Emergent Ising orders of frustrated magnets

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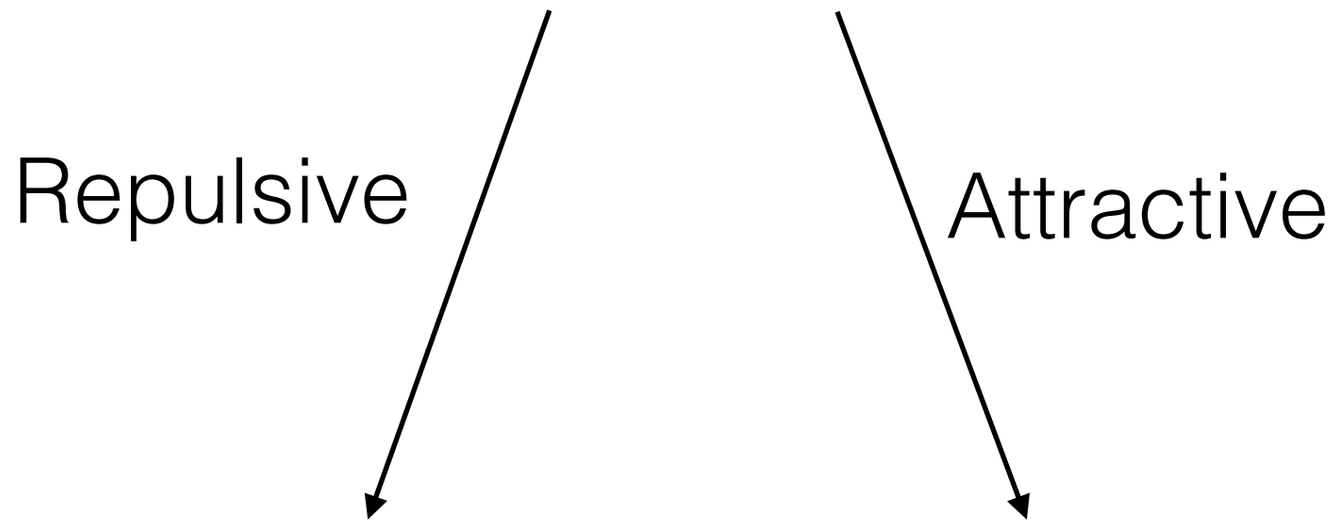
Université Pierre et Marie Curie, Laboratoire de Physique Théorique de la Matiere Condensee, July 3, 2017

Musée des Minéraux, UPMC



Many of the materials are quantum magnets (e.g. linarite)
and many more of them are from Utah :)

Interacting magnons



superfluid

collapse

Anything else?

-> emergence of composite order = nematic

Exotic ordered phases, emergent (Ising) orders

ordered phases



spin nematic composite order



quantum spin liquids

spin nematic

composite order parameter

$$\mathcal{O}^{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j) = \tfrac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \tfrac{1}{3}\delta^{\alpha\beta}\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

LE JOURNAL DE PHYSIQUE TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

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(*Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977*)

Spin nematics

A. F. Andreev and I. A. Grishchuk

Institute of Physics Problems, USSR Academy of Sciences
(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

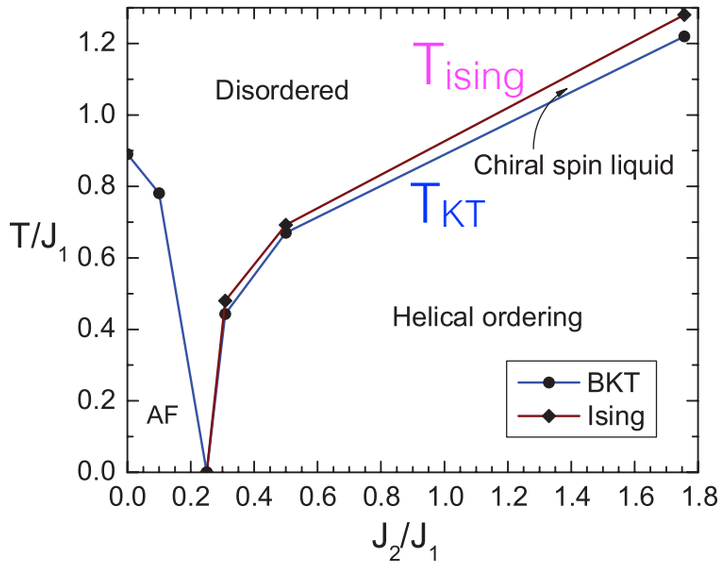
Emergent Ising order parameters

PHYSICAL REVIEW B **85**, 174404 (2012)

Chiral spin liquid in two-dimensional XY helimagnets

A. O. Sorokin[1,*] and A. V. Syromyatnikov[1,2,†]

$$H = \sum_{\mathbf{x}} (J_1 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{a}}) + J_2 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+2\mathbf{a}}) - J_b \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{b}})),$$

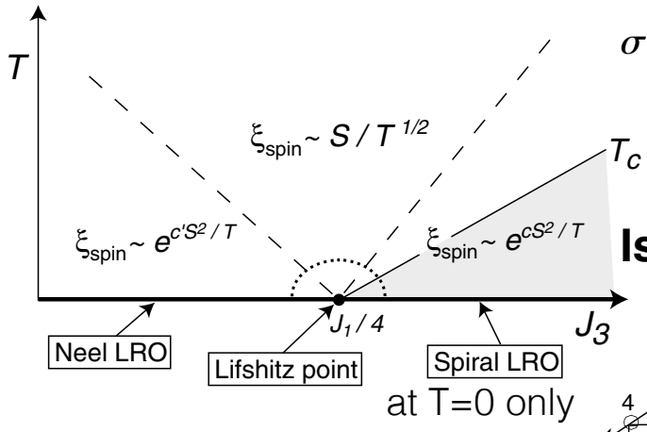


PRL **93,** 257206 (2004) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2004

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

Luca Capriotti[1,2] and Subir Sachdev[2,3]

$$\hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_3 \sum_{\langle\langle i,j \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$



$$\sigma_a = (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_3 - \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{S}}_4)_a,$$

Ising nematic order

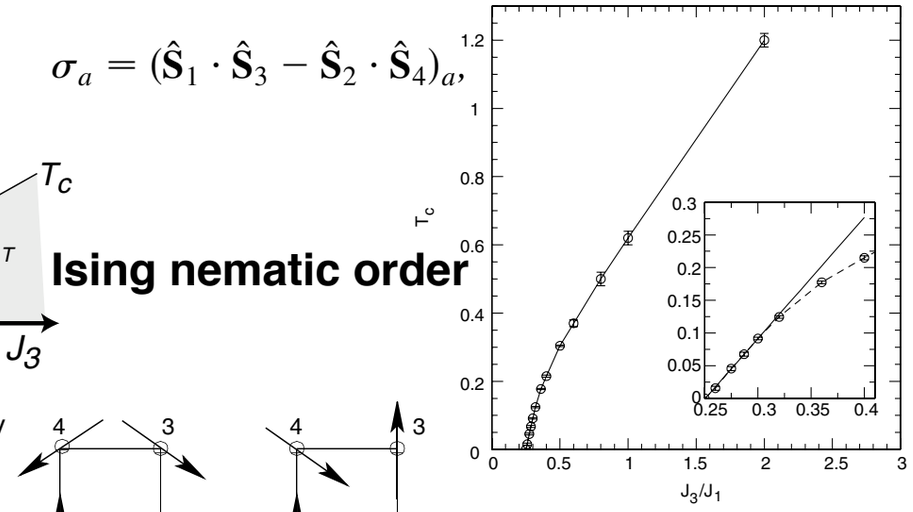


FIG. 2. The two different minimum energy configurations with magnetic wave vectors $\vec{Q} = (Q, Q)$ and $\vec{Q}^{\star} = (Q, -Q)$ with $Q = 2\pi/3$, corresponding to $J_3/J_1 = 0.5$.

PRL **95,** 137206 (2005) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2005

Two-Step Restoration of SU(2) Symmetry in a Frustrated Ring-Exchange Magnet

A. Läuchli,[1] J. C. Domenge,[2] C. Lhuillier,[2] P. Sindzingre,[2] and M. Troyer[3]

[1]*Institut Romand de Recherche Numérique en Physique des Matériaux (IRRMA), PPH-Ecublens, CH-1015 Lausanne, Switzerland*

[2]*Laboratoire de Physique Théorique des Liquides, Université P. et M. Curie, UMR 7600 of CNRS, case 121, 4 Place Jussieu, 75252 Paris Cedex, France*

[3]*Institut für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland*

(Received 23 December 2004; published 22 September 2005)

We demonstrate the existence of a spin-nematic, moment-free phase in a quantum four-spin ring-exchange model on the square lattice. This unusual quantum state is created by the interplay of frustration and quantum fluctuations that lead to a partial restoration of SU(2) symmetry when going from a four-sublattice orthogonal biaxial Néel order to this exotic uniaxial magnet. A further increase of frustration drives a transition to a fully gapped SU(2) symmetric valence bond crystal.

PHYSICAL REVIEW B **72,** 024433 (2005)

Twelve sublattice ordered phase in the J_1–J_2 model on the kagomé lattice

J.-C. Domenge,* P. Sindzingre, and C. Lhuillier

Laboratoire de Physique Théorique de la Matière Condensée, CNRS UMR 7600, Université Pierre et Marie Curie, Boîte Postale 121, 4, Place Jussieu, 75252 Paris Cedex, France

L. Pierre

Bâtiment G, Université Paris-X, Nanterre, 92001 Nanterre Cedex, France

(Received 17 February 2005; revised manuscript received 11 May 2005; published 15 July 2005)

Motivated by recent experiments on an $S=1/2$ antiferromagnet on the kagomé lattice, we investigate the Heisenberg J_1–J_2 model with ferromagnetic J_1 and antiferromagnetic J_2. Classically the ground state displays Néel long-range order with 12 noncoplanar sublattices. The order parameter has the symmetry of a cuboctahedron, it fully breaks $SO(3)$ as well as the spin-flip symmetry, and we expect from the latter a Z_2 symmetry breaking pattern. As might be expected from the Mermin-Wagner theorem in two dimensions, the $SO(3)$ symmetry is restored by thermal fluctuations while the Z_2 symmetry breaking persists up to a finite temperature. A complete study of $S=1/2$ exact spectra reveals that the classical order subsists for quantum spins in a finite range of parameters. First-order spin wave calculations give the range of existence of this phase and the renormalizations at $T=0$ of the order parameters associated to both symmetry breakings. This phase is destroyed by quantum fluctuations for a small but finite $J_2/|J_1| \simeq 3$, consistently with exact spectra studies, which indicate a gapped phase.

Outline

✳ Magnetization process of triangular lattice antiferromagnet

- accidental degeneracy and selection of the magnetization plateau at M=1/3 by quantum (thermal) fluctuations

✳ Instabilities of the plateau

- moving down/up in magnetic field
- spatial anisotropy of exchange interactions

✳ Two-magnon pairing and condensation

- Spin-current (chiral Mott) phase
- Spontaneous generation of DM interaction!

✳ Generic features: (at least) two-fold spectrum degeneracy
strong interactions (in the vicinity of single-particle condensation)

• Mott (PM) to Superfluid (cone) transition
proceeds via intermediate chiral Mott phase
$U(1) \times Z_2$

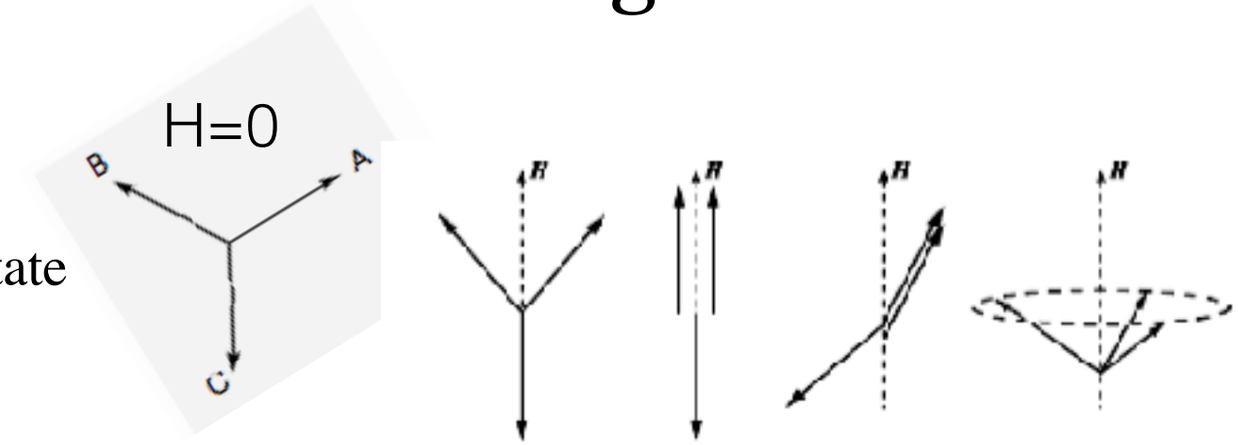
Classical isotropic triangular AFM in magnetic field

- Zero field: co-planar spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

$$H = J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \sum_i \vec{h}\cdot\vec{S}_i$$

$$H = \frac{1}{2}J\sum_{\triangle}\Big(\sum_{i\in\triangle}\vec{S}_i - \frac{\vec{h}}{3J}\Big)^2$$

- all states with $\vec{S}_{i1}+\vec{S}_{i2}+\vec{S}_{i3} = \dfrac{\vec{h}}{3J}$ form the lowest-energy manifold
- Accidental degeneracy
  - O(2) spins: 3 angles, 2 equations => 1 **continuous angle undetermined**
  - O(3) spins: 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about **h**)



120° state co-planar supersolid non-coplanar superfluid

Order-by-disorder: This degeneracy is lifted by thermal/quantum fluctuations.

