Rashba vs Kohn-Luttinger: evolution of p-wave superconductivity in magnetized two-dimensional Fermi gas subject to spin-orbit interaction

Oleg Starykh, University of Utah

with Dima Pesin, Ethan Lake, Caleb Webb

PHYSICAL REVIEW B **93**, 214516 (2016)



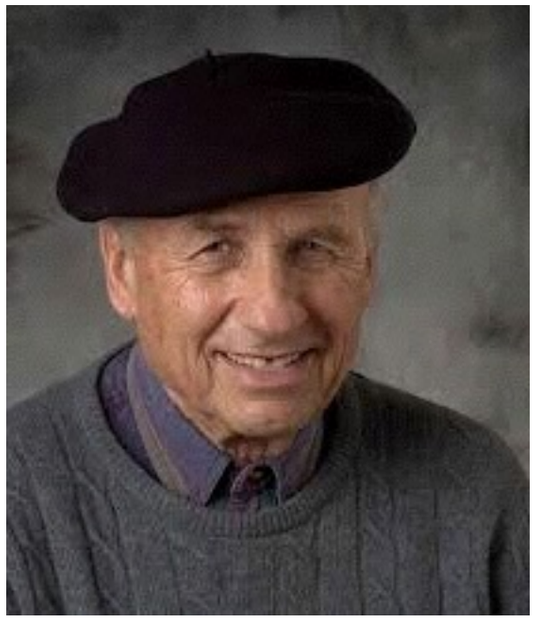
Nordita,
Multi-component and strongly-correlated superconductors,
July 15, 2016



E.I. Rashba



W. Kohn



J. Luttinger



---

under



Ethan Lake



Caleb Webb

grads !

Condensed matter physics in 21 century: the age of **spin-orbit**

✓ spintronics

✓ topological insulators, Majorana fermions

✓ Kitaev's non-abelian spin liquid/toric code

- The key issue: spin-orbit + e-e interaction

Outline

- Spin-orbit + interaction: surprises in 1d
- Kohn-Luttinger (KL) mechanism in 2d
  - magnetized gas
- KL in the presence of spin-orbit

Toy d=1 problem $\vec{B} \perp \vec{SO}$

Important: fully broken spin symmetry

$$H_{B+SO} = \frac{p_y^2}{2m} + \alpha \hat{\sigma}_x p_y - \frac{1}{2} g \mu_B B \hat{\sigma}_z$$

B=0: spin-orbit can be gauged away

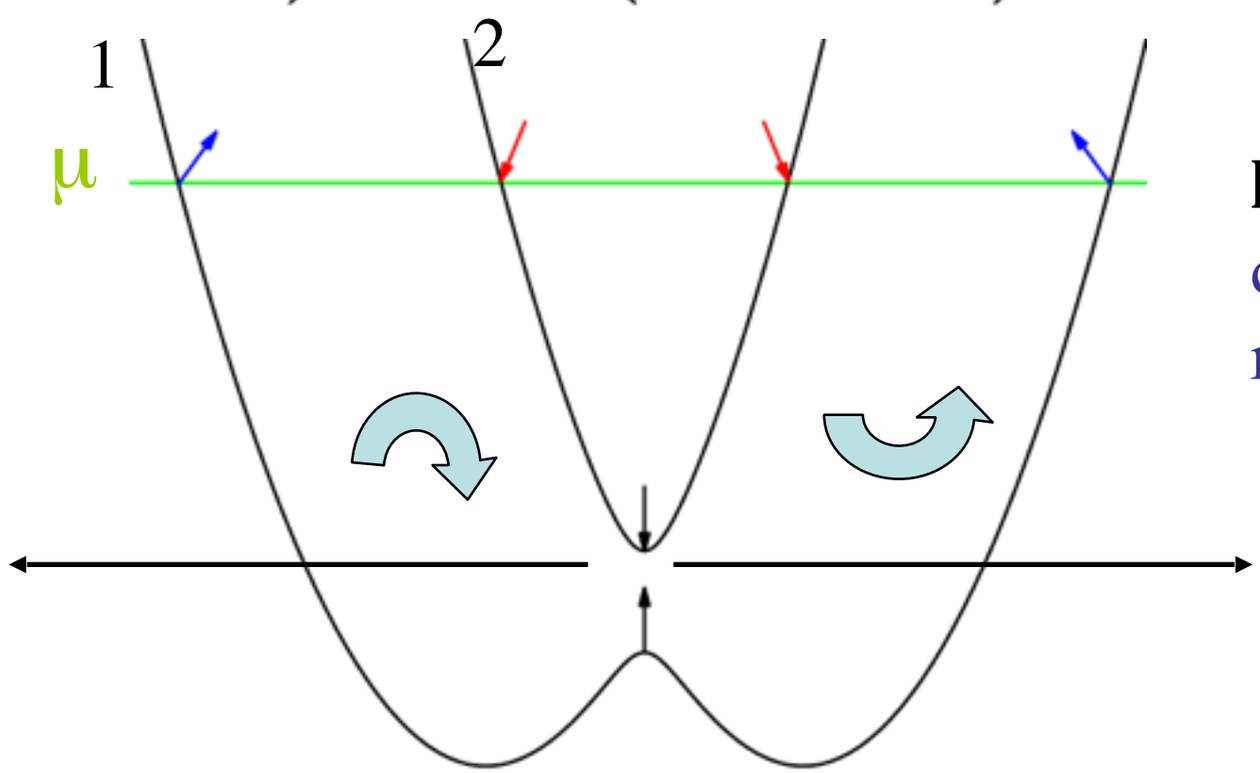
- Eigenvalues $$E_\pm = \frac{p^2}{2m} \pm \sqrt{(\alpha p)^2 + (g\mu B/2)^2}$$

- Eigenstates $$\Psi_\pm = e^{iky}\, \chi_\pm(k)$$

χ_+ and χ_- : orthogonal at the same **k** but not at the same **energy**

spinors $$\chi_+ = \begin{pmatrix} \sin \delta_k/2 \\ \cos \delta_k/2 \end{pmatrix} \quad \chi_- = \begin{pmatrix} \cos \delta_k/2 \\ -\sin \delta_k/2 \end{pmatrix} \qquad \delta_k = \arctan\left[\frac{2\alpha k}{g\mu_B B}\right]$$

k < 0:
clock-wise rotation of spins



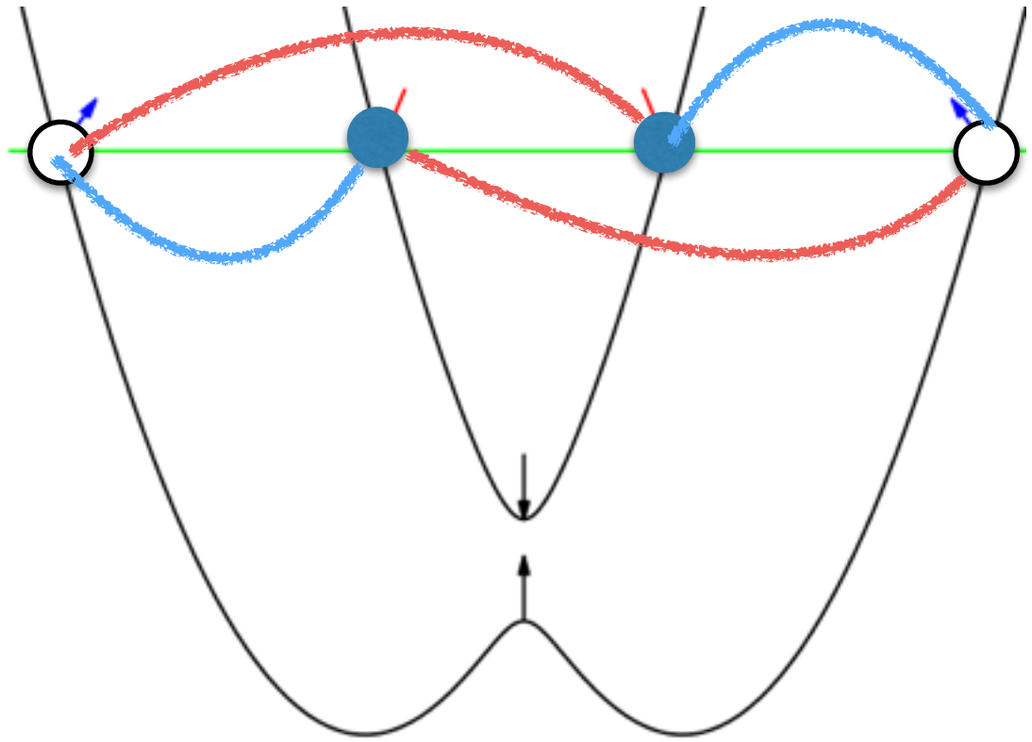
k > 0:
counterclock-wise rotation of spins

E-e interaction adds a new process:

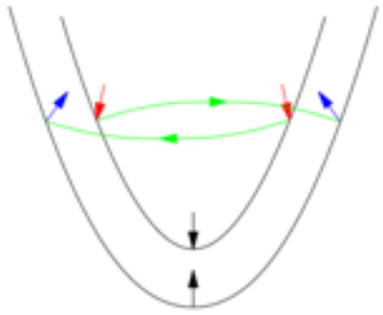
Cooper scattering (inter-band Josephson coupling)

$$R_{1\downarrow}^{\dagger} L_{1\downarrow}^{\dagger} R_{2\uparrow} L_{2\uparrow} + \text{h.c.}$$

- **Cooper channel**: spin non-conserving inter-subband pair tunneling possible due to **Spin-Orbit** only



SDW instability



- Easy limit: $E_F >> g\mu B >> \alpha k_F$

Free charge: $H_c = \frac{v_c}{2K_c}(\partial_x \phi_c)^2 + \frac{1}{2} v_c K_c (\partial_x \theta_c)^2$ **$K_c < 1$**

Interacting spin: $H_s = \frac{v_s}{2K_s}(\partial_x \phi_s)^2 + \frac{1}{2} v_s K_s (\partial_x \theta_s)^2$ **+ Cooper process**

$$H_{\text{Cooper}} = U(2k_F)\left(\frac{\alpha k_F}{g\mu B}\right)^2 \cos[\sqrt{8\pi}\theta_s]$$ **$K_s > 1$** relevant!

- Strong-coupling limit: minimal energy @ $\sqrt{2\pi}\theta_s = (2M+1) \times \frac{\pi}{2}$

Thus θ_s is frozen, hence ϕ_s fluctuates wildly - remember $[\phi,\theta]=i\delta(x-y)$.

