Frustration-driven multi-magnon condensates and their excitations

Oleg Starykh, University of Utah

Leon Balents, KITP, UCSB



Solid State Seminar, ETH Zurich, September 19, 2016

Outline

- Motivation: usual *vs* unusual ordered states
  - emergence of composite orders from competing interactions
- Frustrated ferromagnets and magnon binding
- $LiCuVO_4$
  - ✓ spin nematic: “magnon superconductor”
  - ✓ collinear SDW: “magnon charge density wave”
- Volborthite kagome antiferromagnet
  - experimental status - magnetization plateau
  - UUD, SDW and ***more***

✴ *Field theory of the Lifshitz point*

General motivation: **Exotic** *but* ordered phases



composite order parameter

$$\mathcal{O}^{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j) = \tfrac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \tfrac{1}{3}\delta^{\alpha\beta}\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

LE JOURNAL DE PHYSIQUE TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

J. VILLAIN

Laboratoire de Diffraction Neutronique, Département de Recherche Fondamentale, Centre d'Etudes Nucléaires de Grenoble, 85 X, 38041 Grenoble Cedex, France

(*Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977*)

Spin nematics

A. F. Andreev and I. A. Grishchuk

Institute of Physics Problems, USSR Academy of Sciences
(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

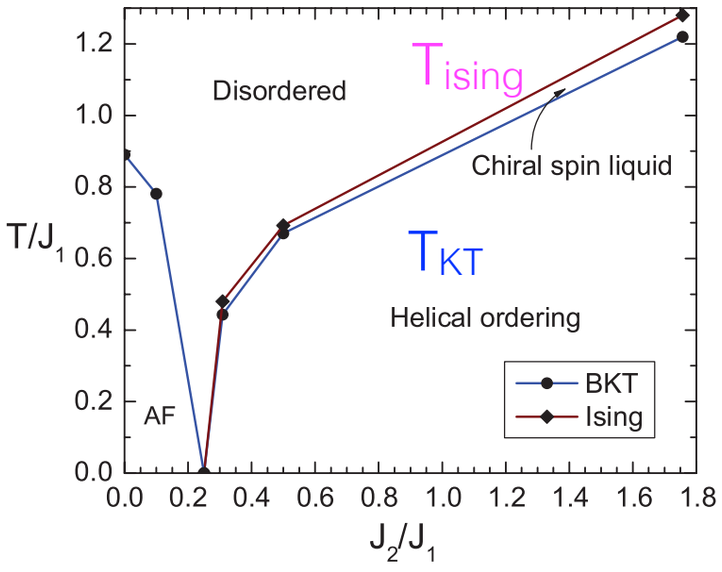
Emergent Ising order parameters

PHYSICAL REVIEW B **85**, 174404 (2012)

Chiral spin liquid in two-dimensional XY helimagnets

A. O. Sorokin[1,*] and A. V. Syromyatnikov[1,2,†]

$$H = \sum_{\mathbf{x}} (J_1 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{a}}) + J_2 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+2\mathbf{a}}) - J_b \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{b}})),$$



Ising order: spin chirality

$$\chi = \sum_{\text{triangle}} \vec{S}_i \times \vec{S}_j$$

Villain 1977

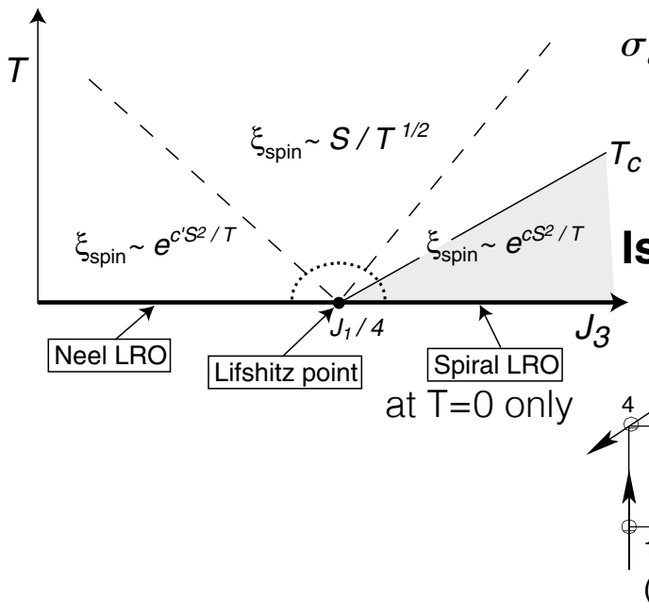
PRL **93,** 257206 (2004) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2004

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

Luca Capriotti[1,2] and Subir Sachdev[2,3]

$$\hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_3 \sum_{\langle\langle i,j \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$

$$\sigma_a = (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_3 - \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{S}}_4)_a,$$



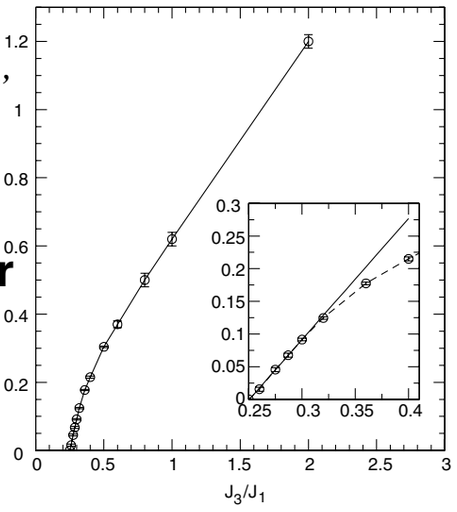


FIG. 2. The two different minimum energy configurations with magnetic wave vectors $\vec{Q} = (Q, Q)$ and $\vec{Q}^{\star} = (Q, -Q)$ with $Q = 2\pi/3$, corresponding to $J_3/J_1 = 0.5$.

Ising nematic in collinear spin system

VOLUME 64, NUMBER 1 PHYSICAL REVIEW LETTERS 1 JANUARY 1990

Ising Transition in Frustrated Heisenberg Models

P. Chandra
Corporate Research Science Laboratories, Exxon Research and Engineering Company, Annandale, New Jersey 08801

P. Coleman and A. I. Larkin [a]
Serin Physics Laboratory, Rutgers University, P.O. Box 849, Piscataway, New Jersey 08854

$$\sigma = \vec{N}_1 \cdot \vec{N}_2 = \pm 1$$

VOLUME 91, NUMBER 17 PHYSICAL REVIEW LETTERS week ending 24 OCTOBER 2003

Ising Transition Driven by Frustration in a 2D Classical Model with Continuous Symmetry

Cédric Weber,[1,2] Luca Capriotti,[3] Grégoire Misguich,[4] Federico Becca,[5] Maged Elhajal,[1] and Frédéric Mila[1]

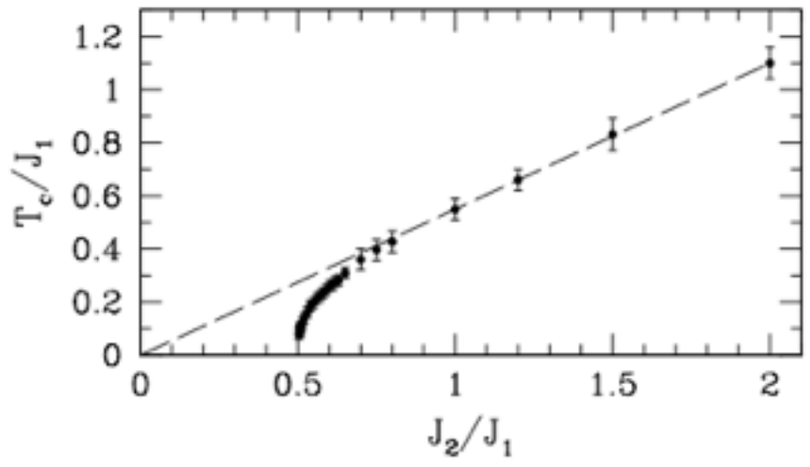


FIG. 4. Monte Carlo results for the critical temperature T_c as a function of the frustrating ratio J_2/J_1. The line is an extrapolation of the large J_2 data down to $J_2 = 0$ (see text).

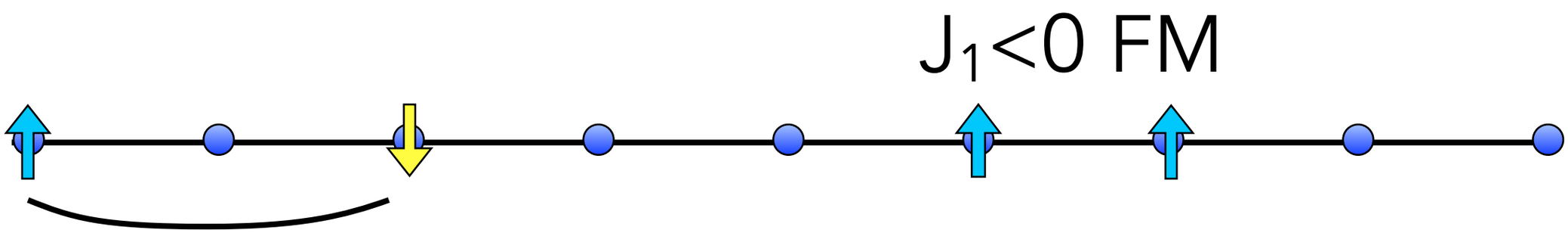
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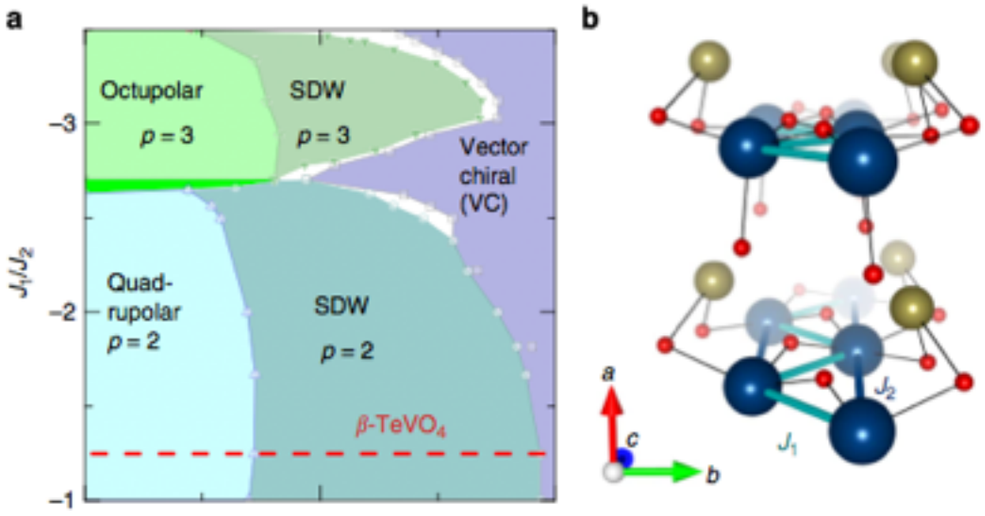
New system: Frustrated ferromagnet

1d S=1/2 chain



J_2>0 AF

$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



$\beta - \mathrm{TeVO_4}$

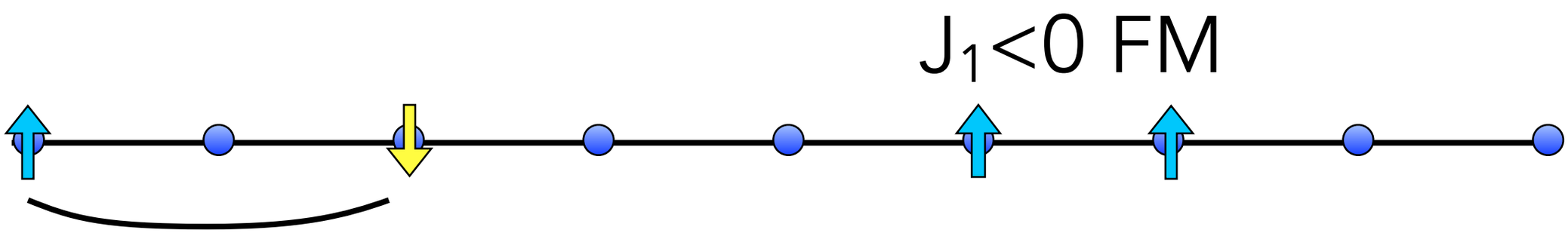
Pregelj *et al.*, Nat.Comm.2015

Compound	J_1, J_2 (K)	∠ Cu-O-Cu (deg)	T_N (K)	H_s (T)
Li_2ZrCuO_4[12, 13]	−151, 35	94.1	6.4	-
$Rb_2Cu_2Mo_3O_{12}$[14, 15]	−138, 51	89.9, 101.8	< 2	14
		91.9, 101.1		
$PbCuSO_4(OH)_2$[16–18]	−100, 36	91.2, 94.3	2.8	5.4
$LiCuSbO_4$[19]	−75, 34	89.8, 95.0	< 0.1	12
		92.0, 96.8		
$LiCu_2O_2$[20–22]	−69, 43	92.2, 92.5	22.3	110
$LiCuVO_4$[23–31]	−19, 44	95.0	2.1	44.4
$NaCuMoO_4(OH)$	−51, 36	92.0, 103.6	0.59	26

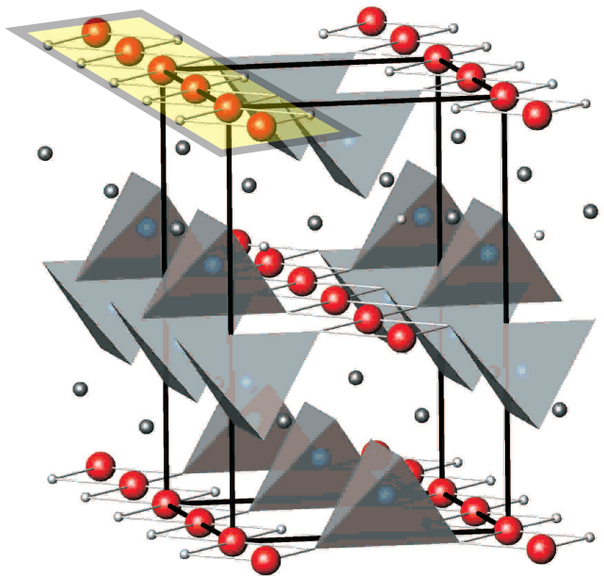
K. Nawa *et al*, arXiv:1409.1310

New system: Frustrated ferromagnet

1d S=1/2 chain



J_2>0 AF



$LiCuVO_4$

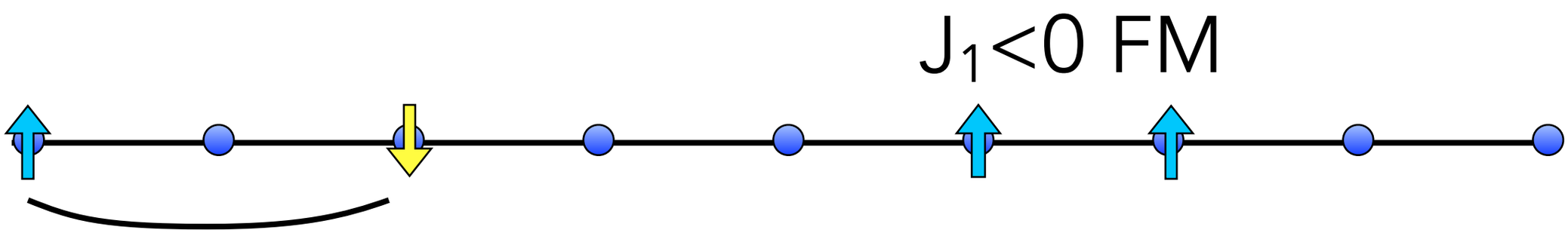
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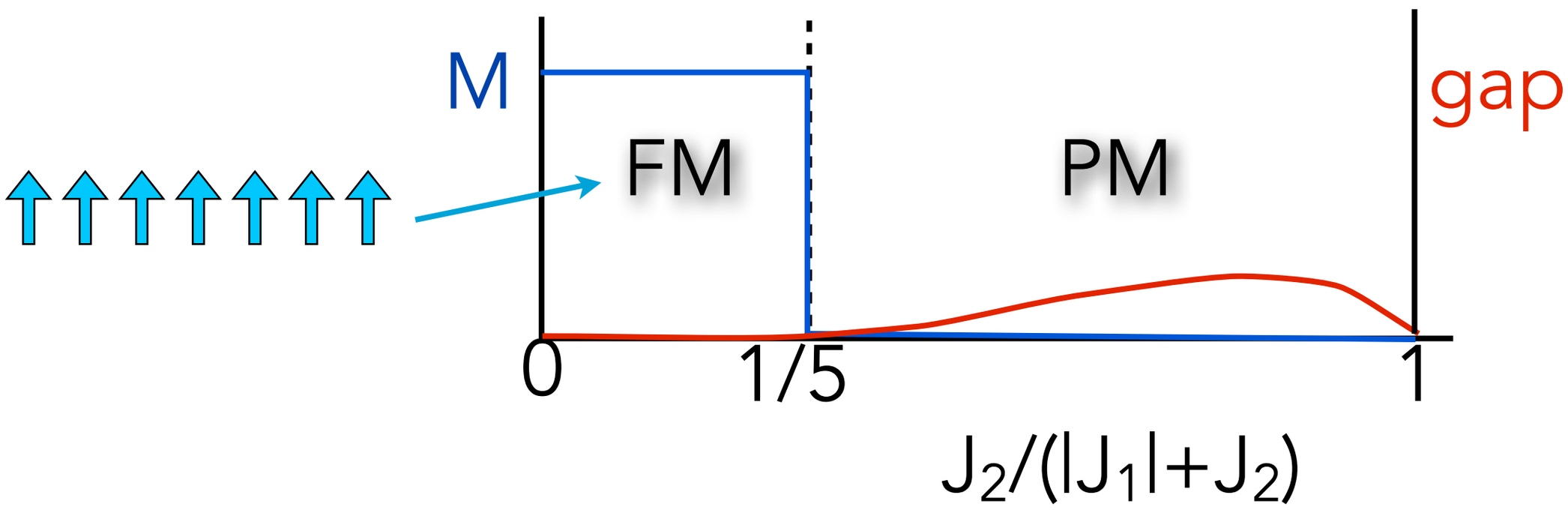
K. Nawa *et al*, arXiv:1409.1310