Phase diagram of the Heisenberg (XXX) model in the field

Journal of the Physical Society of Japan
Vol. 53, No. 12, December, 1984, pp. 4138–4154

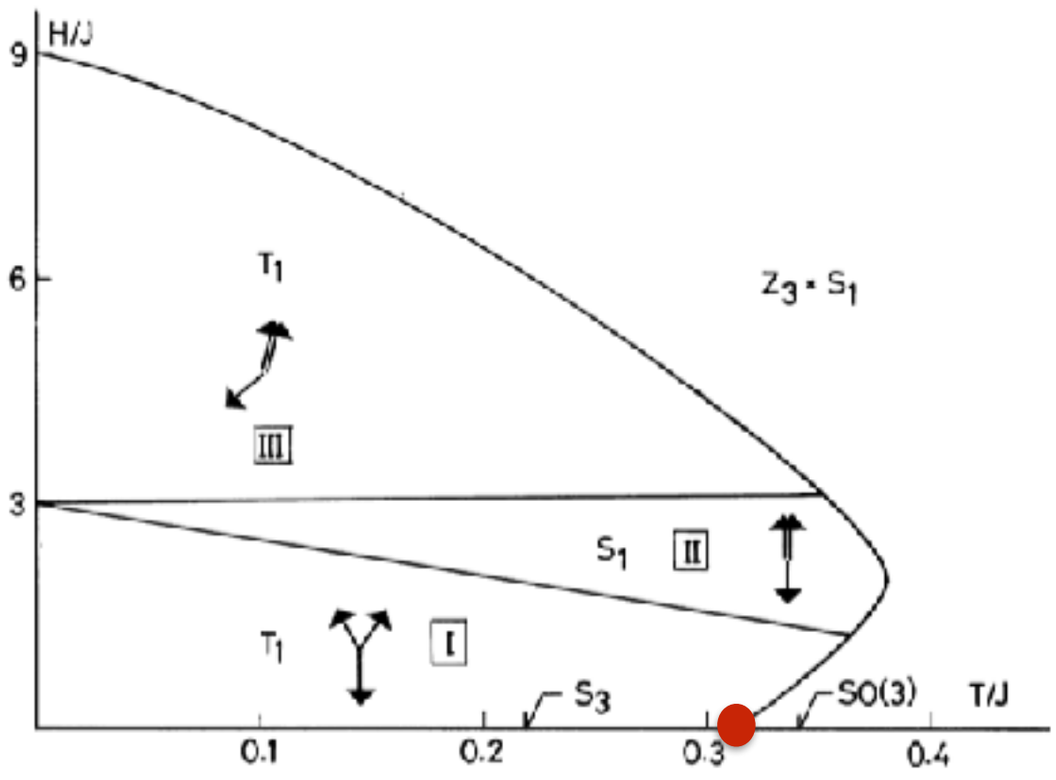
Phase Transition of the Two-Dimensional Heisenberg Antiferromagnet on the Triangular Lattice

Hikaru KAWAMURA and Seiji MIYASHITA†

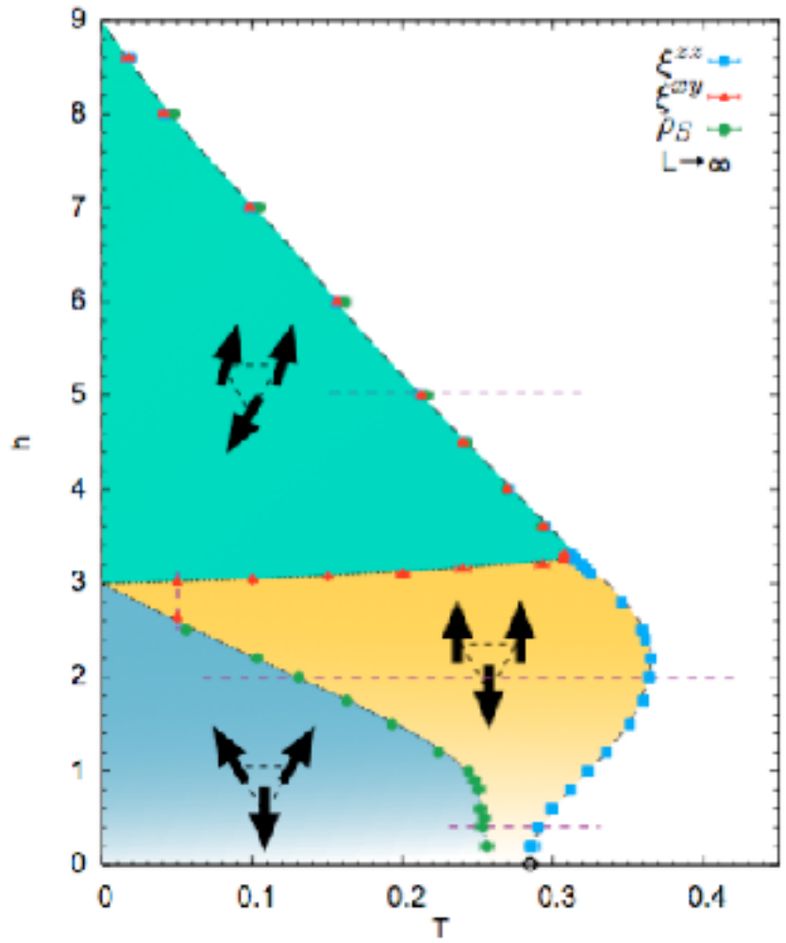
Journal of the Physical Society of Japan
Vol. 54, No. 12, December, 1985, pp. 4530–4538

Phase Transition of the Heisenberg Antiferromagnet on the Triangular Lattice in a Magnetic Field

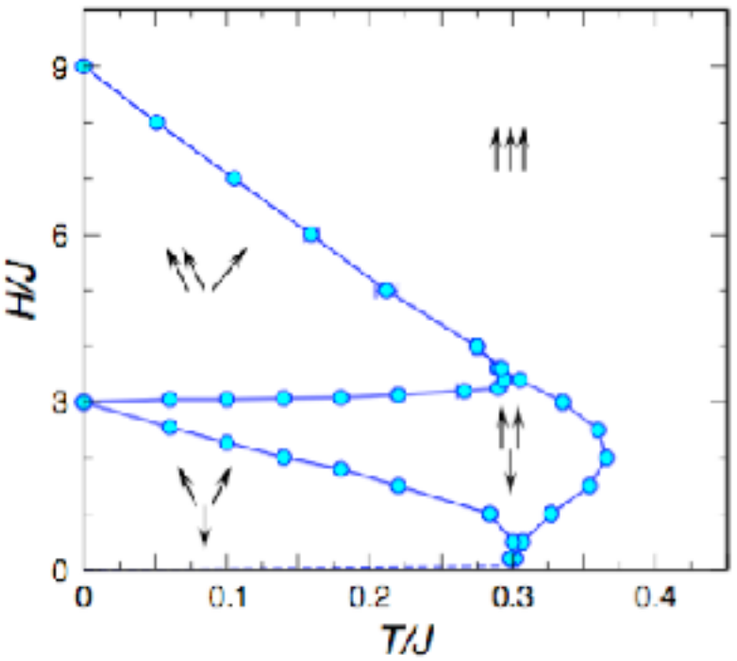
Hikaru KAWAMURA and Seiji MIYASHITA†



Z_2 vortex (chirality ordering) transition



Seabra, Momoi, Sindzingre, Shannon 2011



Gvozdikova, Melchy, Zhitomirsky 2010

Quantum fluctuations, S >> 1, T=0.

J' = **J**: Quantum fluctuations select co-planar and collinear phases

J. Phys.: Condens. Matter 3 (1991) 69–82. Printed in the UK

UUD plateau is due to interactions between spin waves

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

A V Chubukov and D I Golosov
Institute for Physical Problems, USSR Academy of Sciences, 117334 ul. Kosygina 2, Moscow, USSR

Received 9 February 1990

Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and *XY* quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.

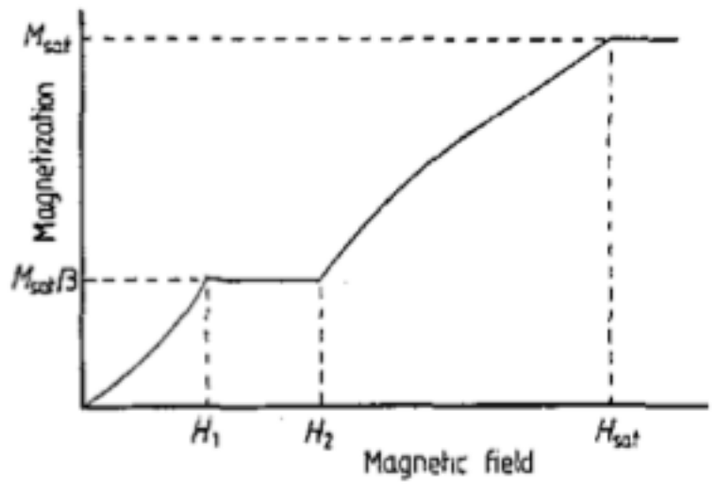


Figure 3. The anticipated behaviour of longitudinal magnetization in 2D Heisenberg AFM on a triangular lattice. The plateau on the magnetization curve results from the stabilization of the collinear phase in the finite region of magnetic fields due to zero-point motion.

$$h_{c2} - h_{c1} = (0.6/2S)\, h_{sat}$$

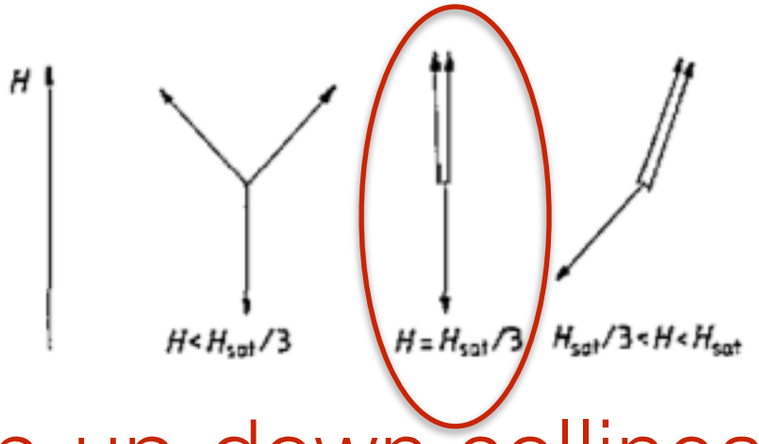


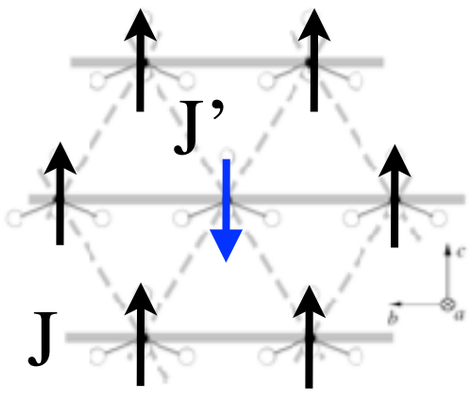
Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_2$ in the vicinity of $H_{sat}/3$.

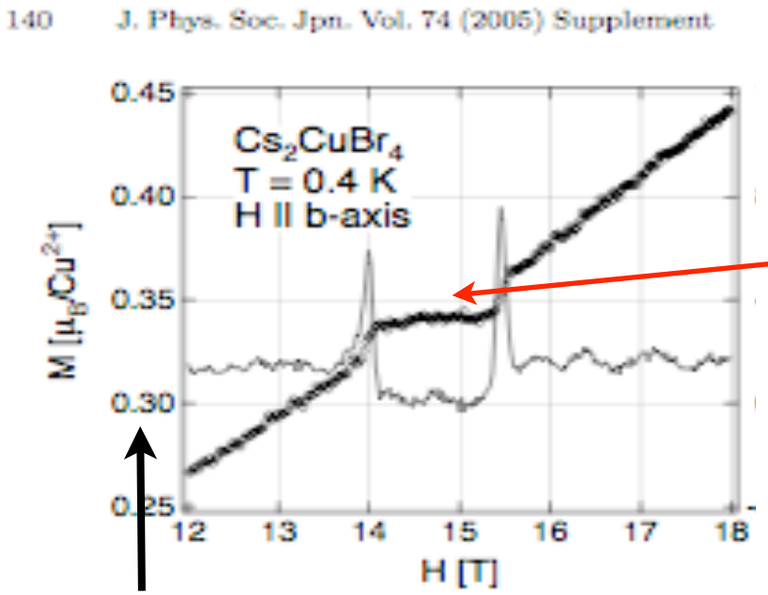
up-up-down collinear state

Exp: M=1/3 magnetization plateau in Cs_2CuBr_4

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007)

S=1/2





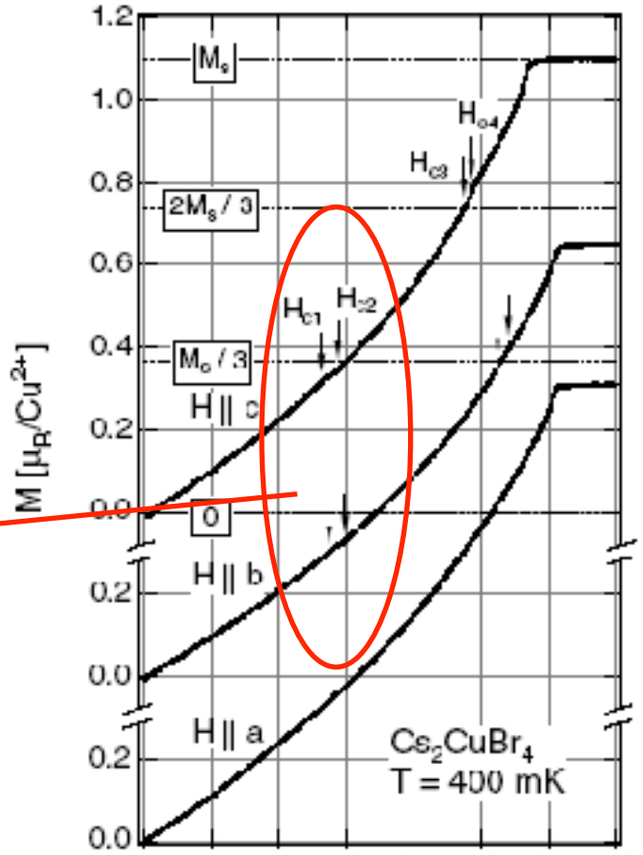
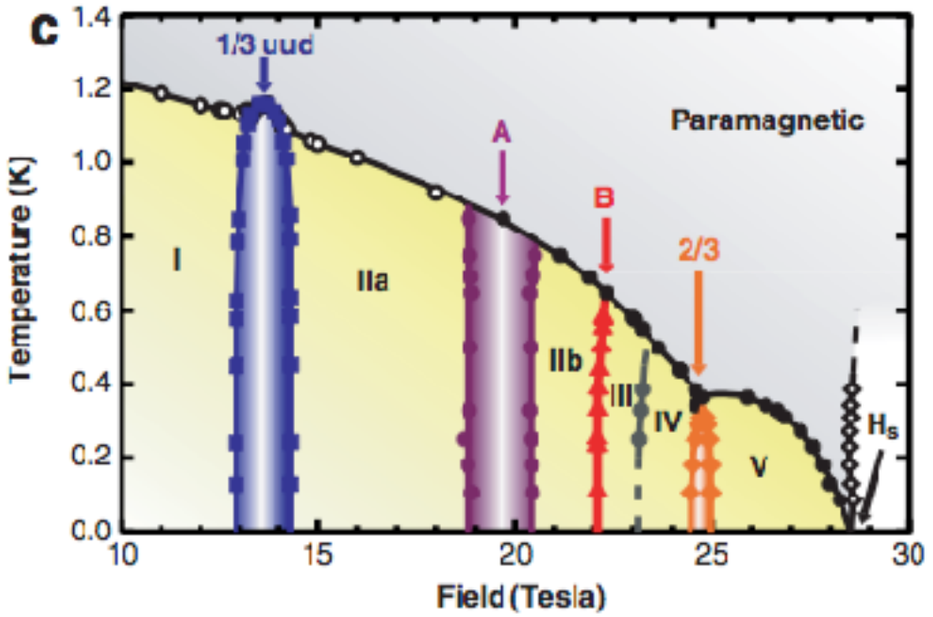


Fig. 8. The magn
measured at T =



★ first observation of "**up-up-down**" state in spin-1/2 triangular lattice antiferromagnet

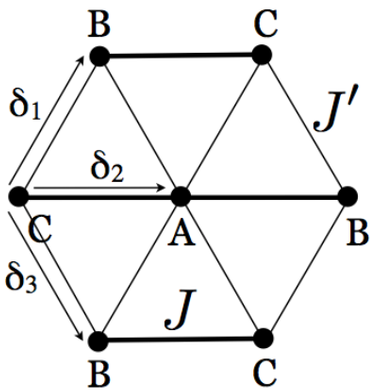
★ total of **9 phases --** instead of 3 expected!

