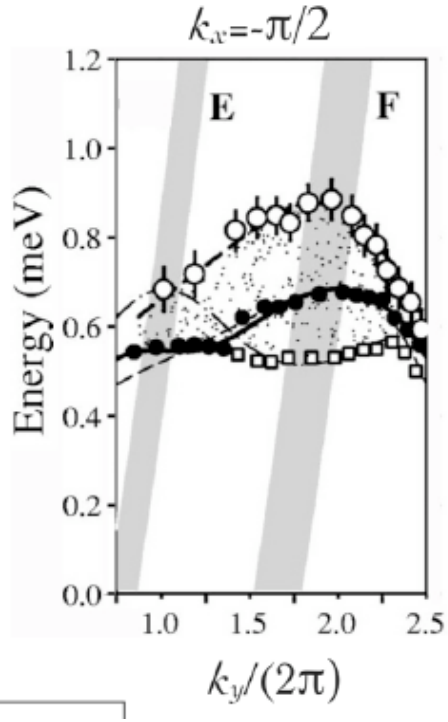
Along the chain



transverse to chain

Nature of continuum: multi-magnon or multi-spinon?

2D theories



- Arguments for 2D:
  - J'/J = 0.3 *not very small*
  - Transverse dispersion
- Exotic theories:

- J.Alicea, O.I.Motrunich & M.P.Fisher: Phys. Rev. Lett. **95**, 247203 (2005).
- S.V.Isakov, T.Senthil & Y.B.Kim: Phys. Rev. B **72**, 174417 (2005).
- Y.Zhou & X.-G.Wen: cond-mat/0210662.
- F.Wang & A.Vishwanath: Phys. Rev. B **74**, 174423 (2006).
- C.-H.Chung, K.Voelker & Y. B. Kim: Phys. Rev. B **68**, 094412 (2003).

- Spin waves:

- M.Y.Veillette, A.J.A.James & F.H.L.Essler: Phys. Rev. B **72**, 134429 (2005).
- D.Dalidovich, R.Sknepnek, A.J.Berlinsky, J.Zhang & C.Kallin: Phys. Rev. B **73**, 184403 (2006).
- R.Coldea, D.A.Tennant & Z.Tylczynski: Phys. Rev. B **68**, 134424 (2003).

- Series expansion: W. Zheng, J. Fjaerestad, R. R. P. Singh, R. H. McKenzie, R. Coldea 2006

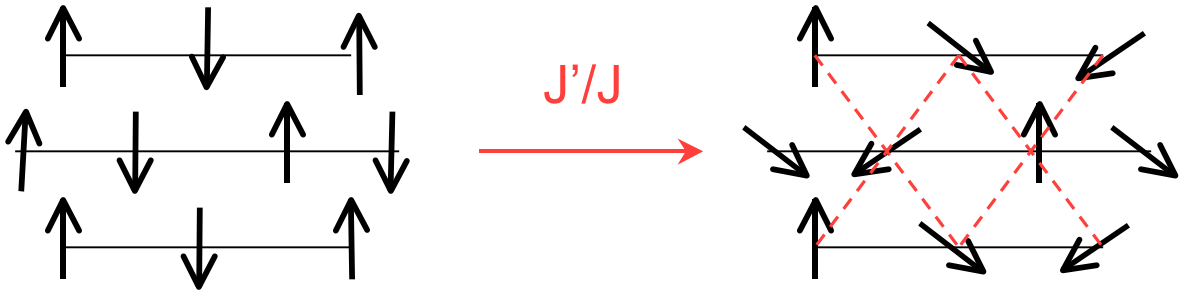
Our strategy

- Approach from the limit of decoupled chains
  (*frustration helps -- it promotes spatial anisotropy*)
  [RPA version: Bocquet, Essler, Tsvelik, Gogolin (2001)]

- Allow for ALL symmetry-allowed inter-chain interactions to develop

- *Most relevant* perturbations of decoupled chains drive ordering

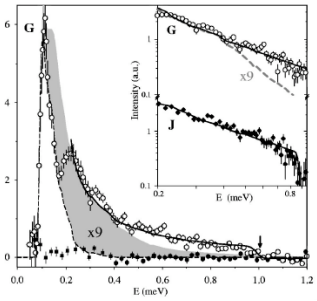
- Study resulting phases and their excitations



Decoupled chains J' =0

Coupled chains J'/J = 0.34

R. Coldea *et al*, 2003

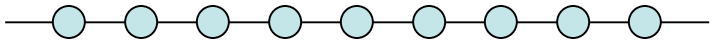


Outline

- Key properties of Heisenberg chain
- Ground states of triangular J-J' model
  - Competition between *collinear* and *dimerized* states (both are generated by quantum fluctuations!)
  - Very subtle order
- Experiments on Cs_2CuCl_4
- Dynamical response of 1D spinons
  - spectral weight redistribution
  - coherent pair propagation
  - Detailed comparison with Cs_2CuCl_4
- Conclusions
- Cs_2CuCl_4 in magnetic field

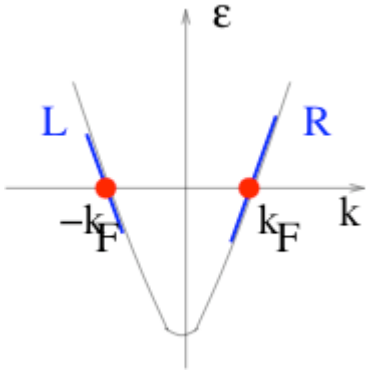
Heisenberg spin chain via free Dirac fermions

- Spin-1/2 AFM chain = half-filled (1 electron per site, $k_F=\pi/2a$) fermion chain



- Spin-charge separation

$$H_{\text{dirac}} = iv \int dx \sum_{s=\uparrow,\downarrow} (\Psi^+_{L,s}\partial_x\Psi_{L,s} - \Psi^+_{R,s}\partial_x\Psi_{R,s})$$



- *q=0 fluctuations: right- and left- spin currents*

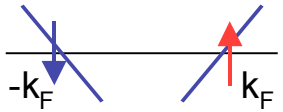
$$\vec{J}_R = \Psi^+_{Rs}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Rs'}\ ,\ \vec{J}_L = \Psi^+_{Ls}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Ls'}$$

- *$2k_F$ (= π/a) fluctuations: charge density wave ε , spin density wave N*

Staggered Magnetization N

$$N^+ \sim \Psi^+_{R\uparrow}\Psi_{L\downarrow} + \text{h.c.}$$
$$N^z \sim \Psi^+_{R\uparrow}\Psi_{L\uparrow} - \Psi^+_{R\downarrow}\Psi_{L\downarrow} + \text{h.c.}$$

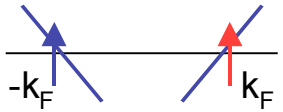
Spin flip ΔS=1



Staggered Dimerization

$$\varepsilon \sim i\left(\Psi^\dagger_{R\uparrow}\Psi_{L\uparrow} + \Psi^\dagger_{R\downarrow}\Psi_{L\downarrow} - \text{h.c.}\right)$$

ΔS=0



$\varepsilon = (-1)^x S_x S_{x+a}$

Susceptibility

1/q $\quad \chi_{1d}(q)$

1/q

1/q

- Must be careful: **often** spin-charge separation must be enforced by hand

Low-energy degrees of freedom

• Quantum triad: uniform magnetization **M** = $\mathbf{J_R} + \mathbf{J_L}$,
staggered magnetization **N** and staggered dimerization $\varepsilon = (-1)^x S_x S_{x+1}$

✓Components of Wess-Zumino-Witten-Novikov SU(2) matrix $G = \varepsilon + i\vec{N}\cdot\vec{\sigma}$

• Hamiltonian H ~ $\mathbf{J_R J_R} + \mathbf{J_L J_L} + \gamma_{bs}\, \mathbf{J_R J_L}$

marginal perturbation

• Operator product expansion $z = v\,\tau - ix$ (similar to commutation relations)

$$\begin{cases} J_R^a(x,\tau)J_R^b(0,0) = \dfrac{\delta^{ab}}{8\pi^2 z^2} + \dfrac{i\varepsilon^{abc}J_R^c(x,\tau)}{2\pi z} \\ J_R^a(x,\tau)N^b(0,0) = \dfrac{-i\delta^{ab}\varepsilon(x,\tau) + i\varepsilon^{abc}N^c(x,\tau)}{4\pi z}\,,\; J_R^a(x,\tau)\varepsilon(0,0) = \dfrac{iN^a(x,\tau)}{4\pi z} \end{cases}$$

• Scaling dimension 1/2 (relevant) $\langle N^a(x,\tau)N^a(0,0)\rangle \sim \dfrac{1}{\sqrt{v^2\tau^2 + x^2}} \sim \langle \varepsilon(x,\tau)\varepsilon(0,0)\rangle$

• Scaling dimension 1 (marginal) $\langle M^a(x,\tau)M^a(0,0)\rangle \sim \dfrac{1}{z^2} + \dfrac{1}{\bar{z}^2}$

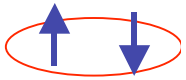
More on staggered dimerization

Measure of bond strength: $\varepsilon(x) = (-1)^x \vec{S}(x) \cdot \vec{S}(x+1)$

Critical correlations: $\langle \varepsilon(x)\varepsilon(0) \rangle = 1/x$ [same as $\langle \vec{N}(x) \cdot \vec{N}(0) \rangle = 1/x$]

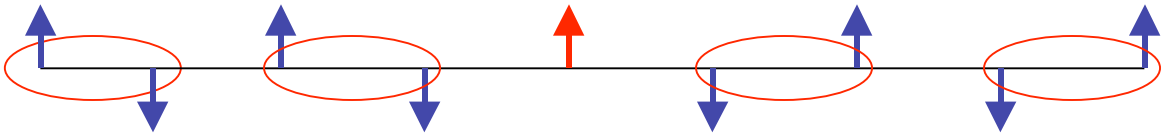
- Lieb-Schultz-Mattis theorem: either critical or doubly degenerate ground state
- Frustration leads to spontaneous dimerization (Majumdar,Ghosh 1970)

  J_1-J_2 spin chain - spontaneously dimerized ground state ($J_2 > 0.24 J_1$)

Spin singlet  $= |\uparrow\downarrow - \downarrow\uparrow\rangle/\sqrt{2}$



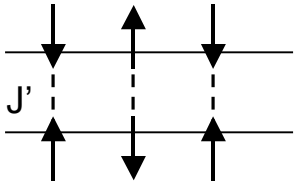
- Kink between dimerization patterns - massive but free S=1/2 spinon!



Shastry,Sutherland 1981

Weakly coupled spin chains (*no frustration*)

- Simple "chain mean-field theory'': adjacent chains replaced by self-consistent field



$$J'(\vec{S}_{x,y+1}+\vec{S}_{x,y-1})\cdot\vec{S}_{x,y}\rightarrow\vec{B}_{\text{eff}}\cdot\vec{N}_y$$

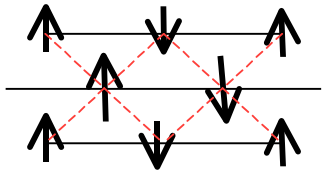
Schulz 1996; Essler, Tsvelik, Delfino 1997; Irkhin, Katanin 2000

- Captured by RPA $\chi_{2d}(q)=\dfrac{\chi_{1d}(q)}{1-J'_{\text{inter}}(q)\chi_{1d}(q)}$
  - $\chi_{1d} \sim 1/q$
  - non-frustrated J': J'(q) = const
  - diverges at some q_0 / energy / Temperature => ***long range order***
  - critical $T_c \sim J'$, on-site magnetization $m \propto \sqrt{J'/J}$

material	J	J'	exp. T_N
$KCuF_3$	406	19	$39K$
Sr_2CuO_3	2600	1.85	$5K$
Ca_2CuO_3	2600	4.3	$11K$

- Weakly coupled chains are generally unstable with respect to Long Range Magnetic Order

Triangular geometry: frustrated inter-chain coupling



- J'<< J: no $N_y N_{y+1}$ coupling between nearest chains (by symmetry)
  - Inter-chain J' is frustrated

$$\vec{S}_x + \vec{S}_{x+1} \approx 0$$

$J'_{inter}(q) \sim J' \sin(q)$, hence
NO divergence for small J',
RPA denominator = 1 - J' sin(q)/q ~ 1-J'

  - naive answer: spiral state with exponentially small gap due to "twist" term $\vec{N}_1 \cdot \partial_x \vec{N}_2$

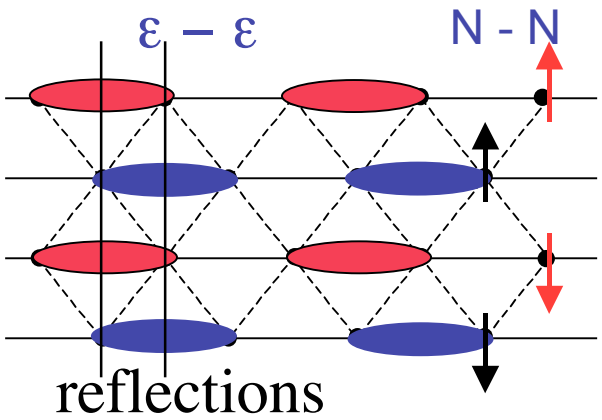
Nersesyan,Gogolin, Essler 1998;
Bocquet, Essler, Tsvelik, Gogolin 2001

  - Spiral order stabilized the **marginal backscattering** (in-chain)

$$\left(1 - J' \sin q \frac{\sqrt{\ln(1/q)}}{q}\right)^{-1}$$

- More **relevant terms** are allowed by the symmetry:
  Involve *next-nearest chains* (e.g. $N_y N_{y+2}$)

OS, Balents 2006



Main steps towards solution

$$H' = \sum_y \int dx \{ \gamma_{\rm bs} \vec{J}_{R,y} \cdot \vec{J}_{L,y} + \gamma_{\rm M} \vec{M}_y \cdot \vec{M}_{y+1} + \gamma_{\rm twist} \vec{N}_y \cdot \partial_x \vec{N}_{y+1} \}$$

- Integrate odd chains: generate non-local coupling between NN chains

- Do Renormalization Group on **non-local action**: generate relevant local couplings

$$H'' = \sum_y \int dx \{ g_N \, \vec{N}_y \cdot \vec{N}_{y+2} + g_\varepsilon \, \varepsilon_y \varepsilon_{y+2} \}$$

- Initial couplings: $g_\varepsilon(\ell_0) = -\frac{3}{2} g_N(\ell_0) \sim \left(\frac{J'}{J}\right)^4 , \ \ell_0 \sim 1$