Frustrated ferromagnet

1d spin-S chain

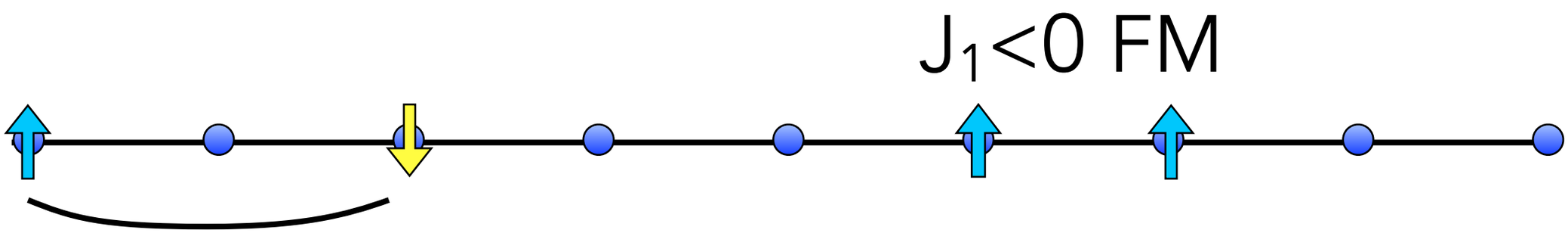


$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

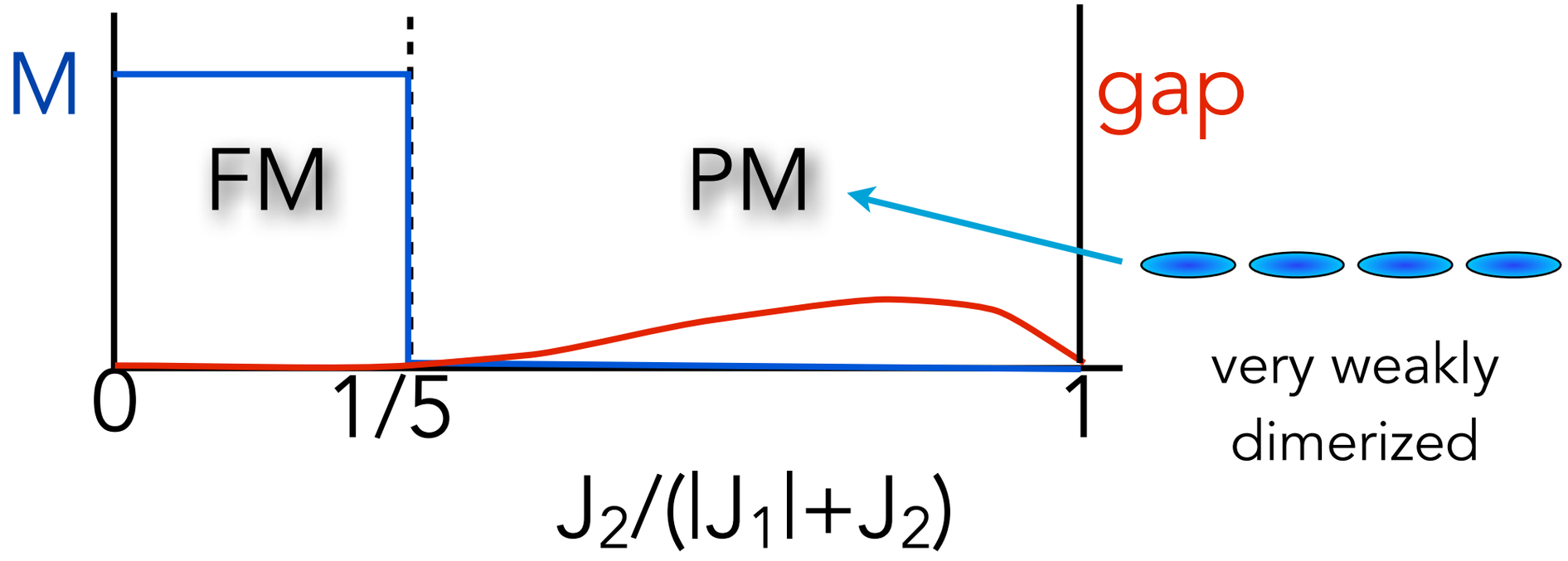


Frustrated ferromagnet

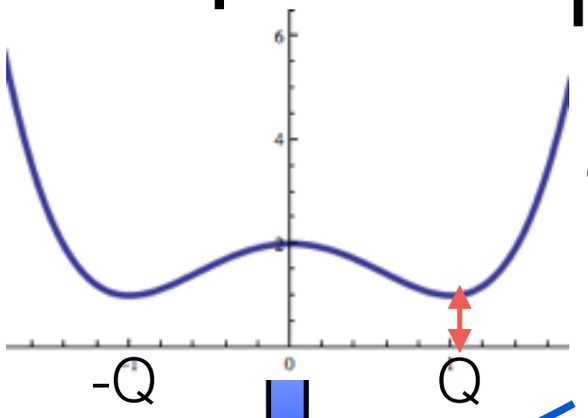
1d S=1/2 chain



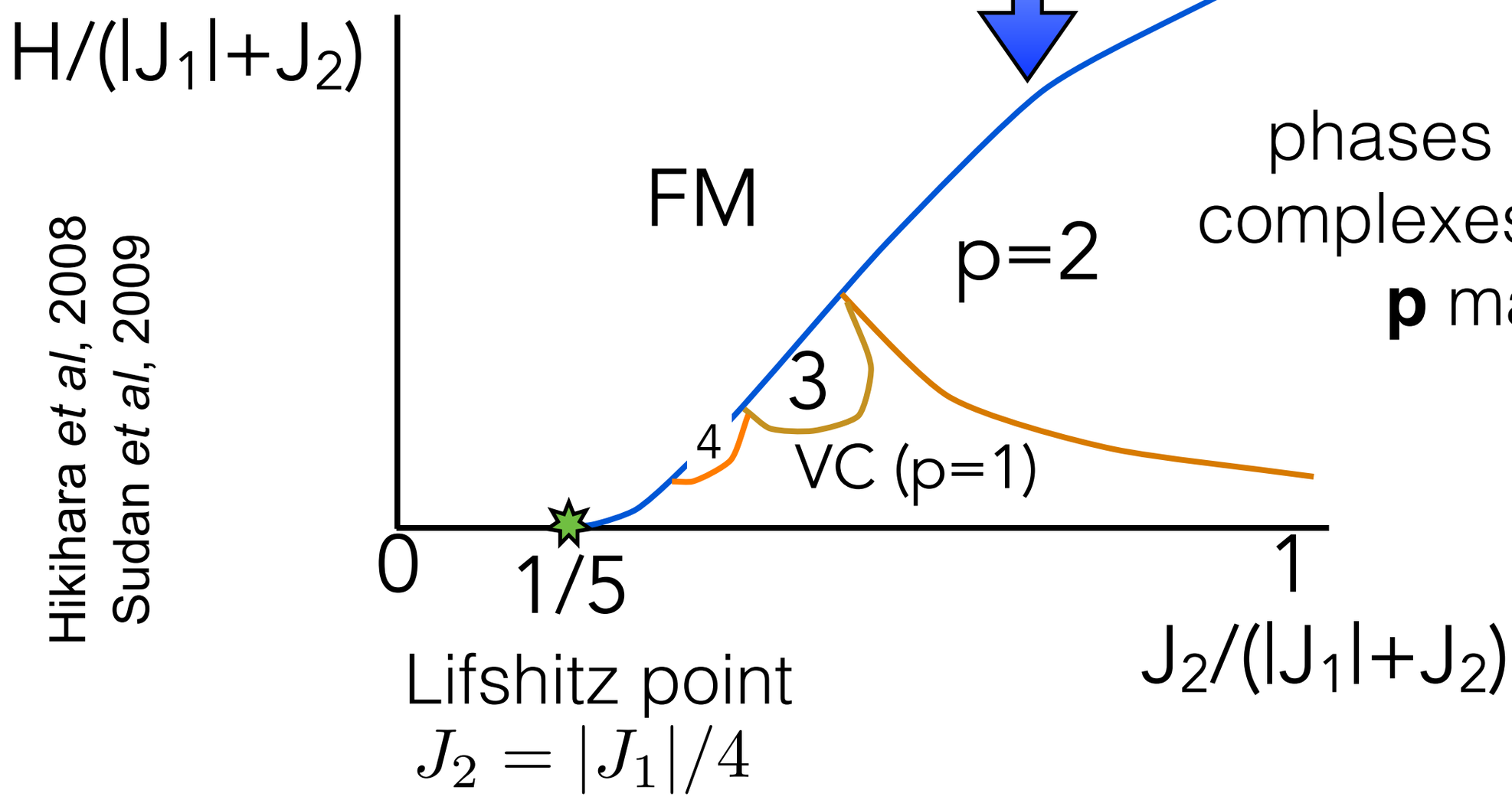
$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



Approaching from fully polarized state — Multipolar phases



$$\omega \sim (k^2 - Q^2)^2 - (h_{\rm sat} - h)$$



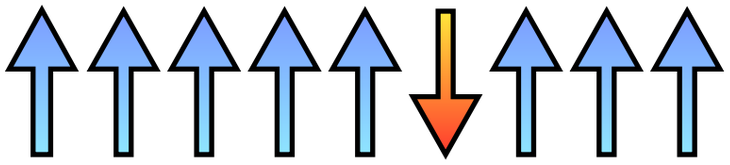
phases with bound complexes made out of **p** magnons

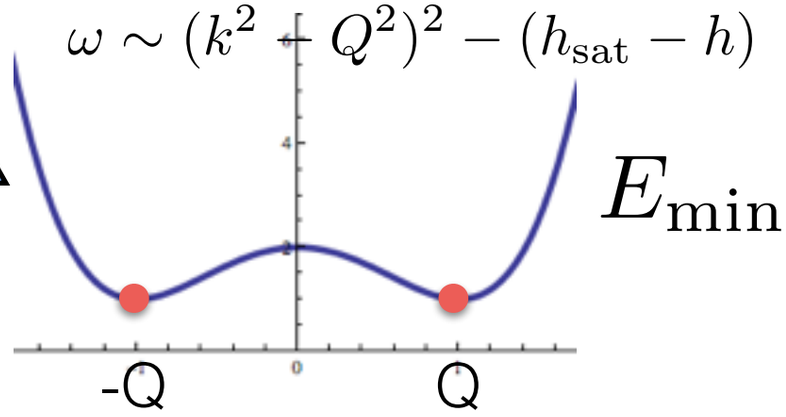
Lifshitz point

$J_2 = |J_1|/4$

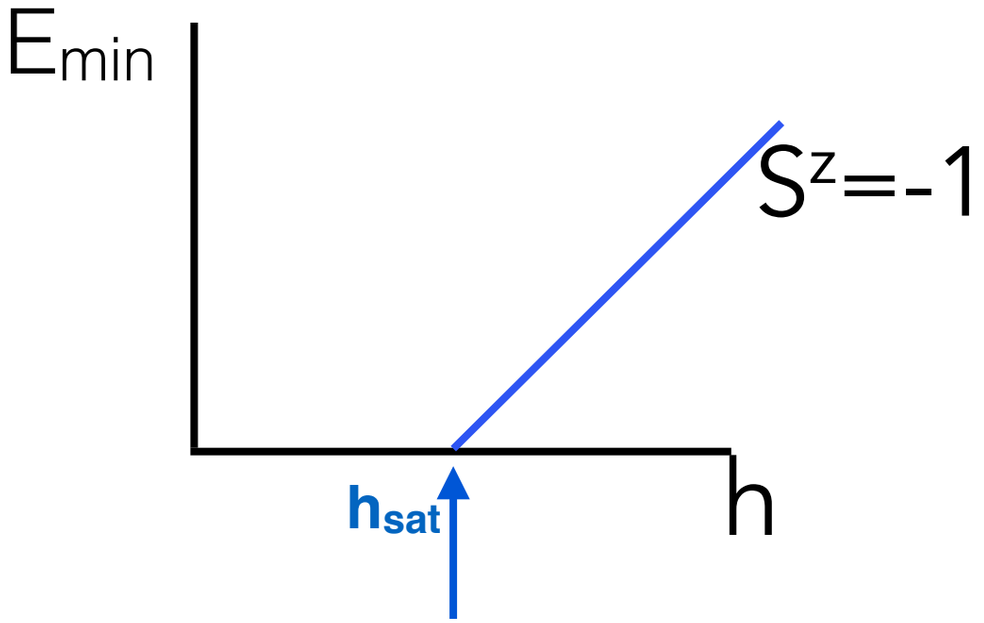
Magnon BEC

1-magnon





$$E_{\rm min} = \epsilon_1(k) + h$$



Condensation:

$$\langle a_k^+ \rangle = \sqrt{N}\ \Psi_Q\ \delta_{k,Q}$$

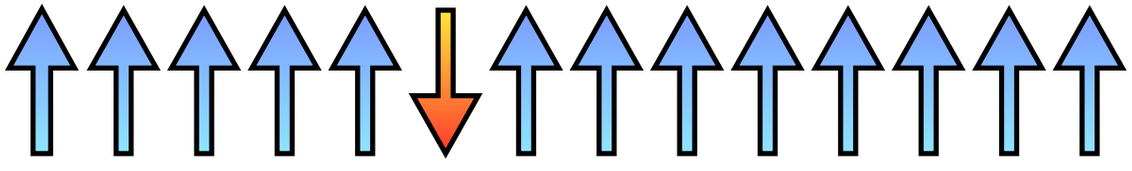
$$\langle S_n^- \rangle \sim \Psi_Q\ e^{iQx_n}$$

Magnon (quasi-)BEC

$$h_{\rm sat} = \frac{S(4J_2 - |J_1|)^2}{4J_2}$$

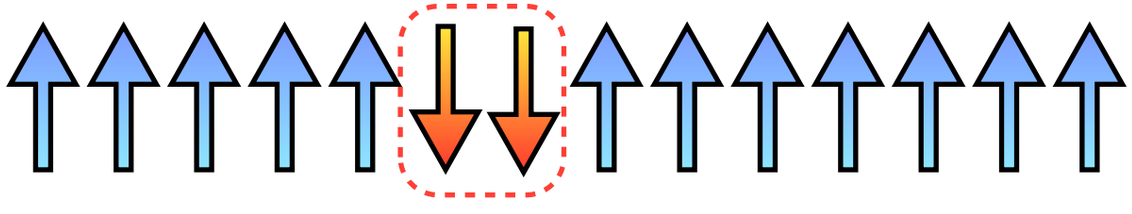
Today: condensation of *magnon pairs*

1-magnon

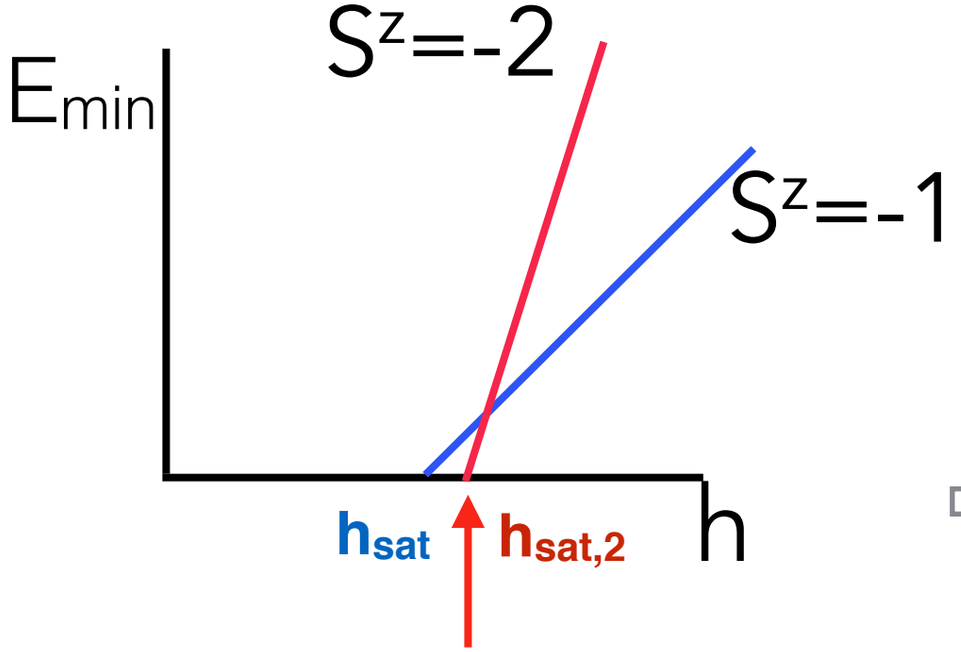


$$E_{\min} = \epsilon_1(k) + h$$

2-magnon bound state



$$E_{\min} = \Omega_2(k) + 2h$$



"molecular" bound state

$$\mathrm{Min}\{\Omega_2\} = 2\epsilon_1(Q) - \Delta_{\mathrm{bound}}$$

$$\Rightarrow h_{\mathrm{sat},2} = h_{\mathrm{sat}} + \Delta_{\mathrm{bound}}/2$$

Formation of molecular fluid

For d>1 at T=0 this is a molecular BEC

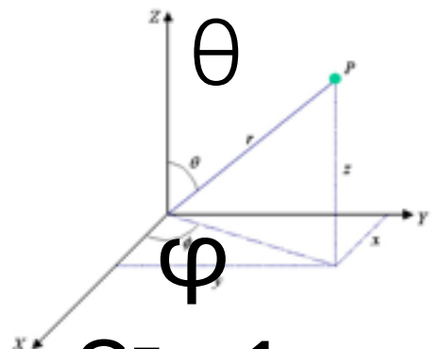
= true spin nematic

Hidden order

No dipolar order

$$\langle S_n^- \rangle = 0$$

$$\langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi}$$



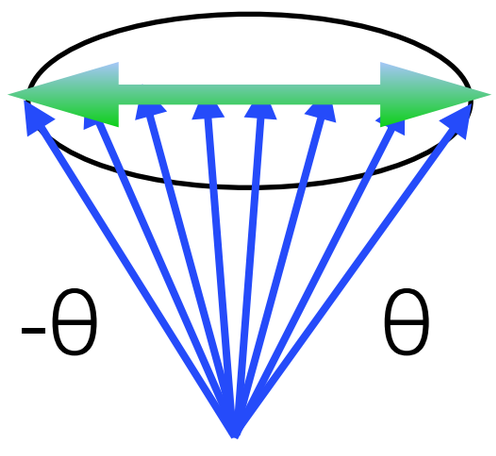
S^z=1 gap

Nematic order

$$\langle S_n^- S_{n+a}^- \rangle = \Phi \neq 0$$

$$\langle S_n^- S_m^- \rangle = \langle S_n^x S_m^x - S_n^y S_m^y - i(S_n^x S_m^y + S_n^y S_m^x) \rangle \sim \sin^2\theta(\cos 2\varphi - i \sin 2\varphi)$$

nematic director



Magnetic quadrupole moment

Symmetry breaking U(1) → Z_2

think of a fluctuating fan state:

φ is constant, while θ fluctuates

in the interval (θ_0, - θ_0)

PHYSICAL REVIEW B **74**, 020403(R) (2006)

Frustrated ferromagnetic spin-$\frac{1}{2}$ chain in a magnetic field: The phase diagram and thermodynamic properties

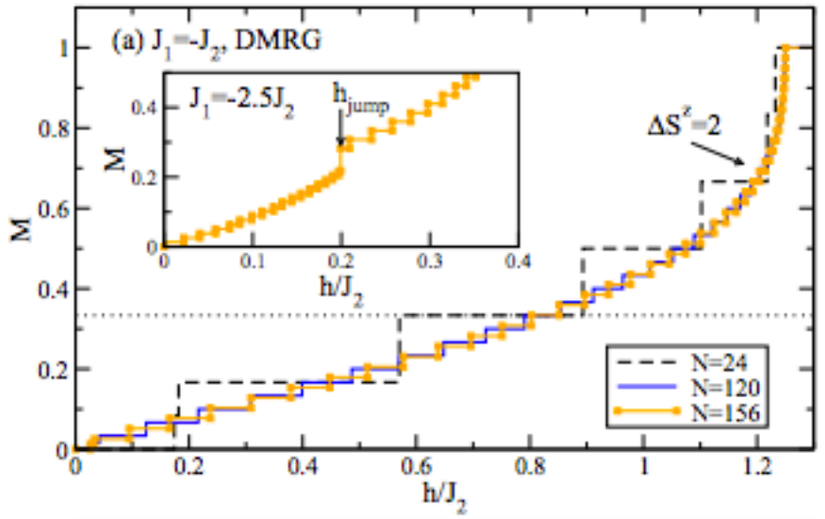
F. Heidrich-Meisner,[1,2] A. Honecker,[3,4] and T. Vekua[5,6]

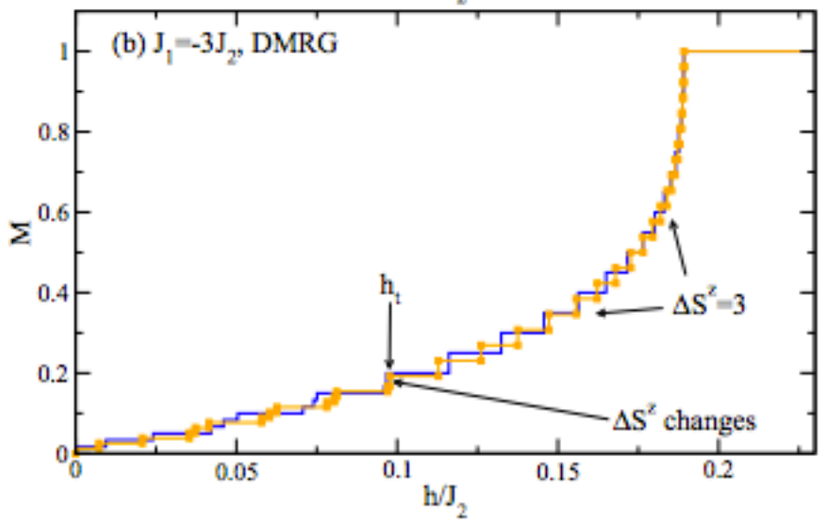
We compute the ground-state energies $E_0(S^z_{\text{total}},h=0)$ in subspaces labeled by S^z_{total} on chains with periodic boundary conditions (PBC) using the Lanczos algorithm. The ground-state energies of substantially larger chains with open boundary conditions (OBC) are calculated with DMRG. Typically, we keep up to $m=400$ states in our DMRG calculations. Then, we include the Zeeman term and obtain the field-dependent ground-state energies

$$E_0(S^z_{\text{total}},h) = E_0(S^z_{\text{total}},h=0) - hS^z_{\text{total}}. \qquad (4)$$

The magnetization curves are constructed by solving the equations $E_0(S^z_{\text{total}},h_{\text{step}})=E_0(S^z_{\text{total}}+s,h_{\text{step}})$, which define those magnetic fields at which the magnetization increases from $M=S^z_{\text{total}}/(N\,S)$ to $M'=(S^z_{\text{total}}+s)/(N\,S)$. Steps larger than $s=1$ may occur.

binding of magnons is easily seen
in the magnetization curve of finite size system:
magnetization changes in steps of **2**
(instead of usual **1**)





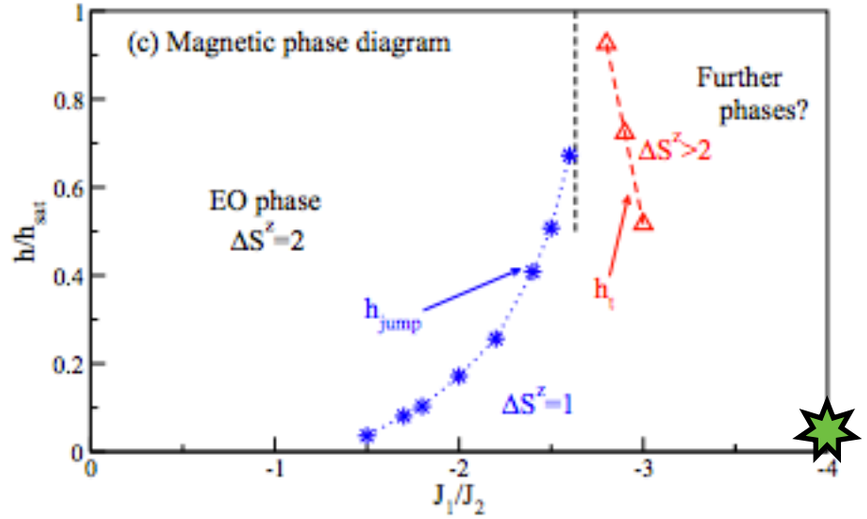


FIG. 1: (Color online) (a), main panel (inset): Magnetization curve $M(h)$ for $J_1 = -J_2$ ($J_1 = -2.5\,J_2$). The horizontal dotted line marks $M = 1/3$. (b): $M(h)$ for $J_1 = -3\,J_2$. (c): Magnetic phase diagram of the frustrated FM chain. The dotted line (with stars) marks the first-order transition between the EO phase and the $\Delta S^z = 1$ region, while the line $h = h_t$ (dashed, triangles) separates the $\Delta S^z = 1$ region from the $\Delta S^z \geq 3$ part. Uncertainties of the transition lines, e.g. due

PHYSICAL REVIEW B **78**, 144404 (2008)

Vector chiral and multipolar orders in the spin-$\frac{1}{2}$ frustrated ferromagnetic chain in magnetic field

Toshiya Hikihara,[1] Lars Kecke,[2,3] Tsutomu Momoi,[2] and Akira Furusaki[2]

s=1/2

magnetization steps of **2, 3** and **4** have been observed (instead of usual **1**)

TABLE I. Number of magnons p and total momentum k of the multimagnon bound states which become gapless at the saturation field.

Parameter range	p	k
$-2.669<J_1/J_2<0$	2	π
$-2.720<J_1/J_2<-2.669$	2	$\pi\pm\delta$ $(\delta>0)$
$-3.514<J_1/J_2<-2.720$	3	π
$-3.764<J_1/J_2<-3.514$	4	π
$-3.888<J_1/J_2<-3.764$	5	π
$-3.917<J_1/J_2<-3.888$	6	π
$-4<J_1/J_2<-3.917$	7	π

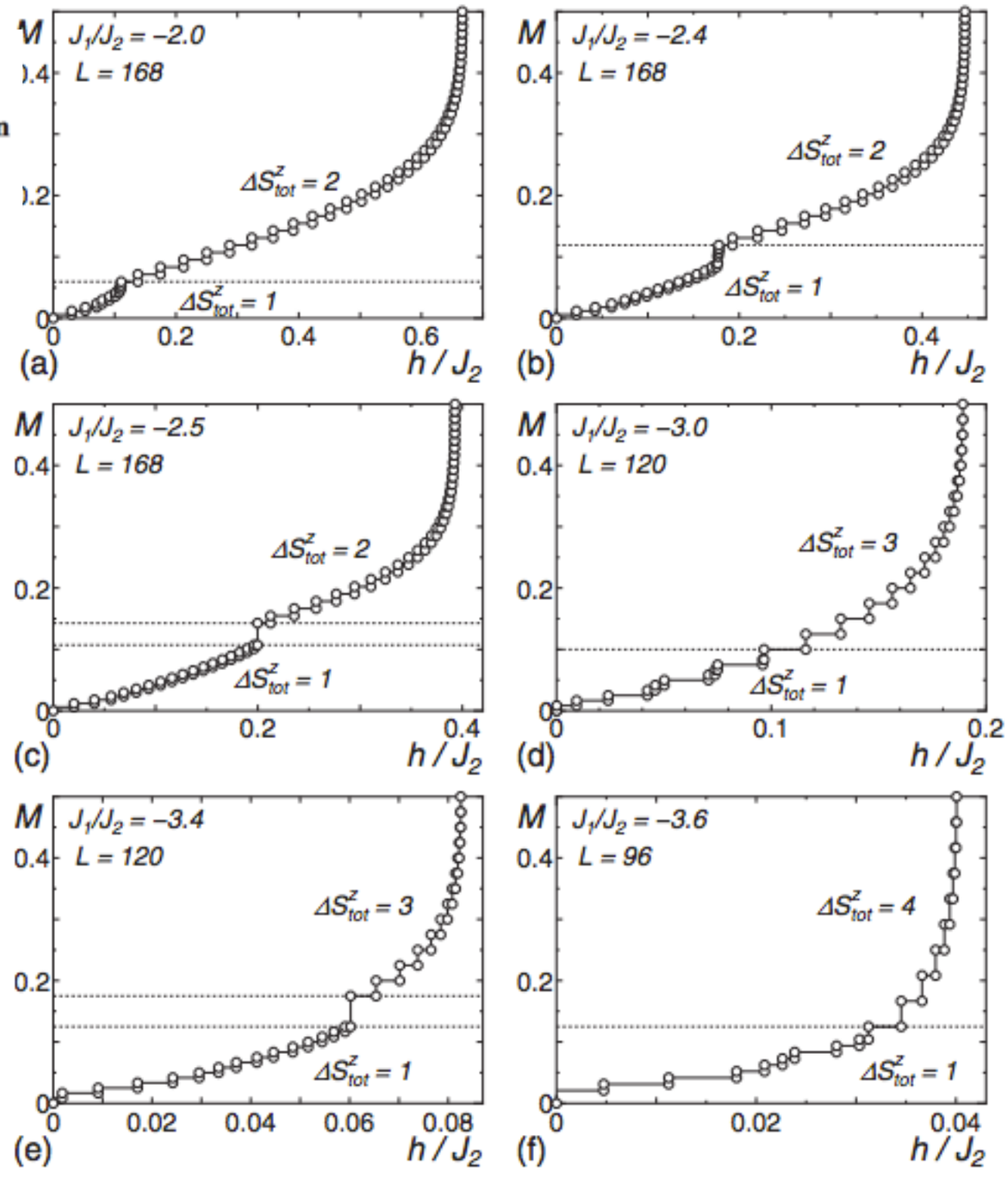


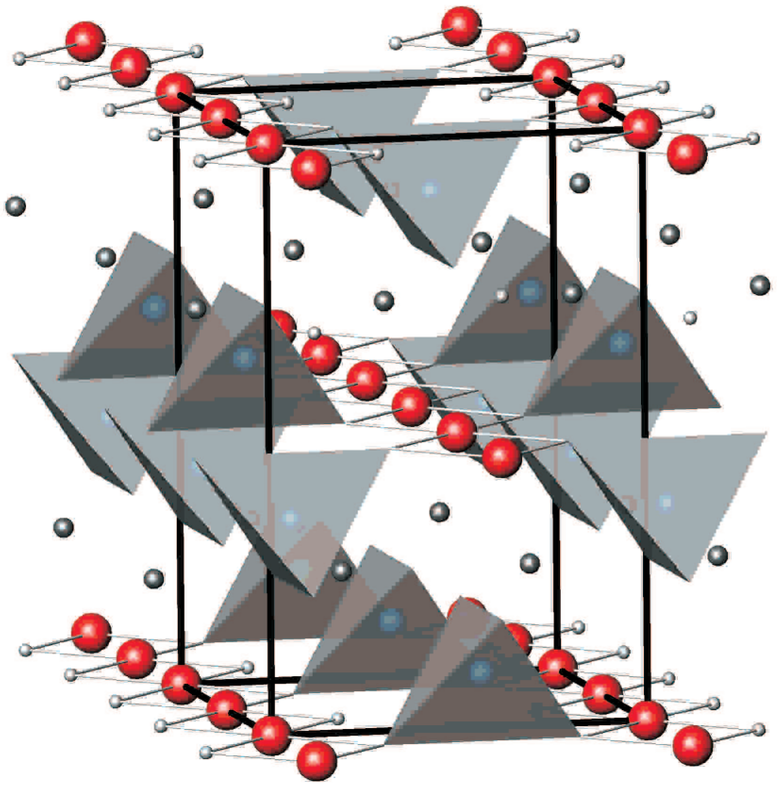
FIG. 9. Magnetization curves for (a) $J_1/J_2=-2.0$, (b) $J_1/J_2=-2.4$, (c) $J_1/J_2=-2.5$, (d) $J_1/J_2=-3.0$, (e) $J_1/J_2=-3.4$, and (f) $J_1/J_2=-3.6$. The dotted lines represent the boundaries of the regions of $\Delta S^z_{\rm tot}=1$ and $\Delta S^z_{\rm tot}\geq 2$.

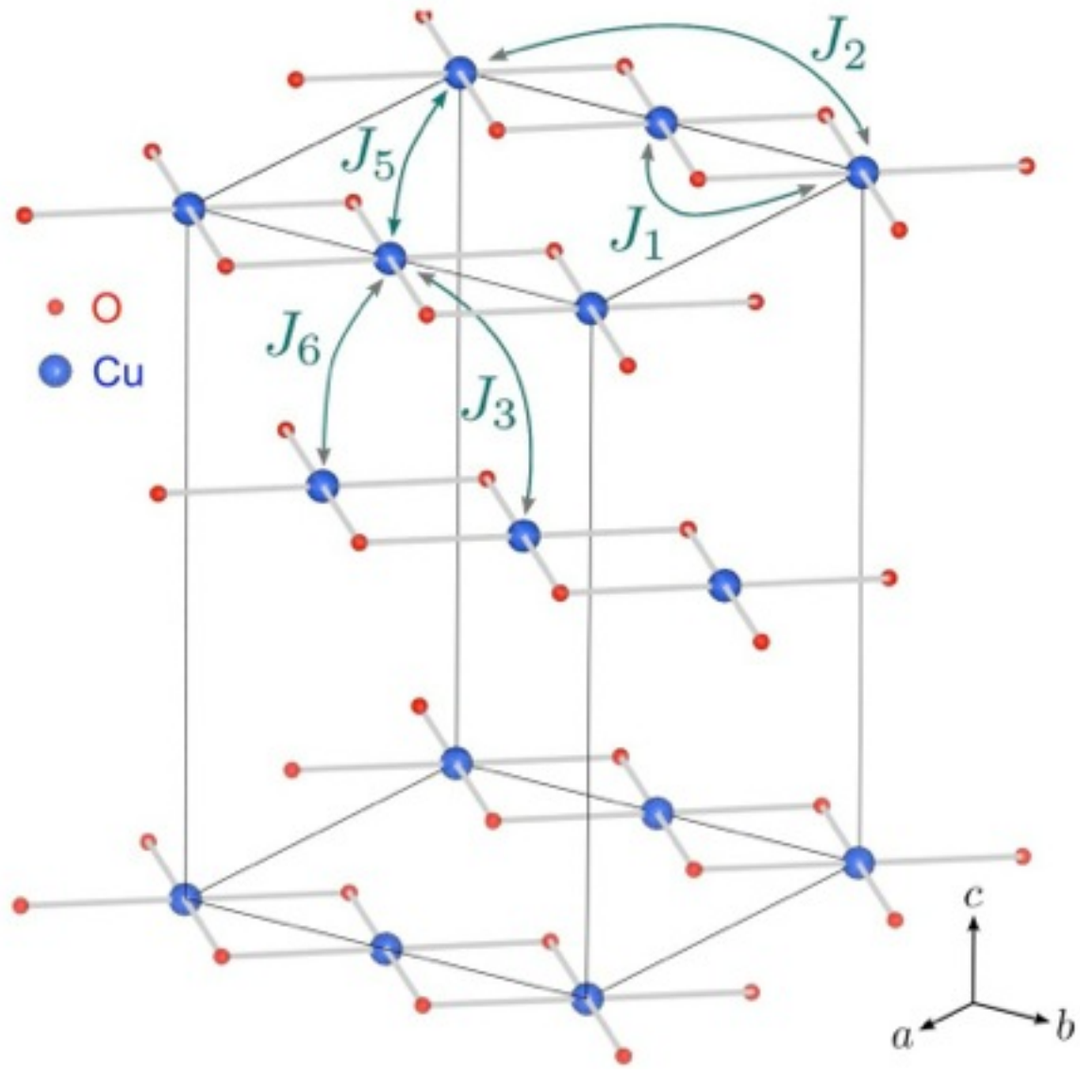
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$LiCuVO_4$: spin nematic?