Important: the lattice is strongly anisotropic

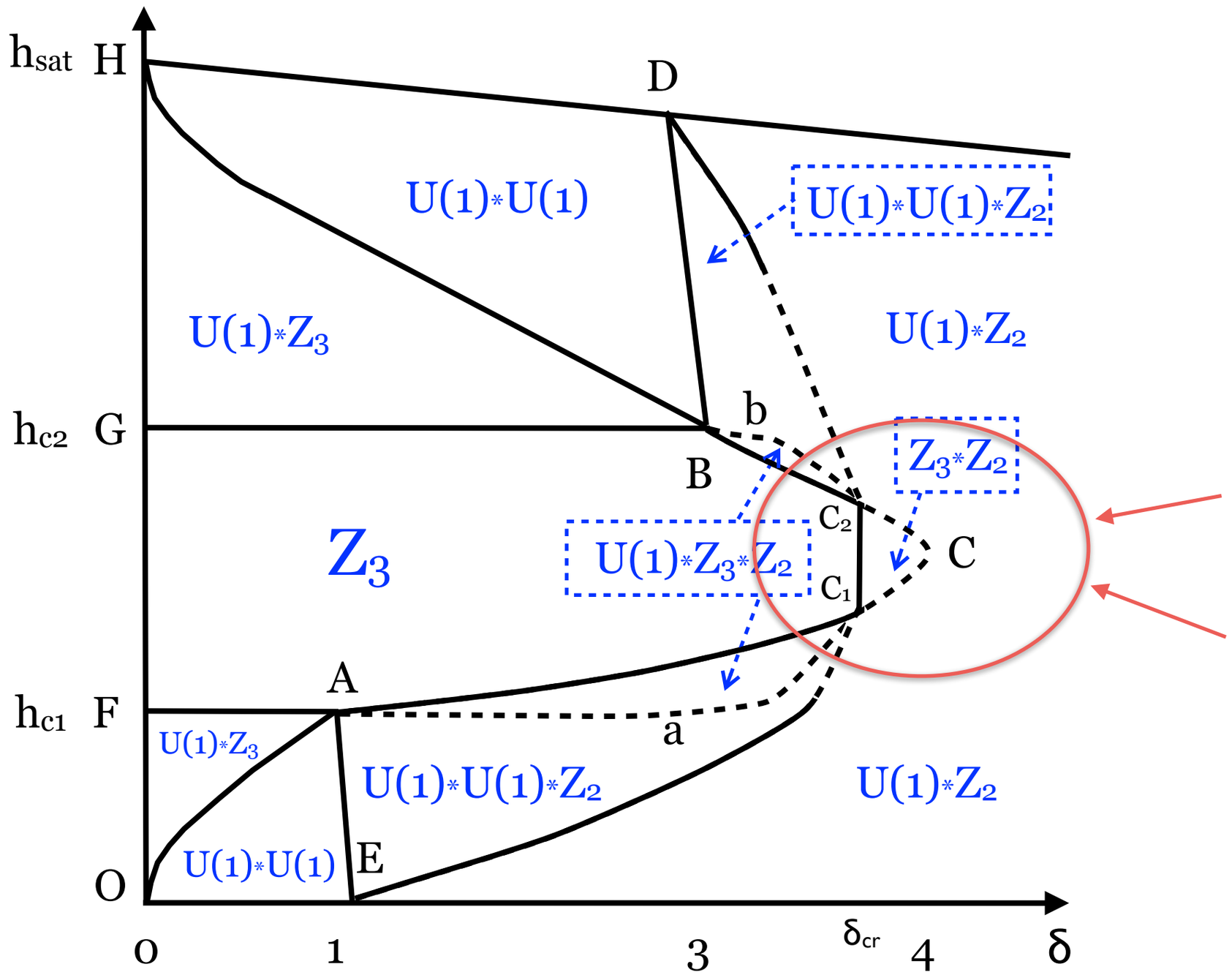
THIS TALK:

Emergent Ising order near the end-point of the 1/3 magnetization plateau

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

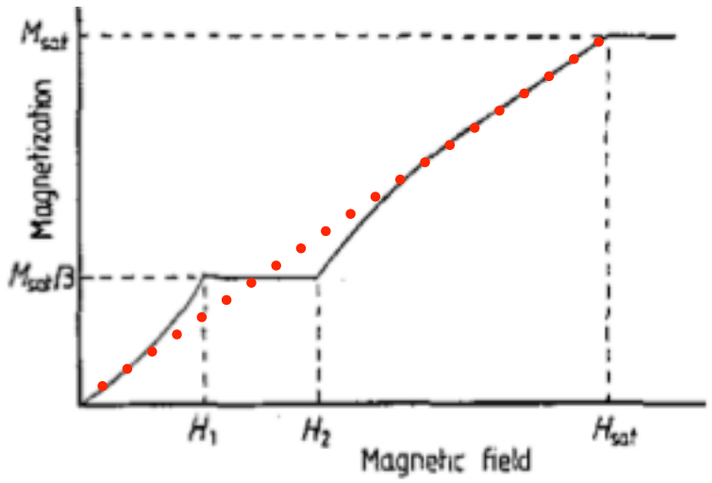


$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$

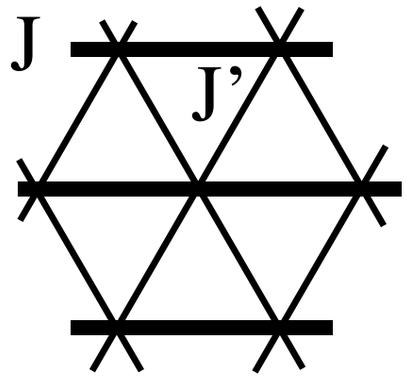


OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

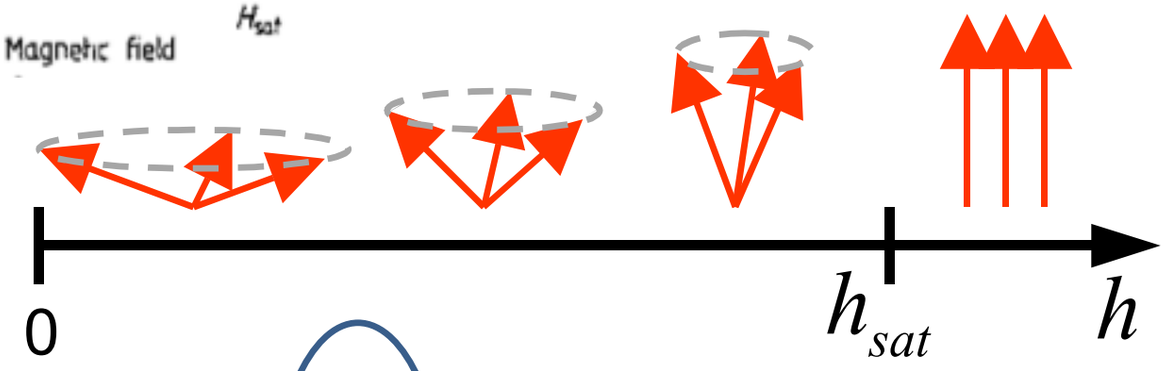
Spatially anisotropic model: classical vs *quantum*



$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



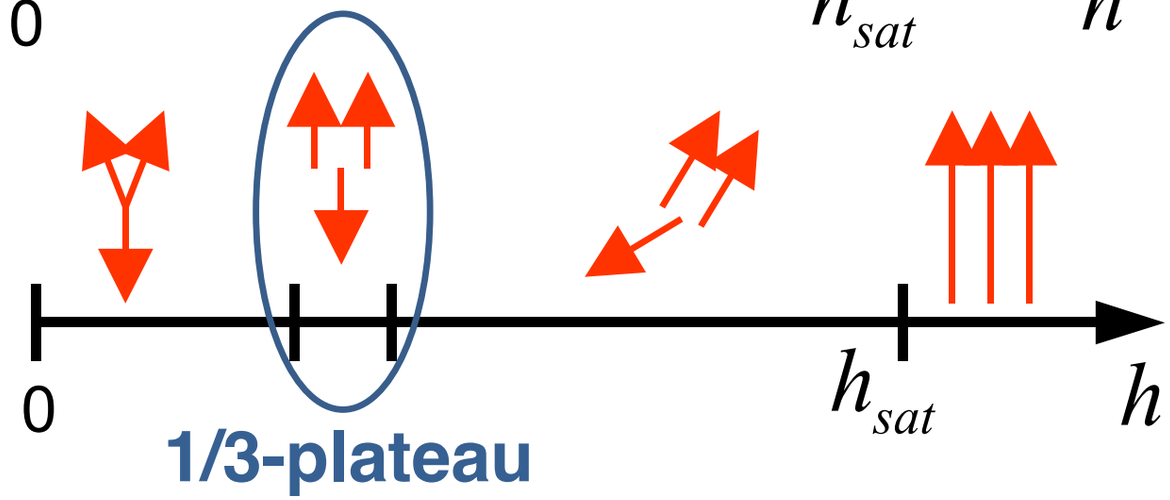
$S = \infty$



Umbrella state:
favored classically;
energy gain $(J\text{-}J')^2/J$

$J' \neq J$

$S = \frac{1}{2}$

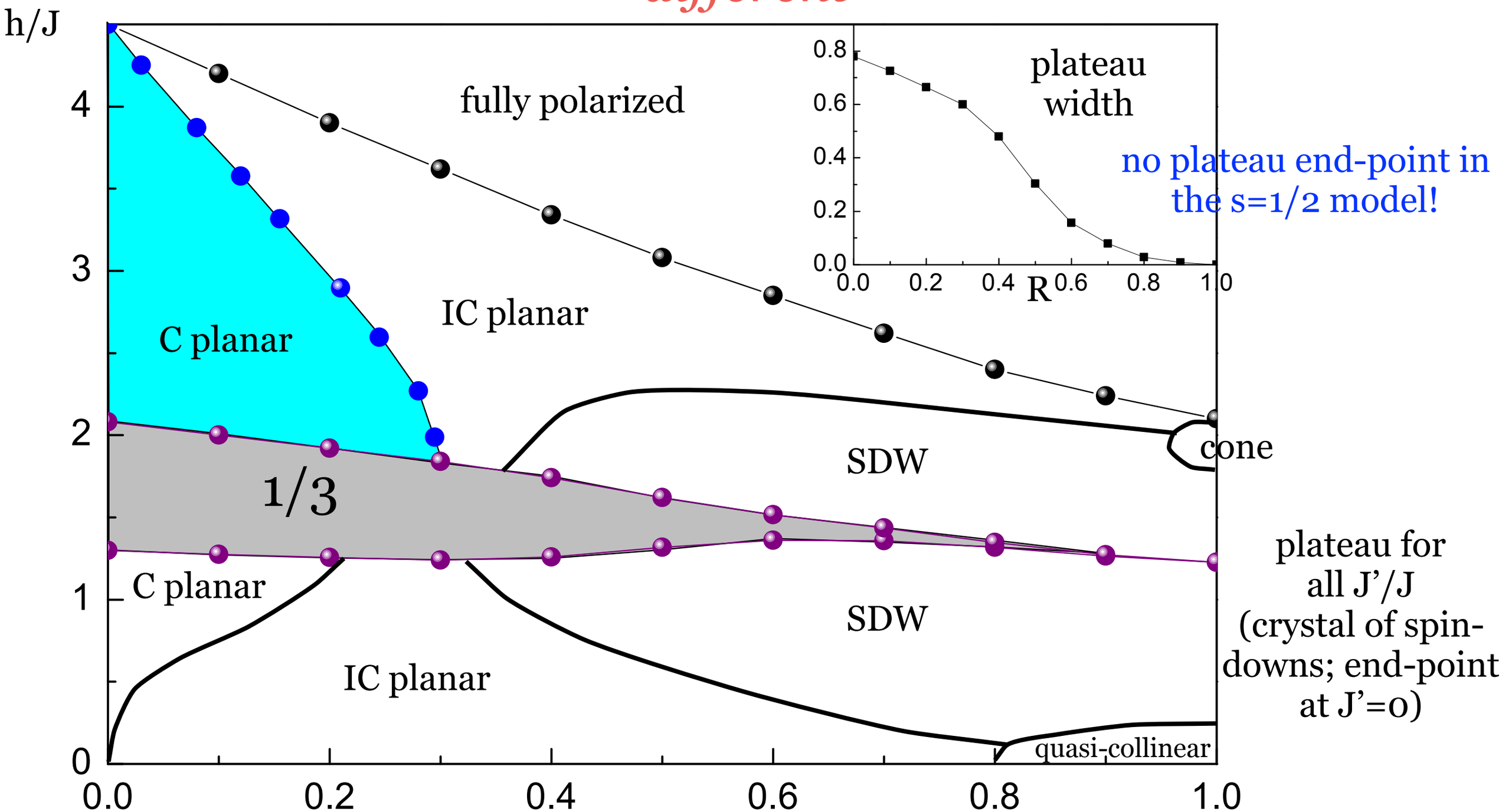


Planar states: favored by
quantum fluctuations;
energy gain J/S

The competition is controlled by
dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

Notable exception: spin-1/2 triangular lattice AFM *is different*



no plateau end-point in the s=1/2 model!

plateau for all J'/J (crystal of spin-downs; end-point at J'=0)

PHYSICAL REVIEW B 87, 165123 (2013)

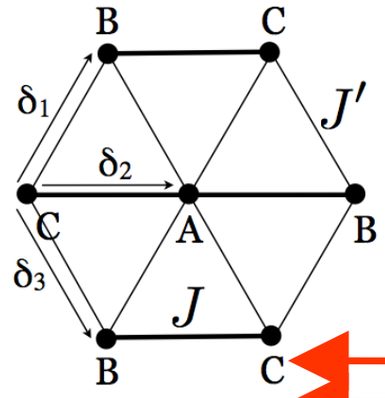
Ground states of spin-$\frac{1}{2}$ triangular antiferromagnets in a magnetic field

Ru Chen,[1] Hyejin Ju,[1] Hong-Chen Jiang,[2] Oleg A. Starykh,[3] and Leon Balents[2]
[1]*Department of Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA*
[2]*Kavli Institute of Theoretical Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA*
[3]*Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA*

S=1/2 vs S=1 difference for the ordered 2d phase!!!