- Run RG keeping track of next-nearest couplings of staggered magnetizations, dimerizations *and* in-chain backscattering

$$\partial_\ell \gamma_{\rm bs} = \gamma_{\rm bs}^2 , \ \partial_\ell g_N = g_N - \frac{1}{4} \gamma_{\rm bs} g_N , \ \partial_\ell g_\varepsilon = g_\varepsilon + \frac{3}{4} \gamma_{\rm bs} g_\varepsilon$$

- Competition between collinear AFM and columnar dimer phases: both are relevant couplings that grow exponentially $g_{N/\varepsilon}(\ell^*) = g_{N/\varepsilon}(\ell_0) \exp[\ell^*] \sim 1$

✓ *Almost* O(4) symmetric theory [Senthil, Fisher 2006; Essler 2007]

Marginally irrelevant backscattering decides the outcome

• (standard) Heisenberg chain: **J_2=0 => large in-chain backscattering γ_{bs}**

$$\frac{\gamma_{bs}(0)}{2\pi v} = \frac{-\Gamma}{\pi J/2} \approx -1.1(0.24 - J_2/J) + 0.86(0.24 - J_2/J)^2 - 1.1(0.24 - J_2/J)^3 + \ldots \to -0.23 \text{ for } J_2 = 0$$

[Eggert 1996]

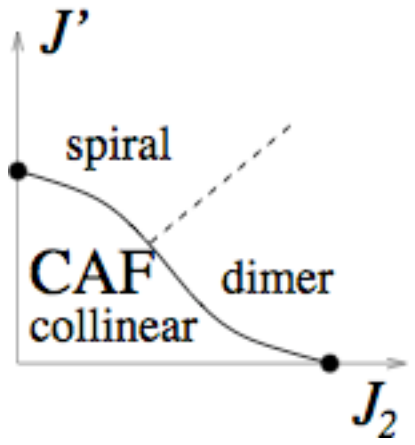
✓Collinear antiferromagnetic (CAF) state

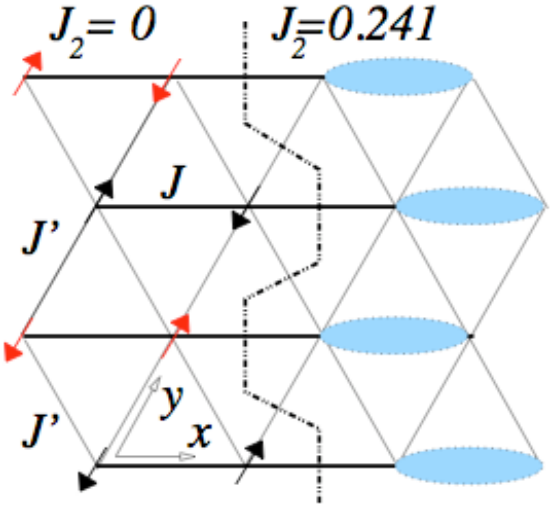
$$\frac{g_N(\ell^*)}{g_\varepsilon(\ell^*)} = \frac{g_N(\ell_0)}{g_\varepsilon(\ell_0)} \frac{1+\Gamma\ell^*}{1+\Gamma\ell_0} \approx \frac{2}{3} \ln(J/J') \gg 1 \text{ for } \ell^* \approx 4 \ln(J/J')$$

▪ *logarithmic enhancement of CAF*

CAF-dimer boundary:

$1/\Gamma \approx \ell^*$





✓Zig-zag dimer phase

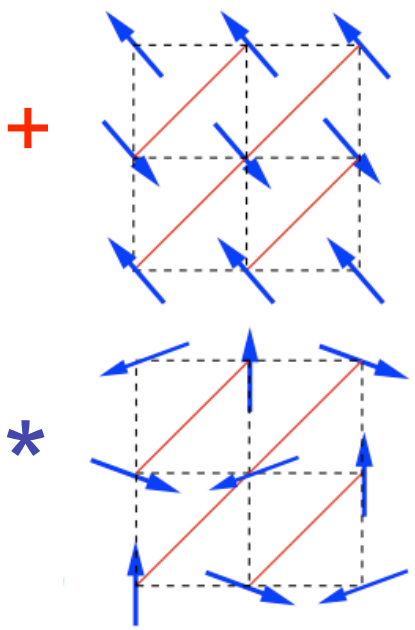
Both CAF and dimerized phases differ from classical *spiral*

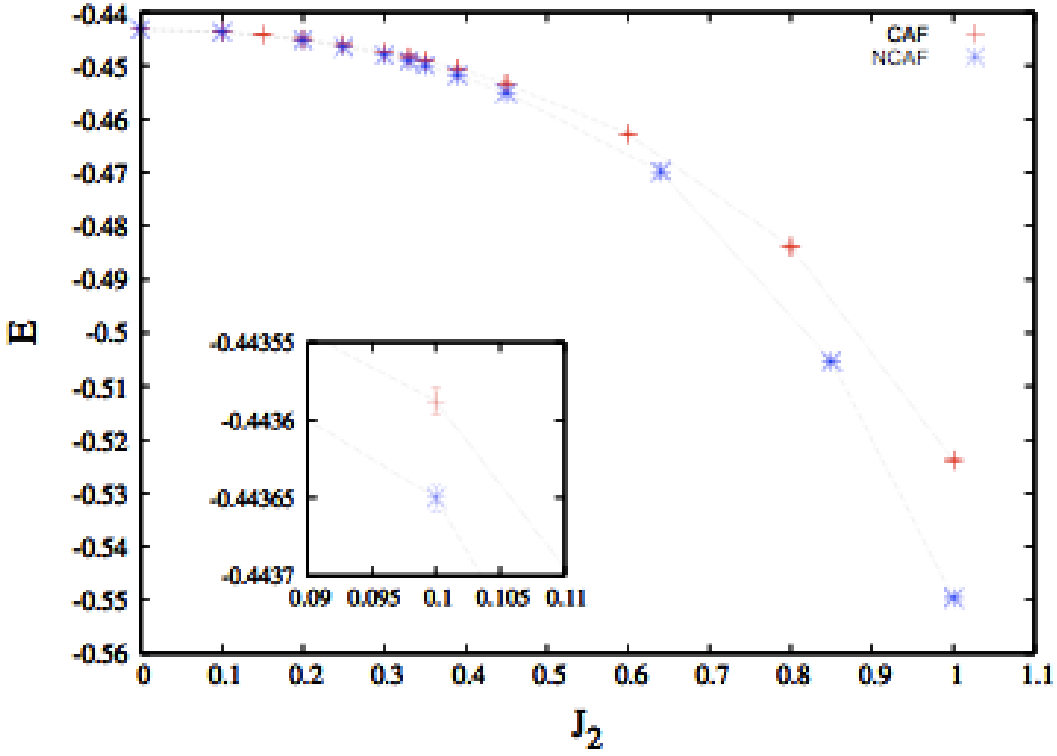
• frustrated chains: **J_2 = 0.24…J => small backscattering**

$$\frac{g_N(\ell^*)}{g_\varepsilon(\ell^*)} = \frac{g_N(\ell_0)}{g_\varepsilon(\ell_0)} \frac{1+\Gamma\ell^*}{1+\Gamma\ell_0} \approx \frac{2}{3} \text{ for } \ell^* \approx 4\ln(J/J') \ll \Gamma^{-1}$$

Numerical Studies

T. Pardini and Rajiv R. P. Singh, [arXiv:0801.3855]





- Non-collinear state (renormalized spiral) has lower energy for J'/J > 0.1

Compare with Cs_2CuCl_4: spiral due to DM interaction

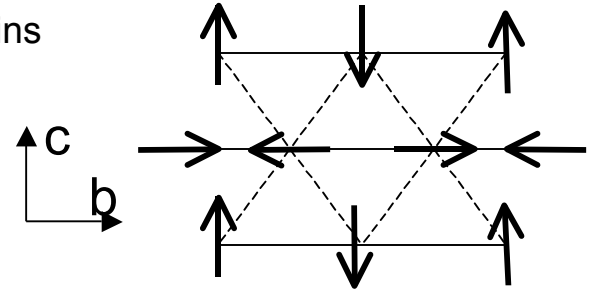
- Even D=0.05 J >> $(J')^4/J^3$ (with constants): DM beats CAF, dimerization instabilities

[y = chain index]

$$\mathcal{H}_{DM} = D\sum_y (-1)^y \hat{z}\cdot \vec{N}(y)\times \vec{N}(y+1)$$

relevant: dim = 1

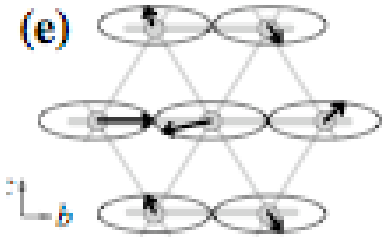
- DM allows *relevant* coupling of N^x and N^y on neighboring chains
  - immediately stabilizes spiral state
    - *orthogonal spins* on neighboring chains



Finite D, but J'=0

  - small J' *perturbatively* makes spiral weakly incommensurate

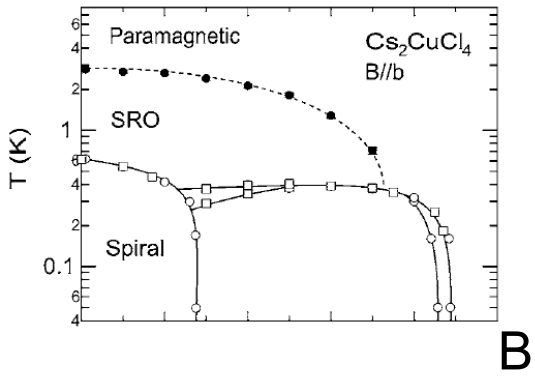
Dzyaloshinskii-Moriya interaction (DM) controls **zero-field phase** of Cs_2CuCl_4 !



Finite D and J'

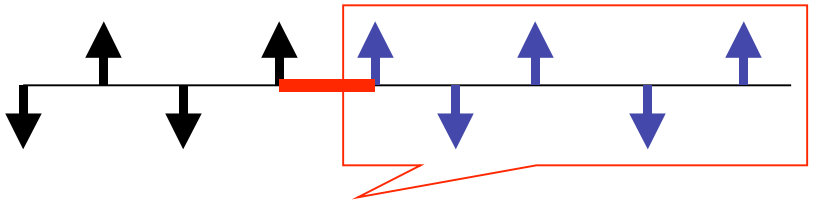
Phase diagram summary

- Frustration promotes one-dimensionality
- Very weak order, via fluctuations generated next-nearest chains coupling $(J'/J)^4 << J'/J$.
- Order (B=0) in Cs_2CuCl_4 is different from theoretical J-J' model due to the crucial Dzyaloshinskii-Moriya interaction.
- *Also worked out:* Phase diagram in magnetic field can be understood in great details as well
  - Sensitive to field orientation
  - *Field induced spin-density wave* to *cone* quantum phase transitions
  - BEC condensation at ~ 9 T



Excitations of S=1/2 chain

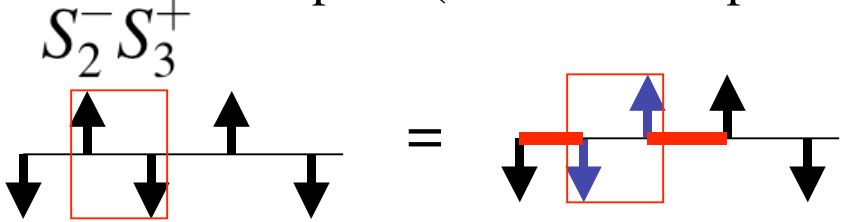
- Spinons = propagating domain walls in AFM background, carries S=1/2



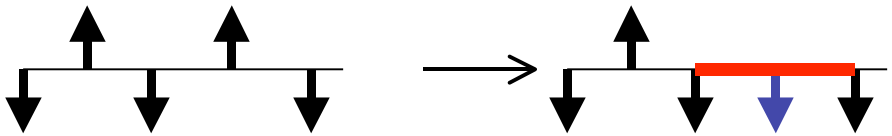
[domain of opposite Neel orientation]

Dispersion $\omega(k) = \frac{\pi J}{2}\sin(ka)$

- Domain walls are created in pairs (but one kink per bond)

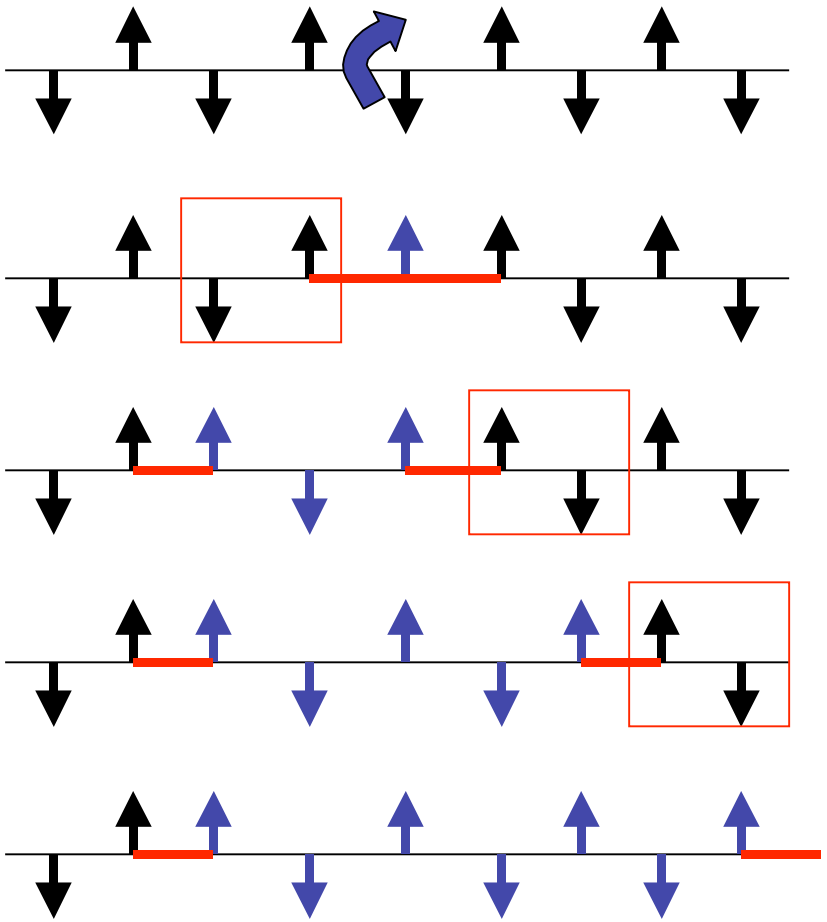


- Single spin-flip = two domain walls: spin-1 wave breaks into pairs of deconfined spin-1/2 spinons.