- **$2k_F$** component of spin operators:

$$S^z_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c]\sin[\sqrt{2\pi}\phi_s] \to 0$$
$$S^y_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c]\cos[\sqrt{2\pi}\theta_s] \to 0$$

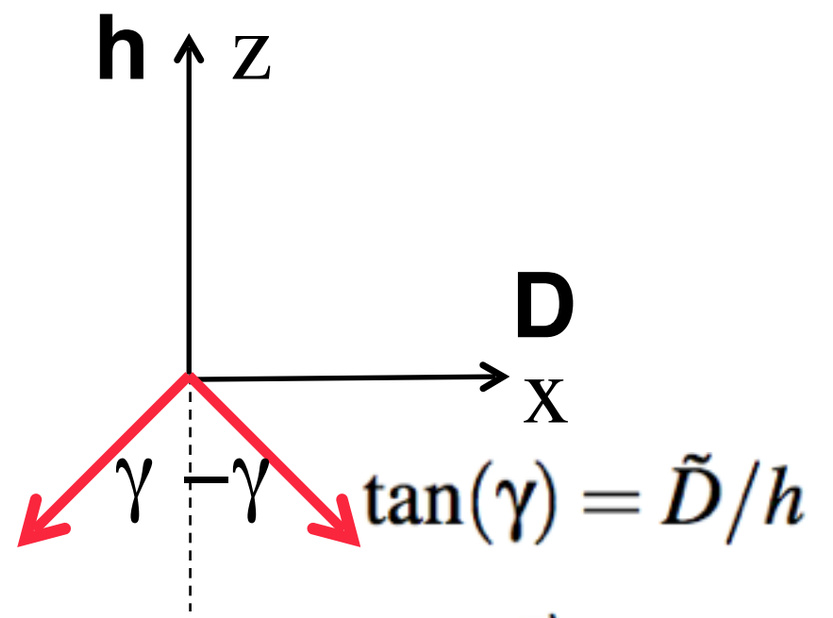
but $$S^x_{2k_F} \sim \cos[\sqrt{2\pi}\phi_c]\sin[\sqrt{2\pi}\theta_s] \to \cos[\sqrt{2\pi}\phi_c]$$

Ising order in the spin sector!

Spin chain with uniform DM term

$$\sum_i D\hat{x}\cdot\vec{S}_j\times\vec{S}_{j+1}\rightarrow\tilde{D}\int_x(J_R^x-J_L^x)$$

$$-h\sum_j S_j^z\rightarrow -h\int_x(J_R^z+J_L^z)$$



•Rotate right (left) current by γ (–γ)

$$R(\gamma):\vec{J}_R\rightarrow M_R^z \qquad R(-\gamma):\vec{J}_L\rightarrow M_L^z$$

$$\tilde{D}(J_R^x-J_L^x)-h(J_R^z+J_L^z)\rightarrow-\sqrt{\tilde{D}^2+h^2}(M_R^z+M_L^z)$$

•Backscattering is modified

$$H_{\text{bs}}=-g\vec{J}_R\cdot\vec{J}_L\rightarrow$$

$$=-gM_R^yM_L^y-g\cos[2\gamma](M_R^xM_L^x+M_R^zM_L^z)-g\sin[2\gamma](M_R^xM_L^z-M_R^zM_L^x)$$

- Magnetic field can now be absorbed

$$-\sqrt{\tilde{D}^2+h^2}(M_R^z+M_L^z)\rightarrow -t\partial_x\varphi$$

- Transverse to field (t) components oscillate

$$M_R^{\pm}\rightarrow M_R^{\pm}e^{\mp itx}\,;\; M_L^{\pm}\rightarrow M_L^{\pm}e^{\pm itx}$$

So that $\quad M_R^+M_L^-\rightarrow M_R^+M_L^-e^{i2tx}\,;\; M_R^+M_L^+\rightarrow M_R^+M_L^+$

- The final Hamiltonian

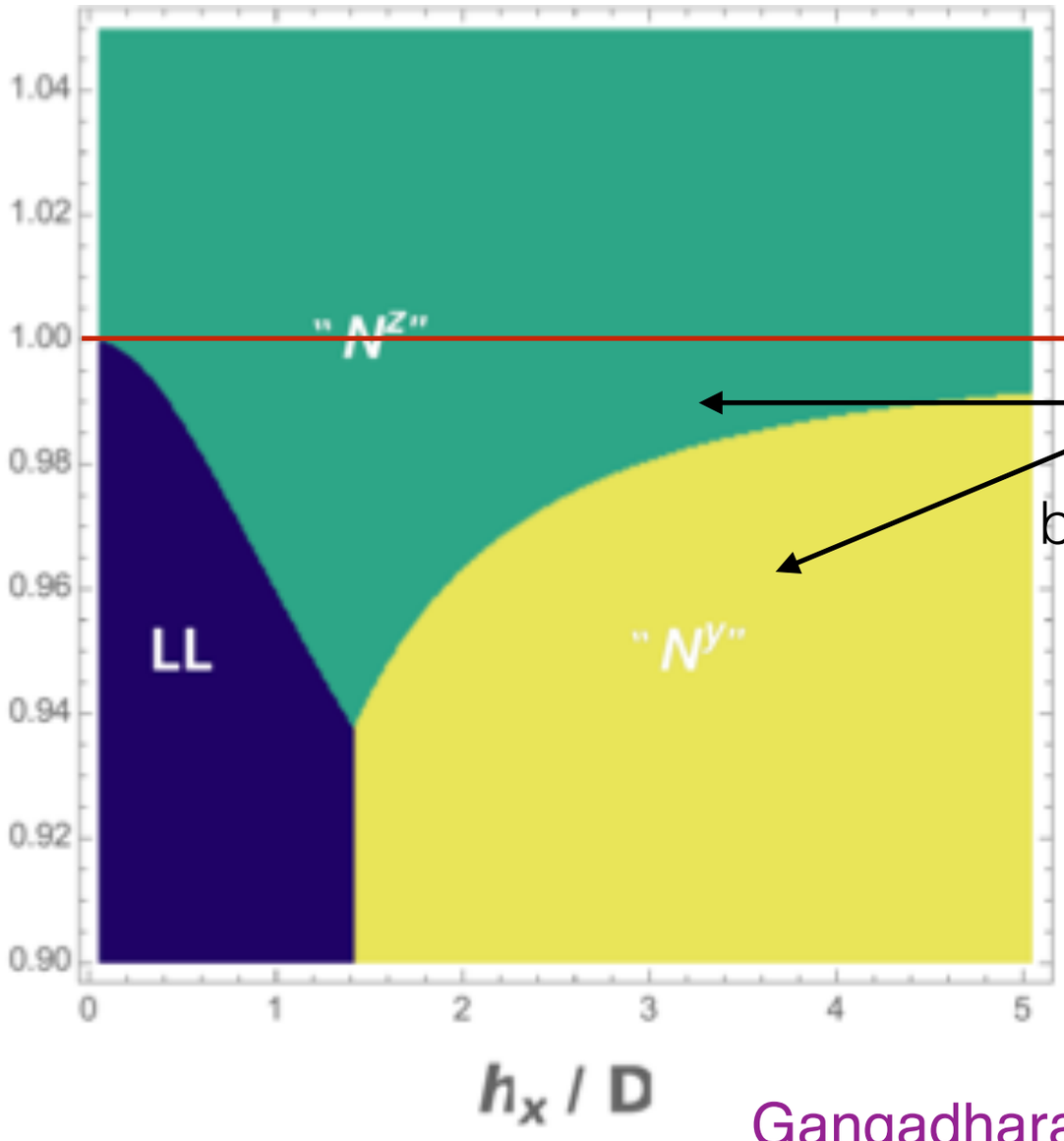
$$H_{\rm bs}=-gM_R^yM_L^y-g\cos[2\gamma](M_R^xM_L^x+M_R^zM_L^z)-g\sin[2\gamma](M_R^xM_L^z-M_R^zM_L^x)$$

$$\rightarrow -g\cos[2\gamma]\underbrace{M_R^zM_L^z}+g\sin^2[\gamma](\underbrace{M_R^+M_L^++M_R^-M_L^-})$$

$$(\partial_x\varphi)^2-(\partial_x\theta)^2 \qquad\qquad \cos[\sqrt{8\pi}\,\theta] \quad \textit{Cooper term}$$

Phase diagram of Heisenberg+uniform DM chain

$$\Delta_{\text{eff}} = \frac{J_z}{J} - \frac{D^2}{2J^2}$$



Lesson:
ordered phases below this line are stabilized by **spin-orbit**+**Zeeman field** + **e-e interaction**

Gangadharaiah, Sun, OS, PRB 2008
Garate, Affleck 2010
Povarov et al, PRL 2011
Karimi, Affleck 2011
Smirnov et al 2015
Wen Jin, OS unpublished

Outline

- Spin-orbit + interaction: surprises in 1d
- Kohn-Luttinger (KL) mechanism in 2d magnetized gas
- KL in the presence of spin-orbit

Kohn-Luttinger mechanism: Superconductivity from repulsion

NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn

University of California, San Diego, La Jolla, California

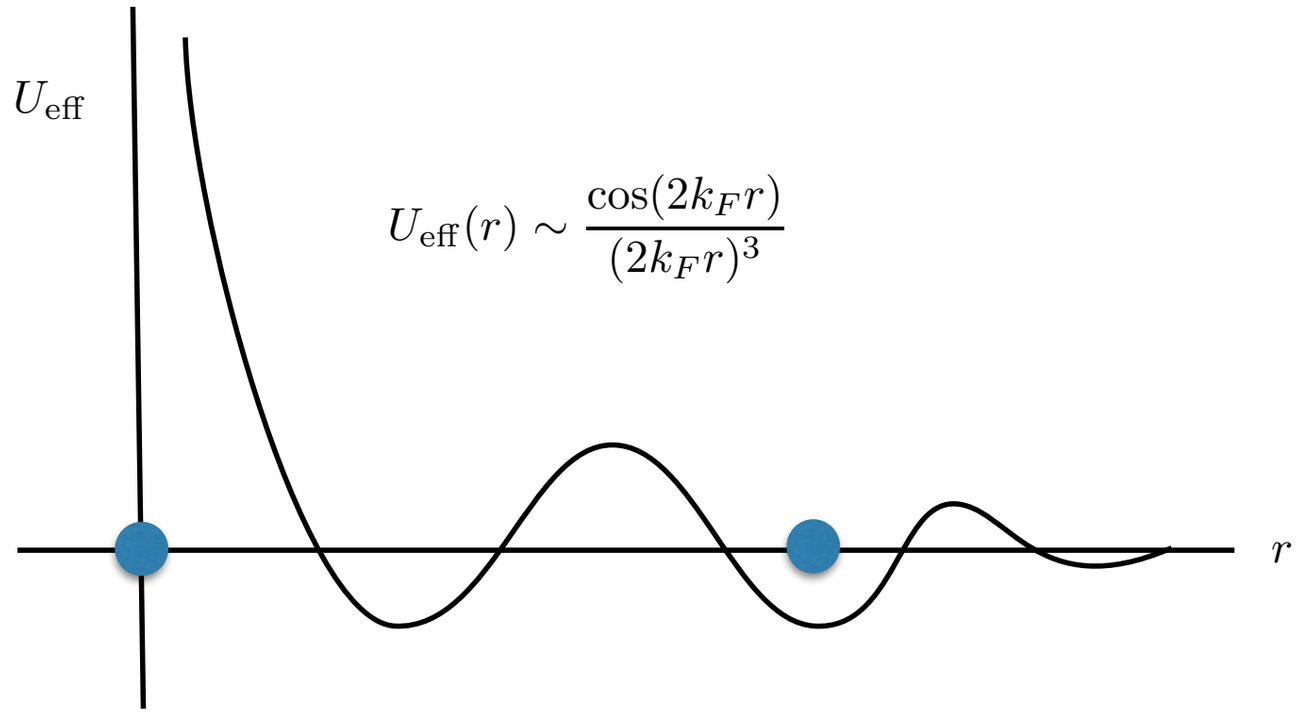
and

J. M. Luttinger

Columbia University, New York, New York

(Received 16 August 1965)