$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

+ weak interchain coupling J_5…

estimates:
J_1 = - 1.6 meV
J_2 = 3.9 meV
J_5 = -0.4 meV

$LiCuVO_4$ experiment: collinear SDW along **B**

Hagiwara, Svistov et al, 2011

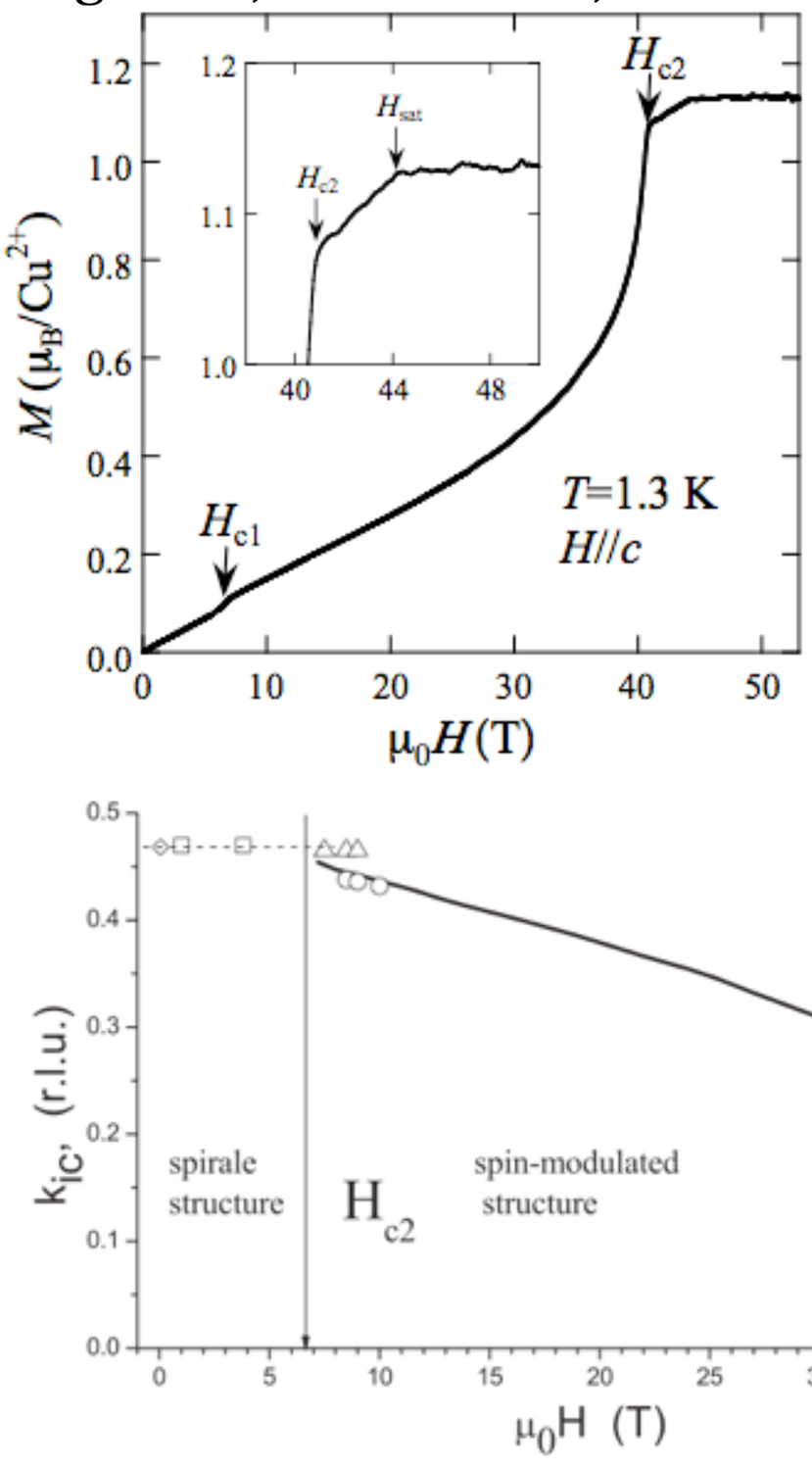
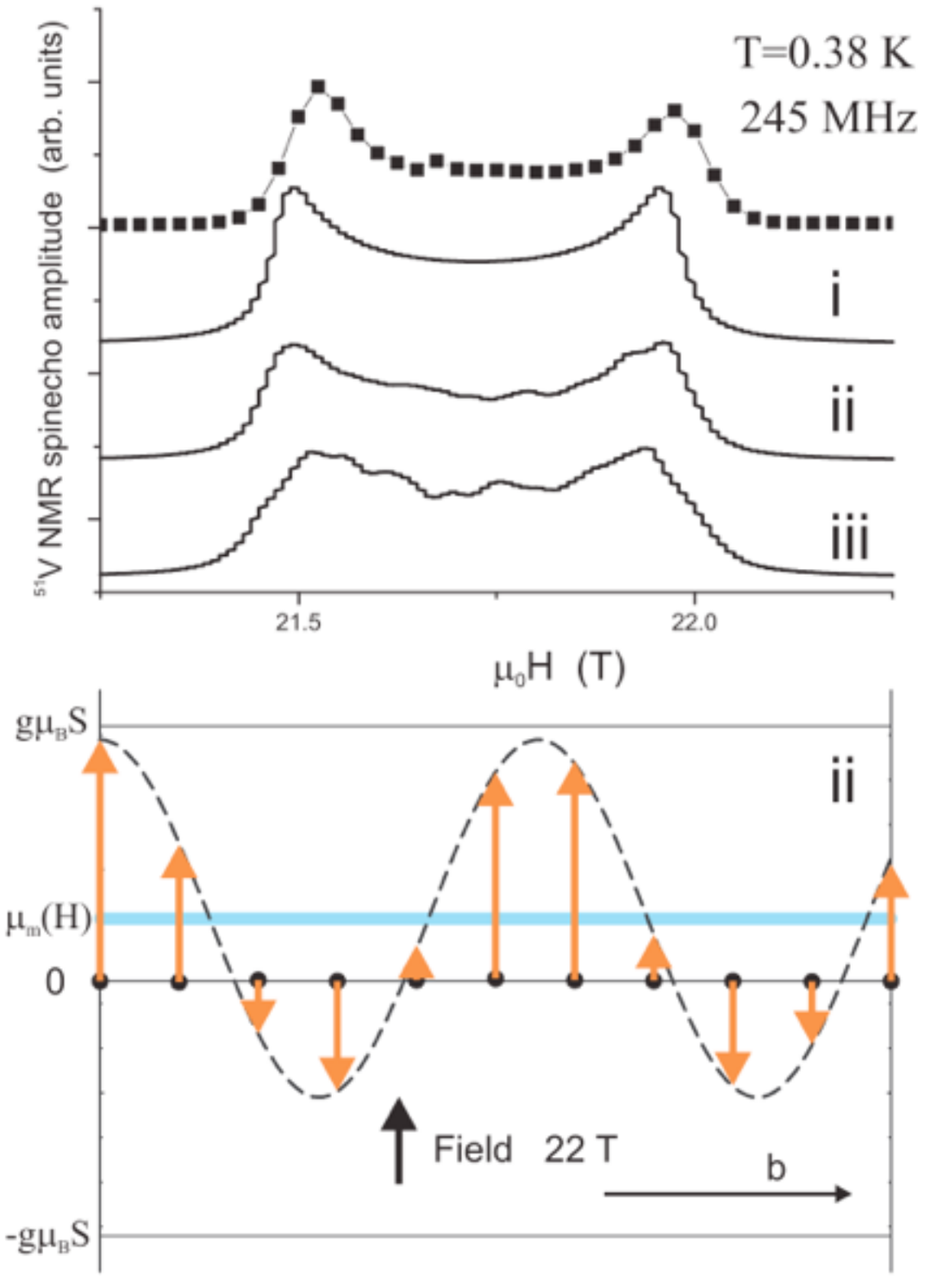


FIG. 2. Field dependence of the incommensurate wave vector k_{ic} for applied magnetic fields **H** ∥ **c** in $LiCuVO_4$. The open symbols

Buttgen et al 2012, 2014

PHYSICAL REVIEW B **85**, 214421 (2012)



longitudinal SDW

Evidence of a Bond-Nematic Phase in $LiCuVO_4$

M. Mourigal,[1,2] M. Enderle,[1] B. Fåk,[3] R. K. Kremer,[4] J. M. Law,[4,*] A. Schneidewind,[5] A. Hiess,[1,†] and A. Prokofiev[6,7]

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, ΔS = 1
NSF = no spin flip, ΔS = 0

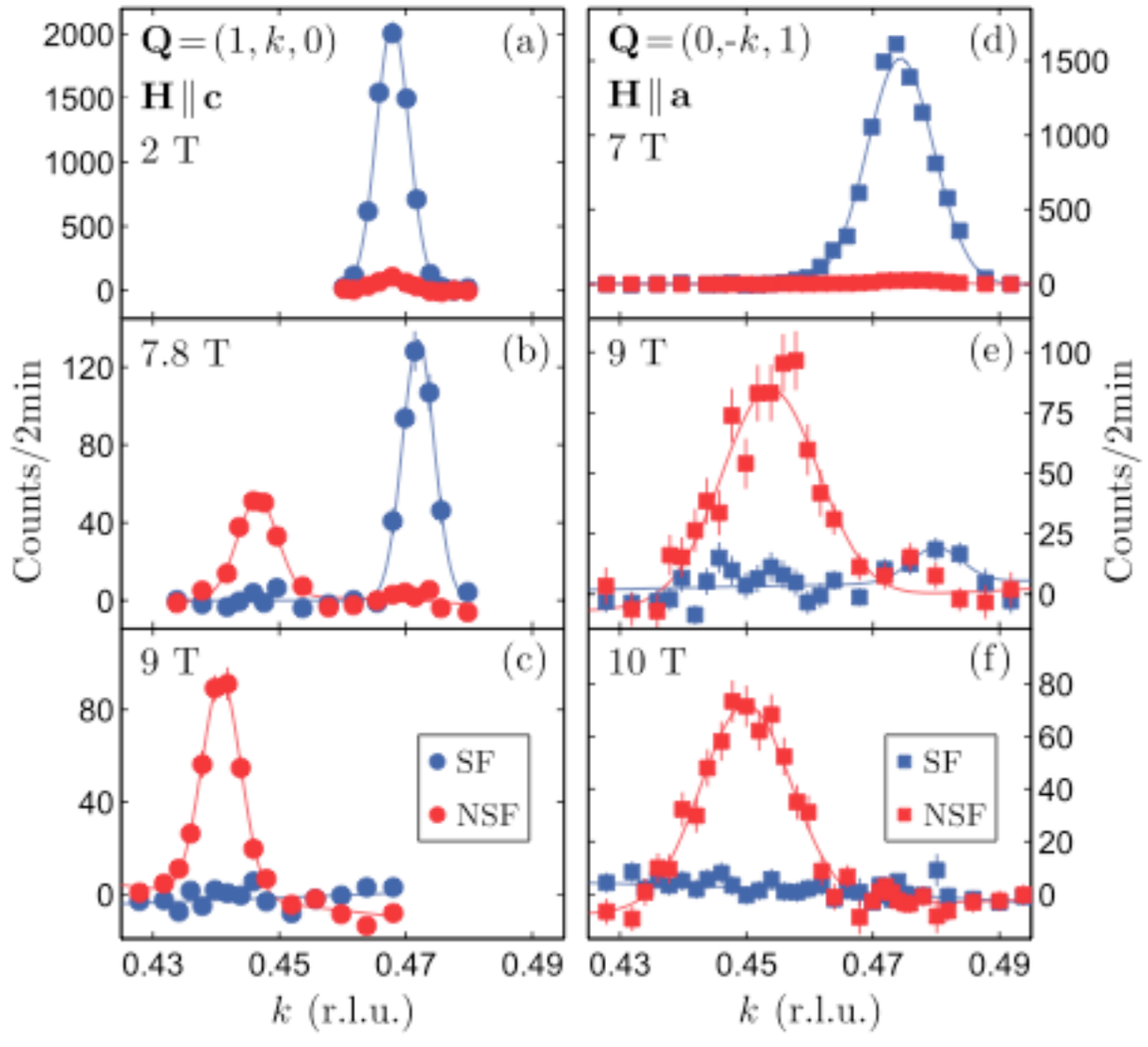
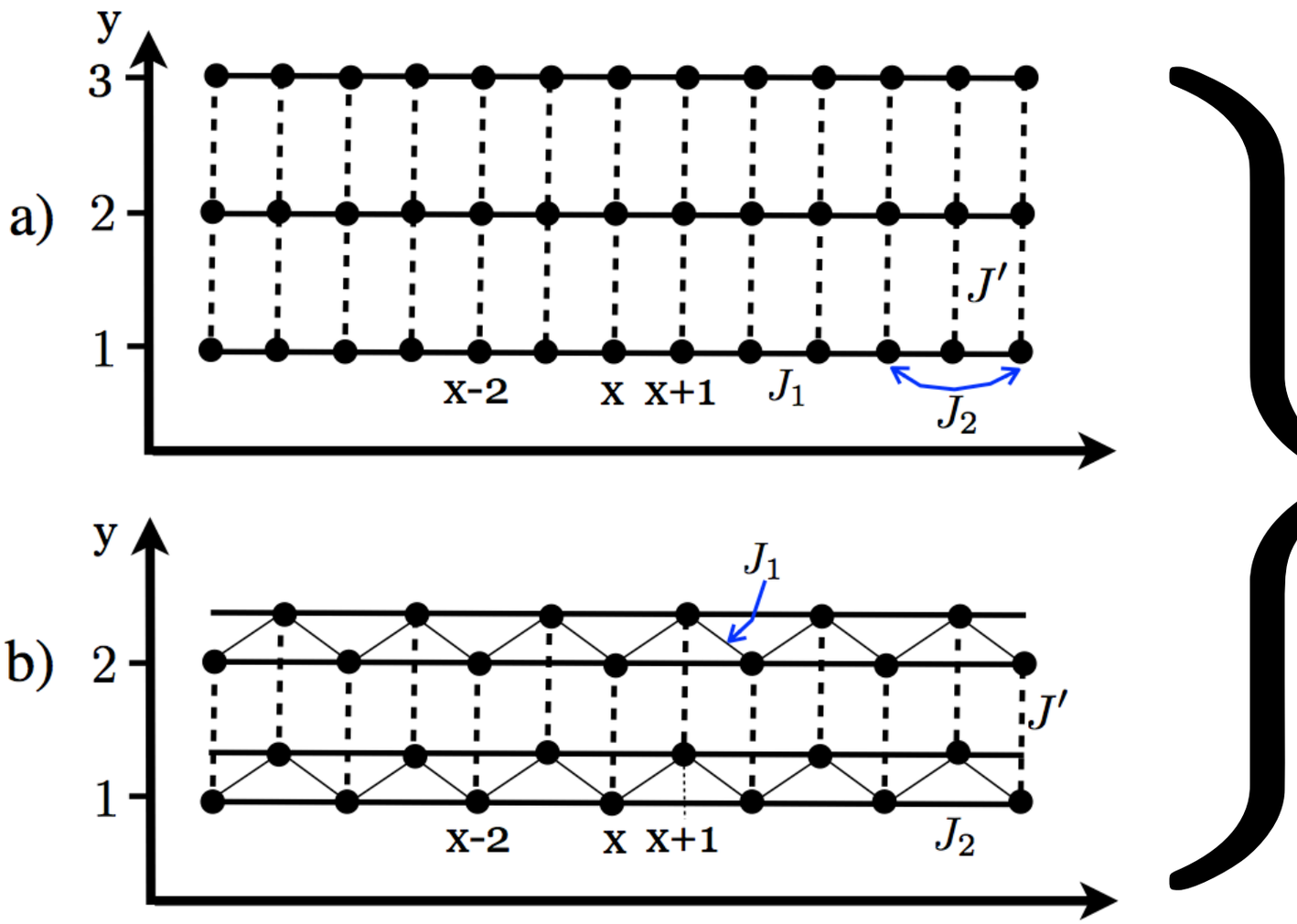
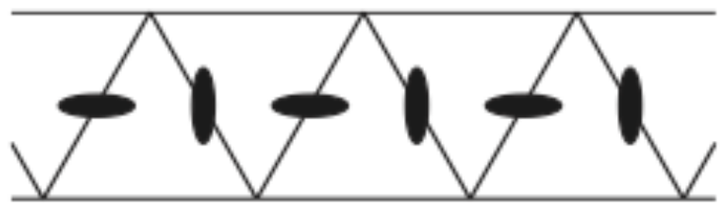


FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $\mathbf{Q} = (1, k_{IC}, 0)$ with $\mathbf{H}\|\mathbf{c}$ [left panels, (a)–(c)] and $\mathbf{Q} = (0, -k_{IC}, 1)$ with $\mathbf{H}\|\mathbf{a}$ [right panels, (d)–(f)].