Breaking the spin waves

Create spin-flip and evolve with $S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+$



Energy cost **J**

time

Segment between two domain walls has opposite to initial orientation.
Energy is size independent.

Two-spinon continuum

Spinon energy $\omega(k_x) = \frac{\pi J}{2}|\sin(k_x)|$

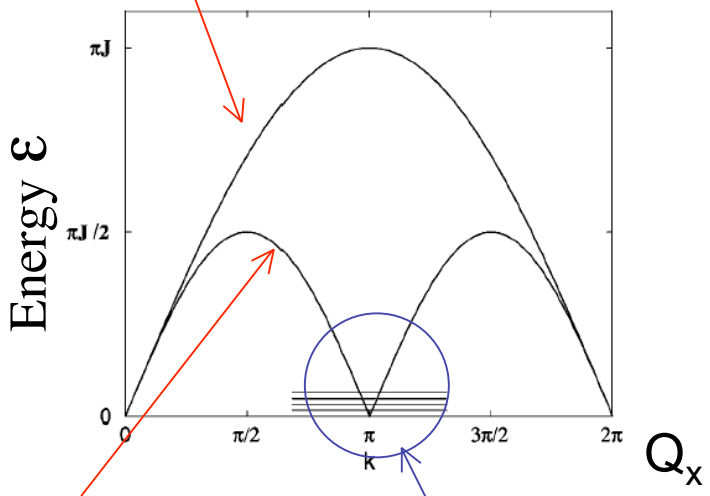
S=1 excitation $\begin{cases} \varepsilon = \omega(k_{x1}) + \omega(k_{x2}) \\ Q_x = k_{x1} + k_{x2} \end{cases}$

$$\varepsilon = \pi J \sin(\frac{Q_x}{2}) \cos(\frac{k_{x1} - k_{x2}}{2})$$

$\omega_{2,\text{upper}} = \pi J \sin(Q_x/2)$

$k_{x1} = k_{x2} = Q_x/2$

Upper boundary



Lower boundary

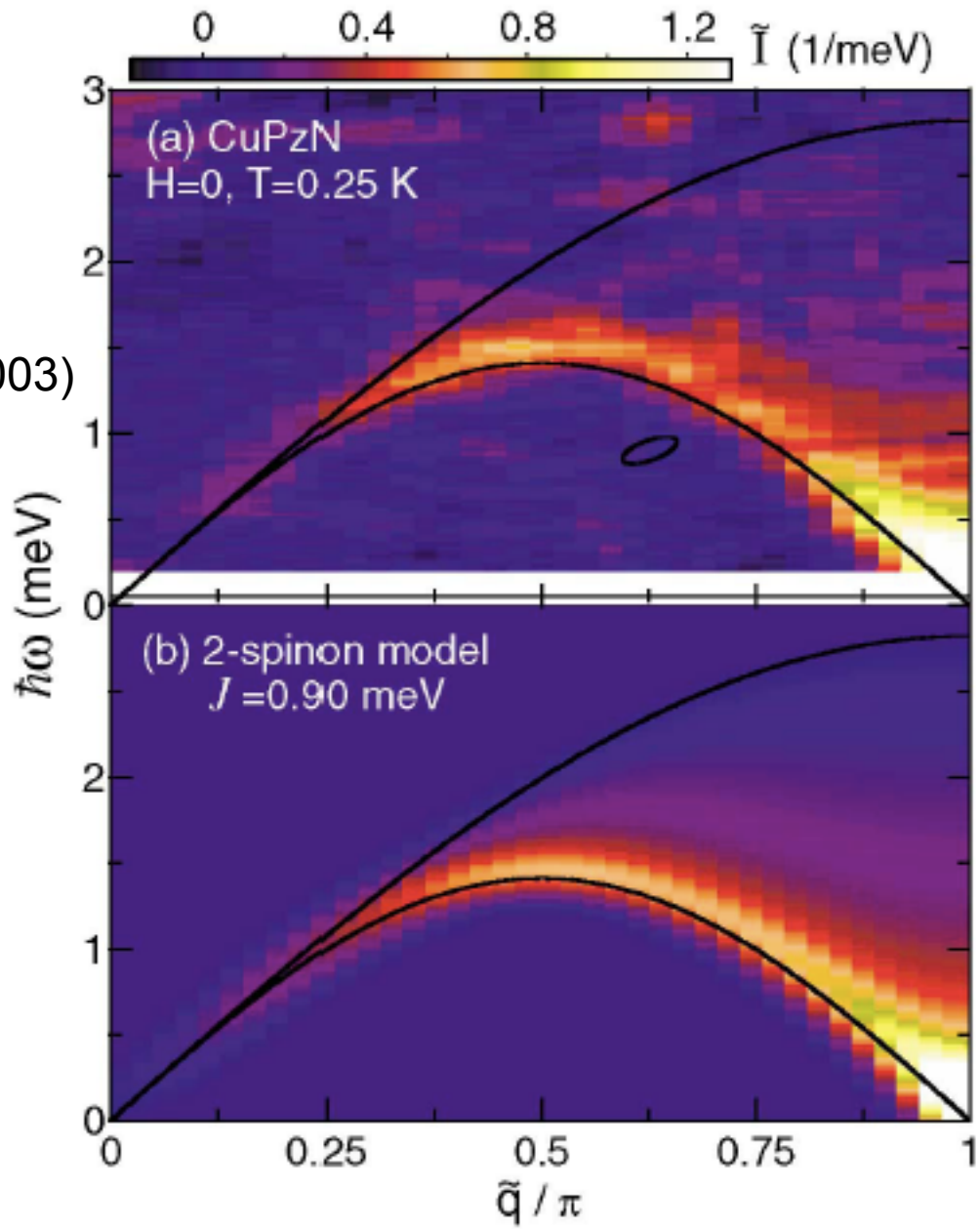
Low-energy sector $Q_{AFM}=\pi$

$\omega_{2,\text{lower}} = \frac{\pi J}{2}\sin(Q_x)$

$k_{x1} = 0, k_{x2} = Q_x$

Dynamic structure factor of copper pyrazine dinitirate (CuPzN)

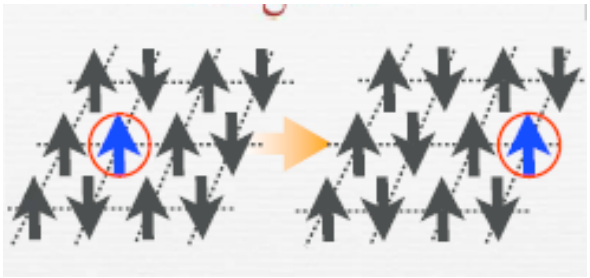
Stone et al,
PRL 91, 037205 (2003)

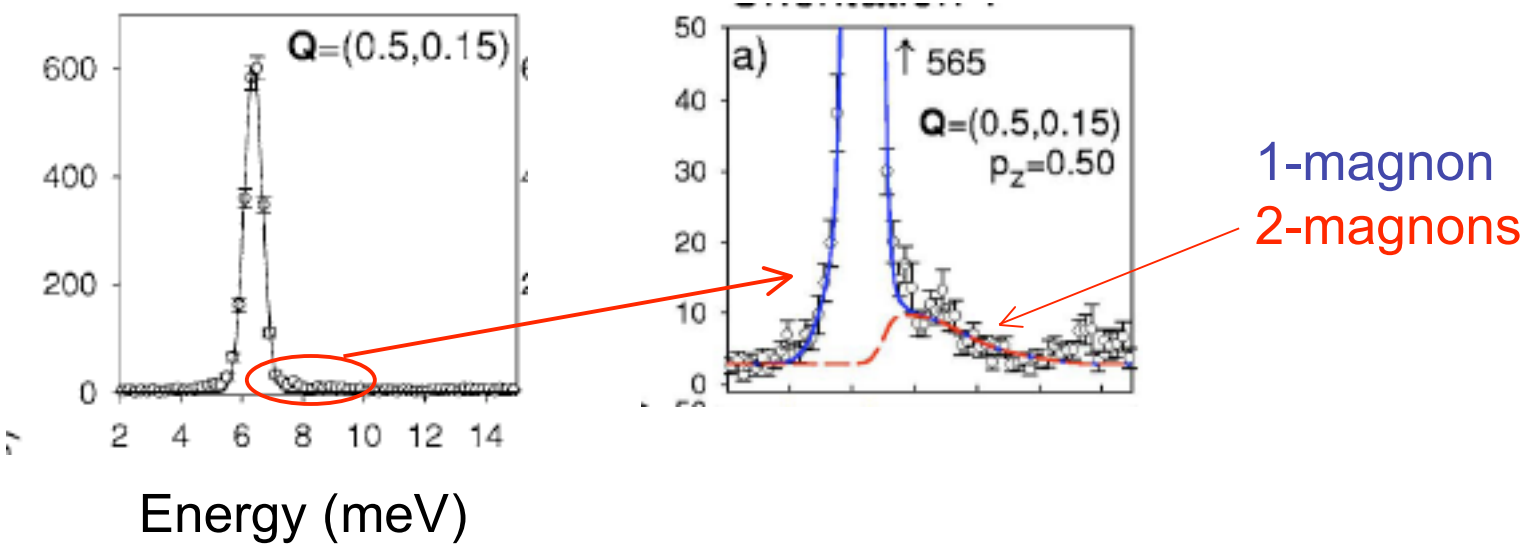


Compare with the “usual” neutron scattering

2D S=5/2 AFM Rb_2MnF_4 , J=0.65meV

Huberman et al. PRB 72, 014413 (2005)





Structure factor is determined by single magnon contribution

$$S(k,\omega) \propto \mathrm{Re}\left\langle S_k^- \delta(\omega - H) S_k^+ \right\rangle \sim Z(k)\delta(\omega - \epsilon(k))$$

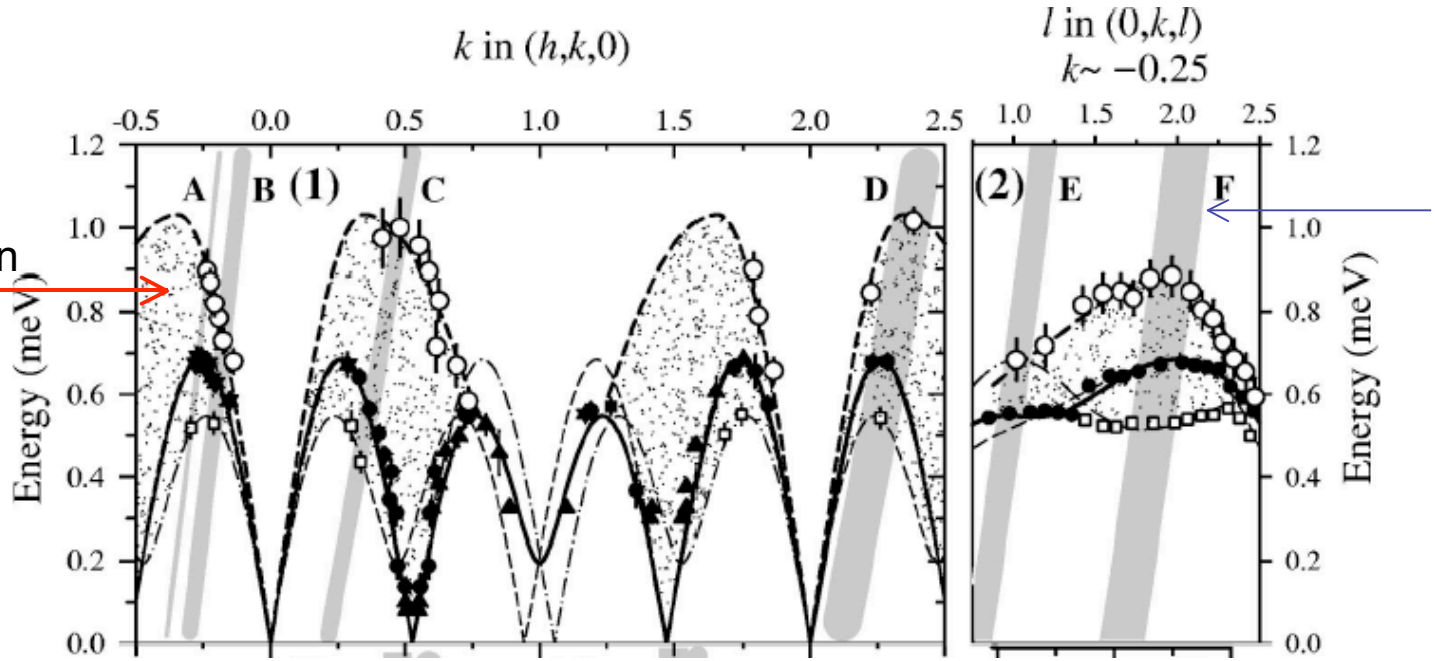
(generally, 2-magnon bound states are rare in antiferromagnets)

PHYSICAL REVIEW B 68, 134424 (2003)

Extended scattering continua characteristic of spin fractionalization in the two-dimensional frustrated quantum magnet Cs_2CuCl_4 observed by neutron scattering

R. Coldea,[1,2,3] D. A. Tennant,[1,3] and Z. Tylczynski[4]

Along the chain



transverse to chain



Probably should try spinons…

Effective Schrödinger equation

- Study two spinon subspace

$$|k_x, k_y; \epsilon\rangle = \sum_y e^{ik_y y} |k_x, \epsilon\rangle_y \otimes_{y' \neq y} |0\rangle_{y'}$$

(two spinons on chain y with S^z=+1)

  - Momentum conservation: 1d Schrödinger equation in ε space

$$\epsilon \psi_{\mathbf{k}}(\epsilon) + \int d\tilde{\epsilon} D_{k_x}(\tilde{\epsilon})\, J'(\mathbf{k}) A^*_{k_x}(\epsilon) A_{k_x}(\tilde{\epsilon})\, \psi_{\mathbf{k}}(\tilde{\epsilon}) = E \psi_{\mathbf{k}}(\epsilon)$$

- Crucial matrix elements known exactly

$$A_{k_x}(\epsilon) \equiv \frac{1}{\sqrt{2}} \langle 0 | S^-_{-k_x, y} | k_x, \epsilon \rangle_y$$

Bougourzi *et al*, 1996

Structure Factor

J.S. Caux *et al*, 2006

- Spectral Representation

$$S(k,\omega) \propto \sum_n \left|\langle n|S_k^+|0\rangle\right|^2 \delta(\omega - E_n)$$