Attraction from Friedel oscillations



More recent history

PHYSICAL REVIEW B VOLUME 48, NUMBER 2 1 JULY 1993-II

Kohn-Luttinger effect and the instability of a two-dimensional repulsive Fermi liquid at $T=0$

Andrey V. Chubukov*

PHYSICAL REVIEW B **81**, 224505 (2010)

Superconductivity in the repulsive Hubbard model: An asymptotically exact weak-coupling solution

S. Raghu,[1] S. A. Kivelson,[1] and D. J. Scalapino[1,2]

PHYSICAL REVIEW B **85**, 024516 (2012)

Effects of longer-range interactions on unconventional superconductivity

S. Raghu,[1,2] E. Berg,[3] A. V. Chubukov,[4] and S. A. Kivelson[1]

PHYSICAL REVIEW B **83**, 094518 (2011)

Superconductivity from repulsive interactions in the two-dimensional electron gas

S. Raghu[1,2] and S. A. Kivelson[1]

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

LETTER

Emergent BCS regime of the two-dimensional fermionic Hubbard model: Ground-state phase diagram

Youjin Deng[1,2], Evgeny Kozik[3,4], Nikolay V. Prokof'ev[2,5] and Boris V. Svistunov[1,2,5]

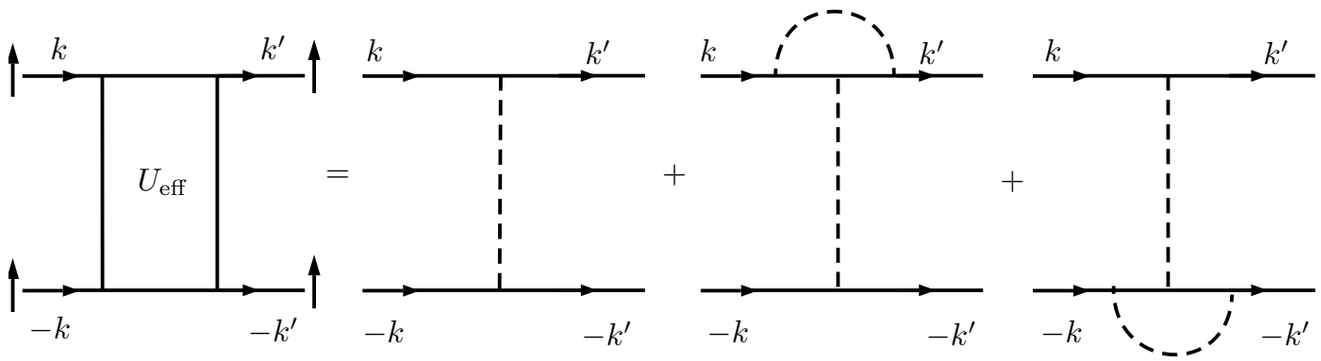
Published 15 June 2015 • Copyright © EPLA, 2015

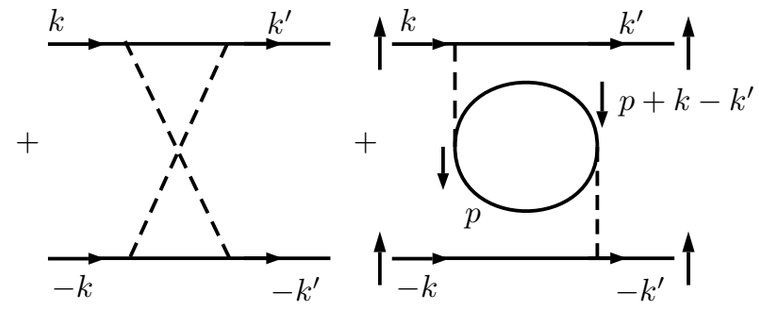
Hamiltonian

$$H_0 = \sum_{\mathbf{k}\sigma\sigma'} E_{\sigma\sigma'}(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma'} \qquad \text{(free part)}$$

$$H_I = U \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1\uparrow} c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{k}_3\downarrow} c_{\mathbf{k}_4\uparrow} \qquad \text{(interaction part)}$$

Diagrams to U^2 order:





Hamiltonian

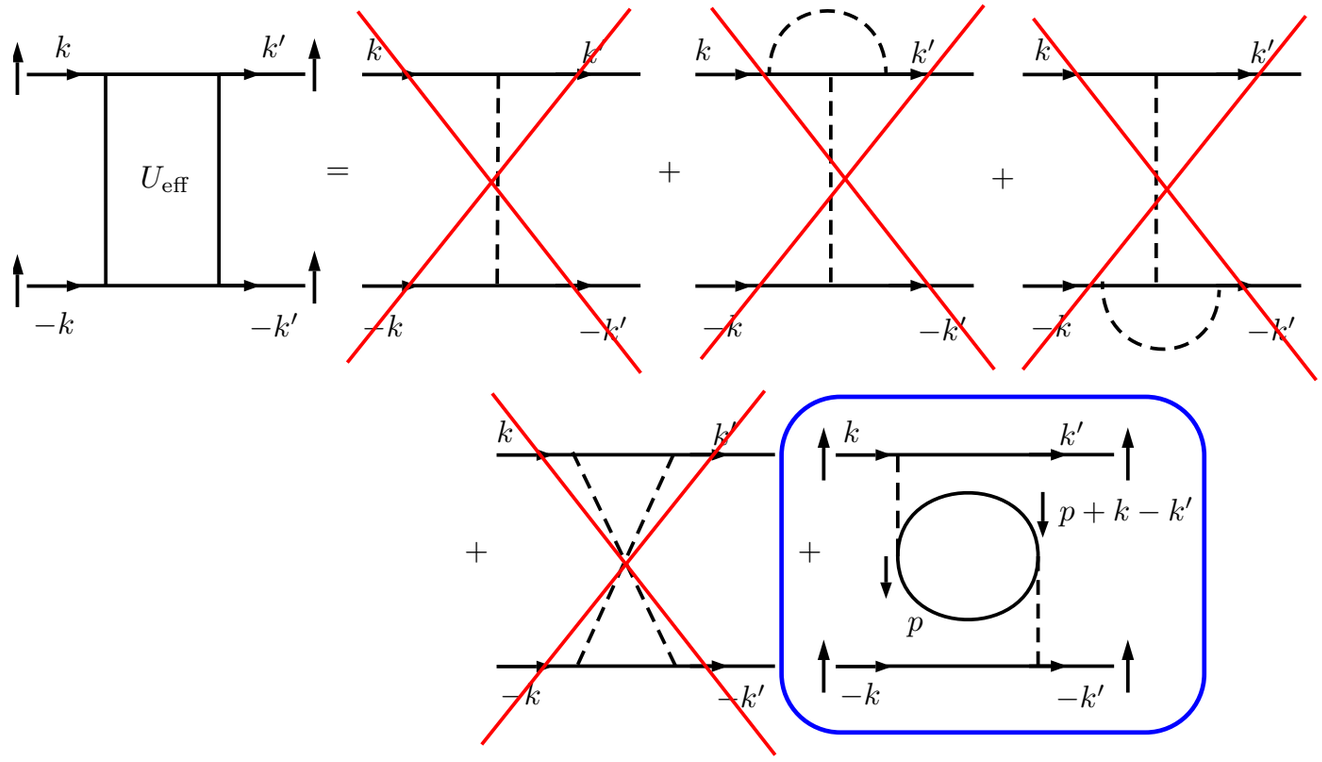
$$H_0 = \sum_{\mathbf{k}\sigma\sigma'} E_{\sigma\sigma'}(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma'} \qquad \text{(free part)}$$

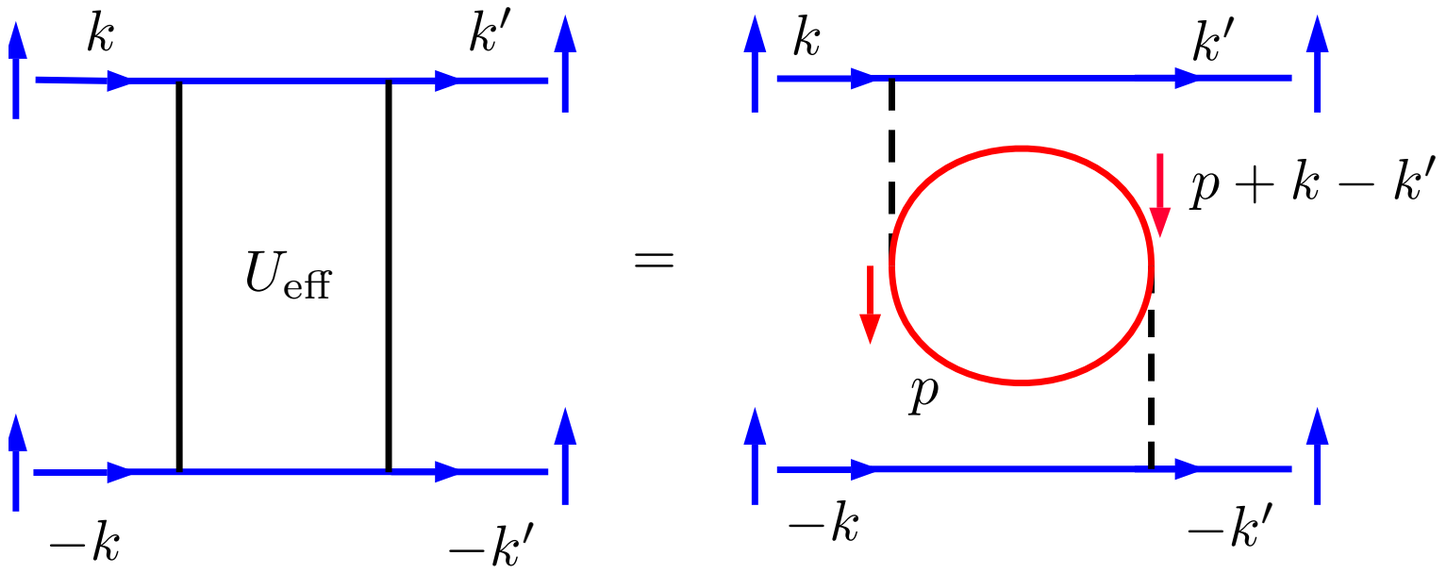
$$H_I = U \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1\uparrow} c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{k}_3\downarrow} c_{\mathbf{k}_4\uparrow} \qquad \text{(interaction part)}$$