Need to study weakly coupled nematic chains



$J_1 < 0$ (ferro)
$J_2 > 0$, $J' > 0$ (afm)
in magnetic field



- No true condensation [U(1) breaking] in d=1.
- Inter-chain interaction is crucial for establishing symmetry breaking in d=2.
- Need to study weakly coupled “superconducting” chains

Sato et al 2013
Starykh and Balents 2014

Inter-chain interaction $H_{\text{inter-chain}} = \sum_y \int dx\ \vec{S}_y \cdot \vec{S}_{y+1} \sim \sum_y \int dx\ S_y^+ S_{y+1}^- + S_y^z S_{y+1}^z$

Superconducting analogy: single-particle (magnon) tunneling between magnon superconductors is strongly suppressed at low energy (below the single-particle gap)

$$H_{\text{inter}}^{\perp} = \sum_y \int dx\ J' \langle S_y^+(x) S_{y+1}^-(x+1) \rangle_{\text{nematic ground state}} \to 0$$

Superconducting analogy: fluctuations generate two-magnon (***Josephson coupling***) tunneling between chains. They are generically weak, ~ $J_1(J'/J_1)^2 << J'$, but responsible for a true **two-dimensional nematic order**

$$H_{\text{nem}} \sim (J'^2/J_1) \sum_y \int dx\ [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

$$T_y^+(x) \sim S_y^-(x) S_y^-(x+1)$$

At the same time, density-density inter-chain interaction does not experience any suppression. It drives the system toward a **two-dimensional collinear SDW order**.

$$S_y^z = M - 2n_{\text{pair}} = M - \tilde{A}_1 e^{i\frac{\sqrt{2}\pi}{\beta}\varphi_y^+(x)} e^{ik_{\text{sdw}}x}$$

$$H_{\text{inter-chain}}^z = H_{\text{sdw}} \sim J' \sum_y S_y^z S_{y+1}^z \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi_y^+ - \varphi_{y+1}^+)]$$

Away from the saturation, **SDW** is more relevant [and stronger, via $J' >> (J')^2/J_1$] than the **nematic interaction**: **coupled** 1d **nematic chains order in a** 2d **SDW state.**

Simple scaling

$$H_{\rm nem} \sim (J'^2/J_1) \sum_y \int dx \; [T_y^+(x) T_{y+1}^-(x) + \text{h.c.}]$$

- describes kinetic energy of magnon pairs, linear in magnon pair density $n_{\rm pair}$

$$H^z_{\rm inter-chain} = H_{\rm sdw} \sim J' \sum_y S^z_y S^z_{y+1} \sim J' \sum_y \int dx \cos[\frac{\sqrt{2}\pi}{\beta}(\varphi^+_y - \varphi^+_{y+1})]$$

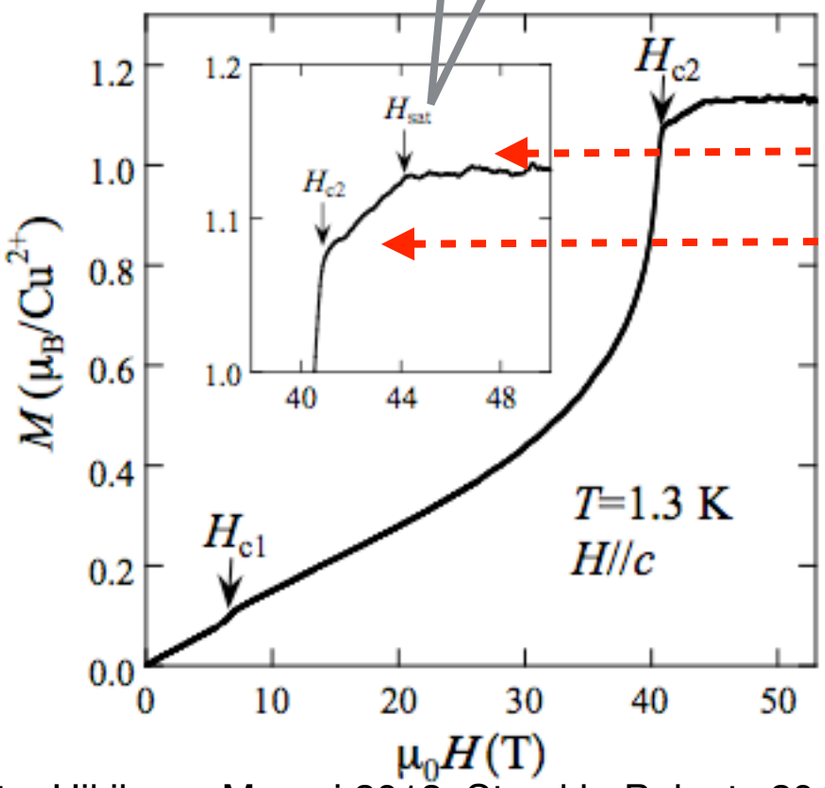
- describes potential energy of interaction between magnon pairs on neighboring chains, quadratic in magnon pair density $n_{\rm pair}$

- Competition $\frac{(J')^2}{J_1} n_{\rm pair} \sim J' n^2_{\rm pair}, \;\; \text{hence} \;\; \boxed{n^*_{\rm pair} \sim J'/J_1}$

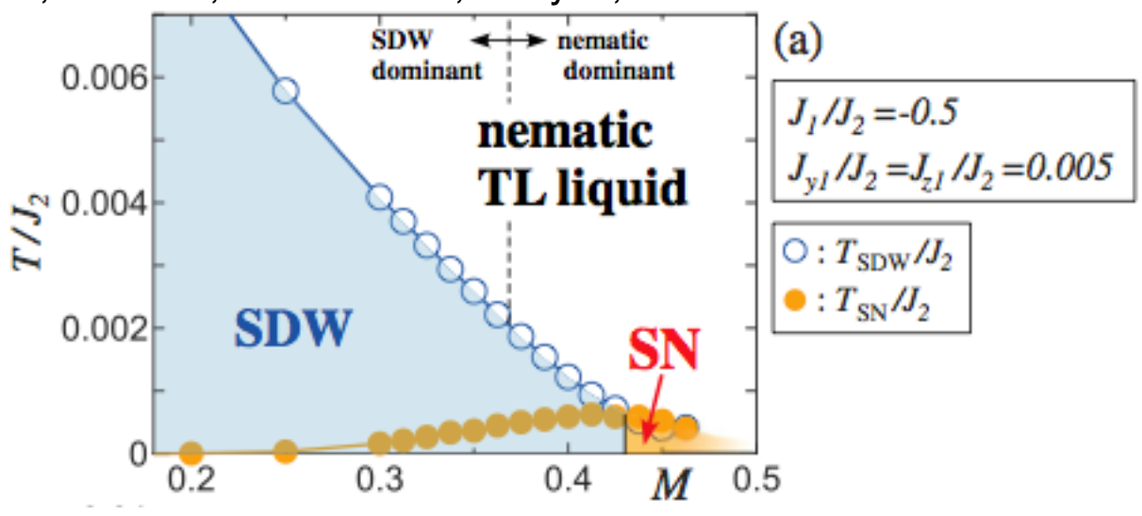
- Hence:
- Spin Nematic *near saturation*, for $n_{pair} < n^*_{pair}$
- SDW for $n_{pair} > n^*_{pair}$

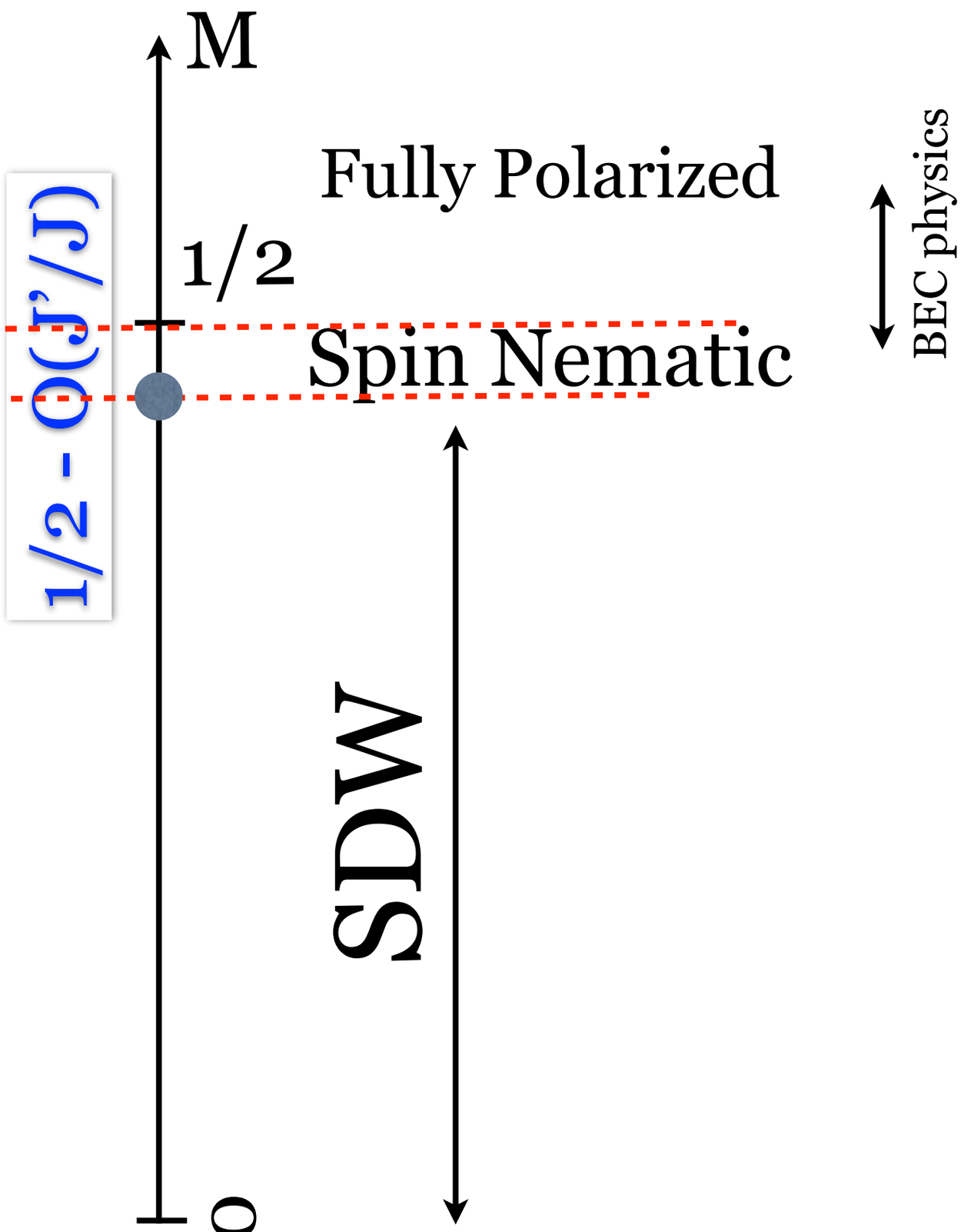
T=0 schematic phase diagram of weakly coupled **nematic** spin chains

Cautionary remark:
an impurity effect !
M. Zhitomirsky 2015



Sato, Hikihara, Momoi 2013; Starykh, Balents 2014





Cold reality

PHYSICAL REVIEW B **90**, 134401 (2014)

Search for a spin-nematic phase in the quasi-one-dimensional frustrated magnet $LiCuVO_4$

N. Büttgen,[1,*] K. Nawa,[2,3] T. Fujita,[4] M. Hagiwara,[4] P. Kuhns,[5] A. Prokofiev,[6] A. P. Reyes,[5] L. E. Svistov,[7,†] K. Yoshimura,[2] and M. Takigawa[3,‡]

"Our results suggest that the theoretically predicted spin-nematic phase, if it exists in LiCuVO4, can be established only within the narrow field range 40.5<H<41.4 T."

- so far, extensive experimental evidence for longitudinal SDW
- Spin Nematic phase is constrained to field interval < 1 T right below the saturation field (of the order 40 T)

Have to look elsewhere

Intermediate Summary

- Interesting magnetically ordered states: SDW and Spin Nematic
  - Gapped $\Delta S=1$ excitations (no usual spin waves!)
  - Linearly-dispersing ***phason*** mode with $\Delta S=0$ in **2d** SDW
  - Linearly-dispersing ***magnon density*** waves in **2d** Spin Nematic

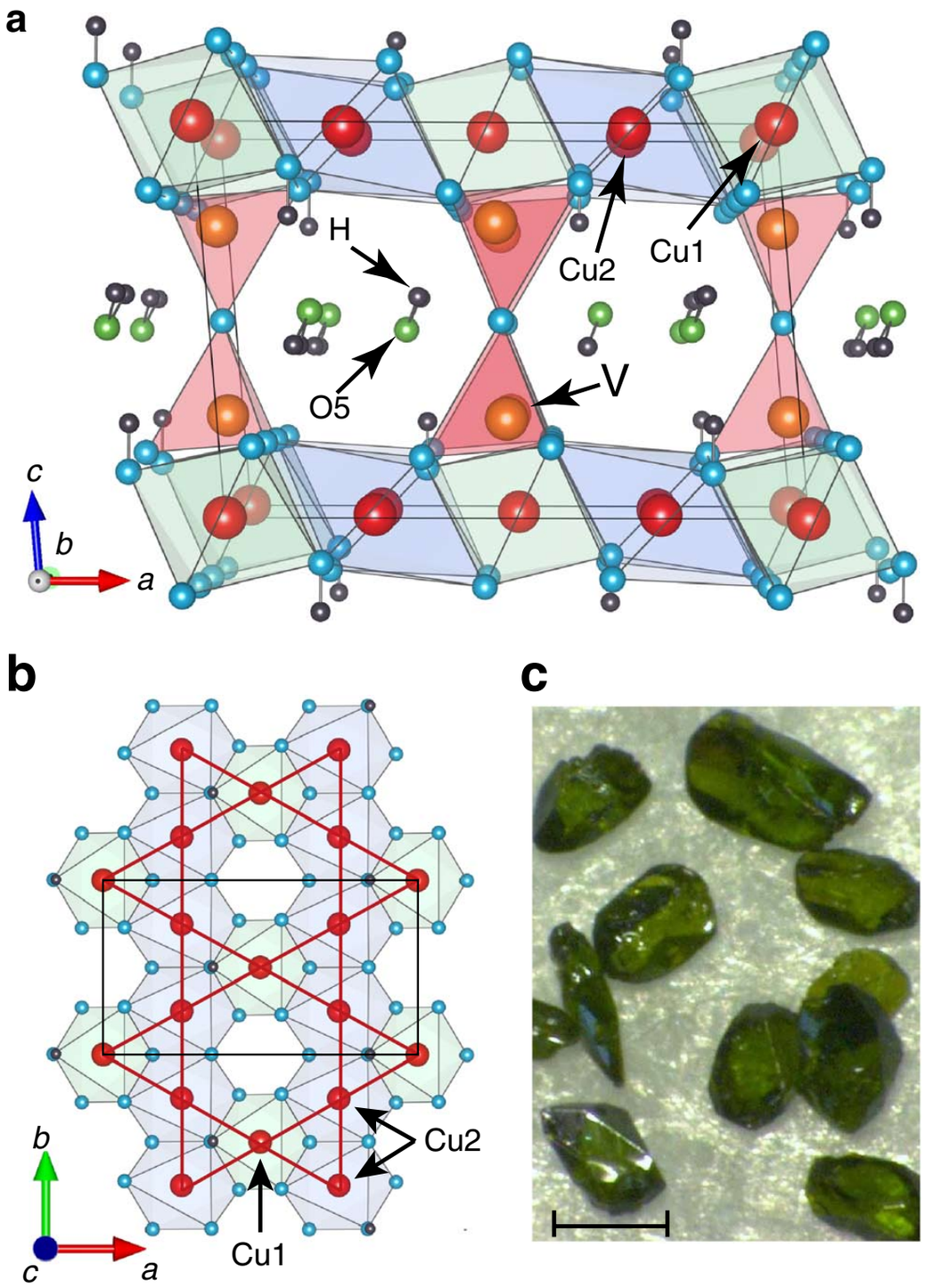
toy problem of "magnon high T_c"

Outline

- Motivation: usual *vs* unusual ordered states
  - emergence of composite orders from competing interactions
- Frustrated ferromagnets and magnon binding
- $LiCuVO_4$
  - ✓ spin nematic: “magnon superconductor”
  - ✓ collinear SDW: “magnon charge density wave”
- Volborthite kagome antiferromagnet
  - experimental status - magnetization plateau
  - UUD, SDW and ***more***

✴ *Field theory of the Lifshitz point*

Volborthite





Volborthite's timeline

Formula: $Cu_3(V_2O_7)(OH)_2 \cdot 2H_2O$

System: Monoclinic **Colour:** Olive-green, ...

Hardness: 3½

Name: Named after Alexander von Volborth (1800–1876), Russian paleontologist, who first noted the mineral.

A secondary mineral found in the oxidized zones of vanadium-bearing hydrothermal deposits.

At least two different monoclinic space-group variants (C2/m, C2/c) seem to be stable at ambient temperature.

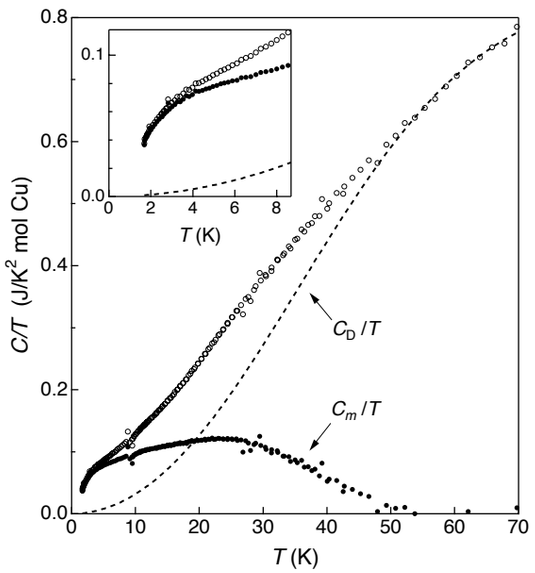
Visually similar to vésigniéite.

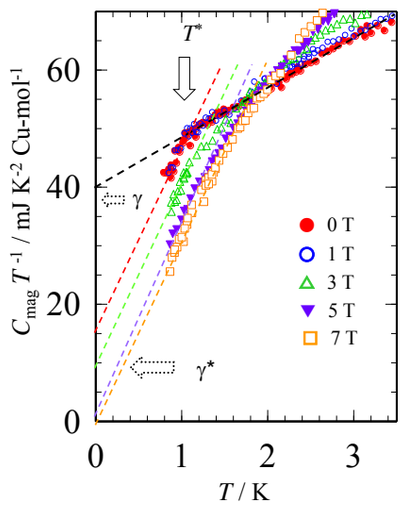
2001 quantum spin liquid?!

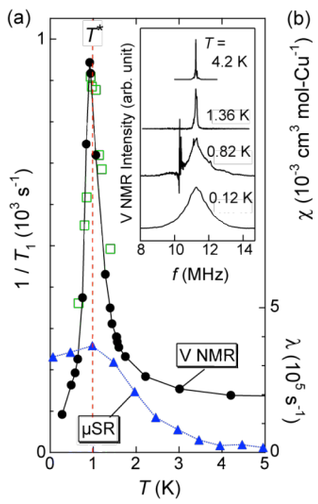
2009 impurity ordering at low T? magnetization steps?