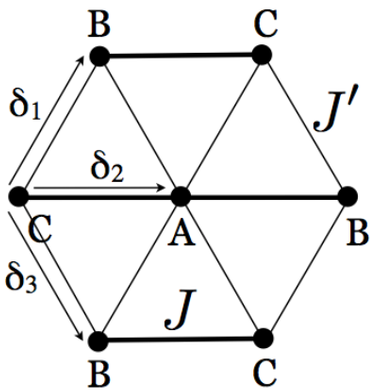
Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at T=0

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

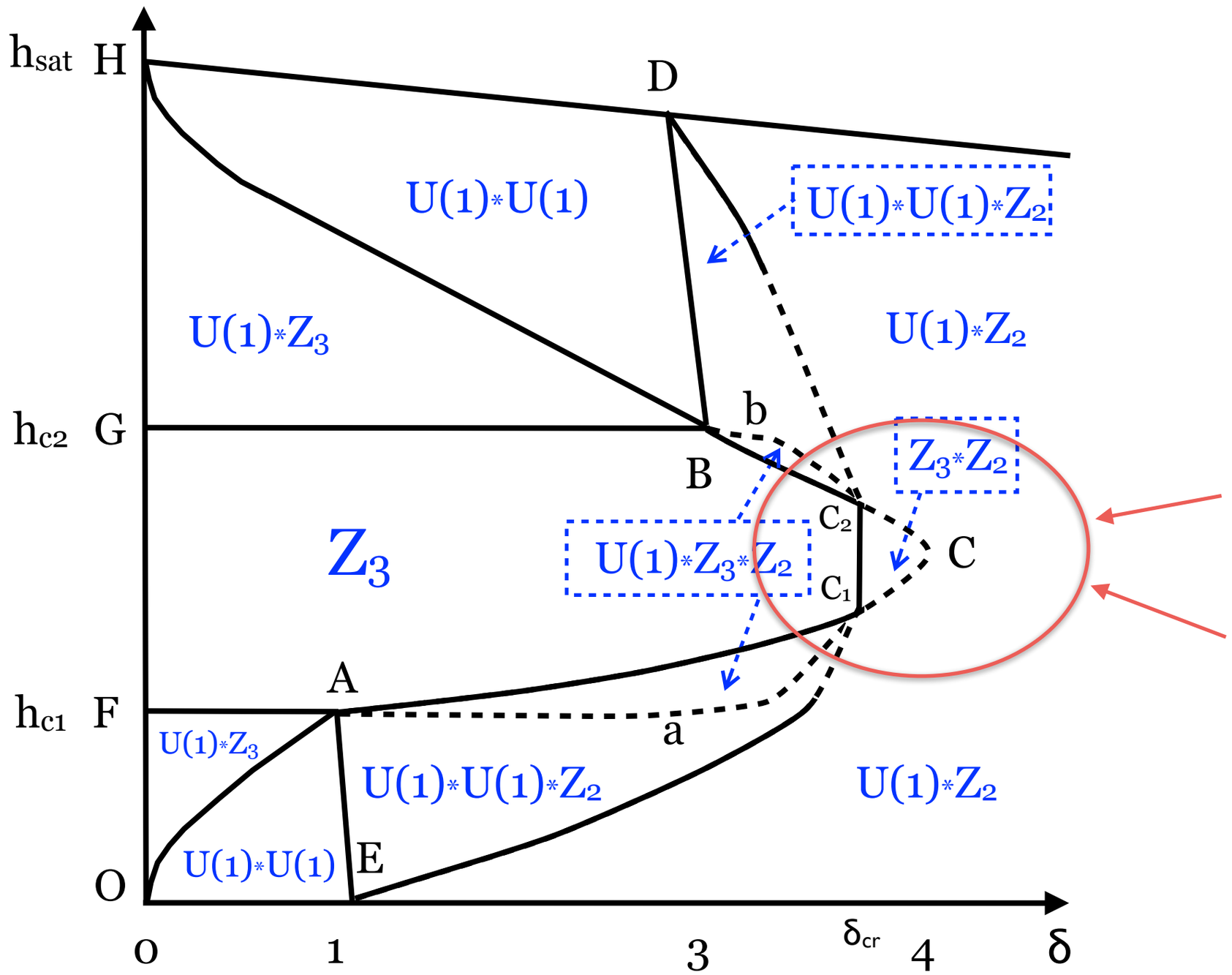


Emergent Ising order near the end-point of the 1/3 magnetization plateau — **the result**

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$

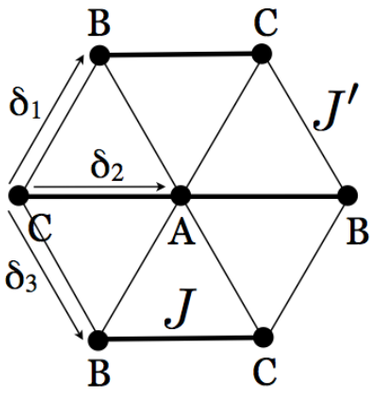


OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

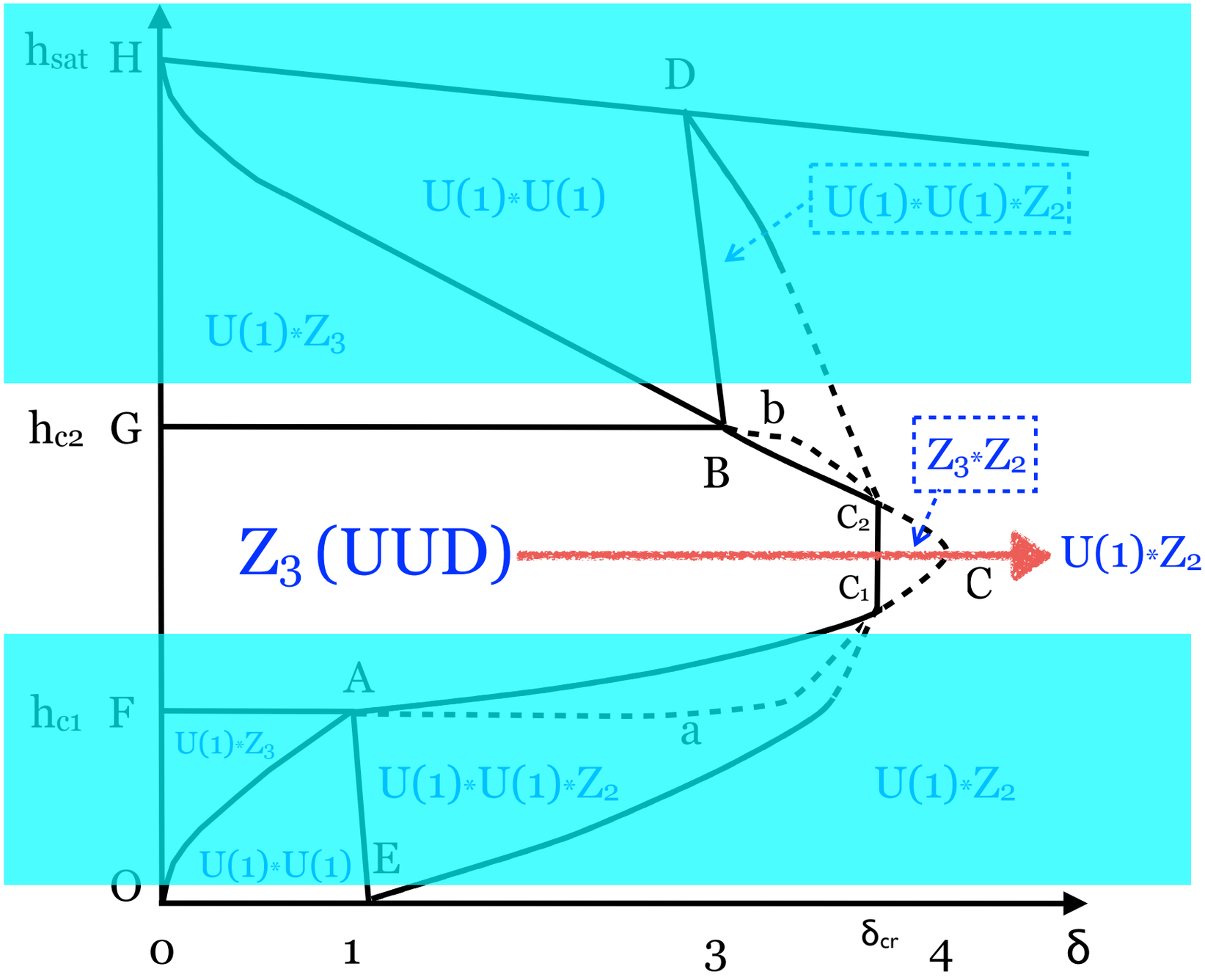
UUD-to-cone phase transition

$$Z_3 \to U(1) \times Z_2 \text{ or } Z_3 \to \text{smth else} \to U(1) \times Z_2 ?$$

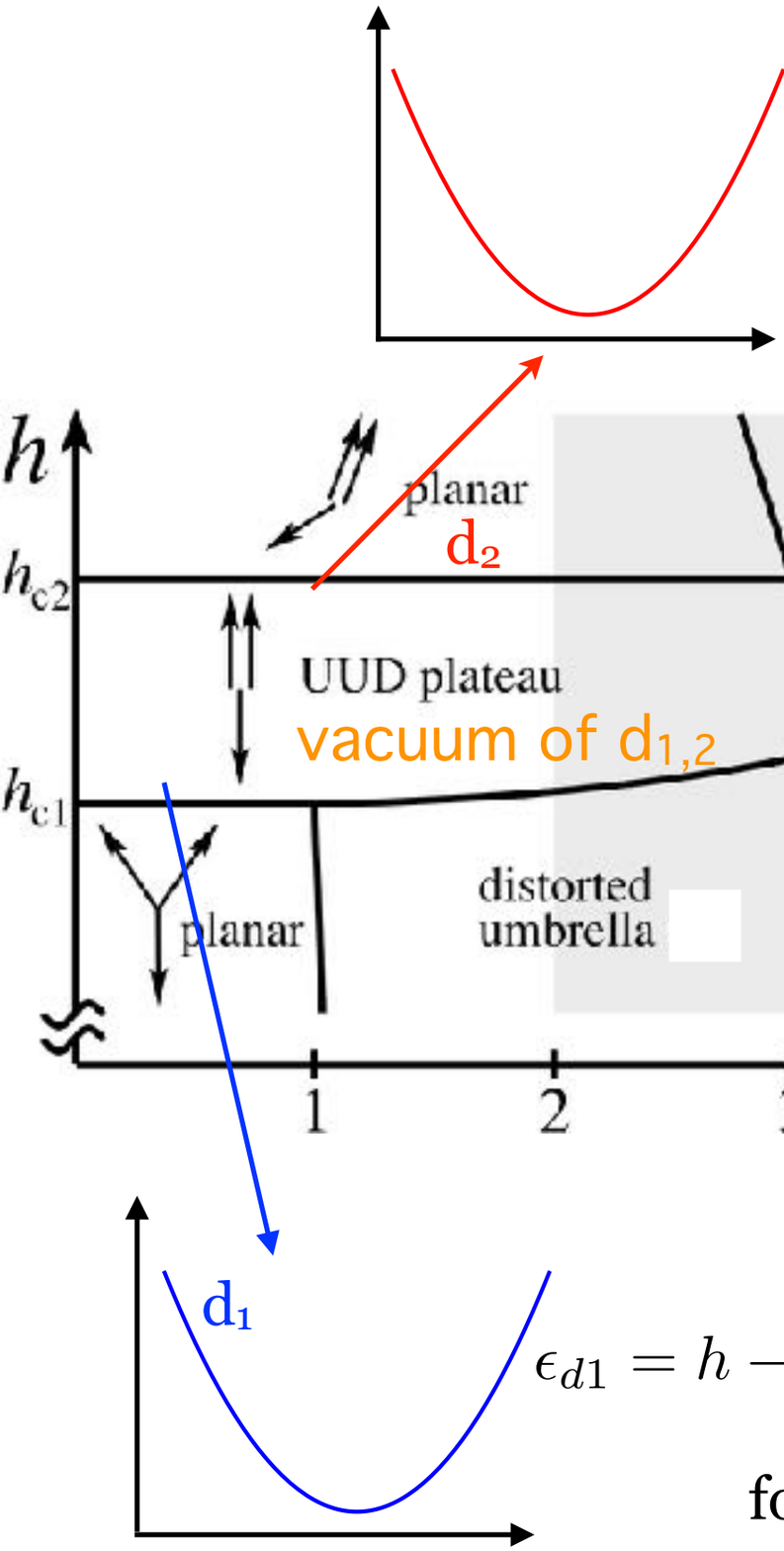
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



Low-energy excitation spectra



$$\epsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4}$$

for δ < 3

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3. Hence, ***very stable***.

$$h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\rm sat} = \frac{0.6}{2S}(9JS)$$

$$\delta = \frac{40\ S}{3}(1 - J'/J)^2$$

$$\epsilon_{d1} = h - h_{c1} + \frac{3Jk^2}{4}$$

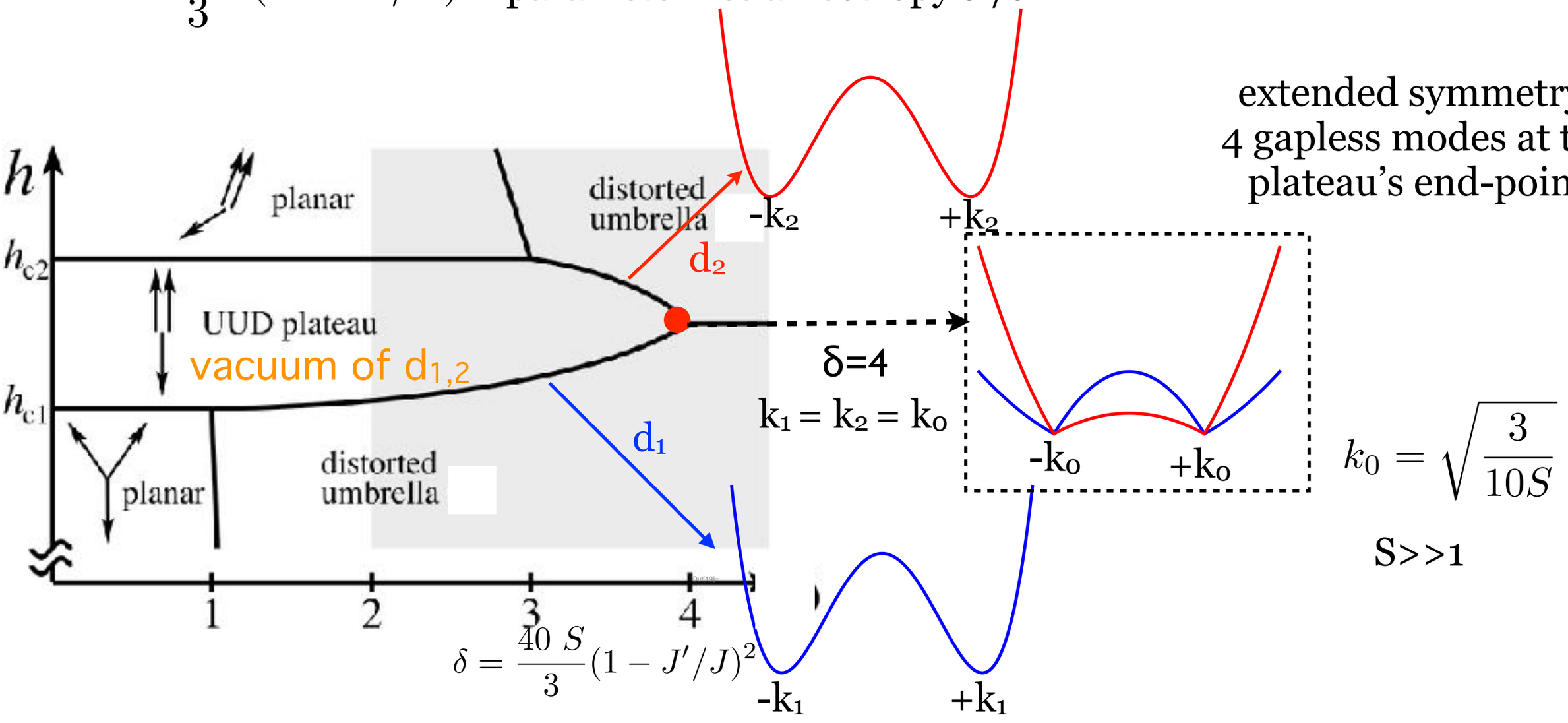
for δ < 1

Bose-Einstein condensation of d_1 (d_2) mode at k =0 leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009

Low-energy excitation spectra near the plateau's end-point

$\delta = \frac{40\ S}{3}(1 - J'/J)^2$ parameterizes anisotropy J'/J



extended symmetry:
4 gapless modes at the plateau's end-point

$k_0 = \sqrt{\frac{3}{10S}}$

S>>1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3.