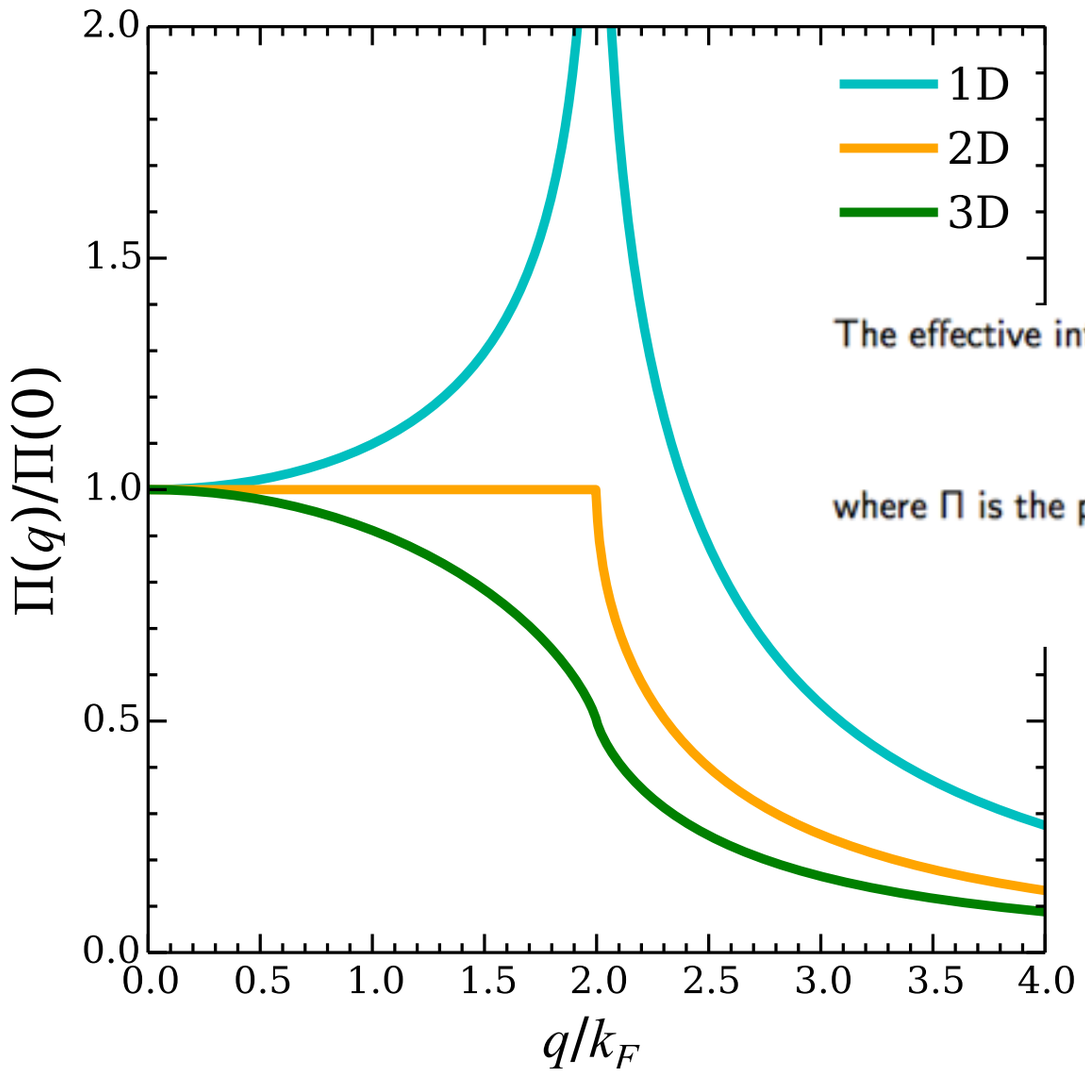
Diagrams to U^2 order:

$$H_I = U n_\uparrow n_\downarrow$$

triplet channel







The effective interaction is, for $\mathbf{q} = \mathbf{k} - \mathbf{k}'$:

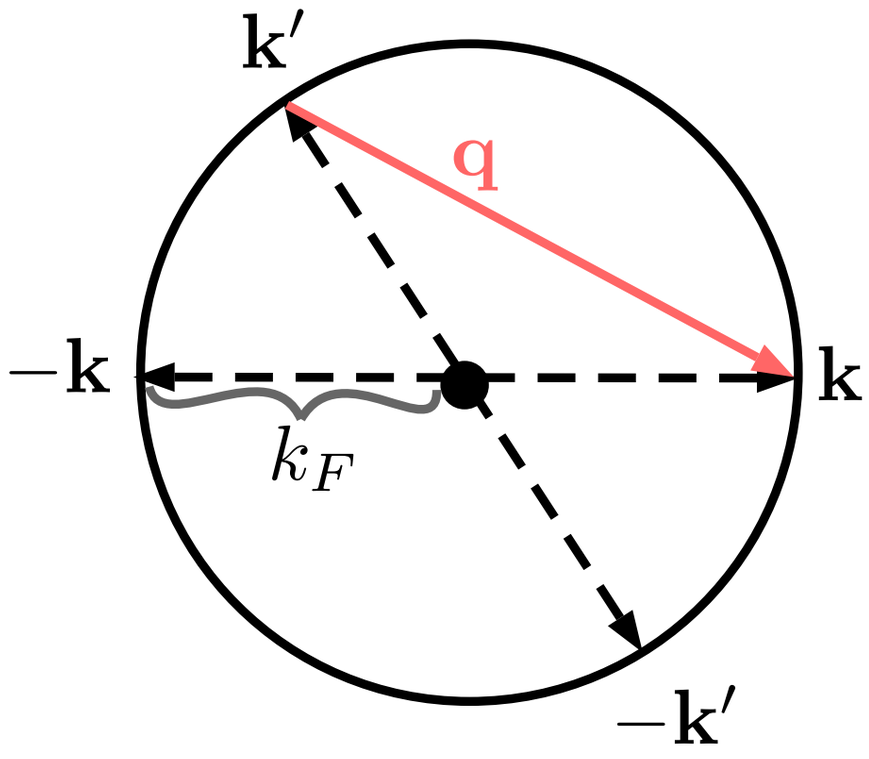
$$U_{\text{eff}}(q) = U^2 \Pi(q),$$

where Π is the polarization operator. In 3D, this has a singular part

$$\Pi_{\text{sing}}(q) \sim (2k_F - q) \ln |2k_F - q|$$

Two dimensions: one-sided singularity

$$U_{2D}^{\text{eff}} \sim U^2 \rho_0 \, \text{Re} \sqrt{1 - \left(\frac{2k_\text{f}}{q}\right)^2}$$

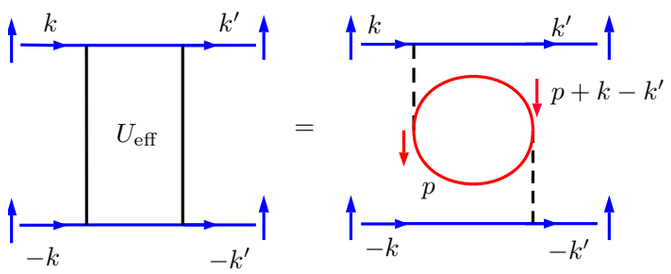


$q \leq 2k_\text{f}$ always

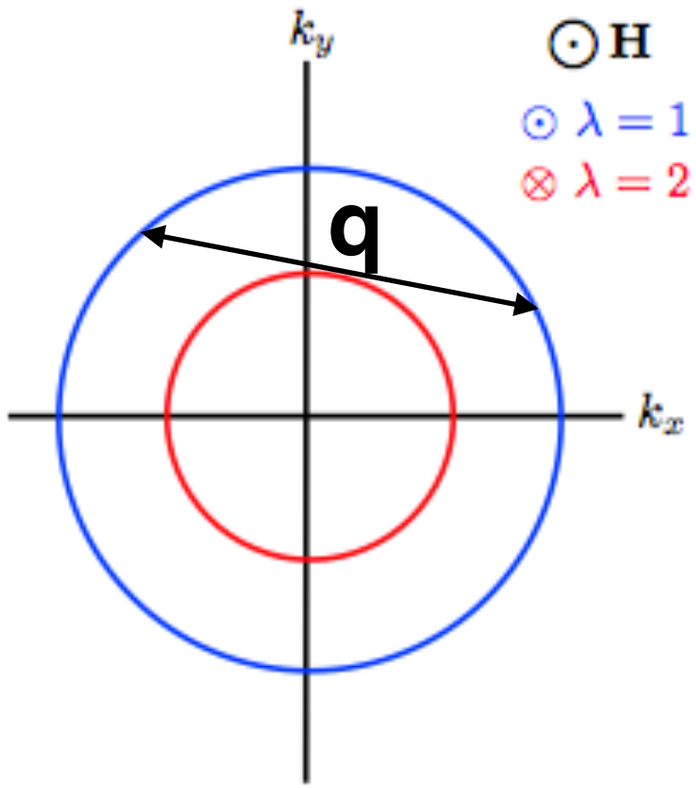
$U_{2D}^{\text{eff}} = 0$

Zeeman field produces superconductivity

Partially polarizing the electron spins solves this problem! (Kagan 91, Raghu & Kivelson 10)



$$U^{\text{sing}}_{2D,\lambda}(q) \sim U^2 \text{Re}\sqrt{q^2 - (2k_{F,-\lambda})^2}$$

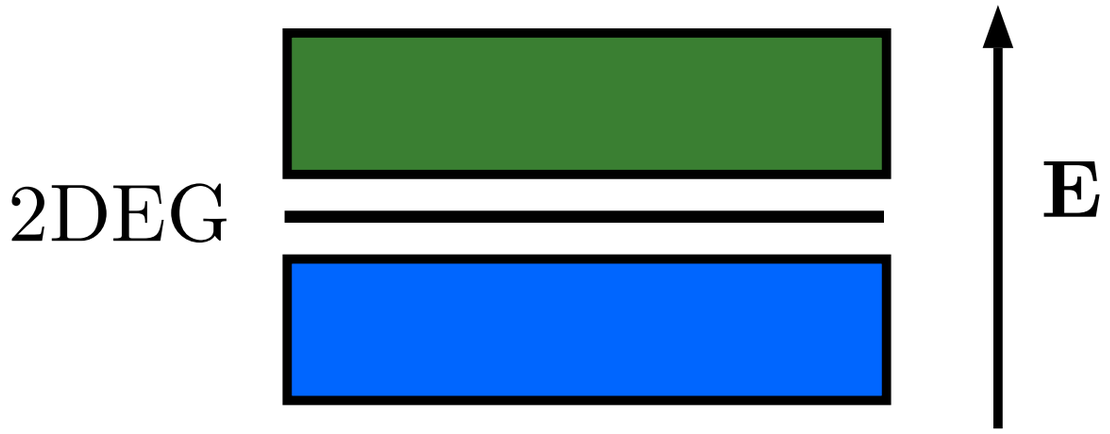


- Spin-up electrons experience effective attraction mediated by spin-down electrons $q \geq 2k_{\text{f},\downarrow}$
- Spin-down electrons remain in the normal state $q \leq 2k_{\text{f},\uparrow}$
- “One-sided” superconductivity