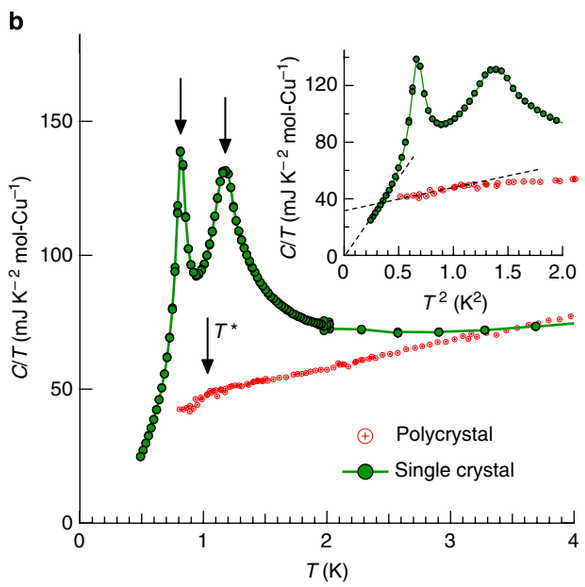
2012 magnetic order !

2014 magnetization plateau









time = material quality

One-Third Magnetization Plateau with a Preceding Novel Phase in Volborthite

H. Ishikawa,[1] M. Yoshida,[1] K. Nawa,[1] M. Jeong,[2] S. Krämer,[2] M. Horvatić,[2] C. Berthier,[2] M. Takigawa,[1] M. Akaki,[1] A. Miyake,[1] M. Tokunaga,[1] K. Kindo,[1] J. Yamaura,[3] Y. Okamoto,[1,4] and Z. Hiroi[1]

[1]*Institute for Solid State Physics, The University of Tokyo, Kashiwa, Chiba 277-8581, Japan*

[2]*Laboratoire National des Champs Magnétiques Intenses, LNCMI-CNRS (UPR3228), UJF, UPS and INSA, B.P. 166, 38042 Grenoble Cedex 9, France*

[3]*Materials Research Center for Element Strategy, Tokyo Institute of Technology, Yokohama 226-8503, Japan*

[4]*Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan*

(Received 23 November 2014; published 1 June 2015)

Huge 1/3 magnetization plateau !

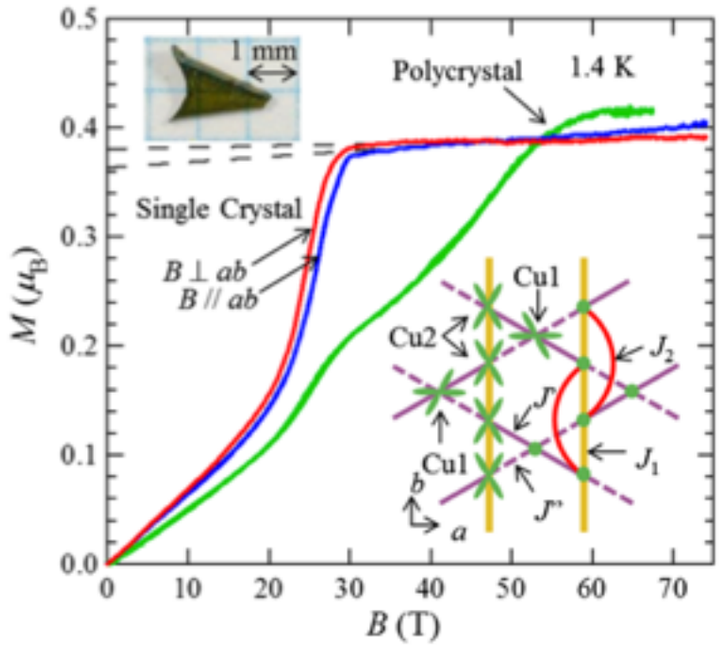


FIG. 1 (color online). Magnetization curves of volborthite measured at 1.4 K on two piles of single crystals in magnetic fields perpendicular (red line) and parallel (blue line) to the *ab* plane, and on a polycrystalline sample (green line) [32]. Shown also are a typical single crystal of volborthite (upper left) and the arrangement of Cu d_{x2-y2} orbitals projected onto the *ab* plane in the low-temperature $P2_1/a$ structure (lower right). J_1 and J_2 represent the NN and NNN interactions in the Cu2 spin chains, respectively. J' and J'' represent the NN interactions between Cu1 and Cu2 spins.

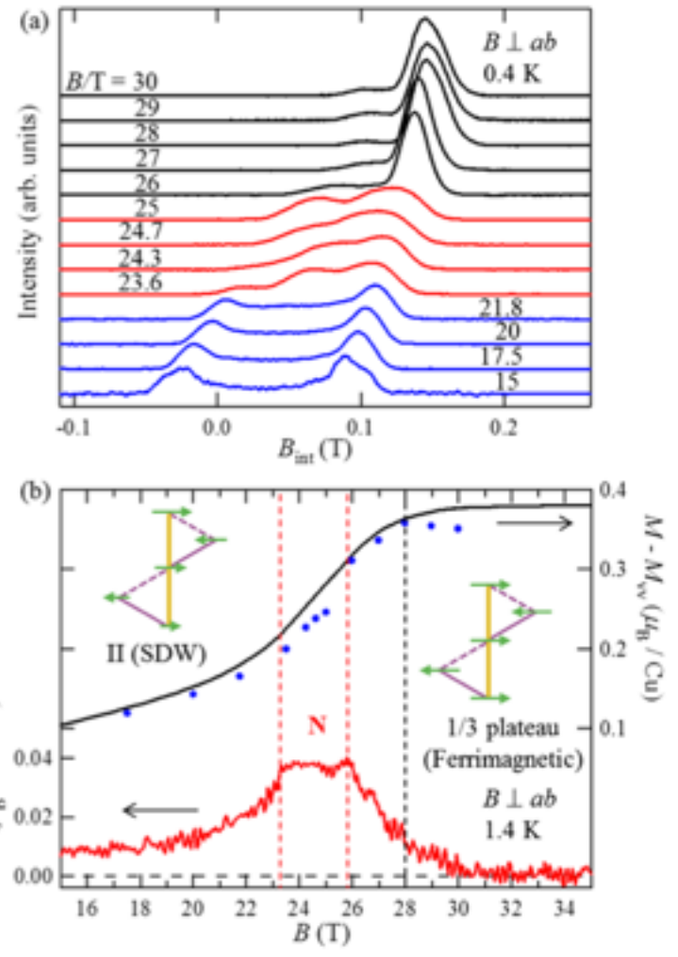


FIG. 2 (color online). (a) ^{51}V NMR spectra measured on a single-domain piece of a crystal in magnetic fields applied perpendicular to the *ab* plane at $T = 0.4$ K. The labeled fields correspond to $B = \nu_0/\gamma(B_{\rm int} = 0)$. (b) Magnetization curve of single crystals (top, black line) and its field derivative (bottom) in $B \perp ab$ at 1.4 K after the subtraction of the Van Vleck paramagnetic magnetization ($M_{\rm VV}$). Magnetization deduced from the center of the gravity of the NMR spectra is also plotted (top, blue circles).

How far in field does the plateau extend?

Meeting abstracts of the Physical Society of Japan 2014

27aAF-9

単結晶 volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ の超強磁場磁化過程

東大物性研，東大院工 A
山下知大 A，宮田敦彦，嶽山正二郎，石川孟，岡本佳比古，広井善二

Magnetization curves of volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ in ultra-high magnetic fields

ISSP, Univ. of Tokyo, Dep. of Appl. Phys., Univ. of Tokyo A
T. Yamashita A, A. Miyata, S. Takeyama, H. Ishikawa, Y. Okamoto, and Z. Hiroi

スピン 1/2 カゴメ格子反強磁性体は、幾何学的フラストレーションによりスピン液体状態が実現する有力な候補と考えられている。最近、その磁化過程の計算が進み、飽和に至るまでに 5 つの磁化プラトーが現れることが示唆された[1]。これら 5 つのプラトーの内、0, 1/9 プラトーはスピン液体状態、1/3, 5/9, 7/9 プラトーは長距離秩序となり、多彩な磁場誘起相の出現が示唆されている。

一方、スピン 1/2 カゴメ反強磁性体を実現する理想的な物質はこれまで見つかっていない。本研究で用いた volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ は、カゴメ格子が歪んでいるが、理想に近い物質の一つとして盛んに研究が行われている。また、近年、volborthite 単結晶試料を用いた磁化測定が 75T まで行われ、幅広い 1/3 プラトーが観測された[2]。

我々は、飽和に至るまでの全磁場誘起相を調べるため、100 T 以上の超強磁場下におけるファラデー回転測定を試みている。図は、波長 532 nm レーザーを用いた 11 K におけるファラデー回転角を 1.3 K における磁化曲線[2]と比較したものである。磁場は単結晶試料の ab 平面に垂直に印加した。ファラデー回転角は磁化の奇数次項で表せ、25 T 近辺における磁化の増加に対応して回転角も増えている。また、1/3 磁化プラトーに対応して回転角も一定になり、この領域は 120 T まで続いている。

本講演では、ファラデー回転測定の結果に加えて、ピックアップコイルを用いた誘導法による磁化測定(<110 T)の結果も合わせて発表する。

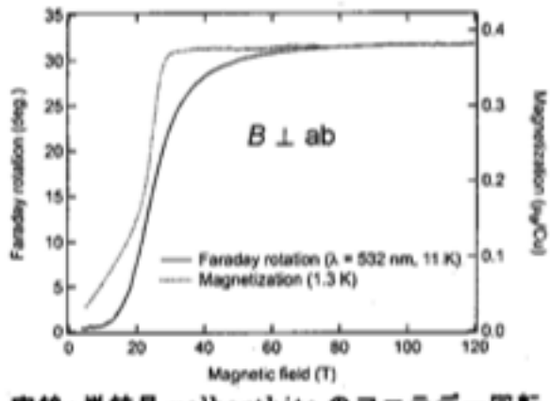


図　実線：単結晶 volborthite のファラデー回転
点線：磁化曲線 [2]

[1] S. Nishimoto, N. Shiba, and C. Hotta, Nature Comm. 4, 2287 (2013).
[2] 石川孟 他，日本物理学会第 69 回年次大会 27aAF-8 (2014).

Translation:
== Intro ==

- S=1/2 kagome antiferromagnet is a known candidate spin liquid.
Under external field, five magnetic plateaus have been theoretically found in S=1/2 antiferromagnet: m=0, 1/9, 1/3, 5/9, and 7/9.
Among them, first two are spin liquids, and the later three are long-range orders (Ref. 1).
- Experimentally, a large 1/3 magnetization plateau was recently observed in Volborthite, which extends above 75T.

== Method & Results ==

- We study the magnetic phases of Volborthite in high-field, above 100T, using Faraday rotation.
- The figure shows rotation angle observed at 11K using a laser of wave length 532nm; it is compared to magnetization curve at 1.3K. We used single crystal samples and the field is applied perpendicular to the ab plane.
- The rotation angle show increase around 25T, where rapid increase in magnetization curve is observed. The rotation angle become nearly constant corresponding to **the 1/3 magnetic plateau, which extends up to 120T**.

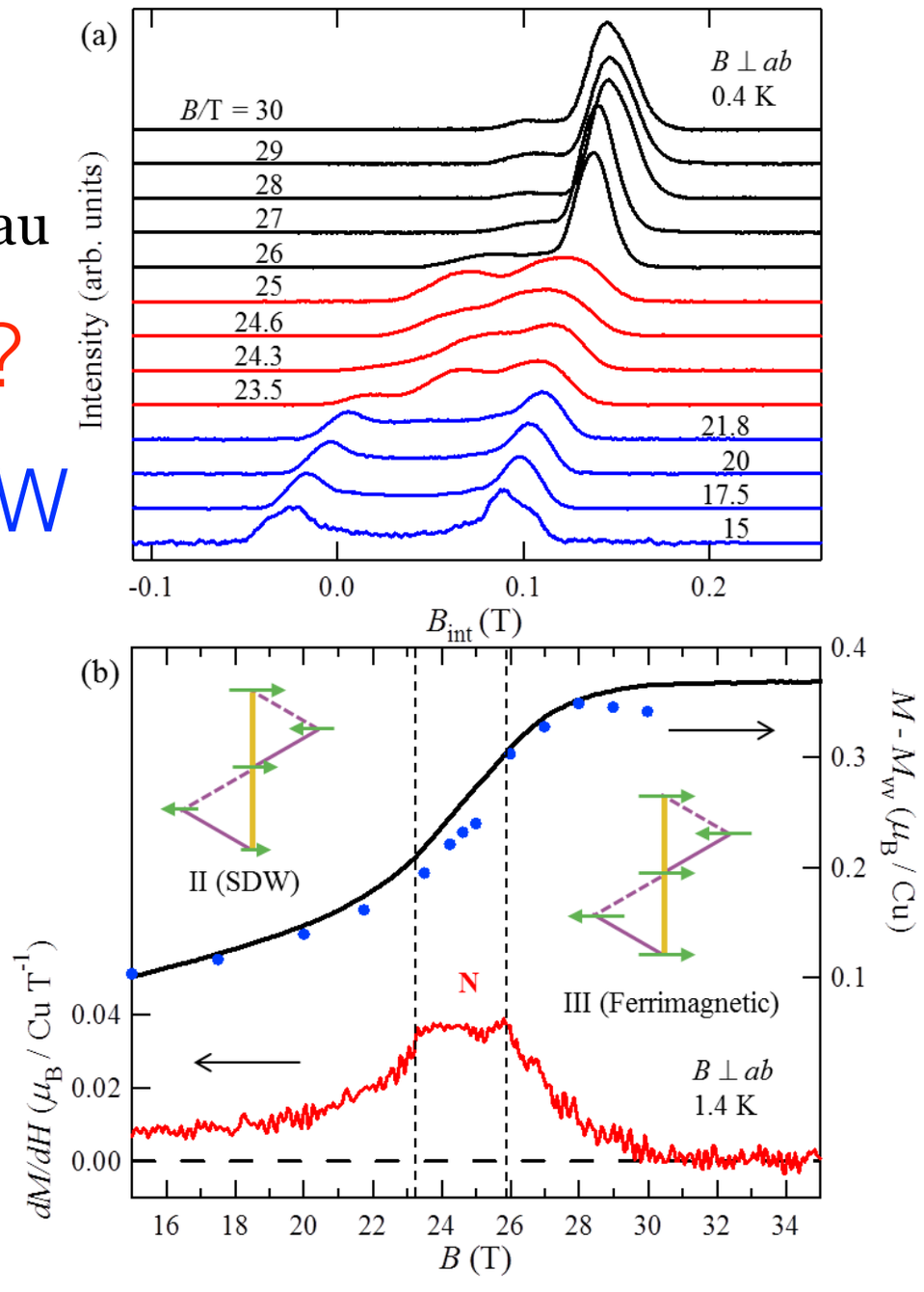
this is the best plateau ever:
28 T < plateau < 120 T

Phase diagram

1/3 plateau

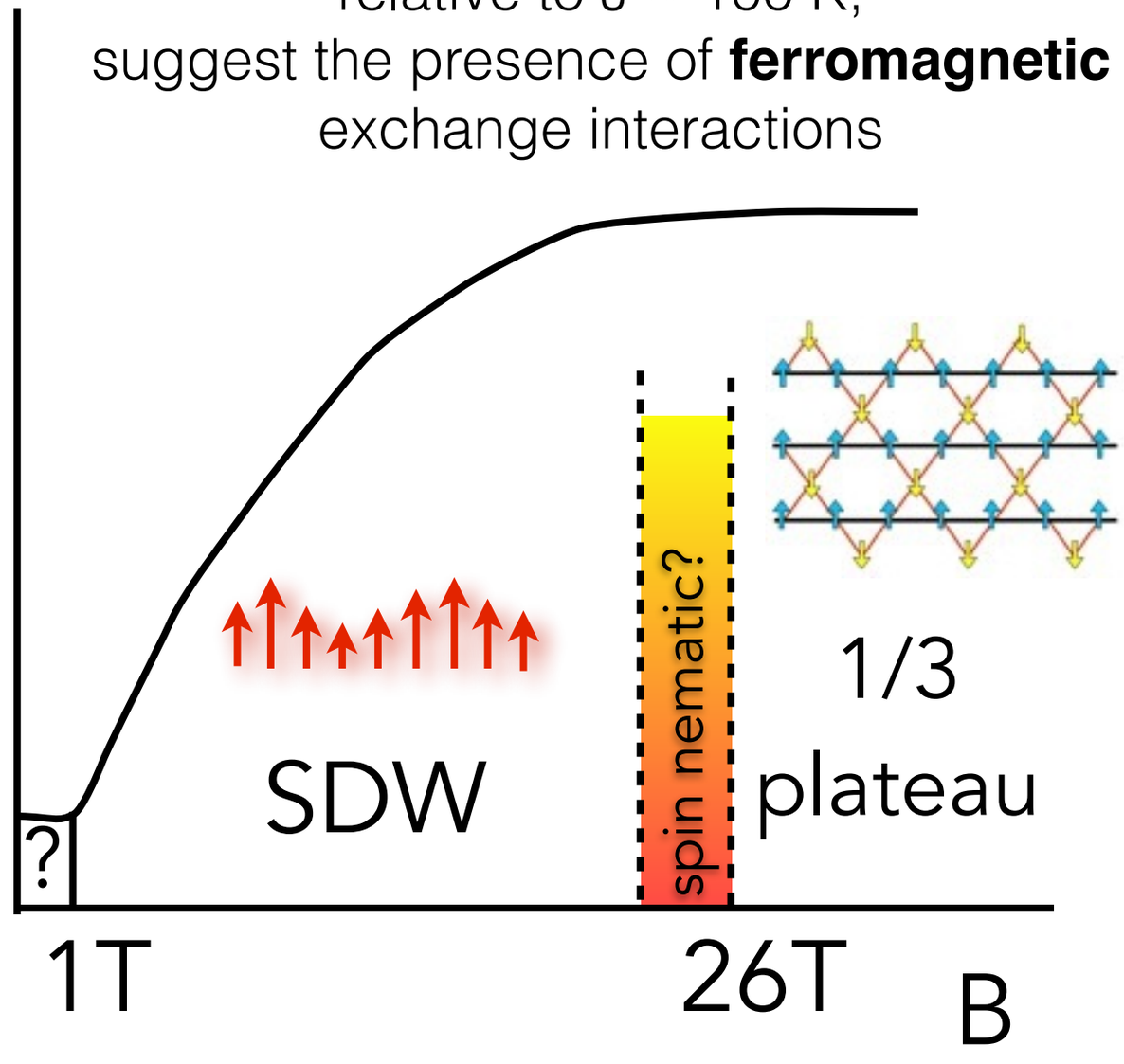
N?