Alicea, Chubukov, OS PRL 2009

Interaction between low-energy magnons

$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\Big(d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}\Big) + \text{h.c.}$$

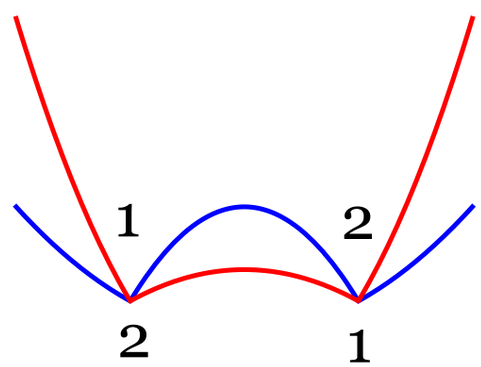
$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

singular magnon interaction

$$\Psi^\dagger_{1,p}\Psi_{2,q} \qquad \Psi^\dagger_{1,p}\Psi^\dagger_{2,q}$$

magnon **pair operators**

$$\begin{cases}\Psi_{1,p} = d_{1,k_0+p}d_{2,-k_0-p}\\ \Psi_{2,p} = d_{1,-k_0+p}d_{2,k_0-p}\end{cases}$$



Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}},\Psi_{2,\mathbf{q}}] = \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}\Big(1 + d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d_{1,\mathbf{k}_0+\mathbf{p}} + d^\dagger_{2,\mathbf{k}_0+\mathbf{p}}d_{2,\mathbf{k}_0+\mathbf{p}}\Big) \to \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state $\langle d^\dagger_1 d_1\rangle_{\text{uud}} = \langle d^\dagger_2 d_2\rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of $\mathbf{d}_{1,2}$ bosons = non-interacting Hamiltonian in terms of $\Psi_{1,2}$ magnon pairs

Chubukov, OS PRL 2013

Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for Ψ - Hamiltonian

$$\langle \Psi^\dagger_{1,\mathbf{p}} - \Psi_{1,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{2,\mathbf{q}} - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi^\dagger_{2,\mathbf{p}} - \Psi_{2,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{1,\mathbf{q}} - \Psi_{1,\mathbf{q}} \rangle$$

`Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4$ - $O(1/S^2)$

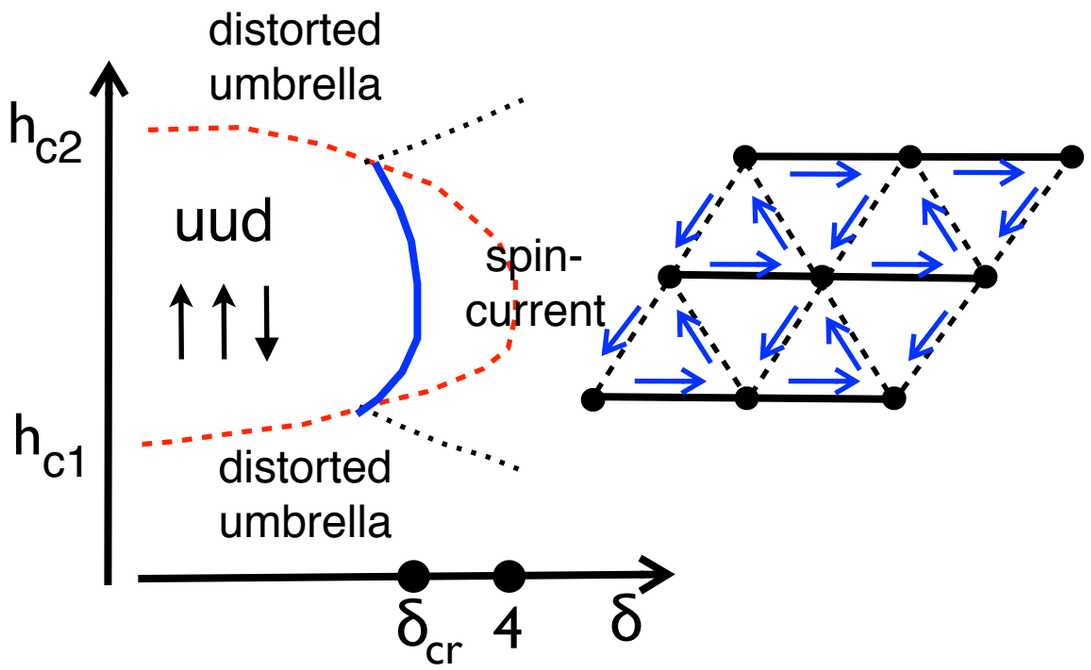
$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1-\delta/4)k_0^2}}$$

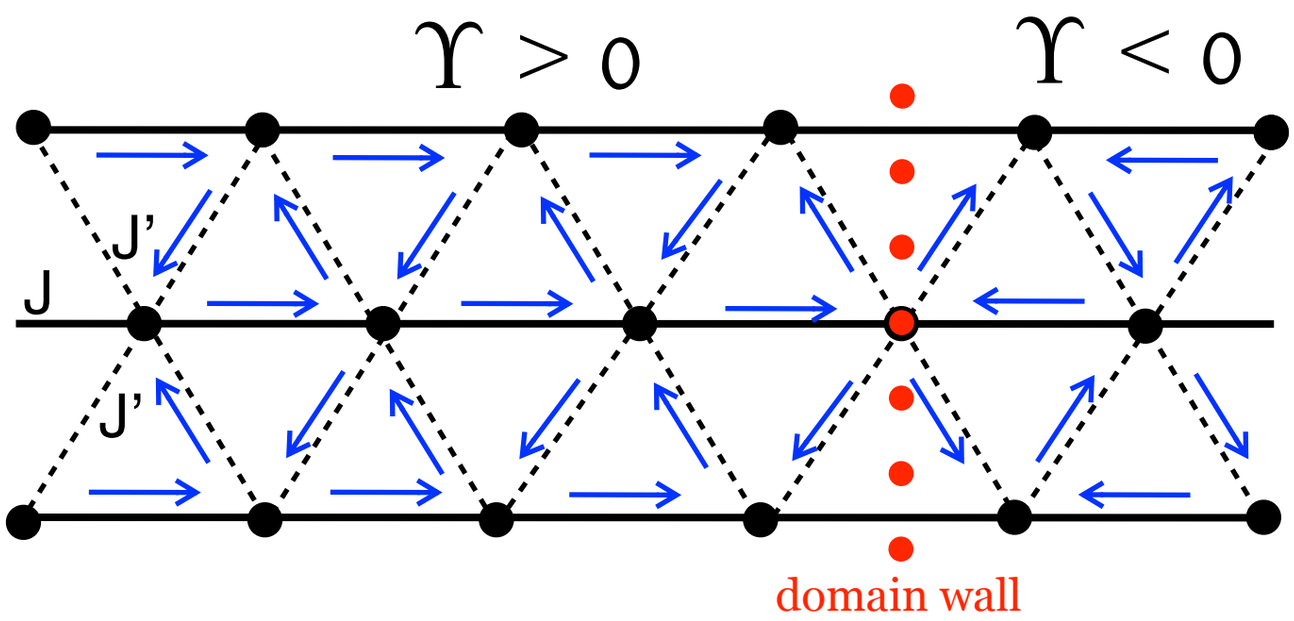
no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

Chubukov, OS PRL 2013

Two-magnon condensate = Spin-current nematic state





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

Finite scalar (and vector) chiralities. Sign of Υ determines sense of spin-current circulation

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

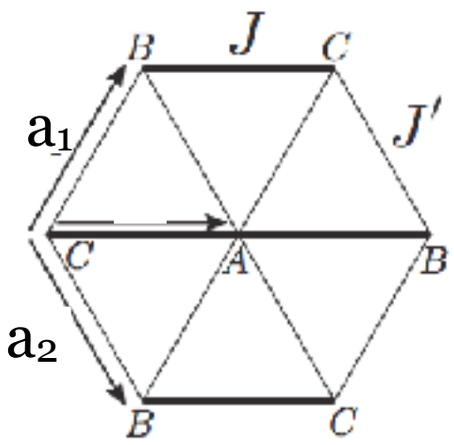
Spontaneously broken Z_2 -- spatial inversion [in addition to broken Z_3 inherited from the UUD state]

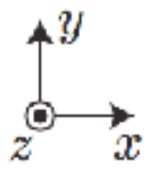
Leads to **spontaneous generation of Dzyaloshisnkii-Moriya interaction**

Chubukov, OS PRL 2013

Spontaneous generation of Dzyaloshinskii-Moriya interaction

$$\mathcal{H}^{(4)}_{d_1 d_2} \propto \frac{1}{N} \sum_{k \in +k_0} \frac{-ik_0}{\sqrt{(k-k_0)^2 + (1-\delta/4)k_0^2}} (d^\dagger_{1,\mathbf{k}} d^\dagger_{2,-\mathbf{k}} - d_{1,\mathbf{k}} d_{2,-\mathbf{k}}) \sum_{p \in -k_0} \frac{-ik_0}{\sqrt{(p+k_0)^2 + (1-\delta/4)k_0^2}} (d^\dagger_{1,\mathbf{p}} d^\dagger_{2,-\mathbf{p}} - d_{1,\mathbf{p}} d_{2,-\mathbf{p}})$$





continuum limit of DM in triangular lattice

$$\sum_{\mathbf{r}} \hat{z} \cdot \mathbf{S_r} \times (\mathbf{S_{r+a_1}} + \mathbf{S_{r+a_2}})$$

B || z

Mean-field approximation:

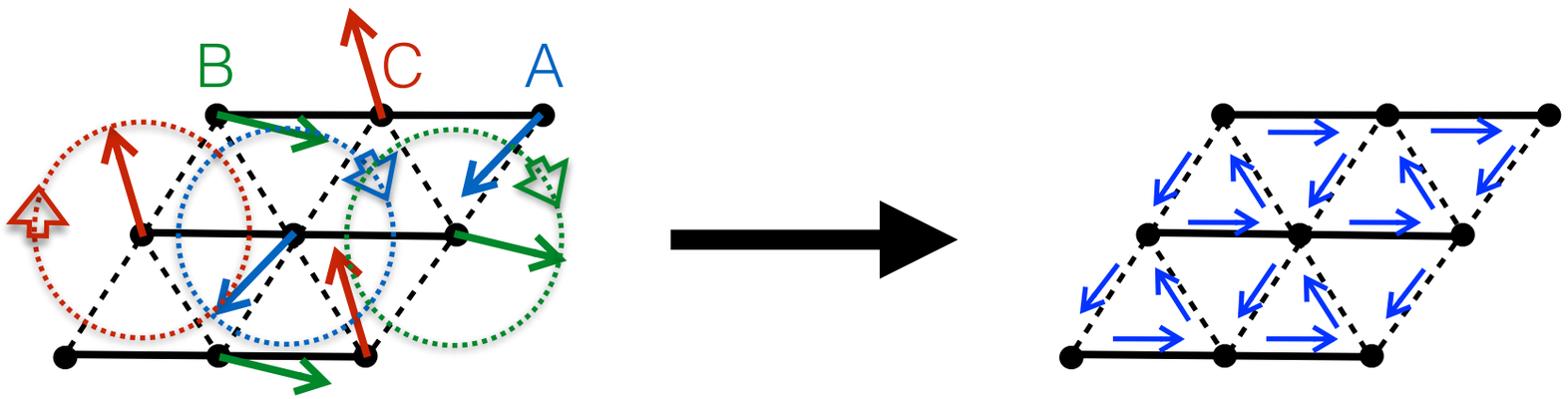
$$\mathcal{H}^{(4)}_{d_1 d_2} \to D \sum_k \left(\frac{k_0}{|k-k_0|} + \frac{k_0}{|k+k_0|}\right)(d^\dagger_{1,\mathbf{k}} d^\dagger_{2,-\mathbf{k}} - d_{1,\mathbf{k}} d_{2,-\mathbf{k}})$$

$$D \sim \Upsilon$$

spin currents appear due to **spontaneously generated DM**

similar to Lauchli et al (PRL 2005) for Heisenberg+ring exchange model;
also ‘chiral Mott insulator’, Dhar et al, PRB 2013; Zaletel et al, 2013

Spin current visualization



Precessing spins on sub lattices A, B, C are phase shifted by 2π/3:

$$\mathbf{S}_A = (\cos[\omega t], \sin[\omega t], M_A), \mathbf{S}_B = (\cos[\omega t \pm \frac{4\pi}{3}], \sin[\omega t \pm \frac{4\pi}{3}], M_B), \mathbf{S}_C = (\cos[\omega t \pm \frac{2\pi}{3}], \sin[\omega t \pm \frac{2\pi}{3}], M_C)$$

Then no dipolar transverse order:

$$\langle \mathbf{S}_{\mathbf{r}}^{x,y} \rangle = 0 \text{ and } \langle \mathbf{S}_A \cdot \mathbf{S}_C \rangle = \langle \mathbf{S}_C \cdot \mathbf{S}_B \rangle = \langle \mathbf{S}_B \cdot \mathbf{S}_A \rangle = \cos[\frac{2\pi}{3}]$$