Outline

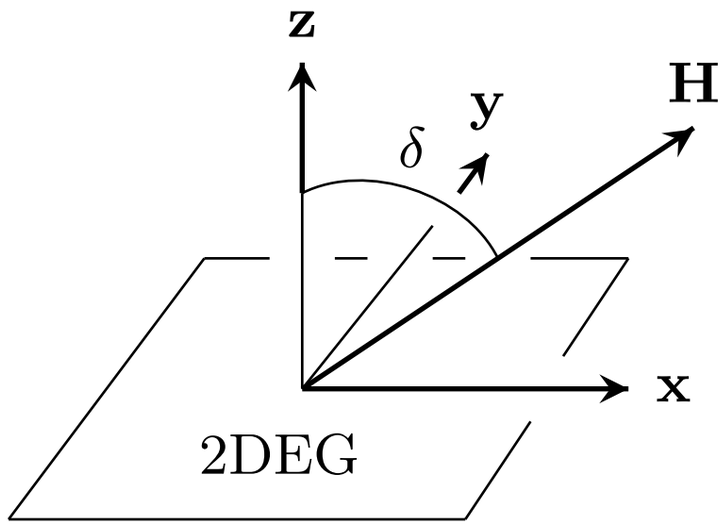
- Spin-orbit + interaction: surprises in 1d
- Kohn-Luttinger (KL) mechanism in 2d magnetized gas
- KL in the presence of spin-orbit

Basic solid state: spin-orbit interaction is unavoidable



$$H_{SOC} = \alpha_R(k_x\sigma^y - k_y\sigma^x)$$

General set-up: arbitrary **H** but Zeeman energy >> Spin-orbit energy



quantization axis along the magnetic field

LETTERS
PUBLISHED ONLINE: 4 SEPTEMBER 2011 | DOI: 10.1038/NPHYS2080
nature physics

Coexistence of magnetic order and two-dimensional superconductivity at $LaAlO_3/SrTiO_3$ interfaces

Lu Li[1], C. Richter[2], J. Mannhart[3] and R. C. Ashoori[1]*

A two-dimensional electronic system forms at the interface between the band insulators[1,2] $LaAlO_3$ and $SrTiO_3$. Samples fabricated until now have been found to be either magnetic or superconducting, depending on growth conditions[3,4]. Combining high-resolution magnetic torque magnetometry and transport measurements, we report here magnetization measurements providing direct evidence of magnetic ordering of the two-dimensional electron liquid at the interface. The magnetic ordering exists from well below the superconducting transition to up to 200 K, and is characterized by an in-plane magnetic moment. Surprisingly, despite the presence of this magnetic ordering, the interface superconducts below 120 mK. This is unusual because conventional superconductivity rarely exists in magnetically ordered metals[5,6]. Our results suggest that there is either phase separation or coexistence between magnetic and superconducting states. The coexistence scenario would point to an unconventional superconducting phase as the ground state.

Superconducting Interfaces Between Insulating Oxides

N. Reyren[1], S. Thiel[2], A. D. Caviglia[1], L. Fitting Kourkoutis[3], G. Hammerl[2], C. Richter[2], C. W. Schneider[2], T. Kopp[2], A.-S. Rüetschi[1], D. Jaccard[1], M. Gabay[4], D. A. Muller[3], J.-M. Triscone[1], J. Mannhart[2,*]

+ Author Affiliations

* To whom correspondence should be addressed. E-mail: jochen.mannhart@physik.uni-augsburg.de

Science 31 Aug 2007:
Vol. 317, Issue 5842, pp. 1196-1199
DOI: 10.1126/science.1146006

Superconducting and Ferromagnetic Phases in $SrTiO_3$ / $LaAlO_3$ Oxide Interface Structures: Possibility of Finite Momentum Pairing

Karen Michaeli, Andrew C. Potter, and Patrick A. Lee
Phys. Rev. Lett. **108**, 117003 – Published 12 March 2012

Expectations

PHYSICAL REVIEW B **83**, 094518 (2011)

Superconductivity from repulsive interactions in the two-dimensional electron gas

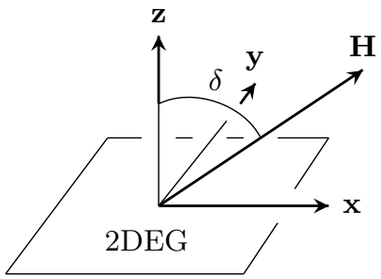
S. Raghu[1,2] and S. A. Kivelson[1]

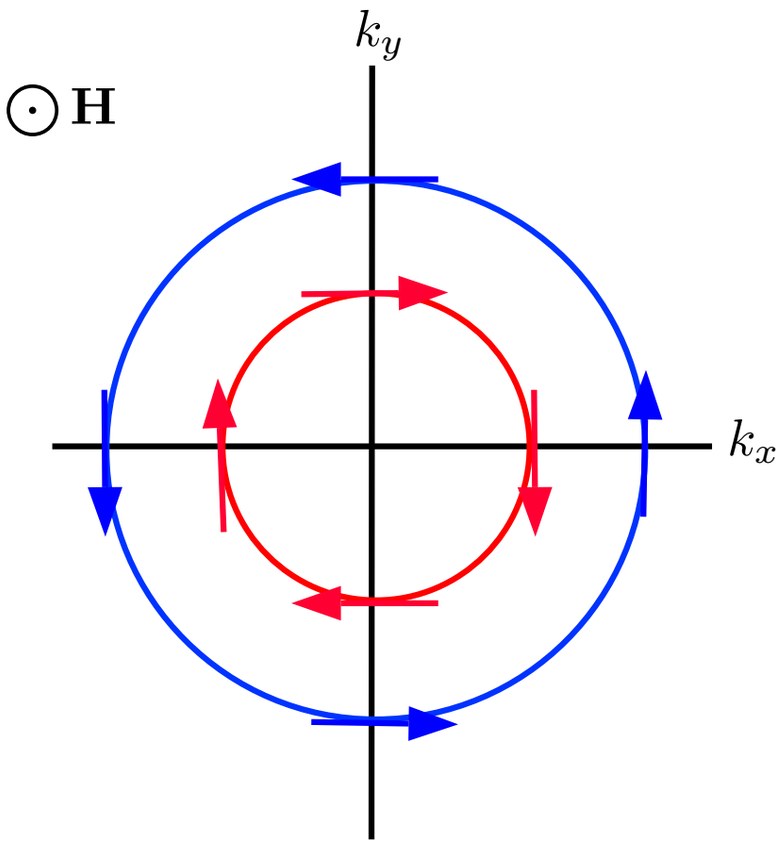
> “ Weak but nonvanishing spin-orbit coupling will generically change this situation, since superconductivity will be induced in the minority fluid by the proximity effect as soon as the majority fluid becomes superconducting. In 2D, this induced superconductivity will generally track the fundamental order parameter. ”

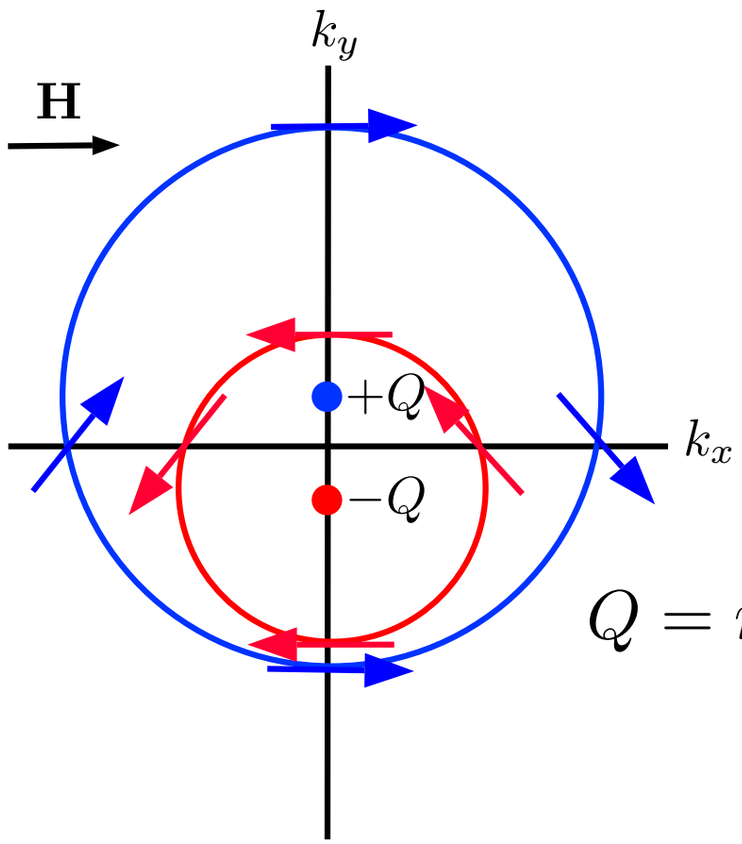
· Nope !