Weight in 1d:
73% in 2 spinon states
99% in 2+4 spinons

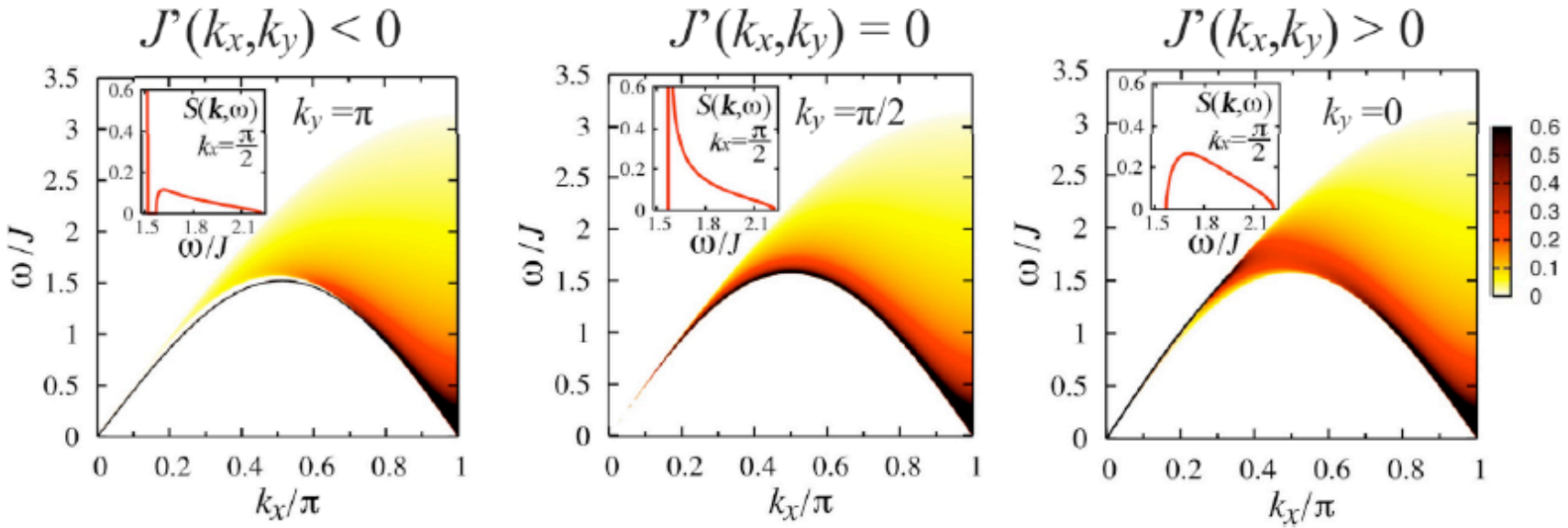
  - Can obtain closed-form "RPA-like" expression for 2d S(k,ω) in 2-spinon approximation

$$S(k,\omega) = \frac{S_{1d}(k,\omega)}{[1 + J'(k)\chi'_{1d}(k,\omega)]^2 + [\pi J'(k) S_{1d}(k,\omega)]^2}$$

$$S_{1d}(k,\omega) = |A_k(\omega)|^2 / \sqrt{\omega^2_{2,\text{upper}} - \omega^2} \qquad \chi'_{1d}(k,\omega) = P\int_0^\infty d\omega' \frac{S_{1d}(k,\omega')}{\omega' - \omega}$$

Types of behavior

- Behavior depends upon spinon interaction $J'(k_x,k_y)$



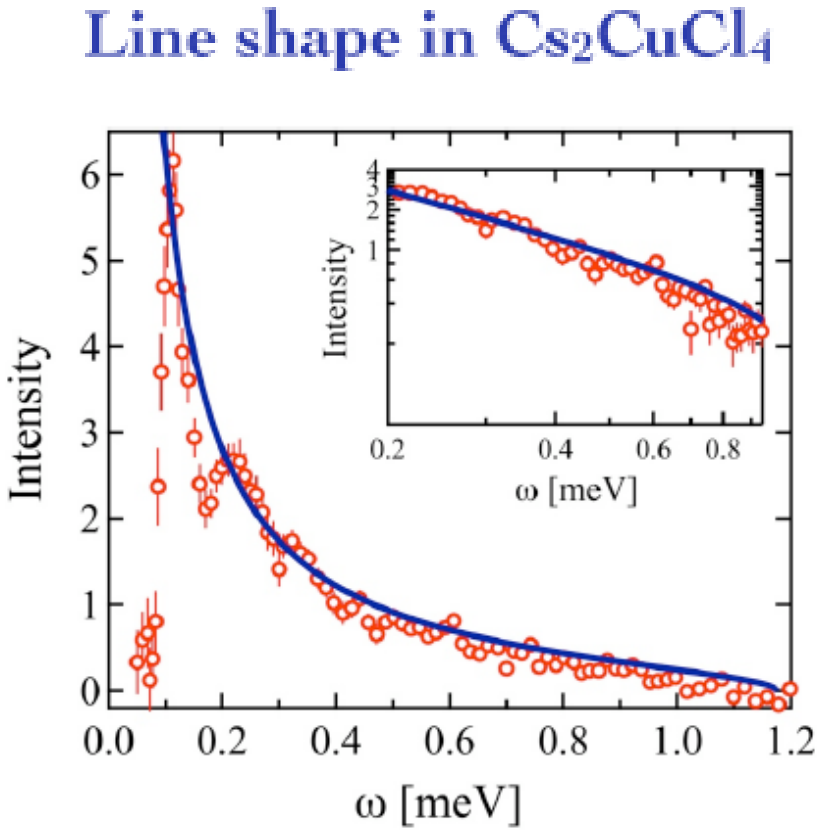
Bound "triplon"

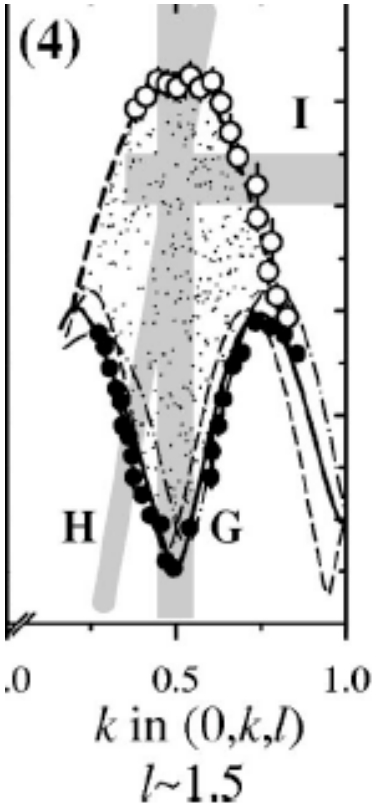
Identical to 1D

Upward shift of spectral weight. Broad resonance in continuum or anti-bound state (small k)

Broad lineshape: "free spinons"

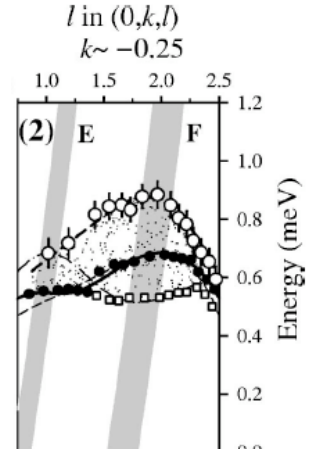
- "Power law" fits well to free spinon result
  - Fit determines normalization



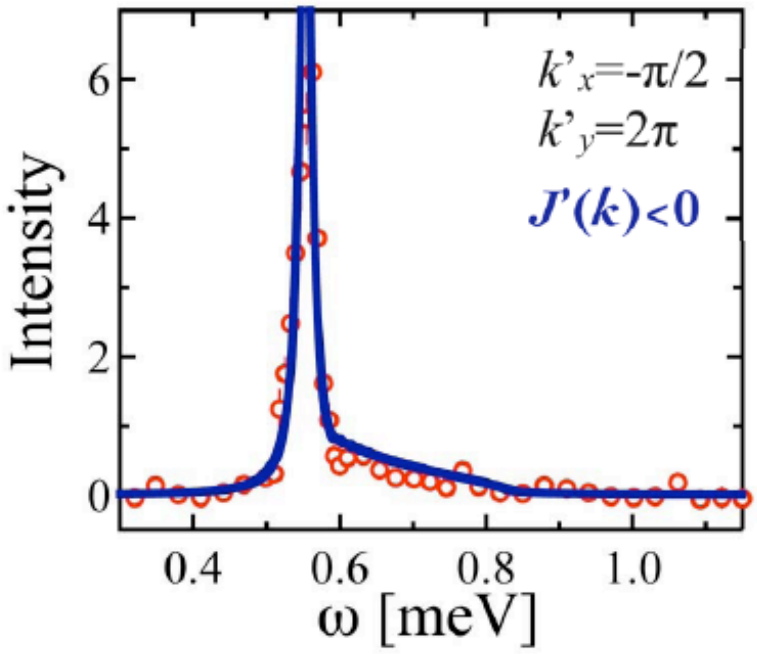


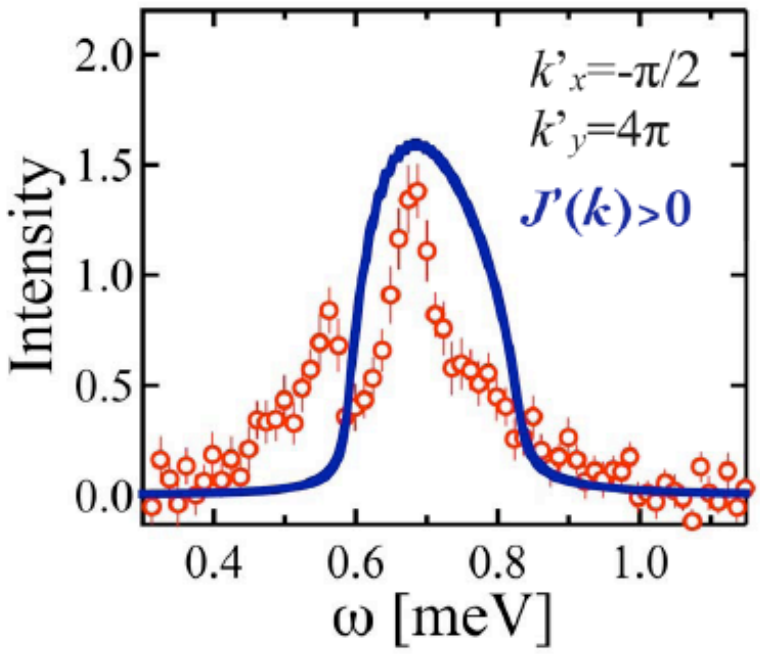
J'(k)=0 here

Triplon: S=1 bound state of two spinons



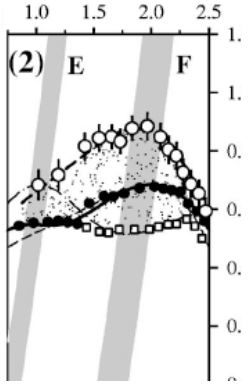
- Compare spectra at J'(k)<0 and J'(k)>0:

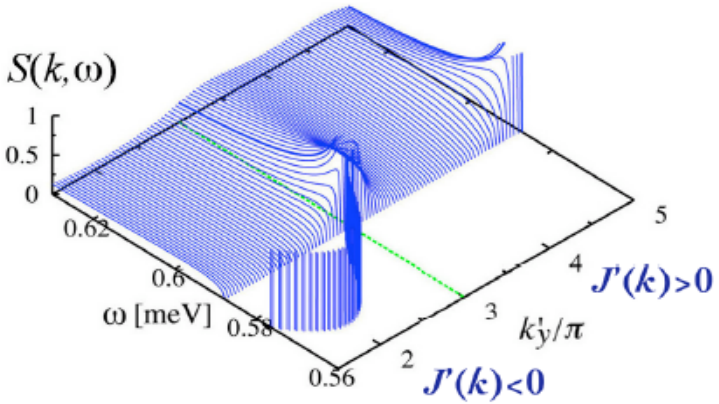




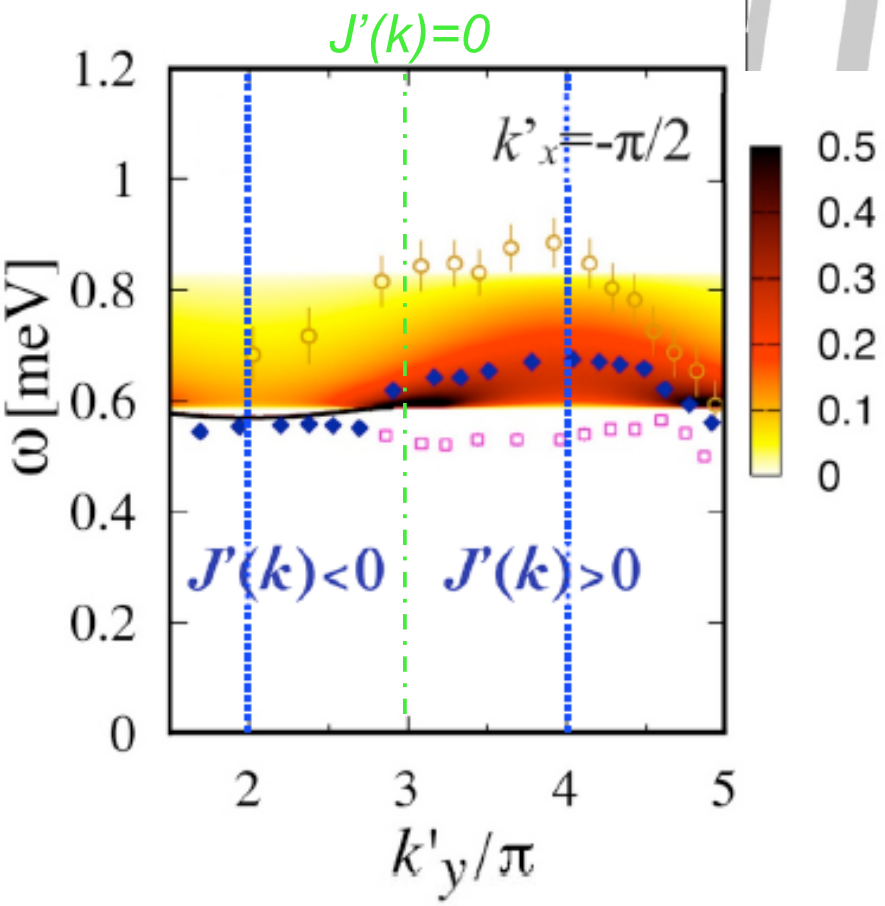
- Curves: 2-spinon theory w/ experimental resolution
- Curves: 4-spinon RPA w/ experimental resolution