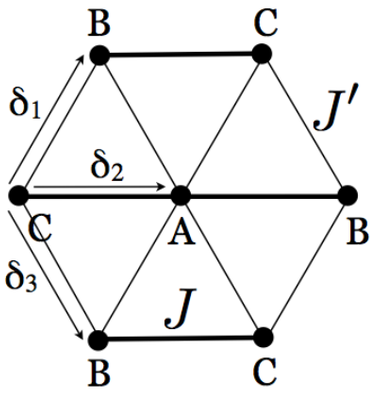
But finite **chirality**, determined by the sign of 2π/3 shift between the sublattices:

$$\langle \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \mathbf{S}_B \times \mathbf{S}_A \rangle = \pm \sin[\frac{2\pi}{3}]$$

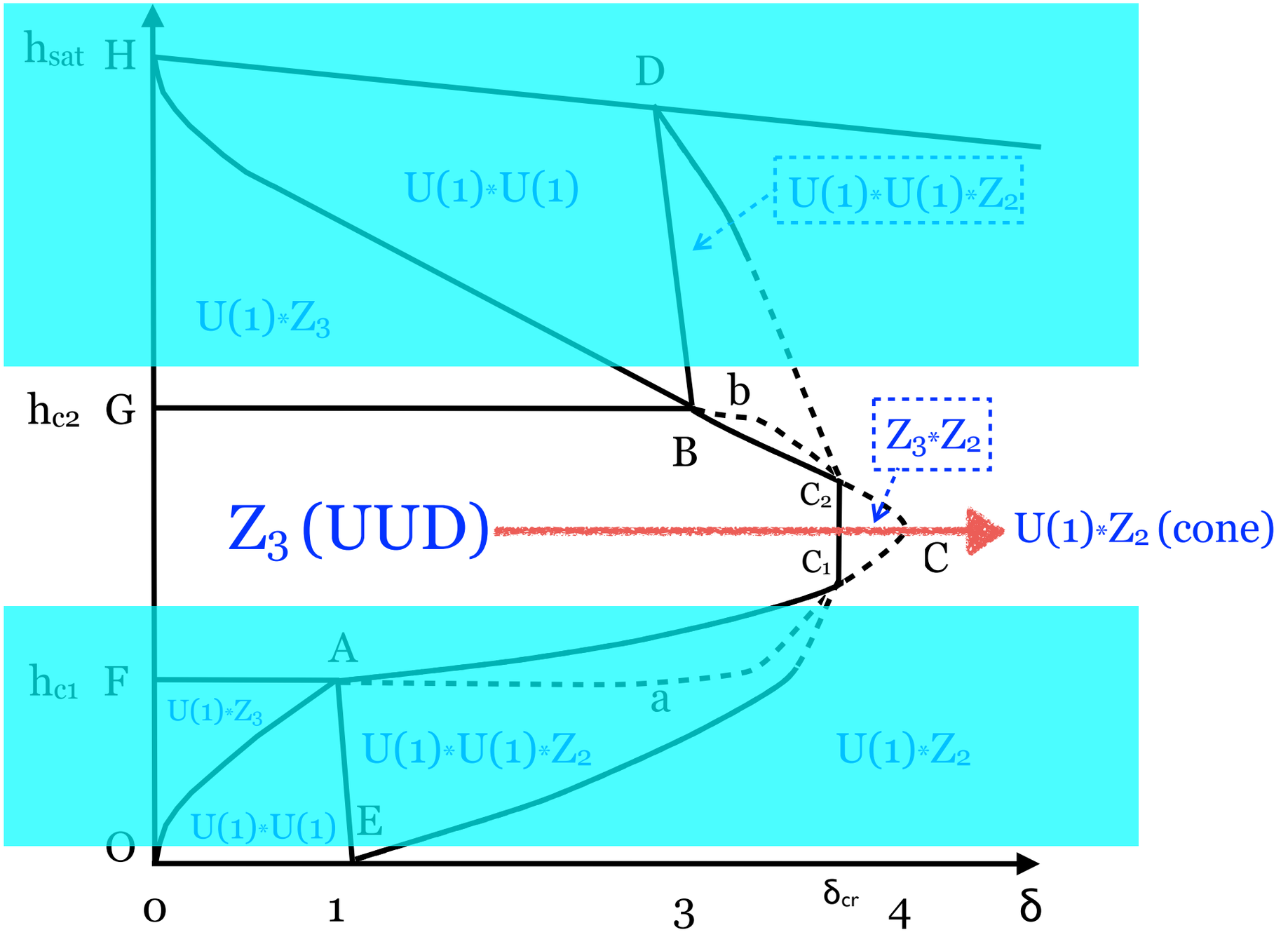
Continuous transitions: plateau ->spin-current —> cone !

$$Z_3 \rightarrow Z_3 \times Z_2 \rightarrow U(1) \times Z_2$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan,[1] Alexander O. Gogolin,[2] and Fabian H. L. Eßler[3]

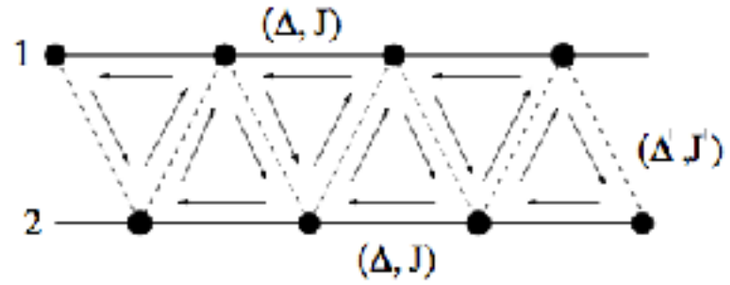


FIG. 3. Structure of the spin currents in the spin nematic phase.

PHYSICAL REVIEW B **87**, 174501 (2013)

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar,[1] Tapan Mishra,[2] Maheswar Maji,[3] R. V. Pai,[4] Subroto Mukerjee,[3,5] and Arun Paramekanti[2,3,6,7]

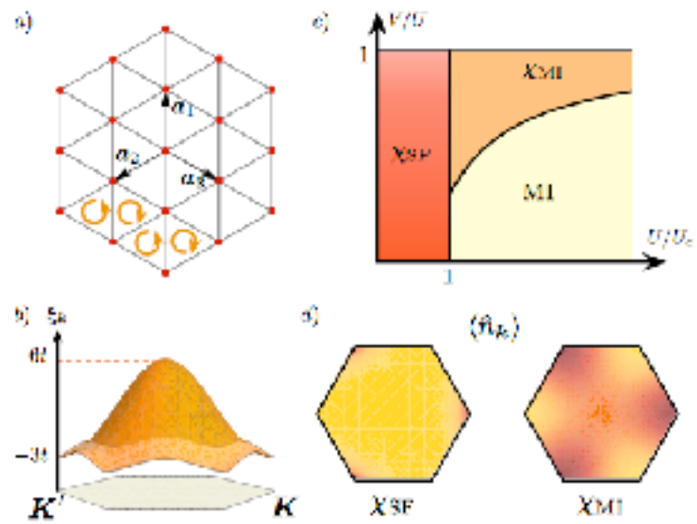


FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion ξ_k, with minima at the K, K' points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution $\langle \hat{n}_k \rangle$ for the chiral phases.

Spin-current phase = chiral Mott insulator

PHYSICAL REVIEW B **89**, 155142 (2014)

Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel,[1] S. A. Parameswaran,[1,2] Andreas Rüegg,[1,3] and Ehud Altman[1,4]

gapped single particles;
but
spontaneously broken time-reversal
= spontaneous circulating
currents

Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

- Spin-1 model with featureless Mott ground state at large D > 0 [$S_r^z = 0$]
- Triangular lattice: two-fold degenerate spectrum, at +Q and -Q

$$H = \sum_{\langle r,r' \rangle} J(S_r^x S_{r'}^x + S_r^y S_{r'}^y + \Delta S_r^z S_{r'}^z) + D \sum_r (S_r^z)^2$$

1. Toy problem of two-spin exciton. Derive Schrodinger eqn for the pair wave function ψ

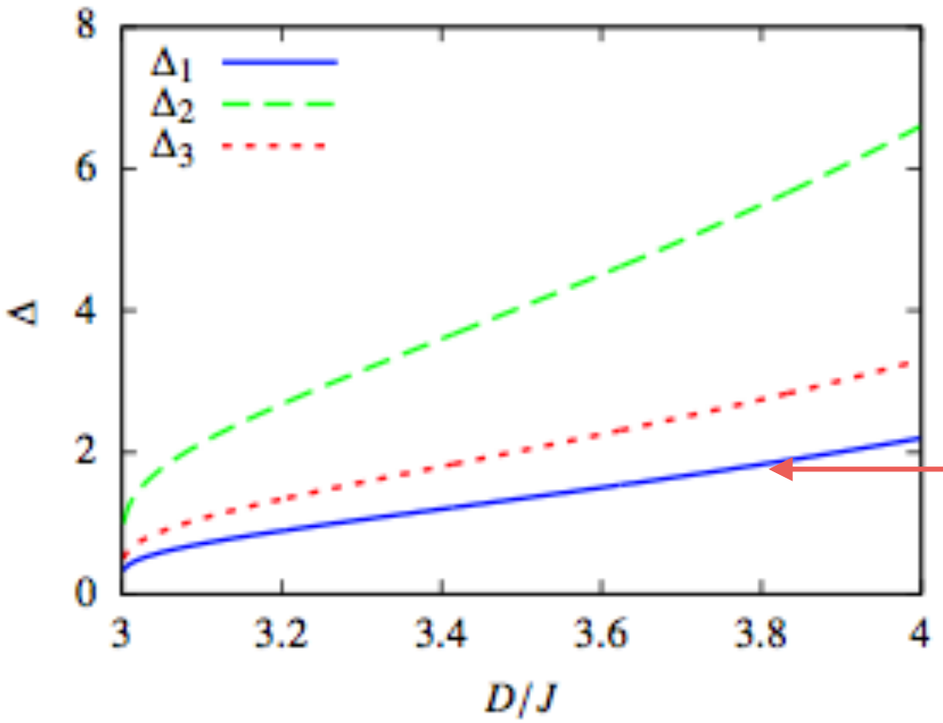
$$|\mathrm{ex}\rangle = \sum_{n \neq m} \psi_{n,m} |n,m\rangle \text{ where } |n,m\rangle = \frac{1}{2} S_n^+ S_m^- \Pi_j |0\rangle_j$$

+ charge, **-** charge

$$B_{\mathbf{g}} = \Delta J \sum_{\mathbf{g}'} M_{\mathbf{g}\mathbf{g}'} B_{\mathbf{g}'}$$

where

$$M_{\mathbf{g}\mathbf{g}'} = \frac{1}{N} \sum_{\mathbf{q}} \frac{e^{i\mathbf{q}\cdot(\mathbf{g}'-\mathbf{g})}}{4J \sum_{j=1}^{3} \cos[\frac{K_j}{2}] \cos[q_j] + 2D - E}$$





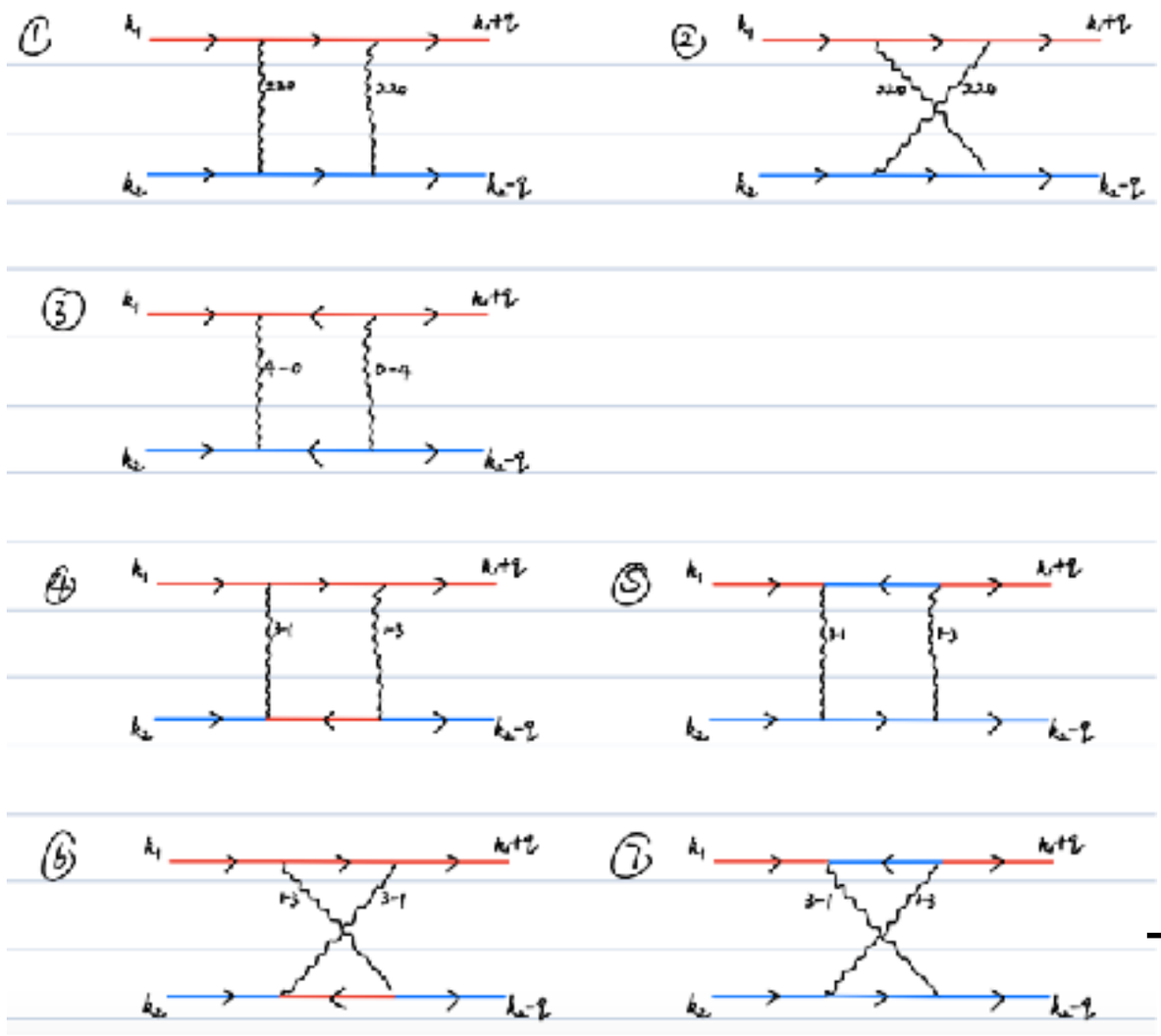
Solution which is **odd** under inversion
is the first instability when approaching from large-D limit.
Indicates chiral Mott phase.
[Single-particle condensation occurs at D=3J.]

Schwinger boson representation of S=1

$$S^z_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^z \mathbf{b}_{\mathbf{r}} = b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}\uparrow} - b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}\downarrow},$$
$$S^+_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^+ \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\downarrow}),$$
$$S^-_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^- \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\uparrow}).$$

Large-D limit: b_0 is condensed, $b_{\uparrow,\downarrow}$ are excitations about the vacuum.