1. Switch to band basis (spin is not conserved)

$$E(\mathbf{k}) = \frac{k^2}{2m}\sigma^0 - \vec{H}\cdot\vec{\sigma} + \alpha_R(k_x\sigma^y - k_y\sigma^x)$$







$$Q = m\alpha_R \sin\delta$$

$$\varepsilon_\lambda(\mathbf{k}) = \frac{k^2}{2m} - \zeta_\lambda\sqrt{\alpha_R^2k^2 + h^2 + 2\alpha_R hk\sin\phi_\mathbf{k}\sin\delta}$$

chirality $\zeta = \pm 1$

2. Project the interaction into band basis

$$\text{keep } U^2, U\alpha_R^2$$
$$\text{drop } U^3, U^2\alpha_R$$

$$H_{\text{eff}} = H_0 + \sum_{\lambda\mu}\sum_{\mathbf{k},\mathbf{k}'} g_{\lambda\mu}(\mathbf{k},\mathbf{k}')a^\dagger_{-\mathbf{k}',\lambda}a^\dagger_{\mathbf{k}',\lambda}a_{\mathbf{k},\mu}a_{-\mathbf{k},\mu},$$

where the full interaction matrix, $g_{\lambda\mu}(\mathbf{k},\mathbf{k}')$, is given by

$$g_{\lambda\mu}(\mathbf{k},\mathbf{k}') = \frac{U\alpha_R^2 k_{\mathrm{f},\lambda}k_{\mathrm{f},\mu}\zeta_\lambda\zeta_\mu}{4h^2}(\cos\phi_{\mathbf{k}} - i\zeta_\lambda\cos\delta\sin\phi_{\mathbf{k}})$$
$$\times(\cos\phi_{\mathbf{k}'} + i\zeta_\mu\cos\delta\sin\phi_{\mathbf{k}'}) + \frac{U^2\Pi_{-\lambda}(\mathbf{k}-\mathbf{k}')}{2}\delta_{\lambda\mu}$$

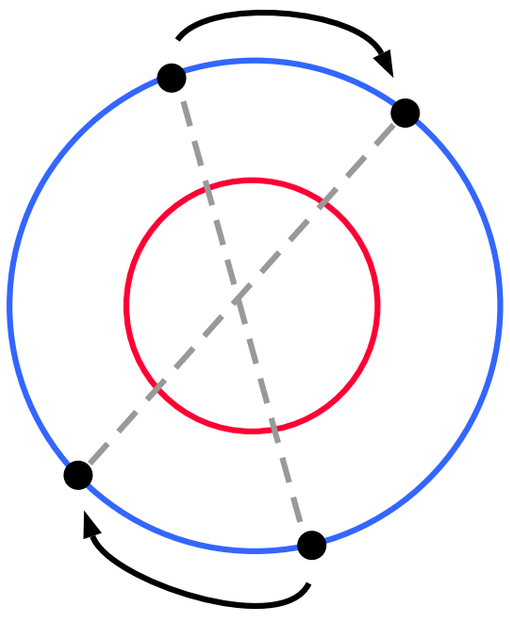
Kohn Luttinger term

where $\zeta_\lambda = \pm 1$ is the helicity of the spins on each band.

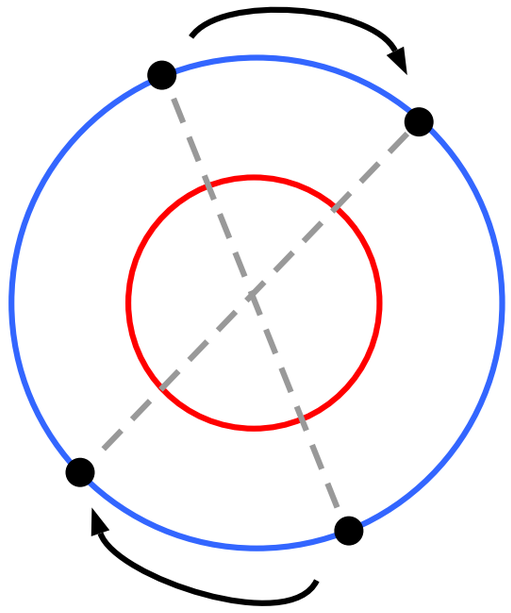
- $\lambda = \mu$: Intra-band interaction, which is *repulsive*.
- $\lambda \neq \mu$: Inter-band interaction (momentum space Josephson coupling), which is *attractive*.

2. Project the interaction into band basis (via Schrieffer-Wolff transformation)

$\Pi < 0$
KL process
$\mathcal{O}(U^2)$

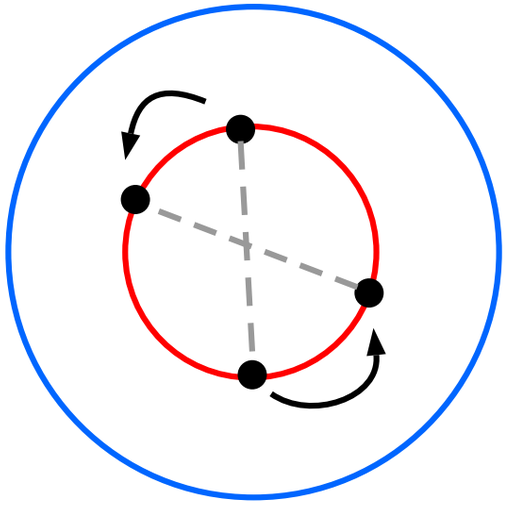


$R_1 > 0$
Repulsion
$\mathcal{O}(U\alpha_R^2)$

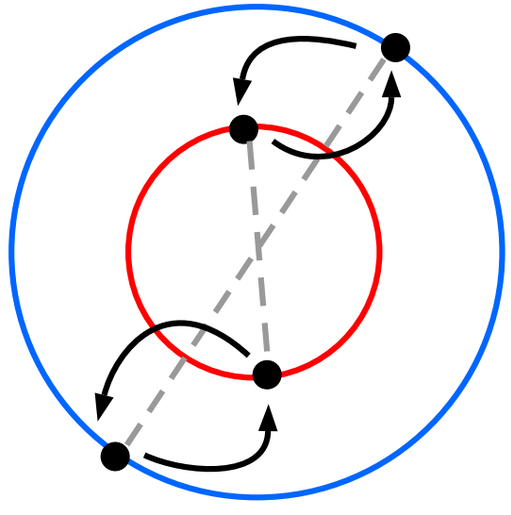


$\lambda = 1$
$\lambda = 2$

$R_2 > 0$
Repuslion
$\mathcal{O}(U\alpha_R^2)$



$J < 0$
Josephson exchange
$\mathcal{O}(U\alpha_R^2)$



Mean-field

order parameter $\Delta_\lambda(\mathbf{k}) = \sum_{\mu\mathbf{k}'} g_{\lambda\mu}(\mathbf{k},\mathbf{k}')\langle a_{-\mathbf{k}'\mu} a_{\mathbf{k}'\mu}\rangle$

dispersion $E_{\mathbf{k}\lambda} = \sqrt{(\varepsilon_{\mathbf{k}\lambda} - \mu)^2 + |\Delta_\lambda(\mathbf{k})|^2}$

Two solutions of self-consistent equations

"Coupled" phase

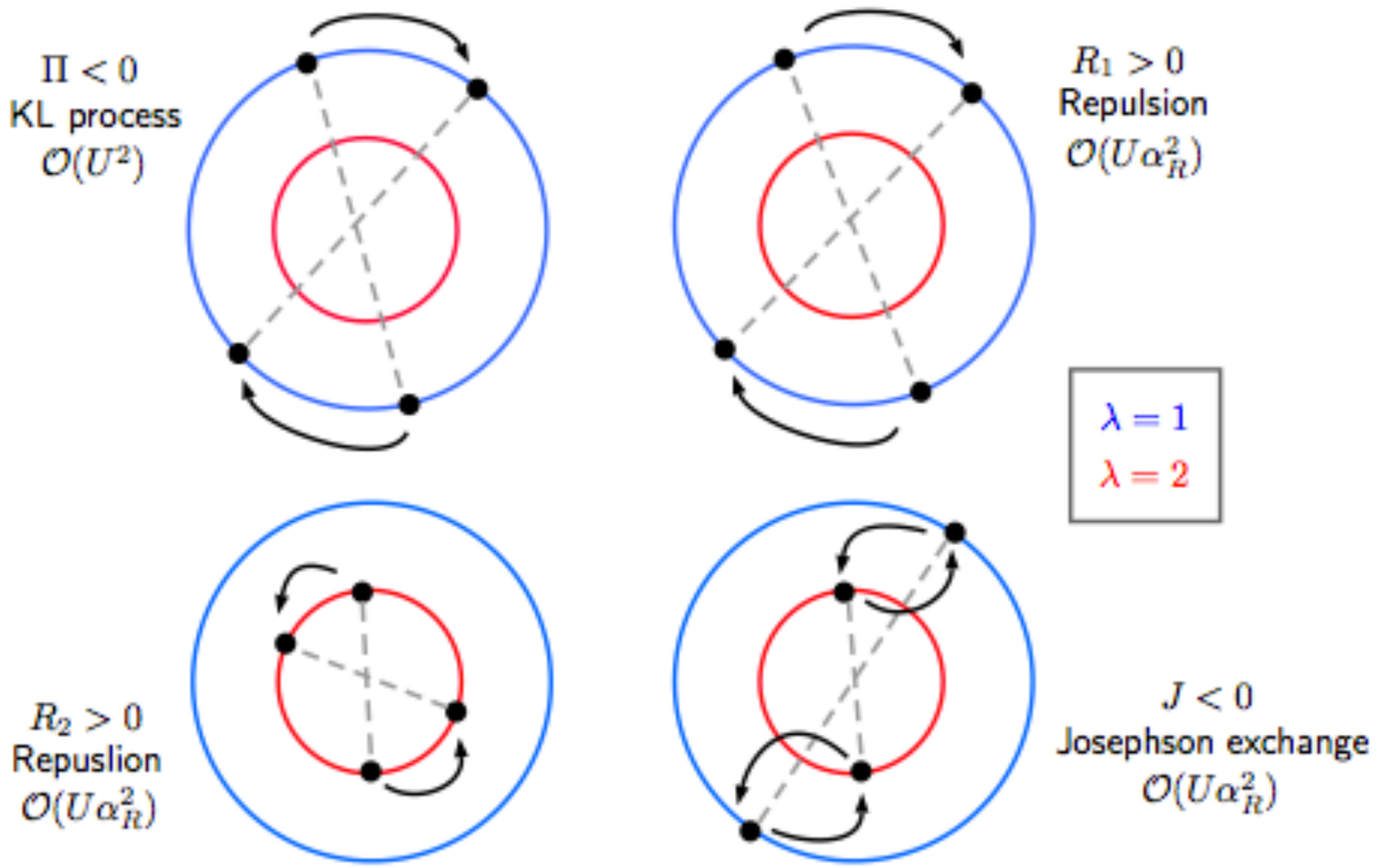
$\Delta_1(\mathbf{k}) = \Delta_1(\cos\phi_\mathbf{k} - i\cos\delta\sin\phi_\mathbf{k})$,
$\Delta_2(\mathbf{k}) = \Delta_2(\cos\phi_\mathbf{k} + i\cos\delta\sin\phi_\mathbf{k})$

"Decoupled" phase

$\Delta_1(\mathbf{k}) = \Delta_1(\cos\delta\cos\phi_\mathbf{k} + i\sin\phi_\mathbf{k})$,
$\Delta_2(\mathbf{k}) = 0$