Transverse dispersion





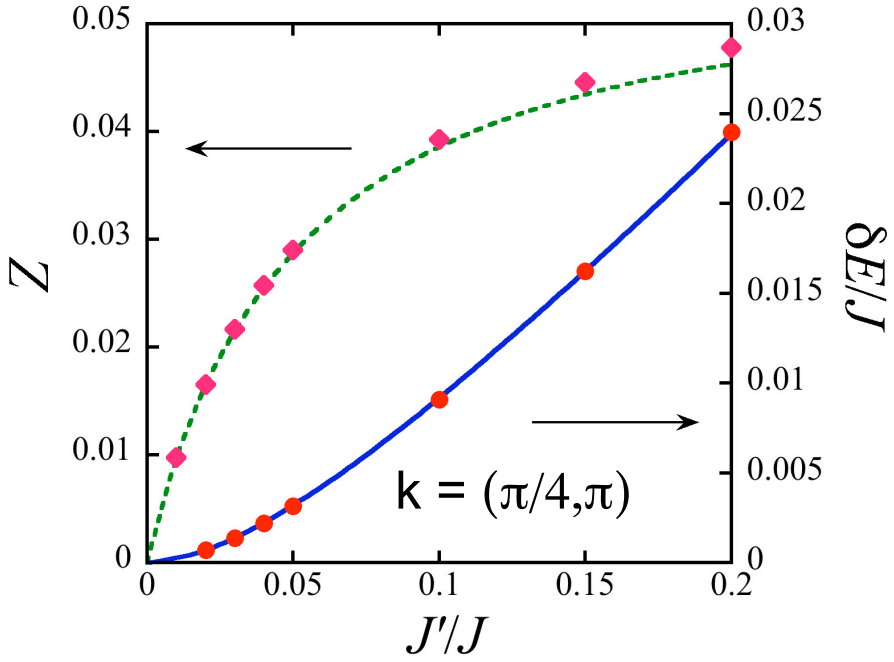
Bound state and resonance

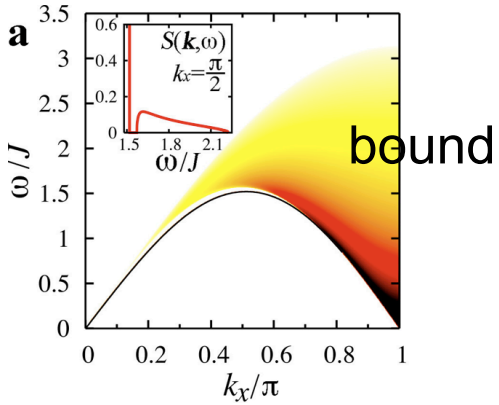


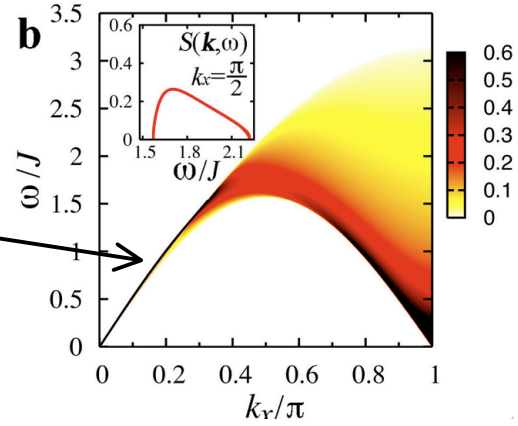
Solid symbols: experiment
Note peak (blue diamonds) coincides with bottom edge only for J'(k)<0

Details of triplon dispersion

- Energy separation from the continuum $\delta E \sim [J'(k)]^2$
- Spectral weight of the triplon pole $Z \sim |J'(k)|$





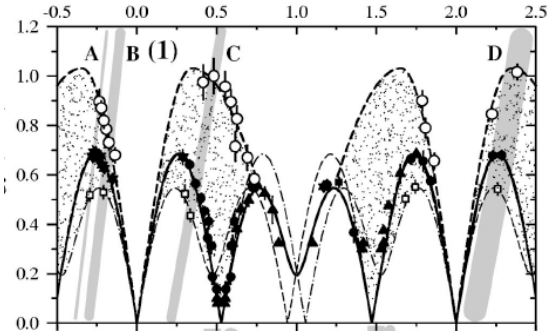


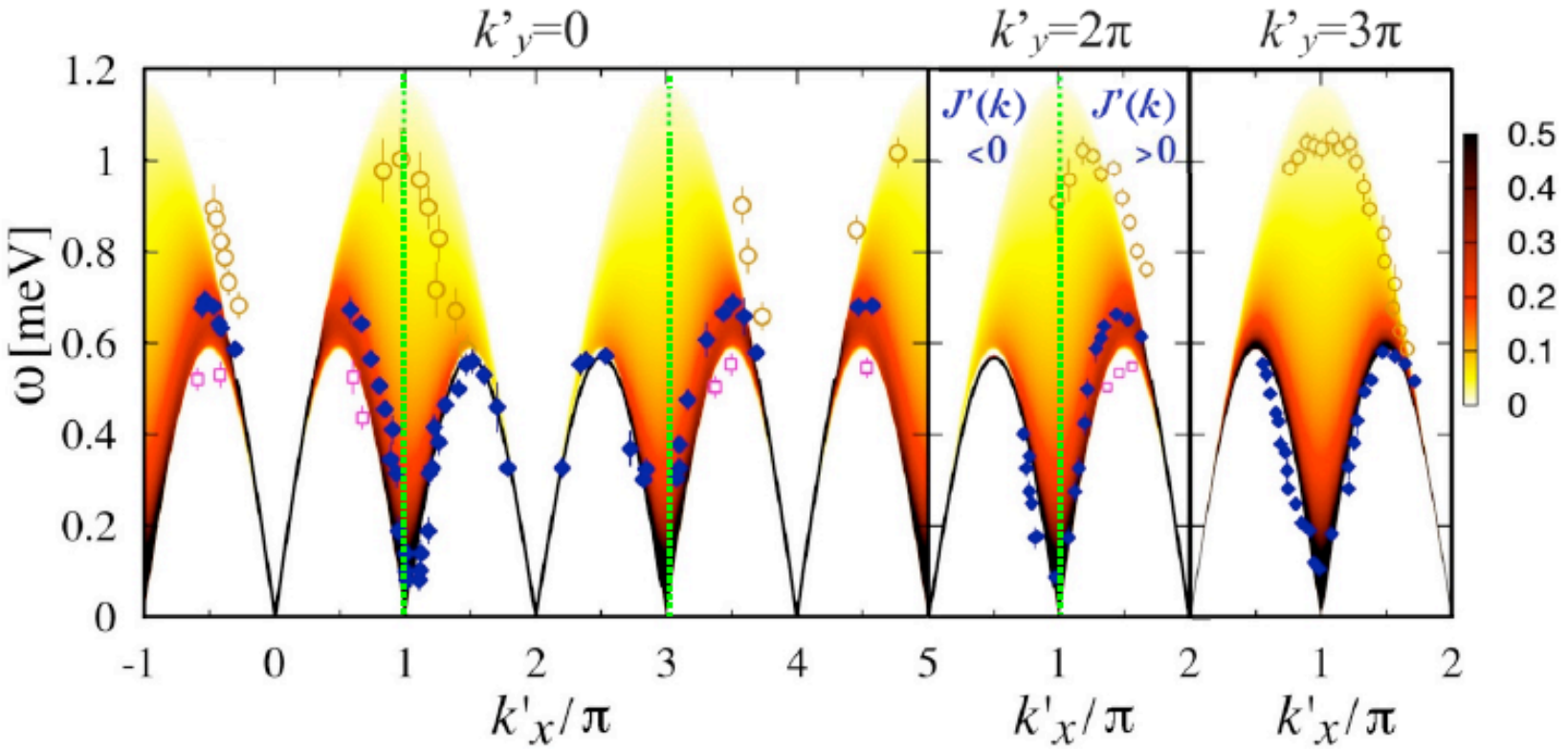
- Anti-bound triplon when $J'(k) > J'_{\text{critical}}(k)$ and

$$1 + J'_{\text{critical}}(k)\chi'(k_x, \omega_{2,\text{upper}}) = 0$$

- Expect at small k~0 where continuum is narrow.
- Always of finite width due to 4-spinon contributions.

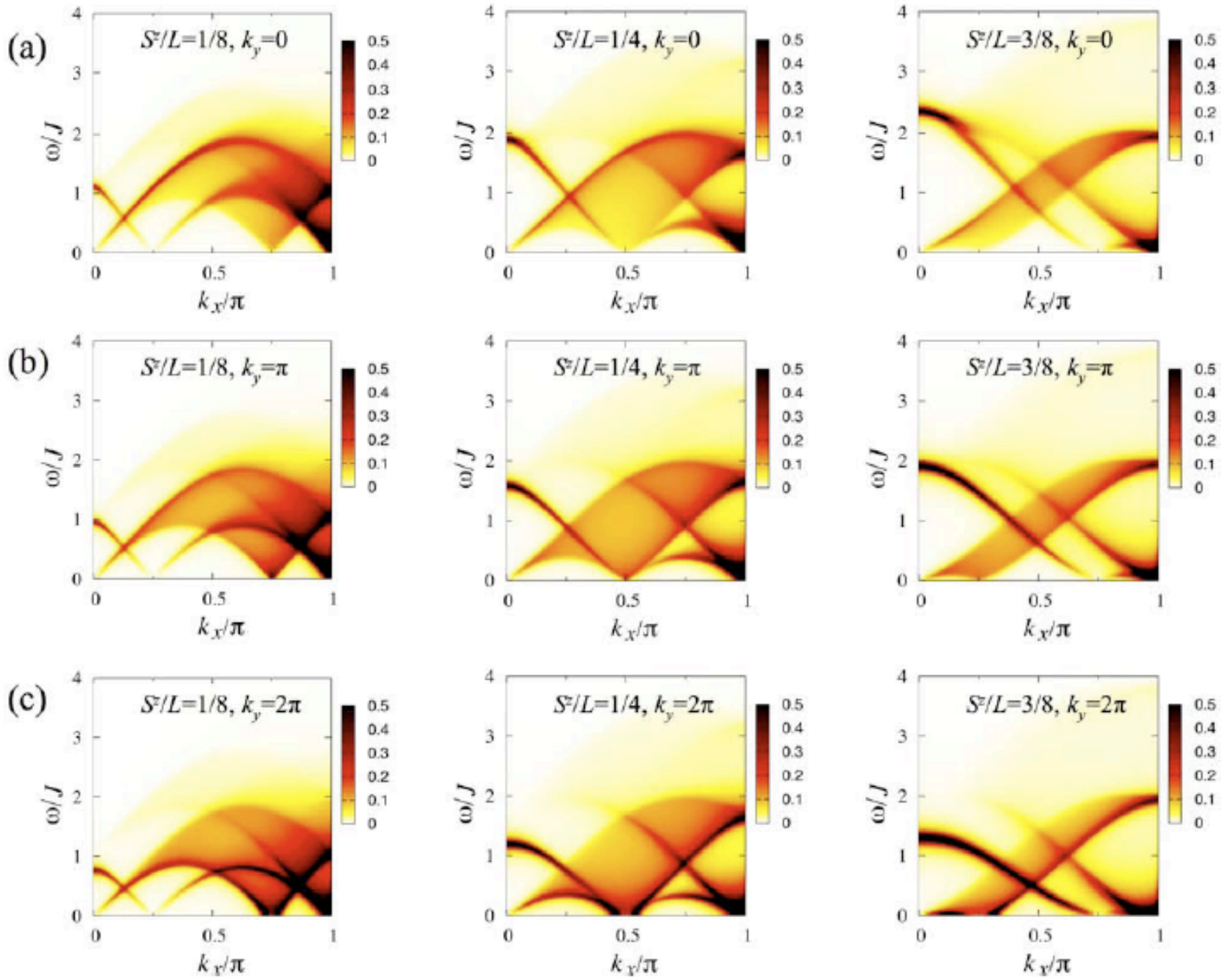
Spectral asymmetry





- Vertical green lines: $J'(k)=4J' \cos[k'_x/2] \cos[k'_y/2] = 0$.