SDW



H. Ishikawa *et al*, PRL 2015

small plateau's onset field of 27 Tesla,
relative to J ~ 100 K,
suggest the presence of **ferromagnetic** exchange interactions



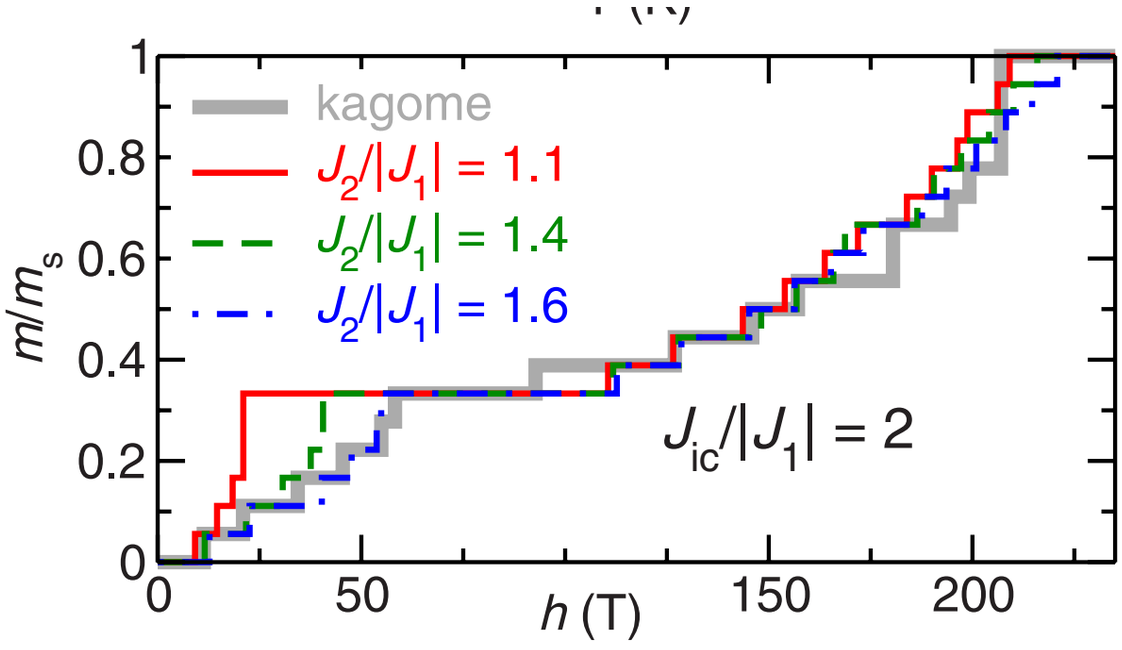
may be a spin nematic??

Frustrated ferromagnetism

PHYSICAL REVIEW B **82**, 104434 (2010)

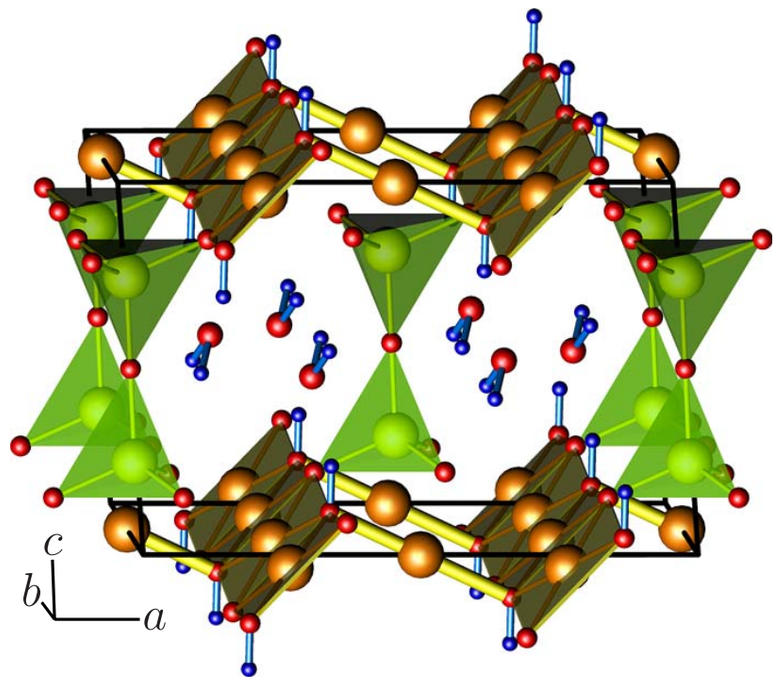
Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

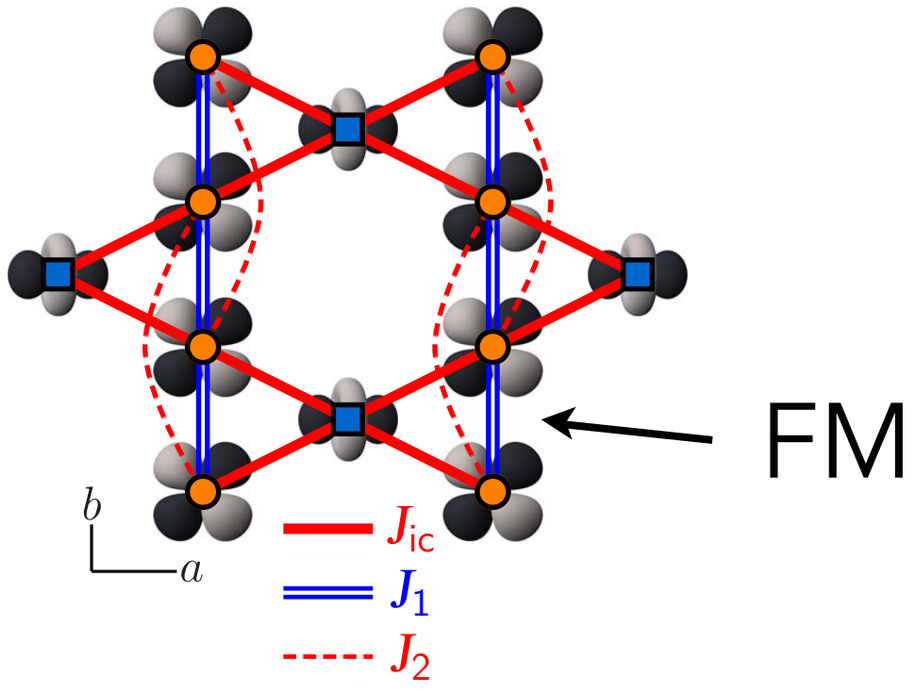
O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Magdeburg, D-39016 Magdeburg, Germany*
[3]*Laboratoire de Physique Théorique de la Matière Condensée, Univ. P. & M. Curie, Paris, France*
(Received 9 August 2010; published 30 September 2010)



DFT gets it right!

$$J_1 < 0, J_2 > 0, J' > 0$$





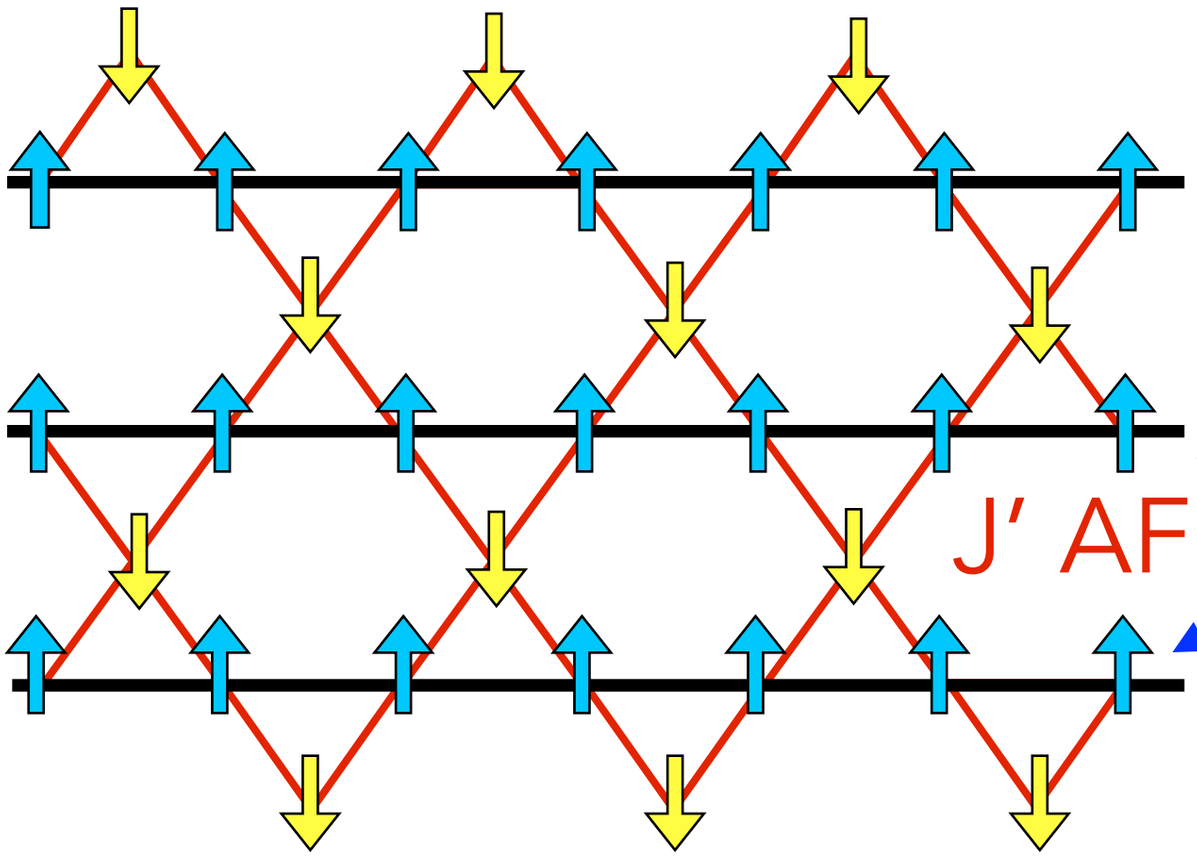
Ferrimagnetic state

PHYSICAL REVIEW B **82**, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$

O. Janson,[1,*] J. Richter,[2] P. Sindzingre,[3] and H. Rosner[1,†]
[1]*Max-Planck-Institut für Chemische Physik fester Stoffe, D-01187 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Magdeburg, D-39016 Magdeburg, Germany*
[3]*Laboratoire de Physique Théorique de la Matière Condensée, Univ. P. & M. Curie, Paris, France*
(Received 9 August 2010; published 30 September 2010)

$$J_1 < 0, J_2 > 0, J' > 0$$



J_1 FM, J_2 AF

J’ AF

polarized chains?!

But the model keeps changing…

1/3 plateau

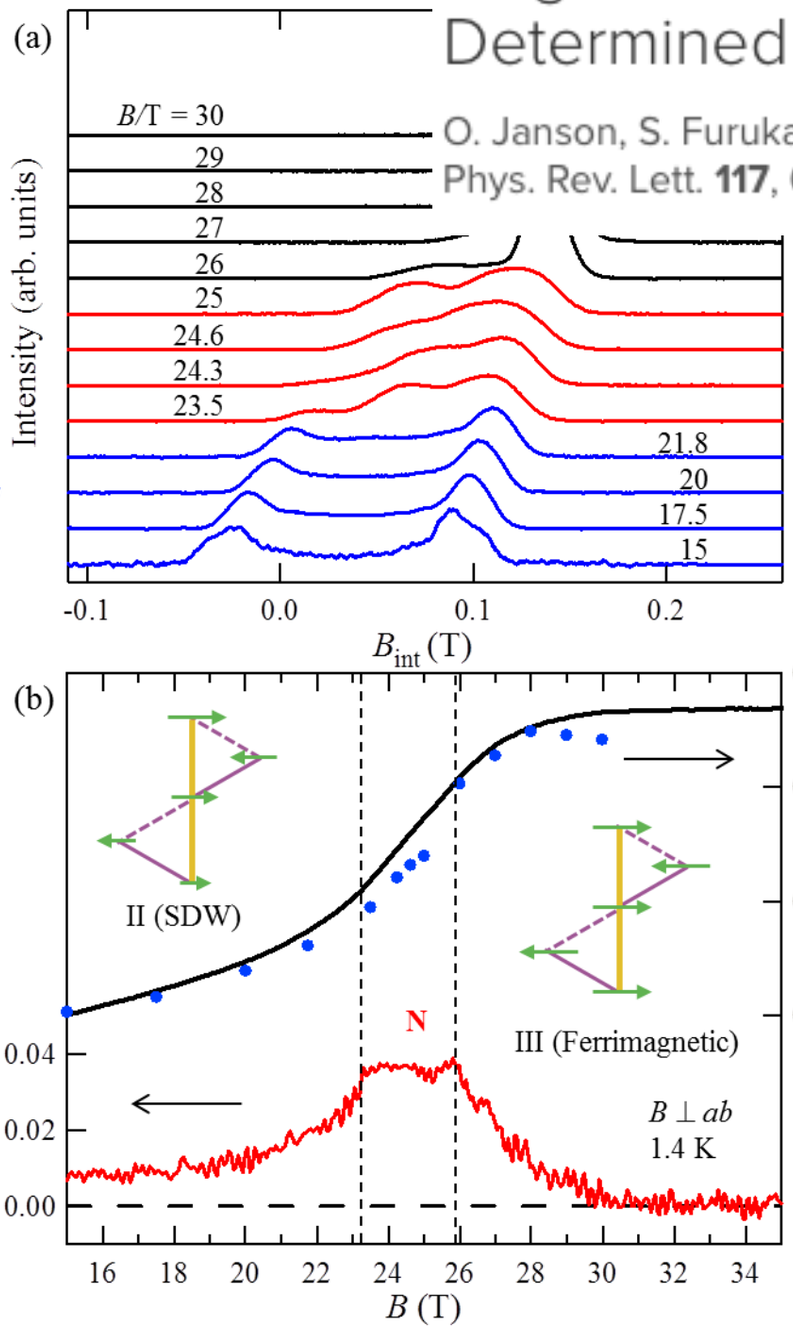
N?

SDW

Magnetic Behavior of Volborthite $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ Determined by Coupled Trimers Rather than Frustrated Chains

O. Janson, S. Furukawa, T. Momoi, P. Sindzingre, J. Richter, and K. Held

Phys. Rev. Lett. **117**, 037206 – Published 12 July 2016



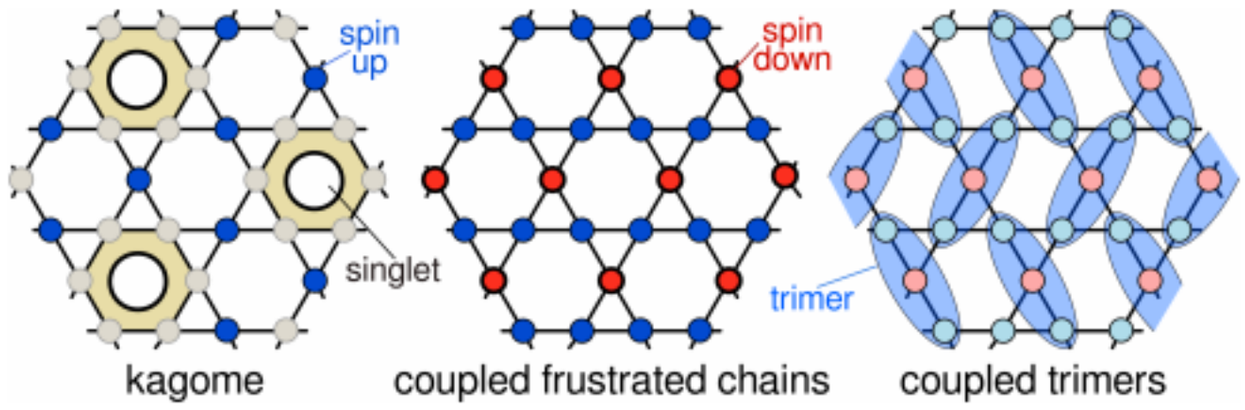


FIG. 1. (Color online) The structure of the $\frac{1}{3}$-magnetization plateau in the kagome model (KHM), coupled frustrated chains (CFC) model from Ref. [23], and the J-J'-J_1-J_2 model.

Most recent development:
the exchange is not uniform in space!?