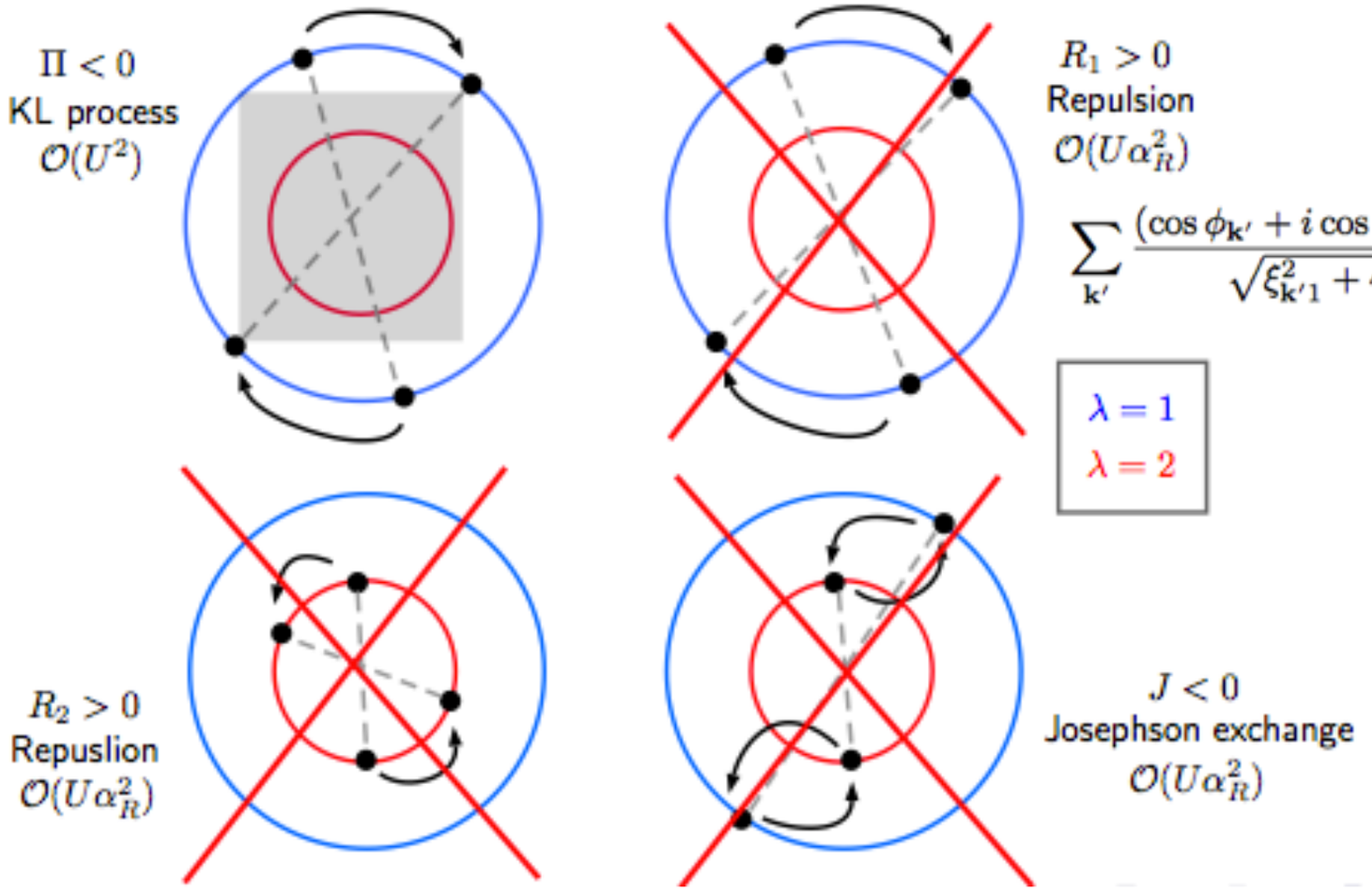
"Coupled" phase

$$\Delta_1(\mathbf{k}) = \Delta_1(\cos\phi_{\mathbf{k}} - i\cos\delta\sin\phi_{\mathbf{k}}),$$
$$\Delta_2(\mathbf{k}) = \Delta_2(\cos\phi_{\mathbf{k}} + i\cos\delta\sin\phi_{\mathbf{k}})$$



"Decoupled" phase

$$\Delta_1(\mathbf{k}) = \Delta_1(\cos\delta\cos\phi_{\mathbf{k}} + i\sin\phi_{\mathbf{k}}),$$
$$\Delta_2(\mathbf{k}) = 0$$



$$\sum_{\mathbf{k}'} \frac{(\cos\phi_{\mathbf{k}'} + i\cos\delta\sin\phi_{\mathbf{k}'})\Delta_1(\mathbf{k}')}{\sqrt{\xi_{\mathbf{k}'1}^2 + 4|\Delta_1(\mathbf{k}')|^2}} = 0.$$

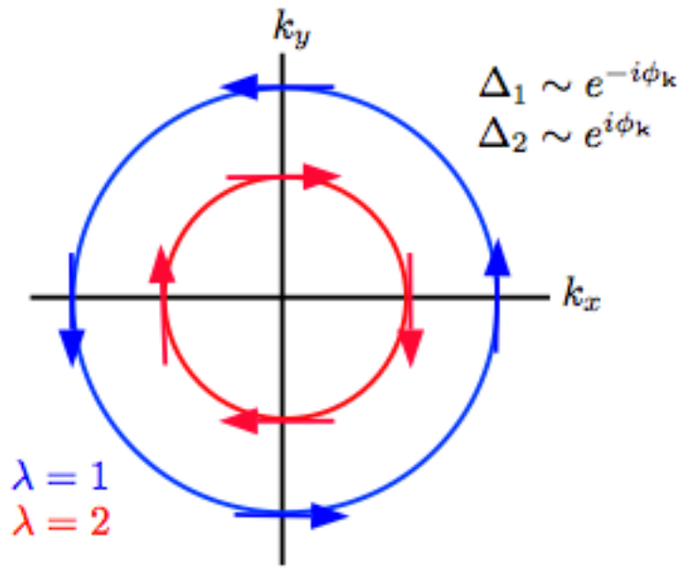
- "Decoupled" phase has lower energy
- "Coupled" phase suffers exponentially from intra-band repulsion $R_{1,2}$

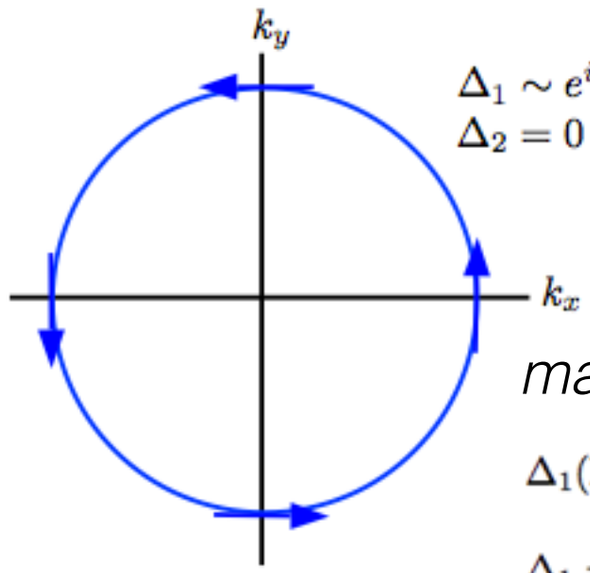
Pairing symmetry

"coupled"

chirality of the order parameter is opposite to that of the band

For out-of-plane fields ($\delta = 0$):





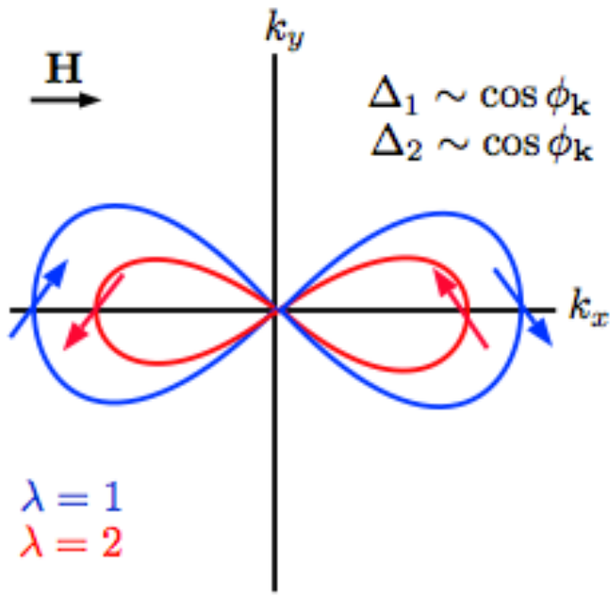
"decoupled"

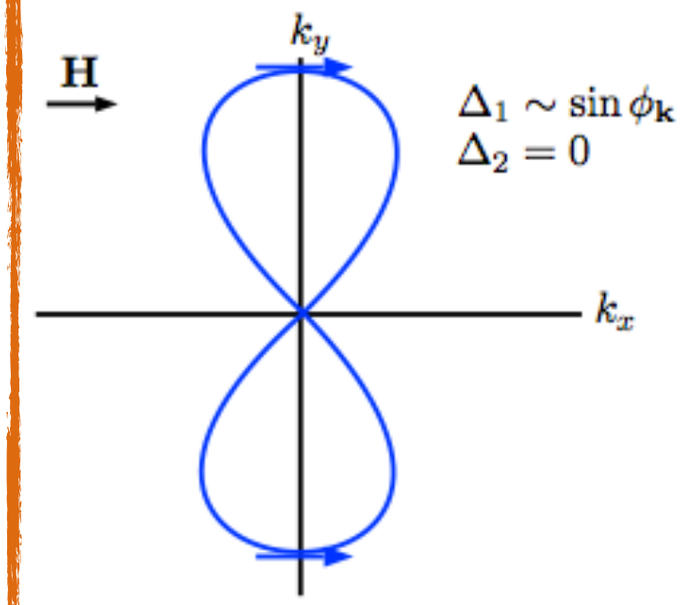
chirality of the order parameter matches that of the band

$$\Delta_1(\mathbf{k}) = \Delta_1\left(\cos\delta\cos\phi_\mathbf{k} + i\sin\phi_\mathbf{k}\right),$$

$$\Delta_1 = \frac{2\omega_c e^{\frac{1-\cos\delta}{2(1+\cos\delta)}}}{1+\cos\delta}\exp[-2/(U^2\rho_1|\chi_2^{(1)}|)]$$

When the field is oriented in-plane ($\delta = \pi/2$), the gap develops nodes along the field direction:





Experimental probe: angle-sensitive specific heat

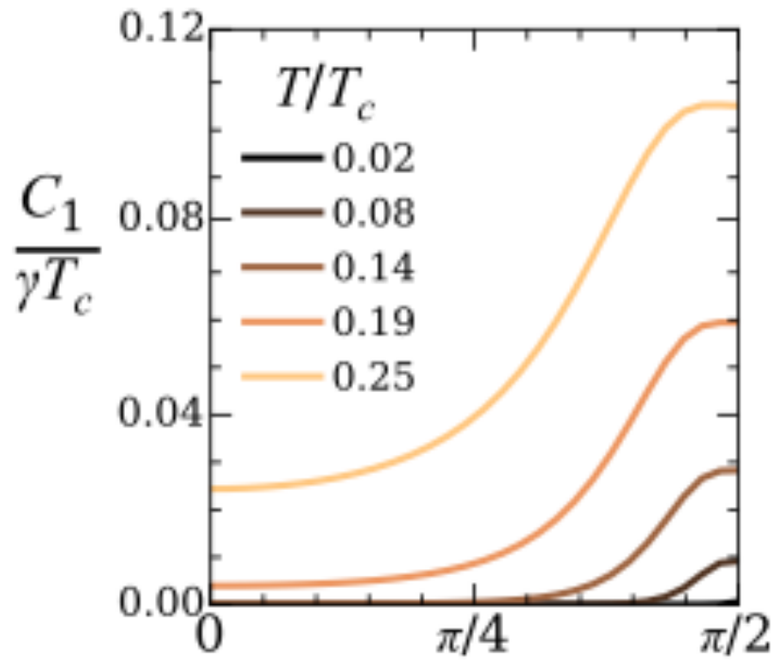
$$C(T) \sim \sum_{\lambda} \int d^2\mathbf{k} \frac{E_{\mathbf{k}\lambda}^2}{T^2[\cosh^2(E_{\mathbf{k}\lambda}/2T)]},$$