Brief preview: magnetic field (M. Kohno, APS March meeting 2008)



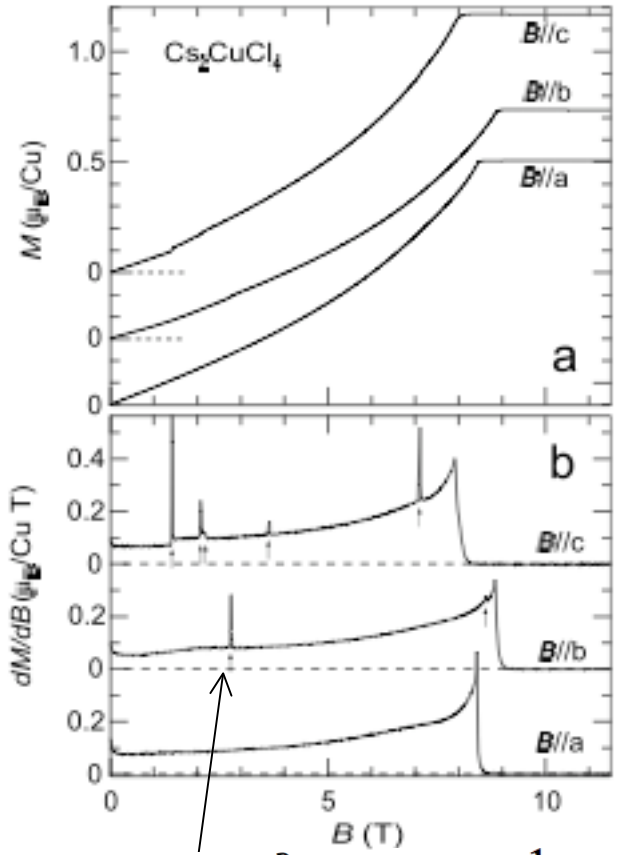
Conclusion

- Phase diagram of spatially anisotropic triangular antiferromagnet: quantum fluctuations promote collinear phase in favor of classical spiral

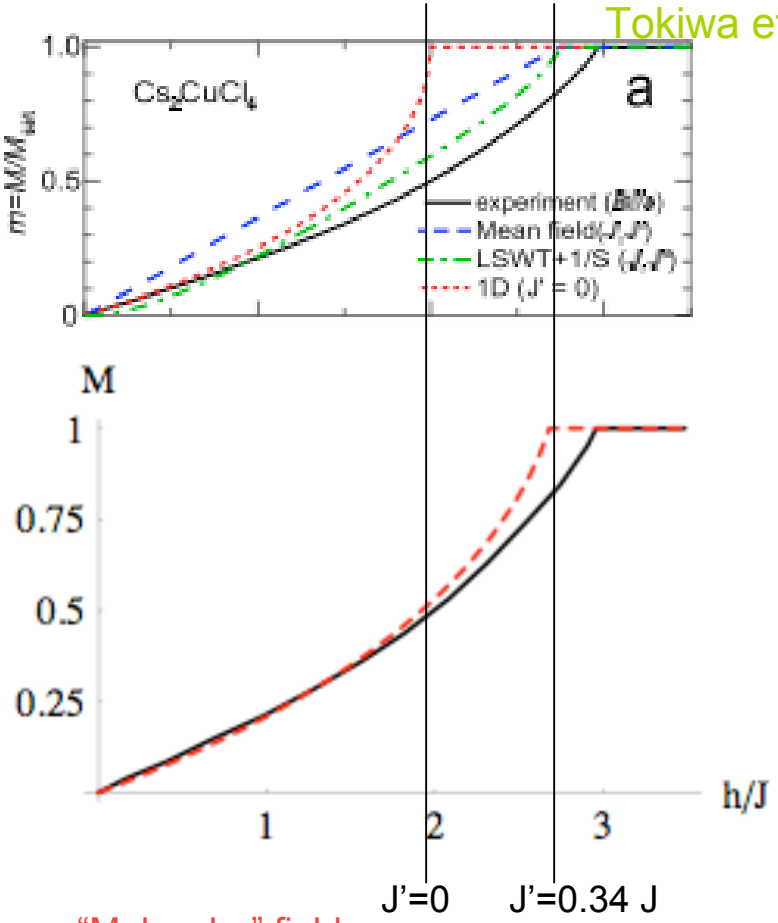
  ("order from order")
- Dynamic response: simple theory works well for frustrated quasi-1d antiferromagnets
  - Frustration actually simplifies problem by *enhancing one-dimensionality* and *suppressing* long range order (even for not too small inter-chain couplings)
- "Mystery" of Cs_2CuCl_4 solved
  - Need to look elsewhere for 2d spin liquids!

Magnetization measurements

- M(h) is smooth: not sensitive to low-energy (long-distance) fluctuations. Determined by *uncorrelated* (but magnetized) chains.



Tokiwa et al, 2006



$$\delta E(M) = 2J'M^2\ ,\ h = \frac{\partial E}{\partial M} \qquad M_{1d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h}}\right)$$

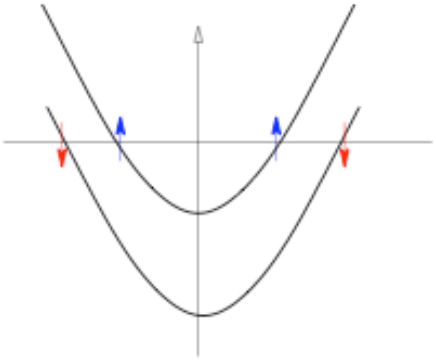
"Molecular" field →

$$M_{2d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h-4J'M_{2d}}}\right)$$

✓Error in saturation field is $(J')^2/2J \approx 2\%$

- dM/dh delineates phase boundaries: divergent derivative = phase transition

S=1/2 AFM Chain in a Field



- Field-split Fermi momenta: $k_{F\uparrow} = \frac{\pi}{2} - \delta \; , \; k_{F\downarrow} = \frac{\pi}{2} + \delta$
  - ✓ Uniform magnetization $M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$
  - ✓ Half-filled condition $n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$

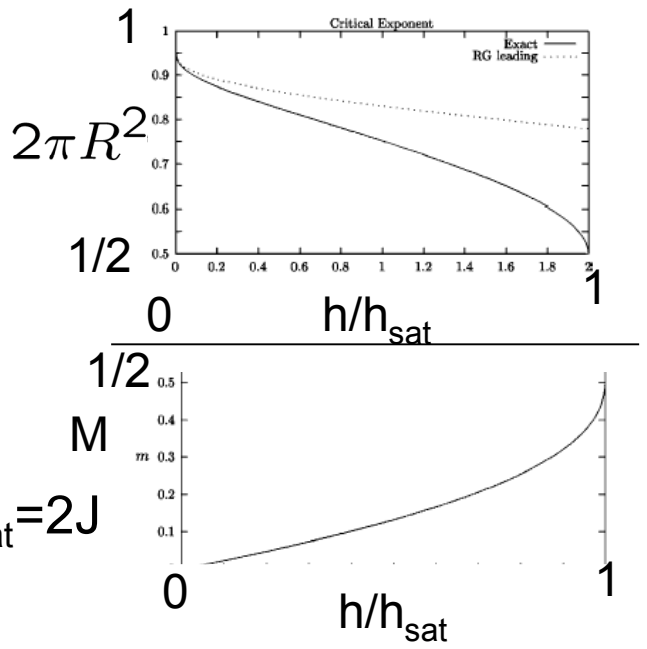
- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$ $S^z_{\pi \pm 2\delta}$ scaling dimension $1/4\pi R^2$ **increases**

- $S^{x,y}$ components ($\Delta S=1$) remain at π $S^{\pm}_{\pi}$ scaling dimension πR^2 **decreases**

- Derived for free electrons but correct always - *Luttinger Theorem*

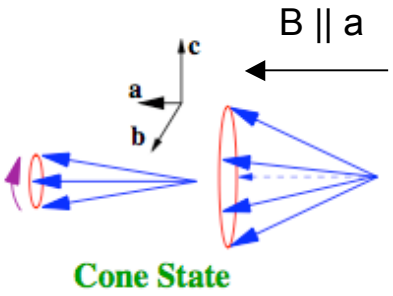
1

Affleck and Oshikawa, 1999



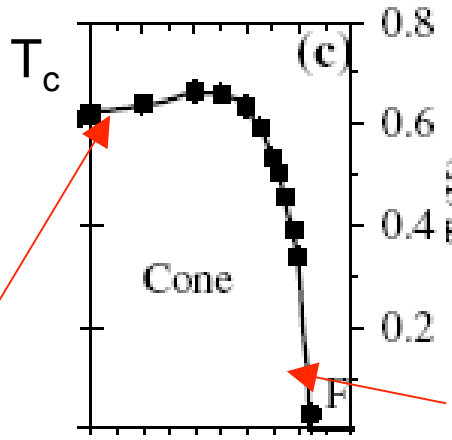
- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

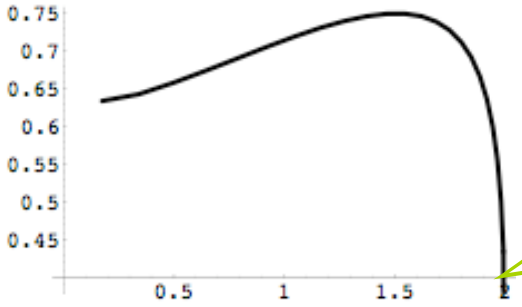
Transverse Field: B ‖ D

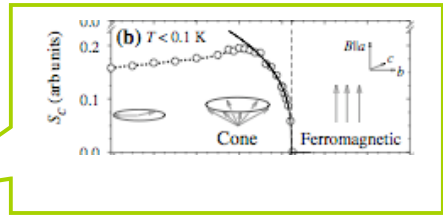


- DM term becomes *more relevant*
- *b-c* spin components (XY) remain commensurate: spin simply tilt in the direction of the field
- Spiral (cone) state just persists for all fields.

Experiment *vs* Theory







Order *increases* with h here due to increasing relevance of DM term

Order *decreases* with h here due to vanishing amplitude as h_{sat} is approached (density of magnons -> 0)

BEC

Coldea et al 2002;
Radu et al 2005;
Veillette et al 2006;
Kovrizhin et al 2006

Longitudinal Field: B $\perp$ D

- DM term involves S^z (at $\pi - 2\delta$) and S^x (at π):
  - ✓ Leads to momentum mis-match for h>0: DM "irrelevant" for **h > D**
    - "averages out"
- With DM killed, sub-dominant instabilities take hold
- Two important couplings for h>0:

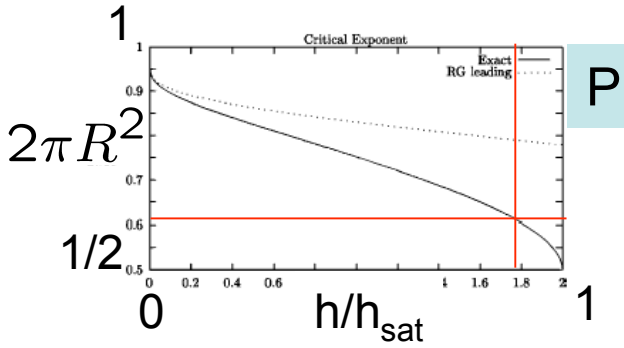
Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J' \left(S^+_\pi(y) \partial_x S^-_\pi(y+1) + \text{h.c.} \right) \right]$$

dim $1/2\pi R^2$: 1 -> 2 — "collinear" SDW

dim $1+2\pi R^2$: 2 -> 3/2 — spiral "cone" state

- "Critical point": $1+2\pi R^2 = 1/2\pi R^2$ gives $2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$

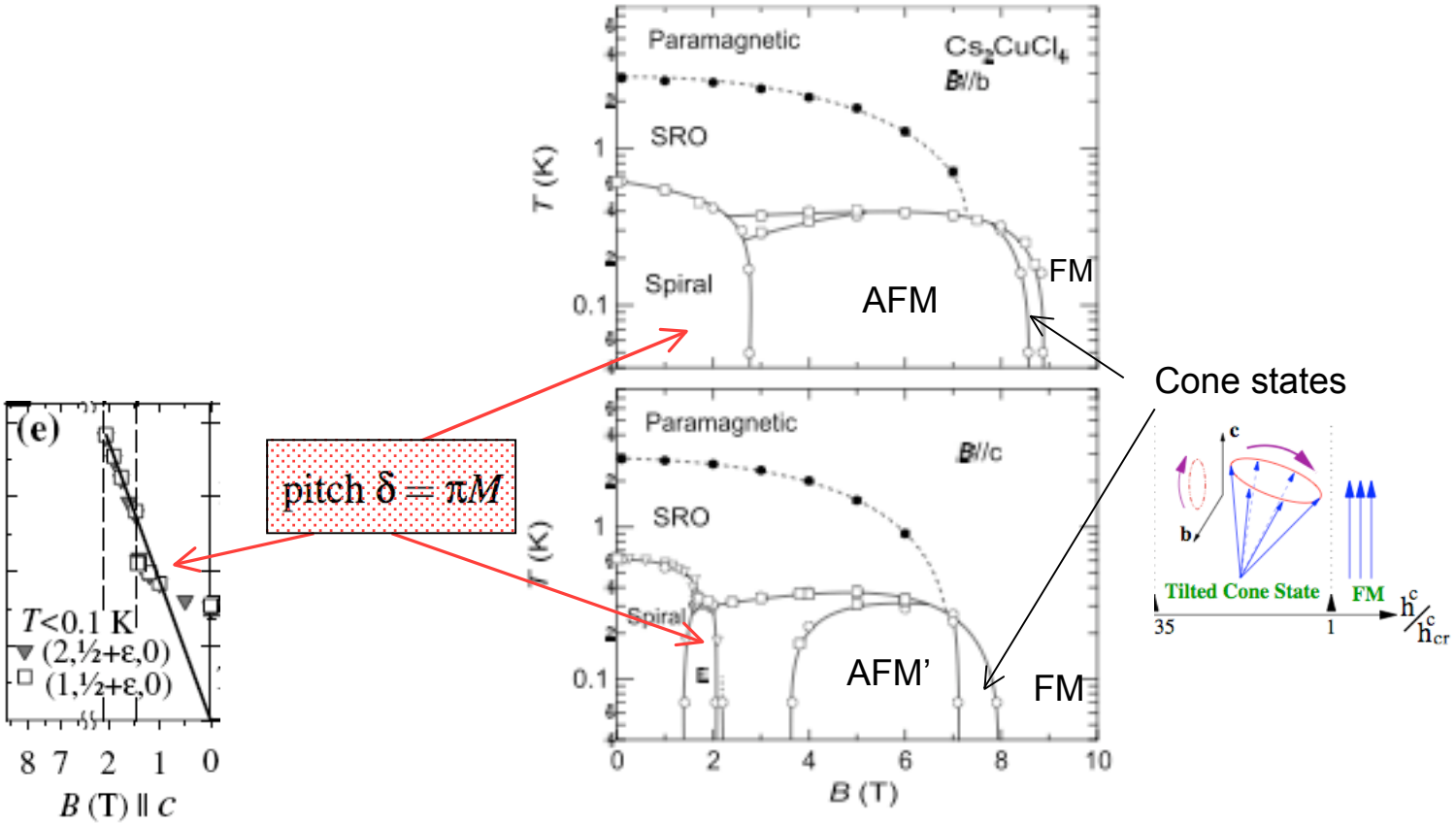


Predicts spiral (cone) state for h>h$_c$ = 0.9 h$_{sat}$ = 7.2 T

observed for h>7.1T

EXPERIMENT: Longitudinal Field ,T_c *vs* B; *SDW, …, cone*

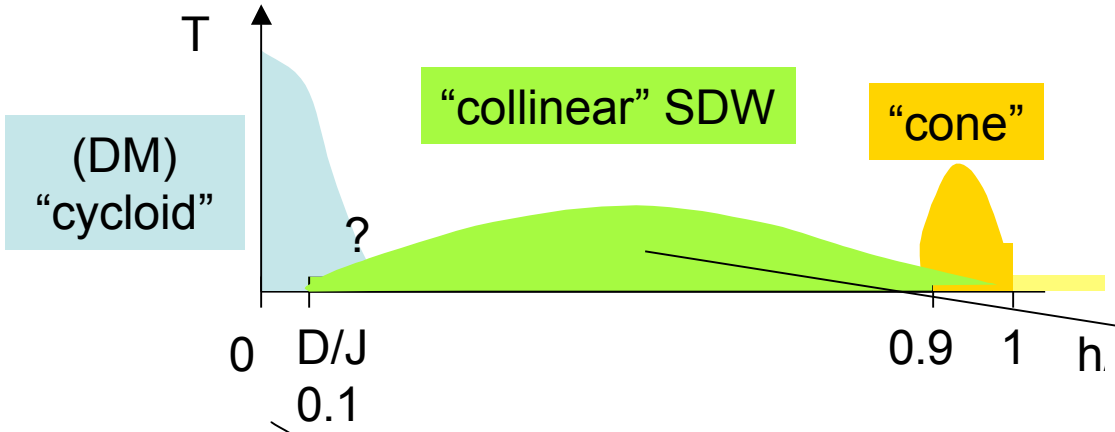
- Very different behavior for B along *b, c* axes (both orthogonal to DM direction *a*)
  - *Additional anisotropies in the problem?*
  - *Nature of AFM and AFM' phases - ?*