Effective interaction is made of a number of 2nd order processes



+ many more…

Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

Spin-1 model with featureless Mott ground state at large D > 0
Triangular lattice

$$H = \sum_{\langle r,r' \rangle} J(S^x_r S^x_{r'} + S^y_r S^y_{r'} + \Delta S^z_r S^z_{r'}) + D \sum_r (S^z_r)^2$$

2. Solve Bethe-Salpeter eqn.

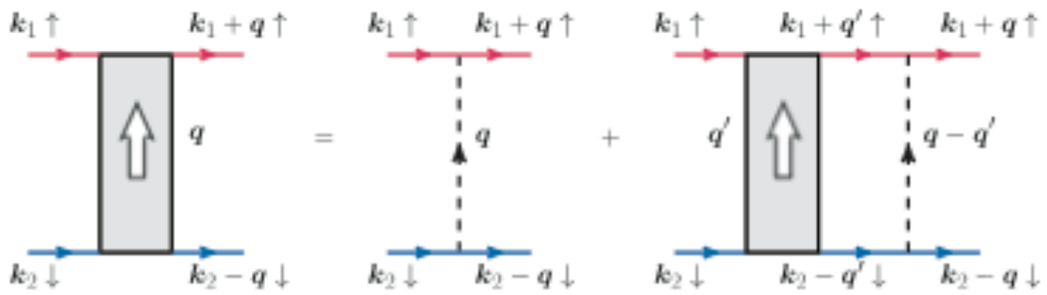
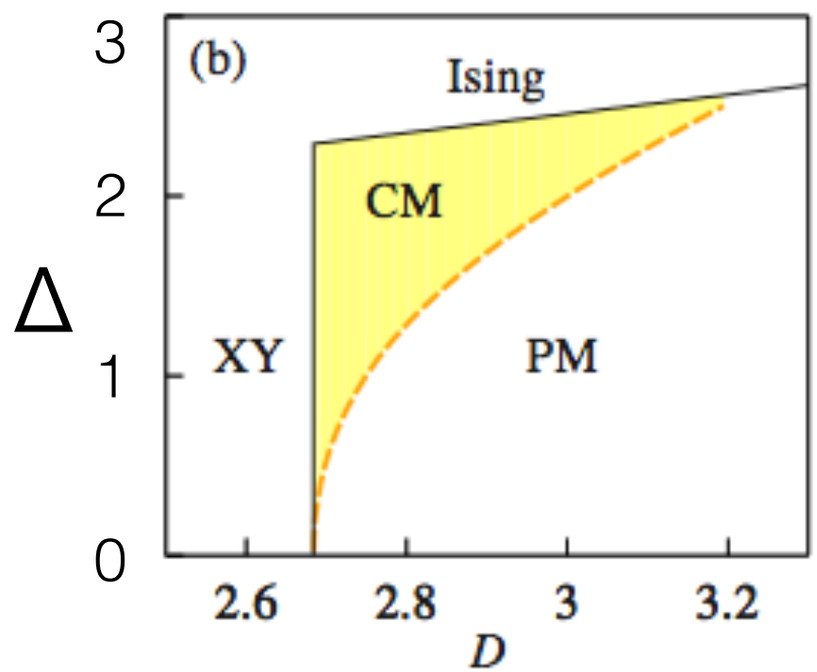


FIG. 2: The two particle propagator is obtained as a sum of ladder diagrams.

Preliminary result





Conclusions

Mott -> **superfluid** transition in frustrated lattice requires U(1) x Z_2 breaking.

This proceeds via intermediate **spin-current** (**chiral Mott**) phase (breaking Z_2 only).

Spontaneously breaks spatial inversion.

All single particle excitations are gapped.

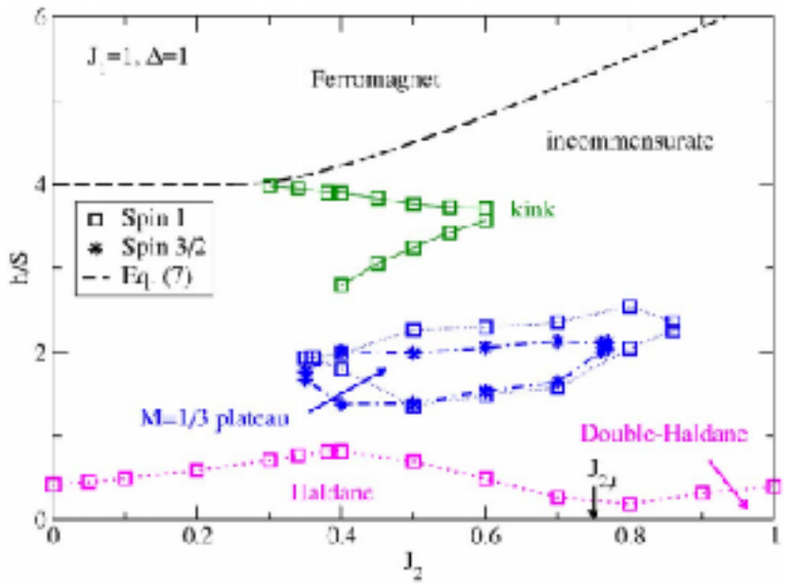
Thank you!

Side remark: J1-J2 spin-S chain

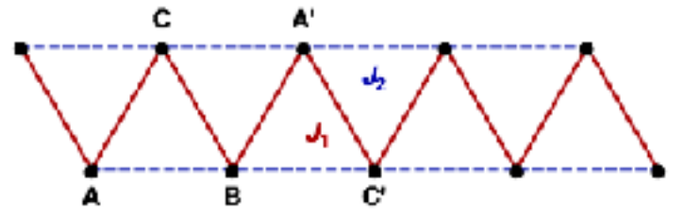
PHYSICAL REVIEW B 75, 064413 (2007)

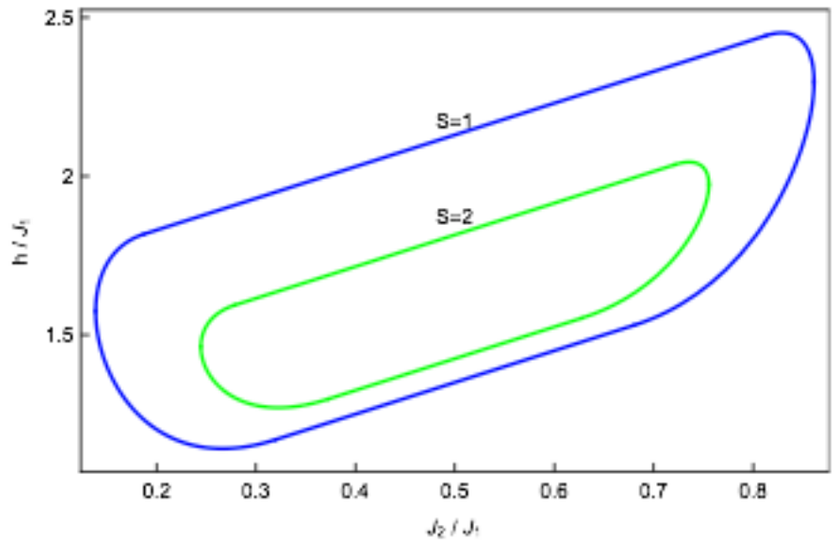
Universal emergence of the one-third plateau in the magnetization process of frustrated quantum spin chains

F. Heidrich-Meisner,[1] I. A. Sergienko,[1] A. E. Feiguin,[2] and E. R. Dagotto[1]









The end-plateau 2-magnon instability occurs at $\delta \sim 1/\sqrt{S} >> 1/S^2$

Should be checked by numerics!

Wen Jin, OAS

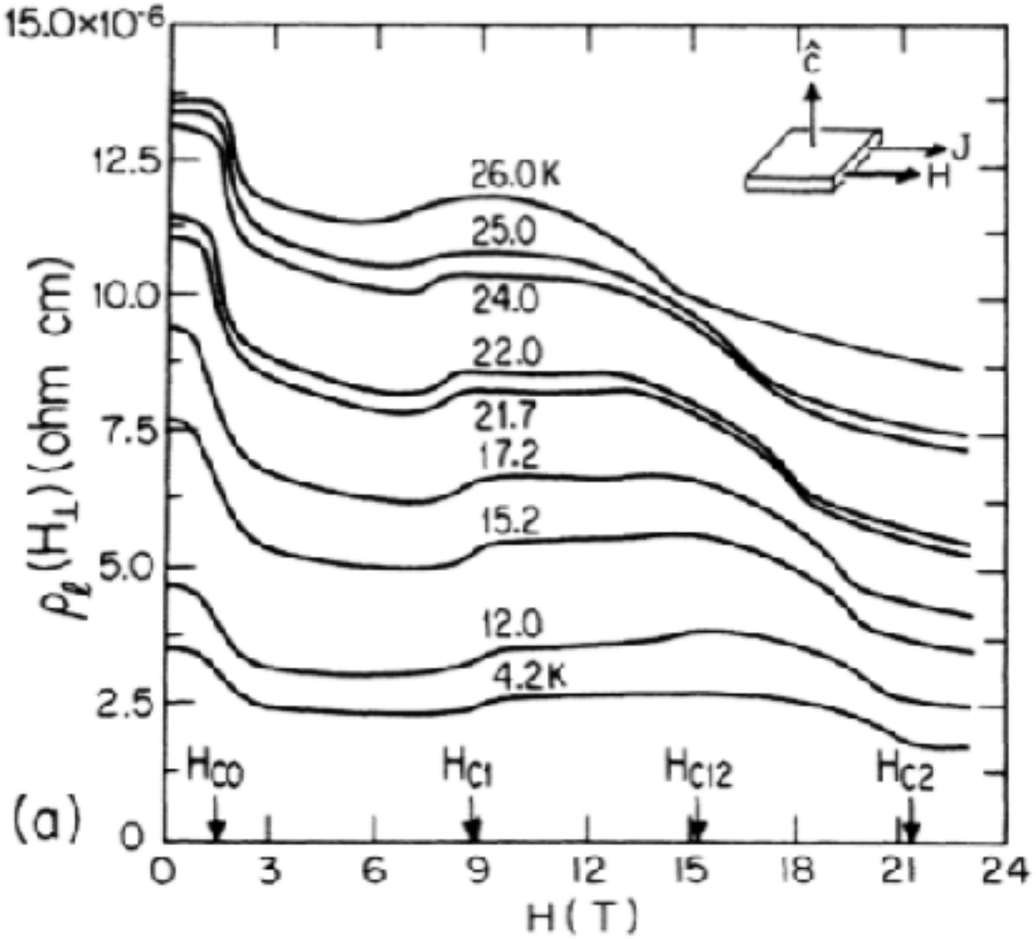


Magneto-resistivity of EuC_6

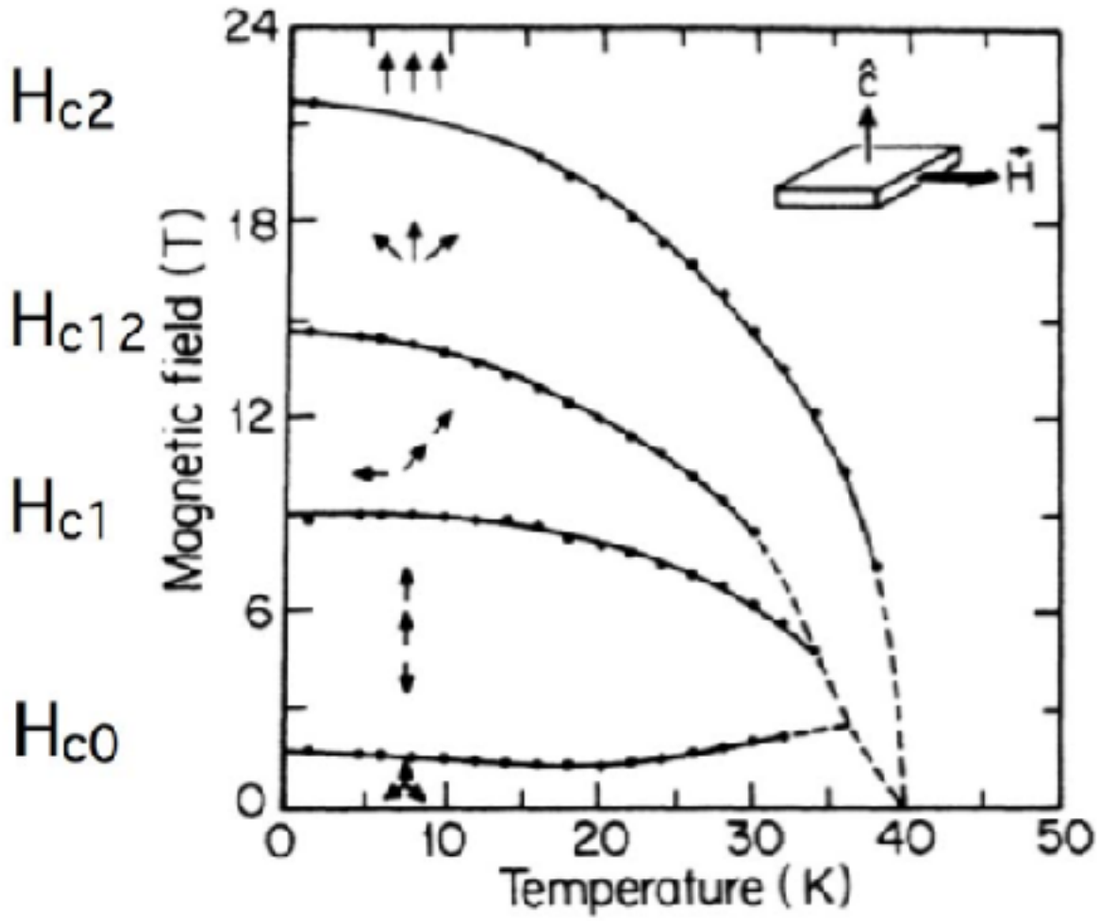


Magnetic phase transitions are detected electrically!