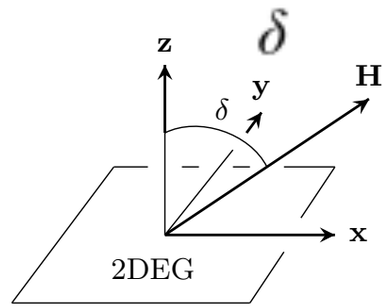
$$E_{\mathbf{k}\lambda} = \sqrt{\varepsilon_{\mathbf{k}\lambda}^2 + |\Delta_\lambda(\mathbf{k})|^2}$$

$$C(T)_{\mathbf{H}||\hat{\mathbf{z}}} \sim \frac{\rho_1 \Delta_1^{5/2}}{T^{3/2}} e^{-\Delta_1/T} + \rho_2 T,$$

$$C(T)_{\mathbf{H}\perp\hat{\mathbf{z}}} \sim \frac{\rho_1 T^2}{\Delta_1} + \rho_2 T.$$

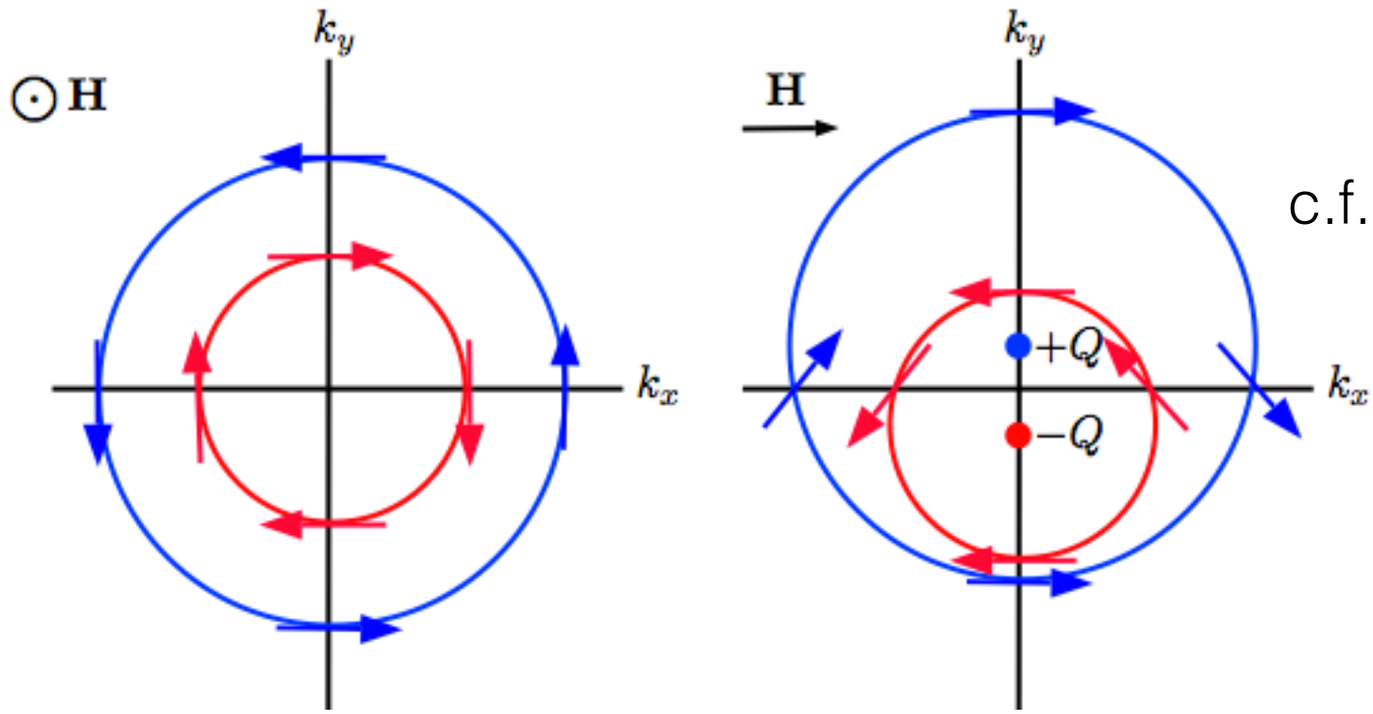
from the node of the order parameter





Unexpected bonus: finite-momentum pairing

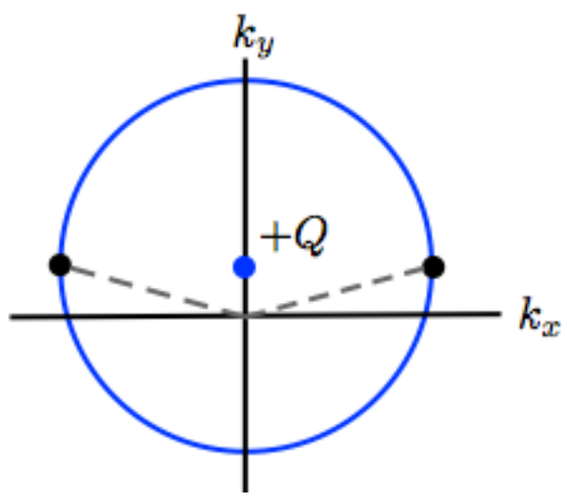
When the field has an in-plane component, the Fermi surfaces are displaced from their centers along directions orthogonal to **H**:



c.f. D. F. Agterberg and R. P. Kaur, 2007

Can be seen from 2-particle problem: $|\Psi\rangle = \sum_{\lambda \mathbf{k}} \Phi^{Q}_{\lambda}(\mathbf{k}) a^{\dagger}_{\mathbf{k}+\mathbf{Q},\lambda} a^{\dagger}_{-\mathbf{k}+\mathbf{Q},\lambda} |0_{\lambda}\rangle$

Turns out that $Q = m\alpha_R \sin\delta$ is energetically favorable.

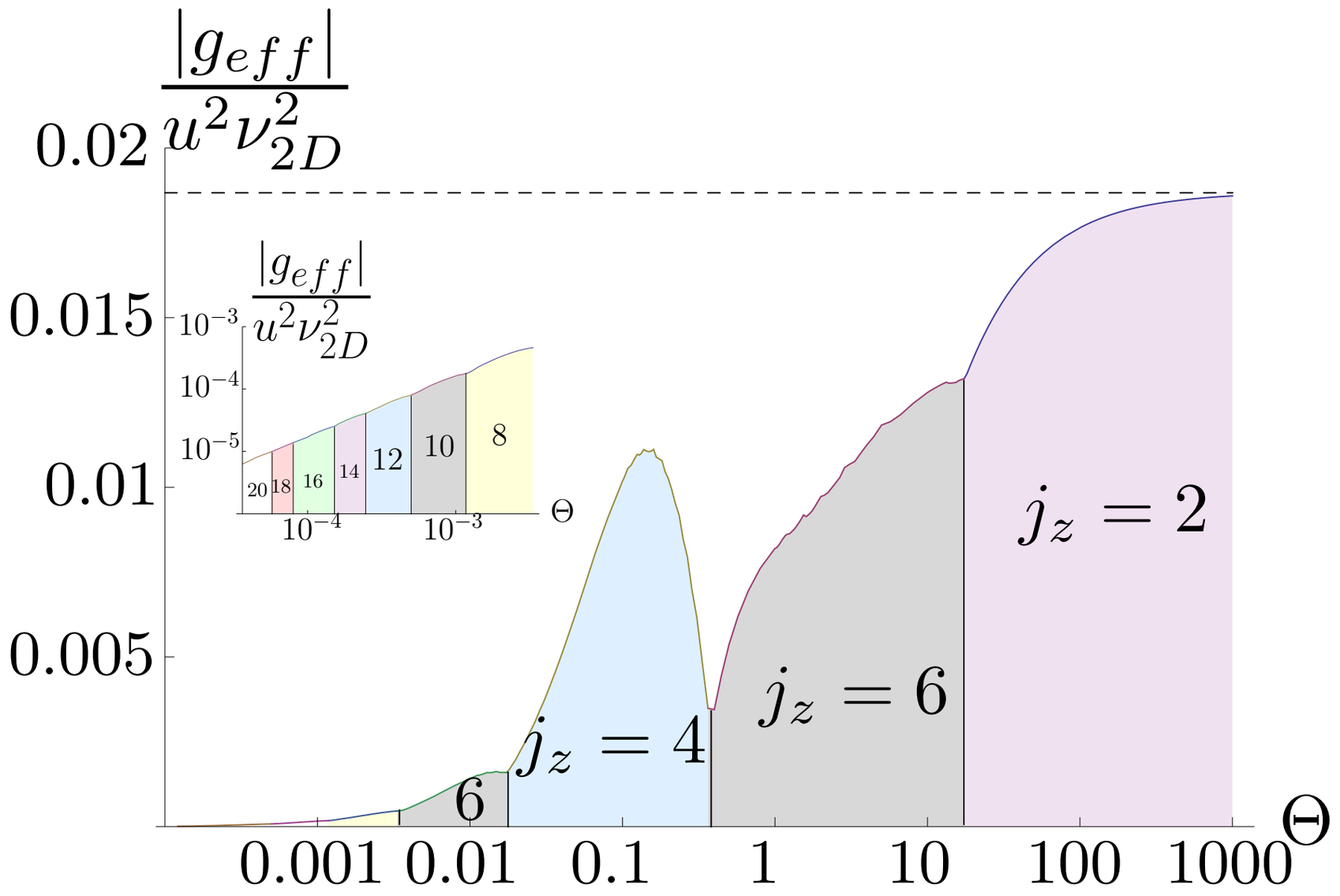


Conclusions

- The Kohn-Luttinger mechanism creates a superconducting instability from *purely repulsive* electronic interactions
- Magnetized 2DEGs with SOC condense into a p-wave superconducting state via the KL mechanism
- In the small spin-orbit regime, the majority band always superconducts, while the minority band never does – this is a spinless *"half-superconductor"*

d=1: spin-orbit stabilized phases
d=2: spin-orbit only relieves degeneracy

Previous studies: no mag. field



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