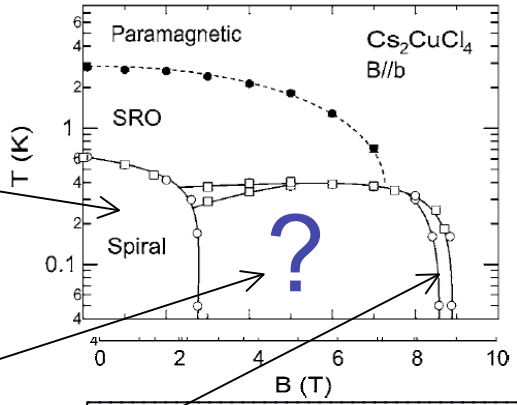


R. Coldea *et al*, 2001;
T. Radu *et al*, 2005;
Y. Tokiwa *et al*, 2006

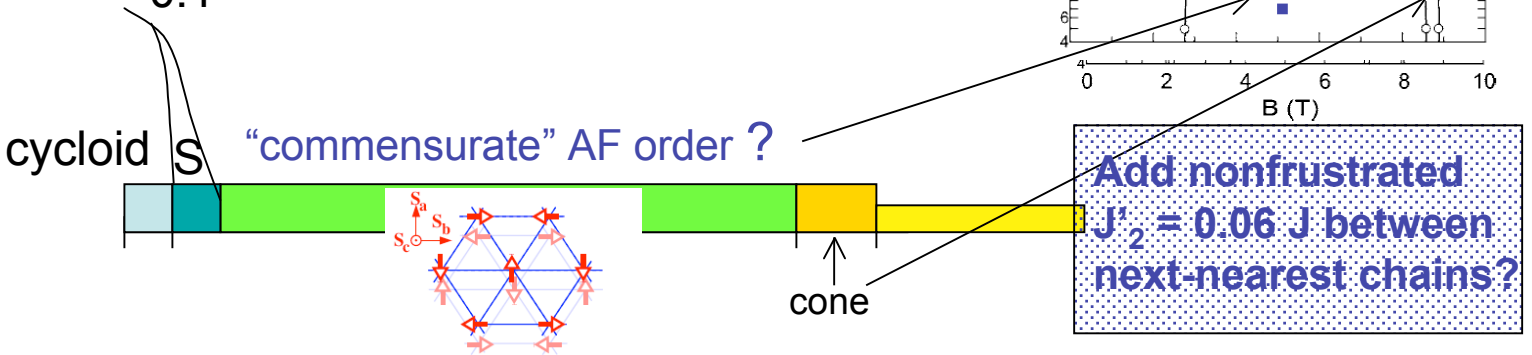
(somewhat naïve) Phase Diagram

Theory



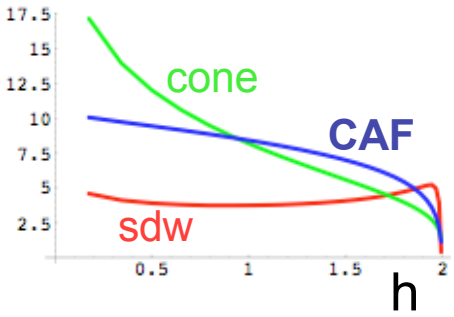


Experiment (on same scale)



? "Commensurate Collinear" order of some sort has been observed recently:
- seems to have *orthogonal spins* on neighboring chains (Coldea // Veillette Chalker 2006)

RG length:
shortest wins
sdw / **cone**

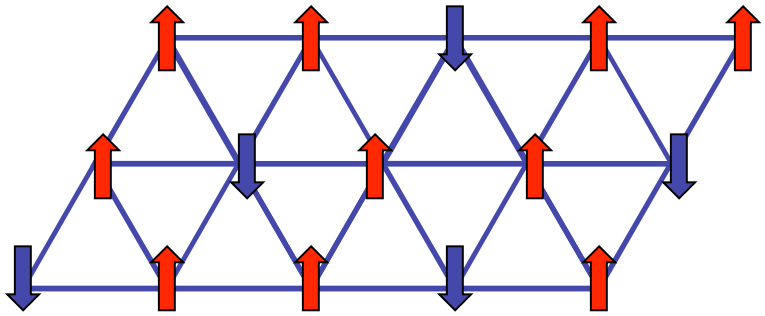


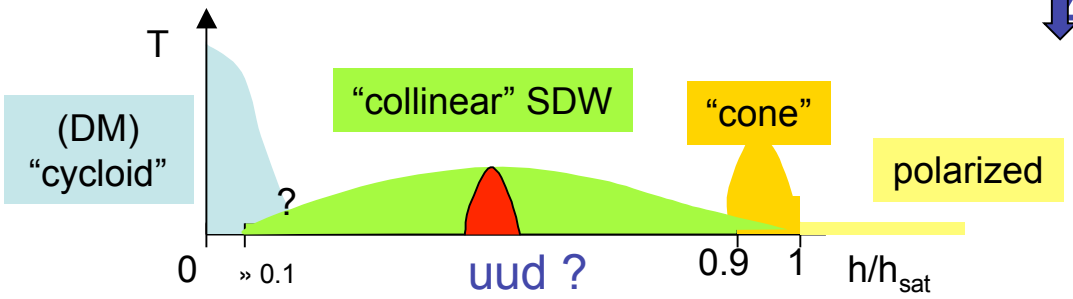
Our theory does predict fluctuations-induced term of right sign, but *too small an amplitude*

$$+ \; c\frac{(J')^4}{J^3}\sum_y S^+_\pi(y)S^-_\pi(y+2) + \text{h.c.}$$

Beyond the naïve

- "Collinear" SDW state *locks* to the lattice at low-T
  - -"irrelevant" (1d) umklapp terms become relevant once SDW order is present (when *commensurate*)
  - -strongest locking is at M=1/3 M_{sat}
    - ***down***-spins at the centers of hexagons





✓Observed in Cs_2CuBr_4 (Ono 2004)

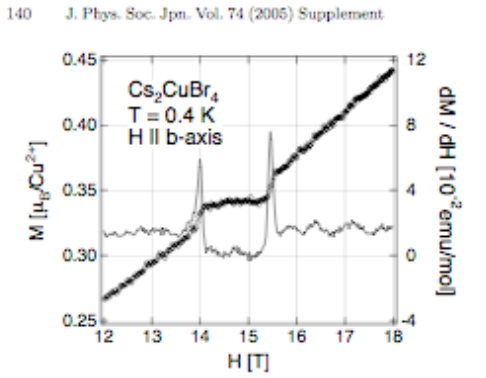


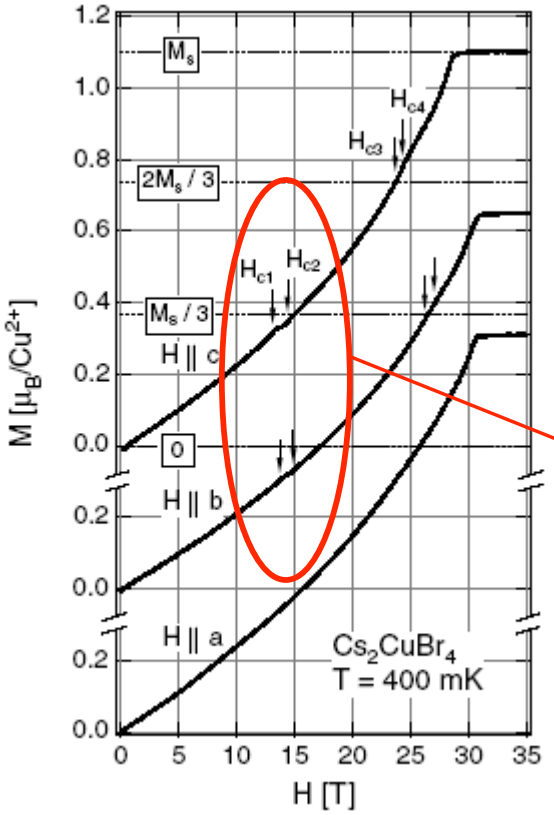
Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel b$ measured at $T = 0.4$ K in magnetic fields up to 20 T.

- Also: "*up-up-down*" state predicted by large-S expansion (Chubukov, Golosov 1992) and Algebraic Vortex Liquid (Alicea, Fisher 2006)

Cs_2CuBr_4

- Isostructural to Cs_2CuCl_4 but believed to be less quasi-1d: J'/J = 0.5

T. Ono *et al*, 2004



- Magnetization plateau at M=1/3 M_{sat} observed for longitudinal but not transverse fields

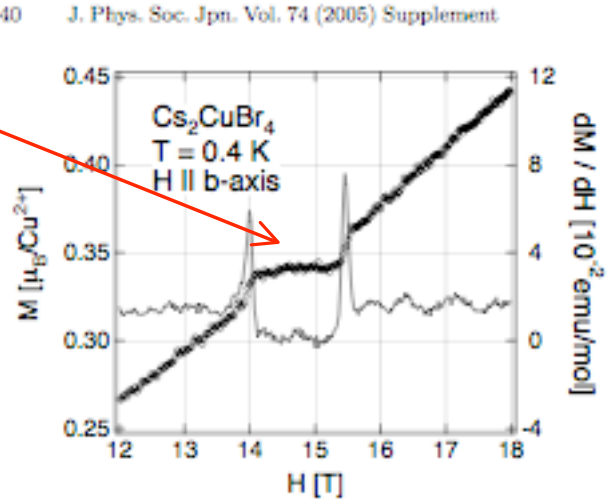
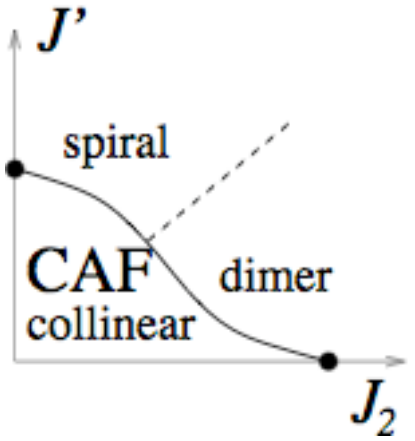


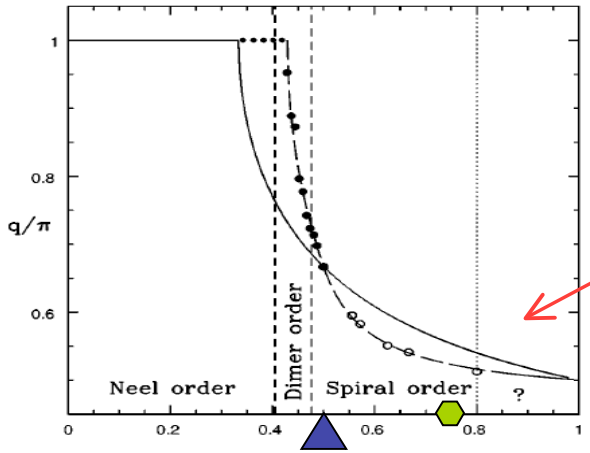
Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel b$ measured at $T = 0.4$ K in magnetic fields up to 20 T.

Alicea, Chubukov, Starykh 2008

Compare with numerics



- Generated interchain couplings: $(J'/J)^4 << J'/J$
- Very weak order: $(J'/J)^2 << (J'/J)^{1/2}$
- The order is *collinear*

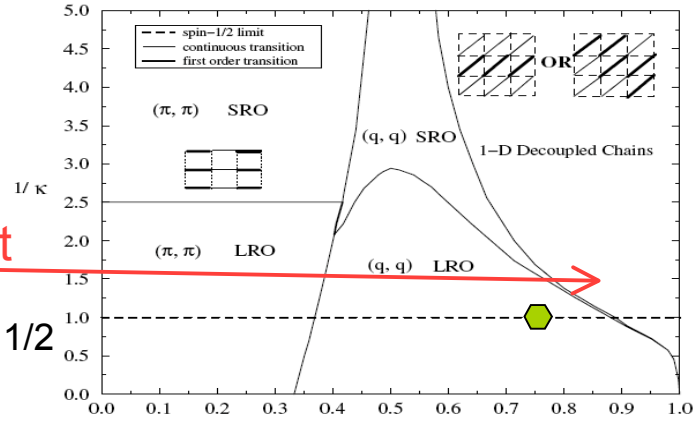


Isotropic triangular lattice

1D limit

"spin" 1/2

J/(J + J')

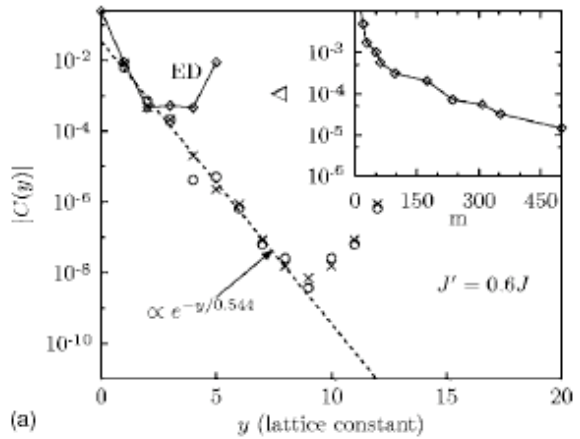


= J/(J+J') = 3/4

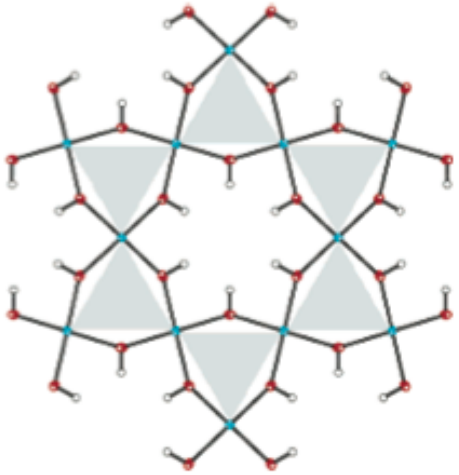
- Series and Large-N expansion: spiral order but possible dimerized states (Fjaerestad 2006-07; Chung et al 2001)