Magneto-resistivity



Phase Diagram



Magnetoresistivity and Monte Carlo studies of magnetic phase transitions in C_6Eu

S. T. Chen, M. S. Dresselhaus, G. Dresselhaus, H. Suematsu, H. Minemoto, K. Ohmatsu, and Y. Yosida
Phys. Rev. B **34**, 423 – Published 1 July 1986

Other systems

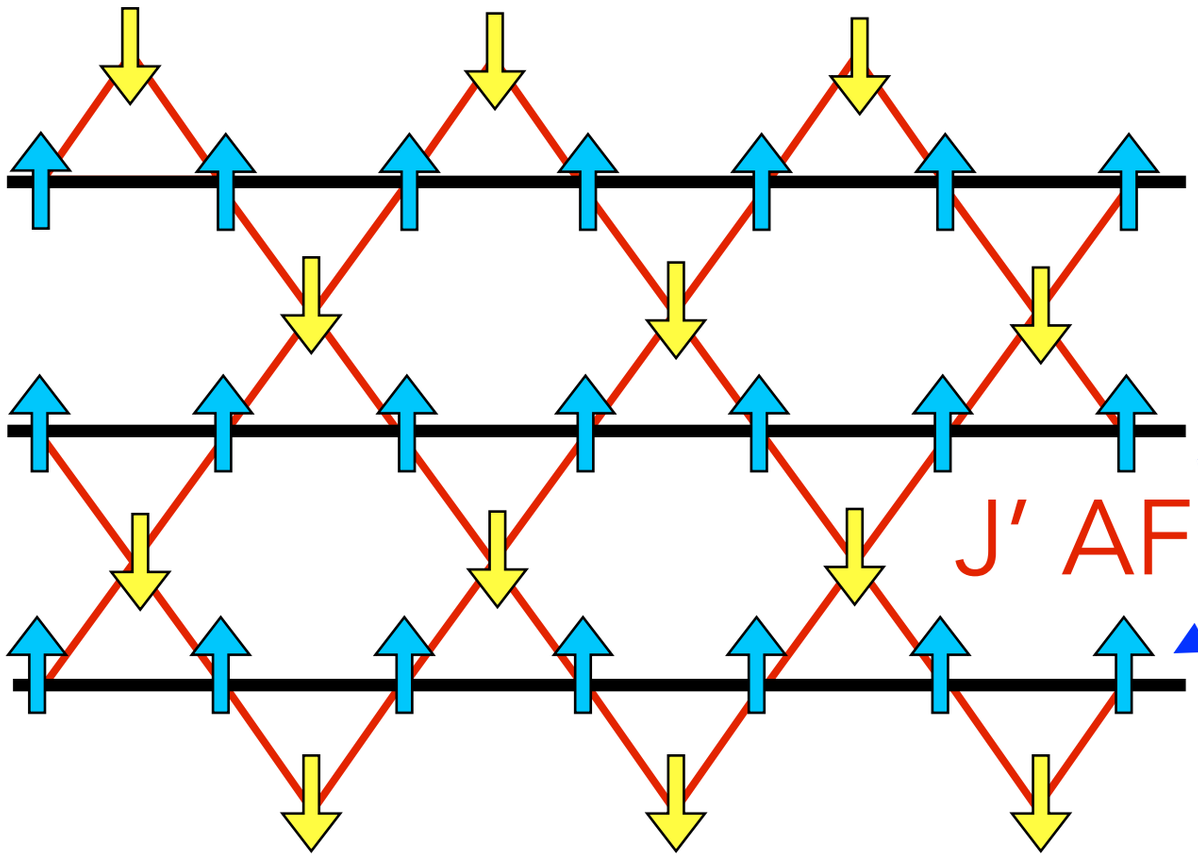
Ferrimagnetic state of kagome (anti)ferromagnet volborthite @ 1/3 magnetization plateau

PHYSICAL REVIEW B **82**, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Magdeburg, D-39016 Magdeburg, Germany*
[3]*Laboratoire de Physique Théorique de la Matière Condensée, Univ. P. & M. Curie, Paris, France*
(Received 9 August 2010; published 30 September 2010)

$$J_1 < 0, J_2 > 0, J' > 0$$

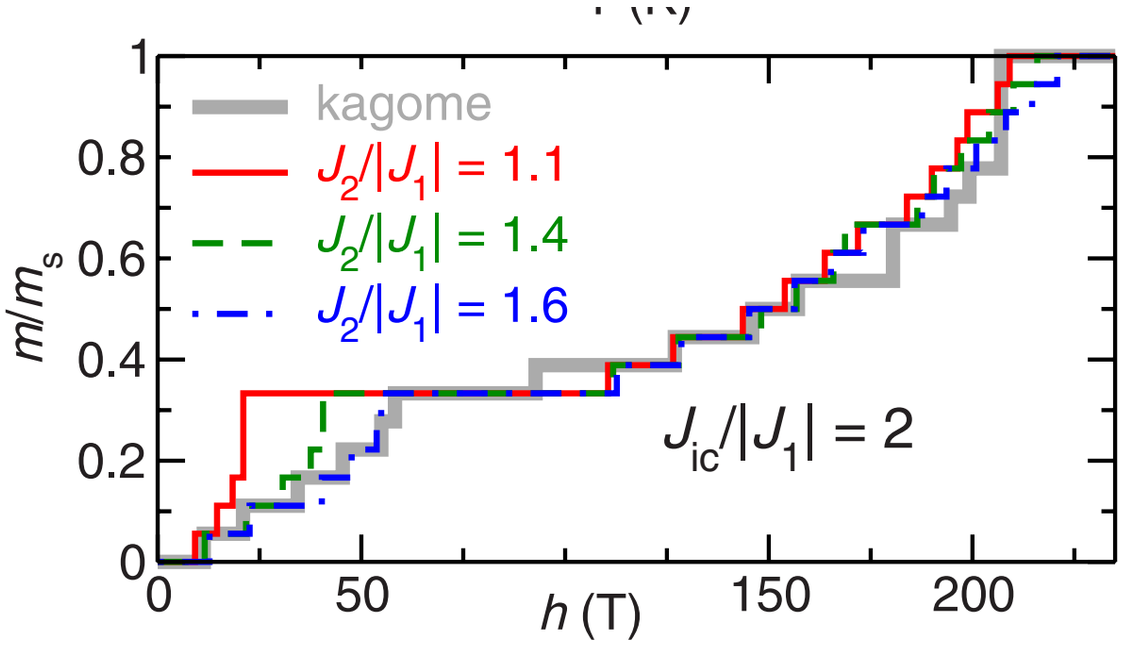


Frustrated ferromagnetism

PHYSICAL REVIEW B **82**, 104434 (2010)

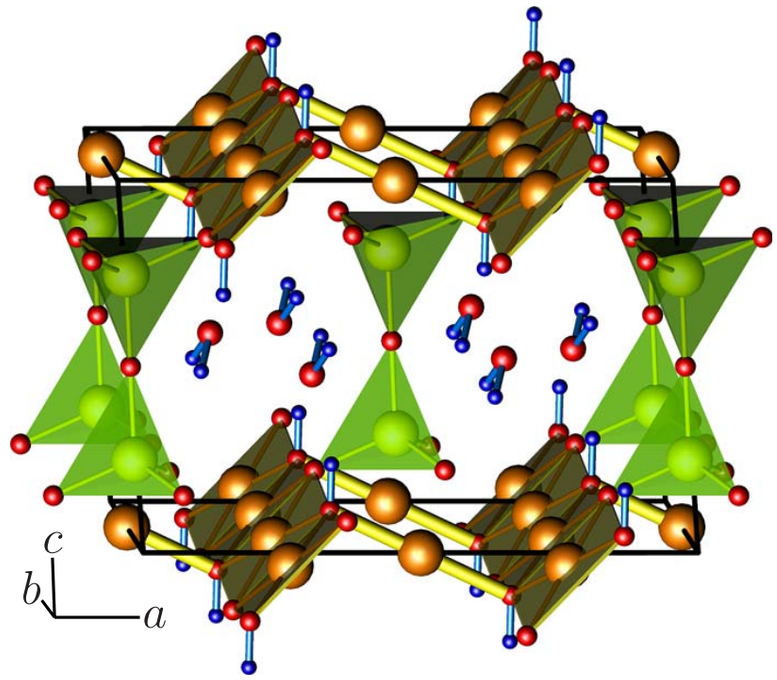
Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

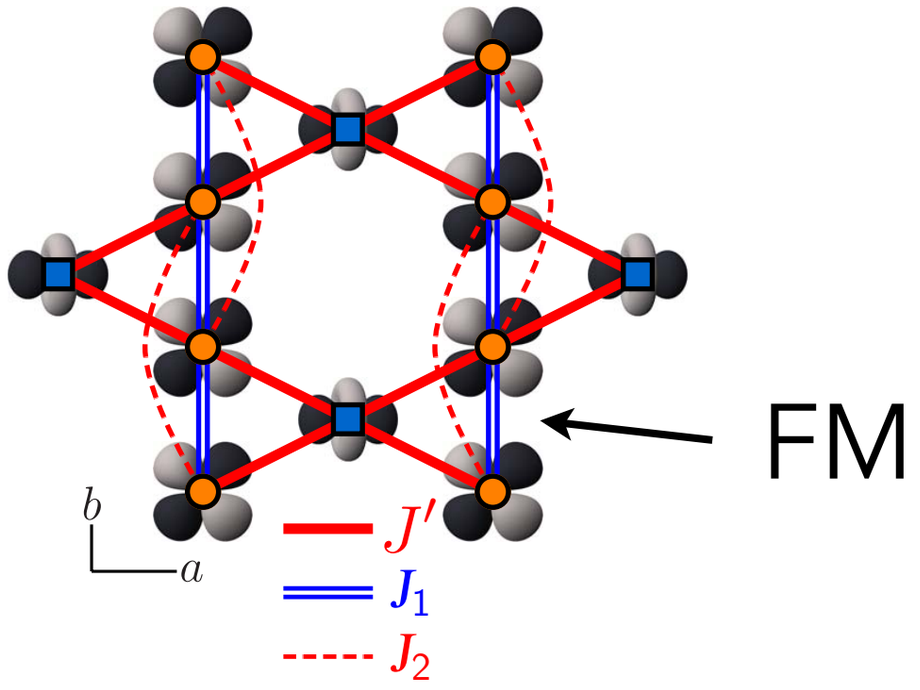
O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Magdeburg, D-39016 Magdeburg, Germany*
[3]*Laboratoire de Physique Théorique de la Matière Condensée, Univ. P. & M. Curie, Paris, France*
(Received 9 August 2010; published 30 September 2010)



DFT gets it right!

$$J_1 < 0, J_2 > 0, J' > 0$$





Other systems

Semiclassical analysis of a magnetization plateau in a 2D frustrated ferrimagnet

Edward Parker*
Department of Physics, University of California, Santa Barbara, CA 93106

Leon Balents
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106

(Dated: November 9, 2016)

arXiv:1610.03135

Kagome geometry

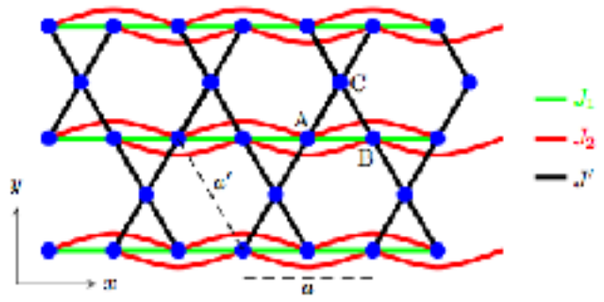


FIG. 1. Proposed Hamiltonian for volborthite. The blue dots represent spin-1/2 copper ions and the line segments represent Heisenberg couplings. $J_1 < 0$ is ferromagnetic while $J_2 > 0$ and $J' > 0$ are antiferromagnetic. The distances between adjacent unit cells is slightly anisotropic, with $a = 5.84$ Å and $a' = 6.07$ Å [10]. Capital letters label the sublattices.

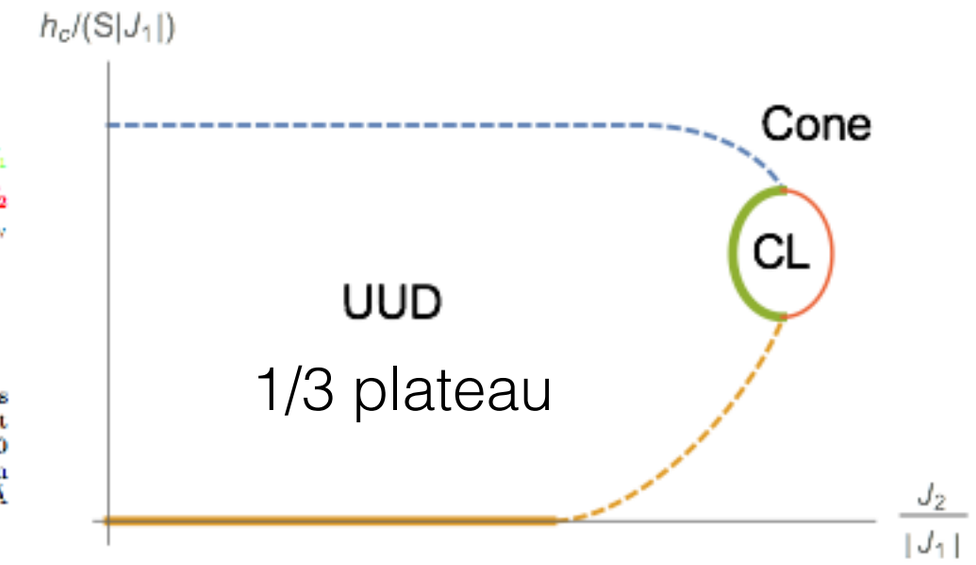


FIG. 6. Schematic quantum phase diagram at $J' = 0.5|J_1|$. The UUD state breaks no symmetries, the gapped chiral liquid (CL) phase only breaks chiral symmetry, and the gapless cone state breaks both chiral and a $U(1)$ symmetry combining translation and spin rotation. The thick solid lines and dashed lines represent first- and second-order transitions respectively. We did not investigate the nature of the transition between the chiral liquid and cone phases represented by the red line. In a 3D phase diagram like that of Fig. 4 that includes the applied field, the chiral liquid phase would appear as a thin tube around the stabilization curve where the two sheets meet.

Spin-current pattern

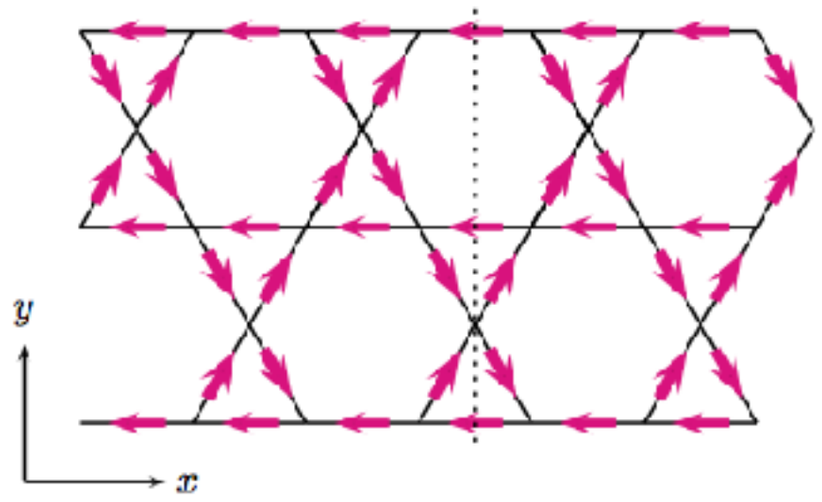


FIG. 7. Spin-current configuration in the chiral liquid phase. The magenta arrows indicating the direction of spin current flow; in the orthogonal ground state the flow is reversed. The spin current on the diagonal bonds, represented by thicker arrows, is larger by a factor of $\sqrt{2}$ and determines the net current flow. The ground state is chiral and spontaneously breaks the lattice symmetry of reflection about the dotted line.