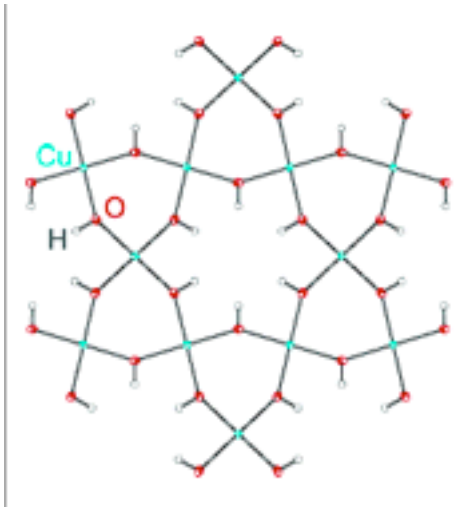
- Exact diagonalization (6x6): extended spin-liquid J'/J ~ 0.7 (Weng et al 2006)

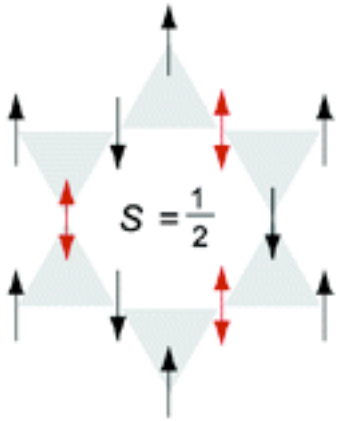
  - ✓ Exponentially weak inter-chain correlations

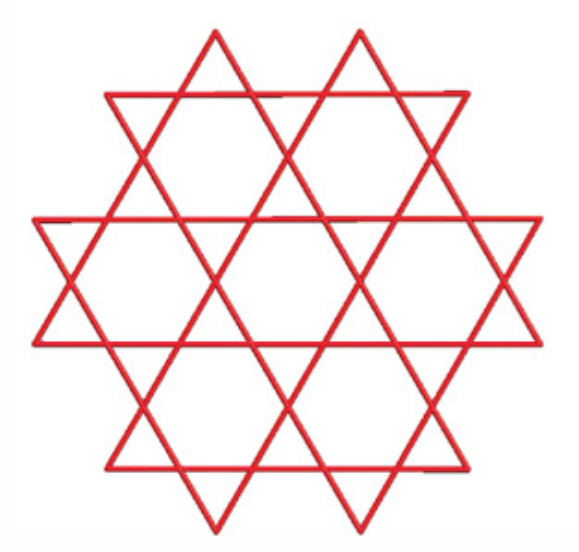


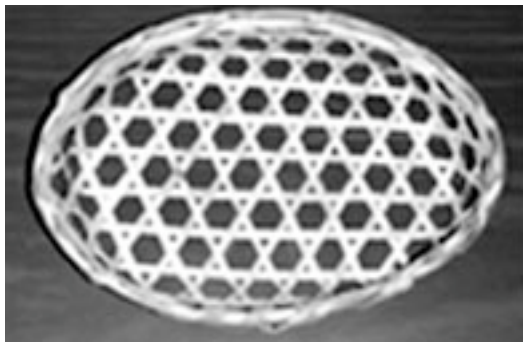
Flood of spin liquids: Kagome



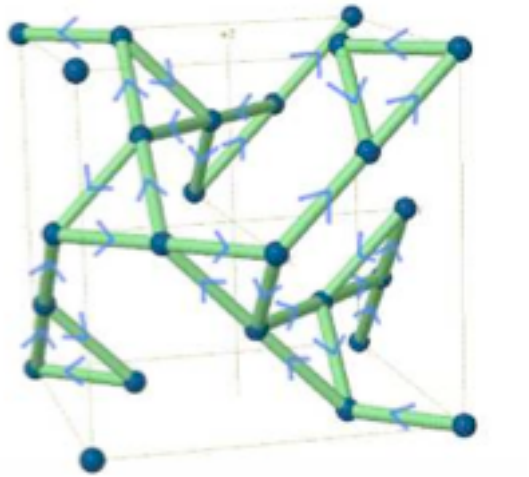


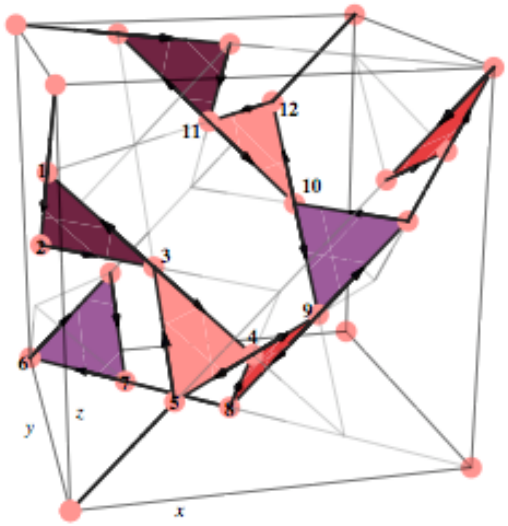


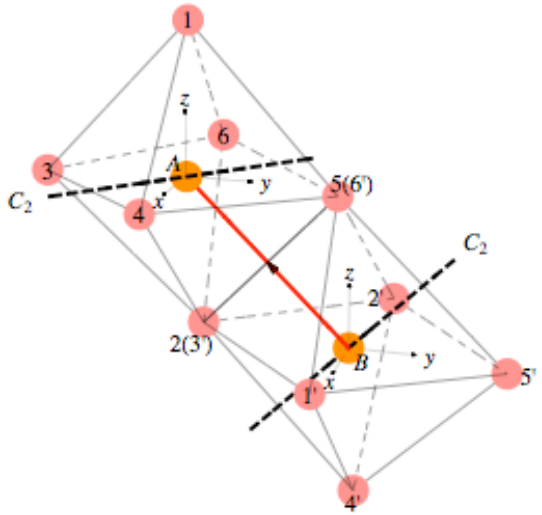




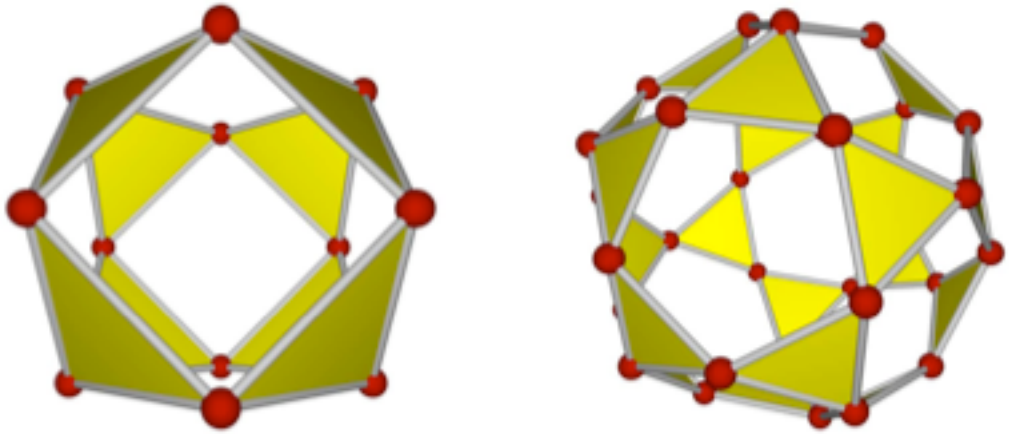
Hyper Kagome $Na_4Ir_3O_8$: 3D lattice of corner-sharing triangles





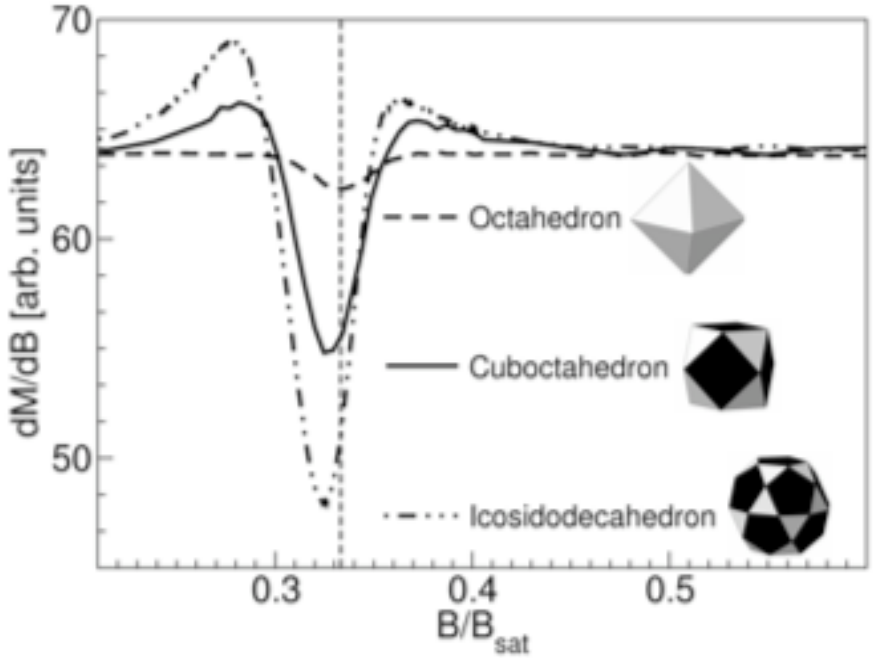


Magnetic molecules with triangular motiff



$Mo_{72}Fe_{30}$

S=5/2 Heisenberg



Triangular lattice in k-$(ET)_2X$: from spin-liquid to superconductor

Side view

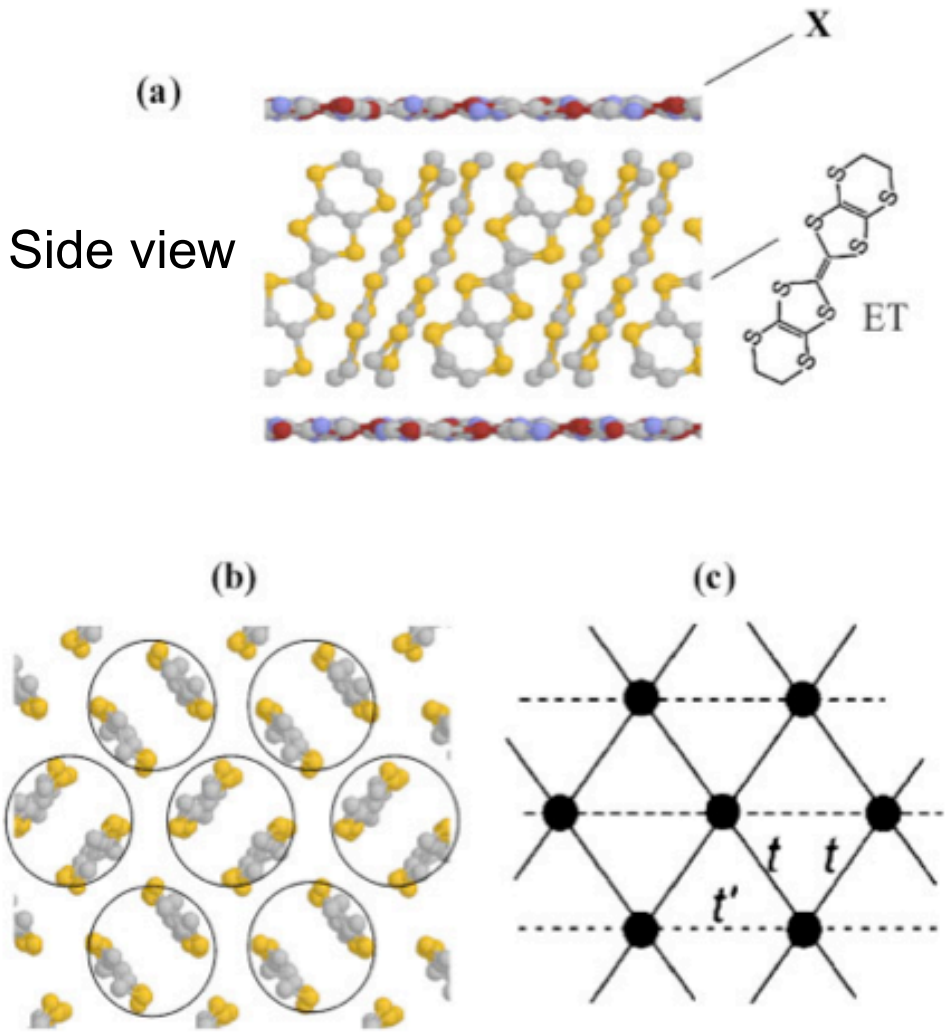
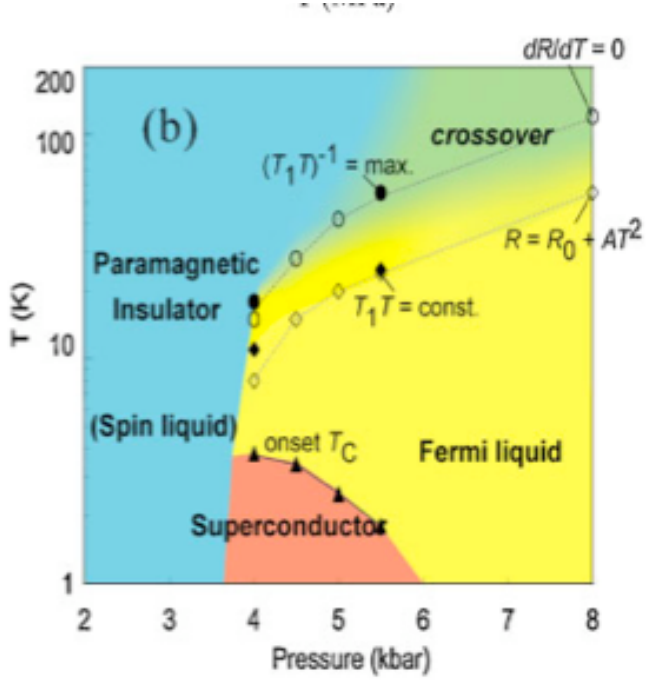


Fig. 1. (Color online) Structure of κ-$(ET)_2X$: (a) side view and (b) top view of the layer and (c) isosceles triangular lattice modelling in-plane structure with two types of transfer integral.



Kanoda 2006