H. Ishikawa *et al*, PRL 2015

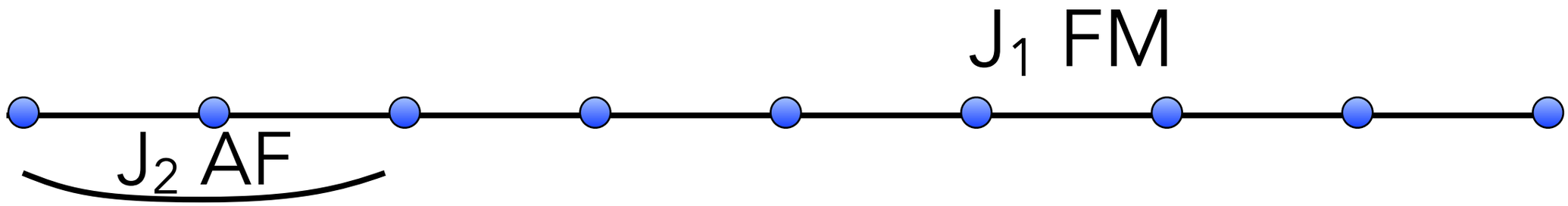
Outline

- Motivation: usual *vs* unusual ordered states
  - emergence of composite orders from competing interactions
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  - ✓ spin nematic: “magnon superconductor”
  - ✓ collinear SDW: “magnon charge density wave”
- Volborthite kagome antiferromagnet
  - experimental status - magnetization plateau
  - UUD, SDW and ***more***

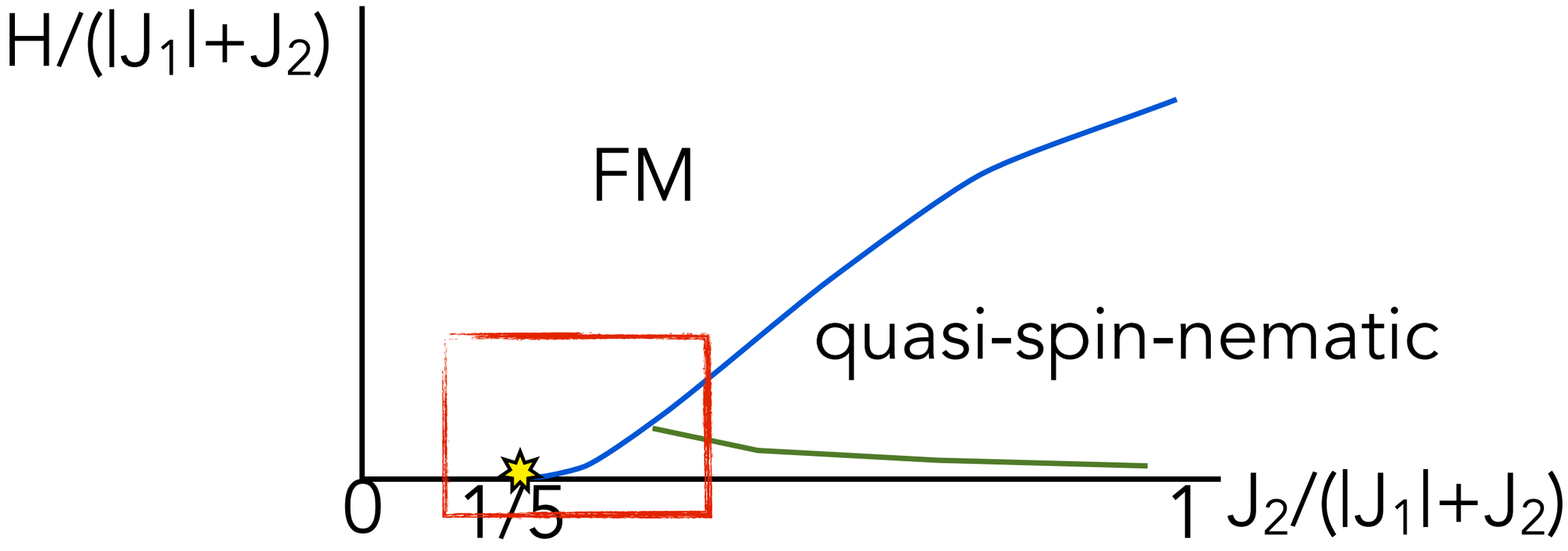
✴ *Field theory of the Lifshitz point*

Spin chain redux

Frustrated ferromagnetic chain

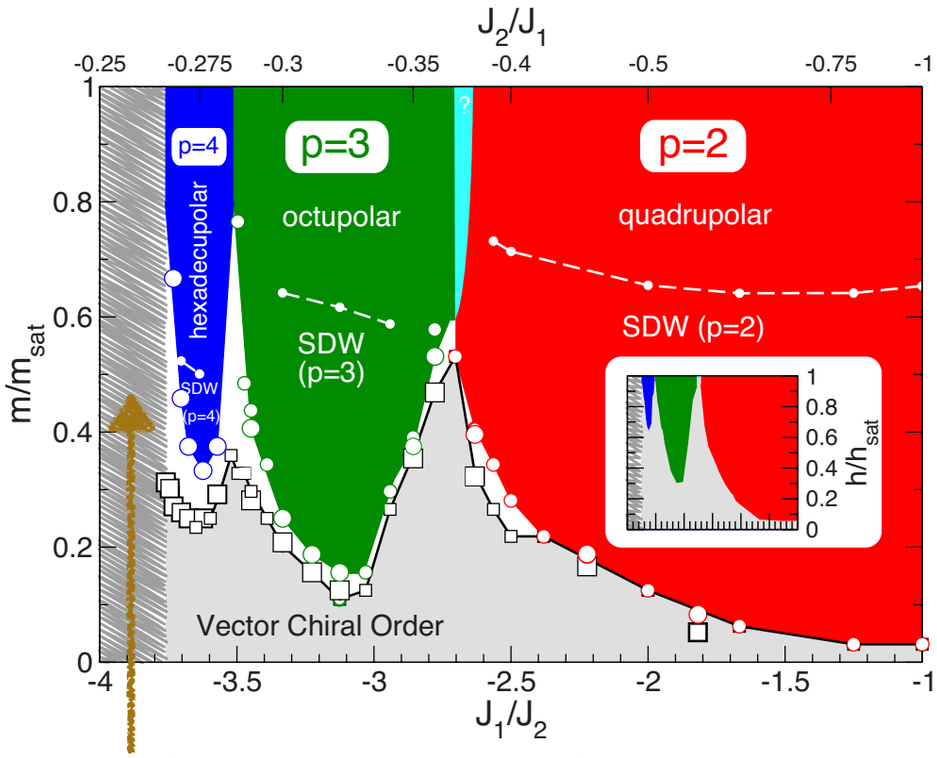


$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



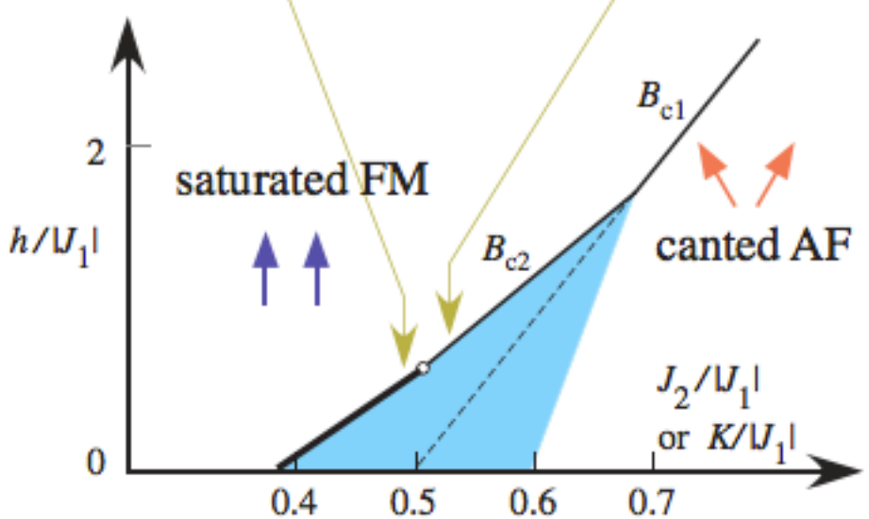
Numerics: nematicity and 1st order seem connected?

1d chain



Sudan et al, 2009

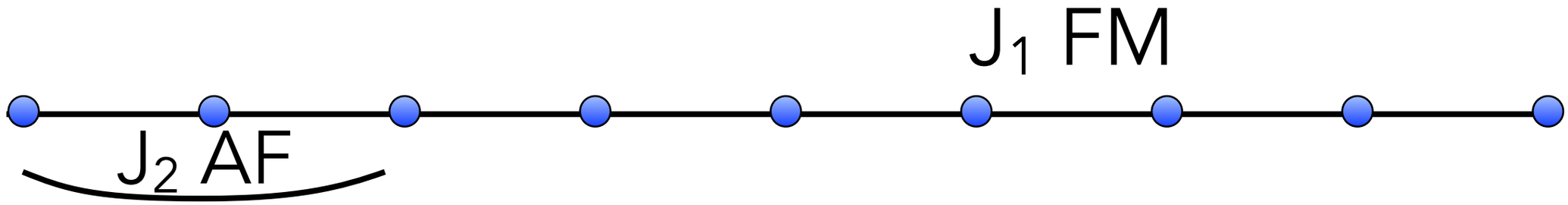
2d square lattice



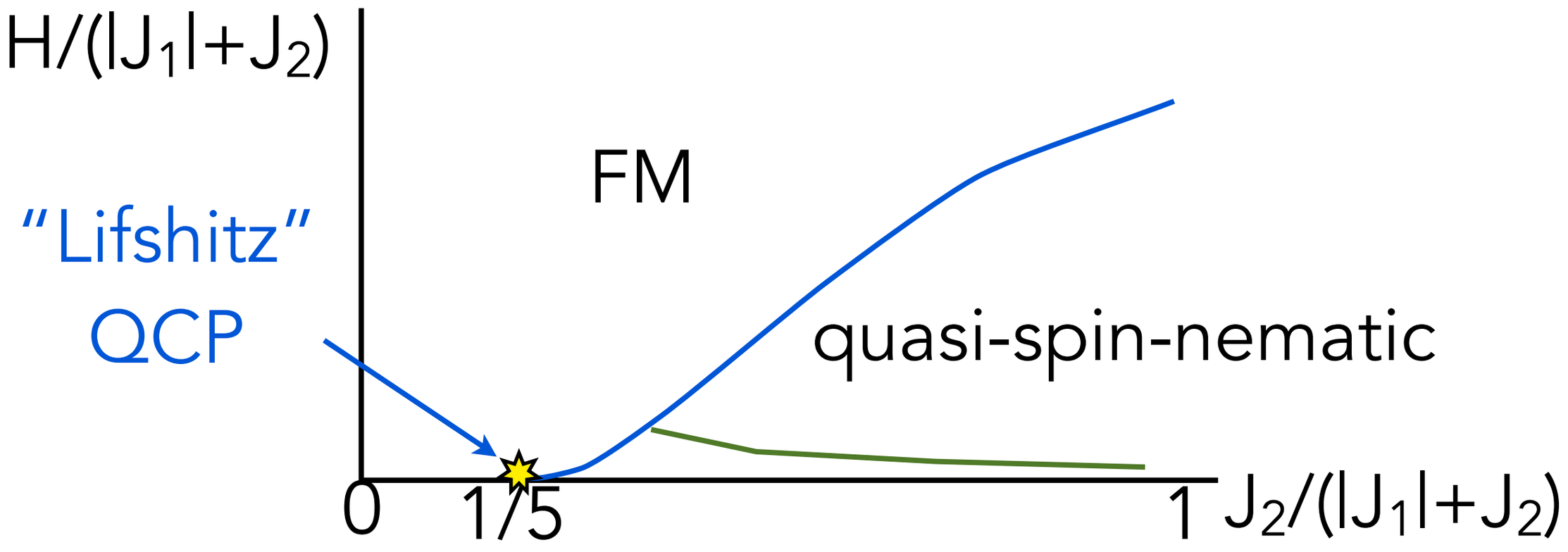
Shannon, Momoi, Sindzingre PRL 2006

Quantum Lifshitz point

Frustrated ferromagnetic chain

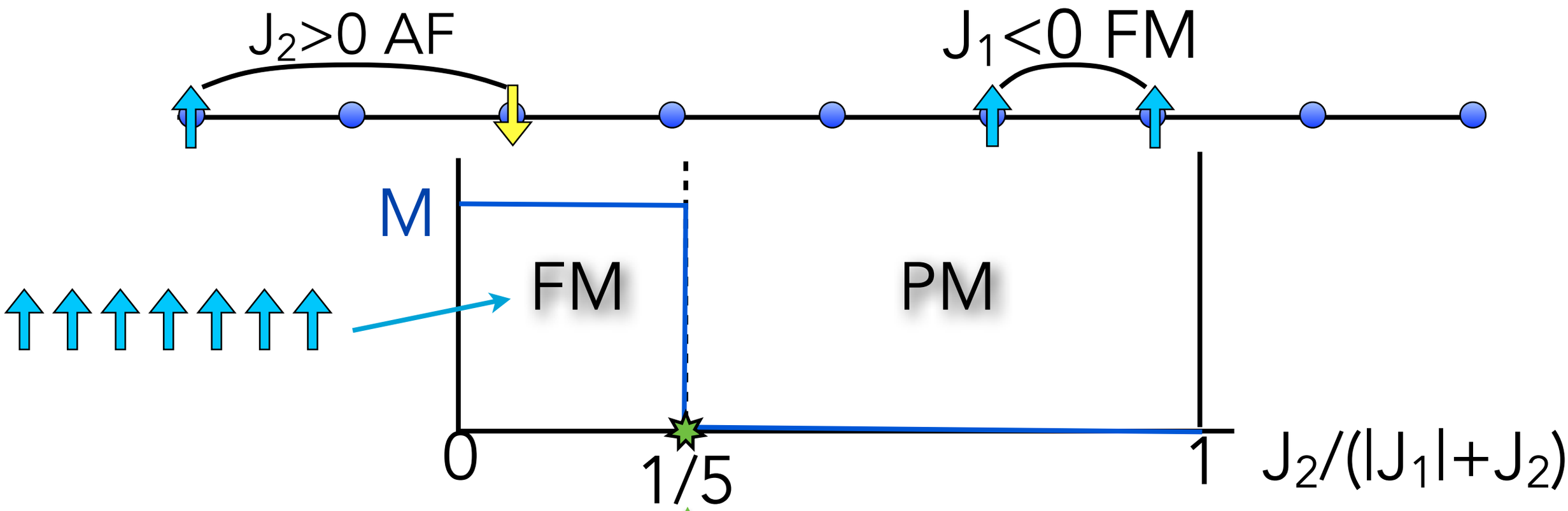


$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

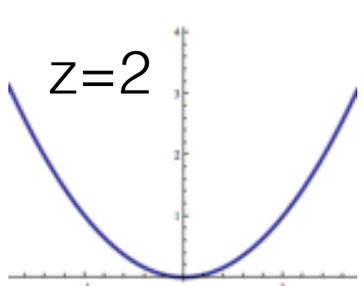


Frustrated ferromagnet

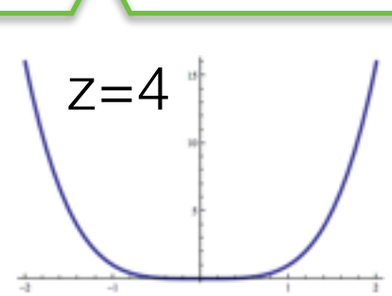
1d S=1/2 chain $$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



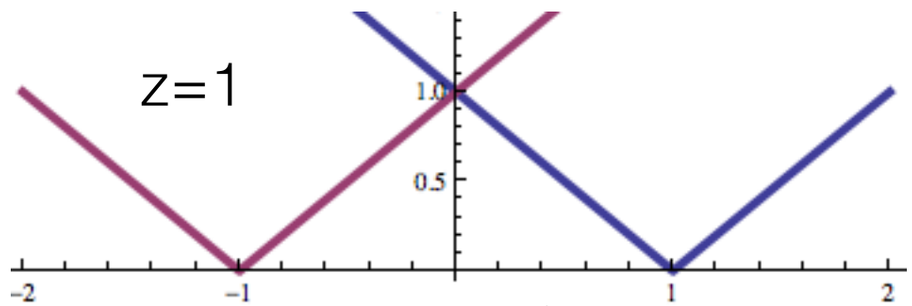
(spin-wave) dispersion



$\omega \sim k^2$



$\omega \sim k^4$



$\omega \sim (4J_2 - |J_1|)^{3/2} |k - K_i|$

Lifshitz Point

Balents, Starykh arXiv:1510.07640

- Unusual QCP: order-to-order transition
- Effective action - NLσM for unit vector **m**

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$$\mathcal{A}_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}$$

Berry phase term

tunes QCP

$\delta \propto |J_1| - 4J_2$

two symmetry allowed interactions at $O(q^4)$

All properties near Lifshitz point obey "one parameter universality" dependent upon u/K ratio

Lifshitz Point

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau$$

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Large parameter:
saddle point!

$$v = \frac{u}{K} \qquad \overline{h} = \frac{hK}{\delta^2}$$

Lifshitz point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

v derives from quantum fluctuations

Need it be positive?

$$\hat{m} \cdot \hat{m} = 1 \quad \Rightarrow \quad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}|$$

Theory is stable for v>-1

In fact, v<0

- Semiclassical large s limit: $v \sim -3/2s$

Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

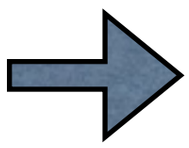
$$\hat{m} = \begin{pmatrix} |\Psi|\cos(qx+\phi) \\ \pm|\Psi|\sin(qx+\phi) \\ \sqrt{1-|\Psi|^2} \end{pmatrix}$$



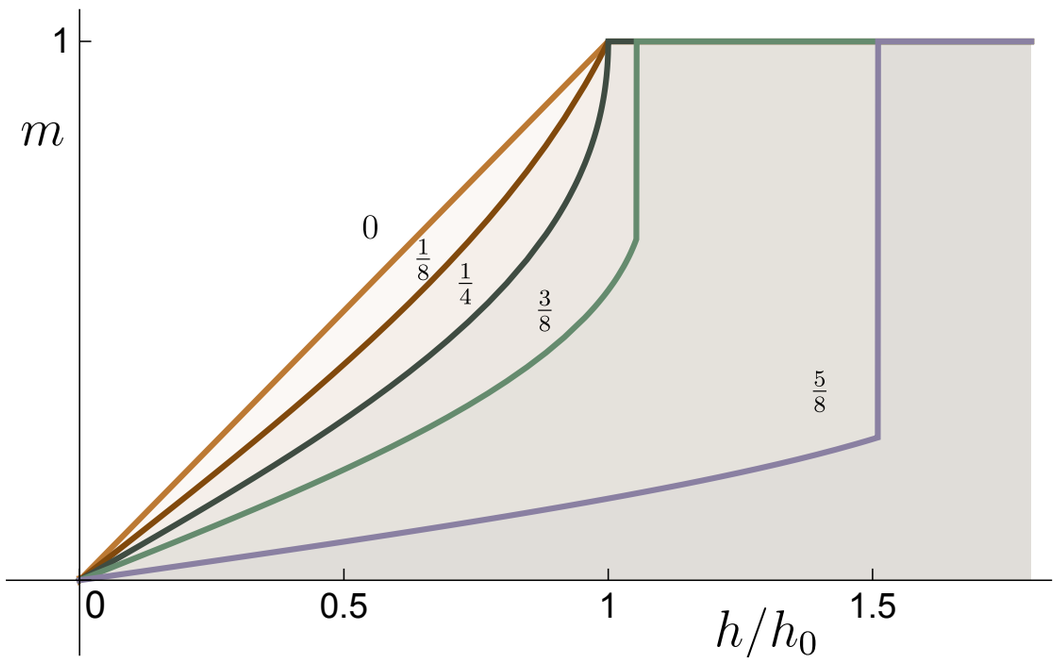
cone state

Obtain

q, Ψ versus h, v, δ



many physical quantities



Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

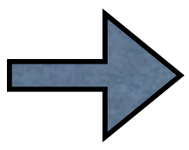
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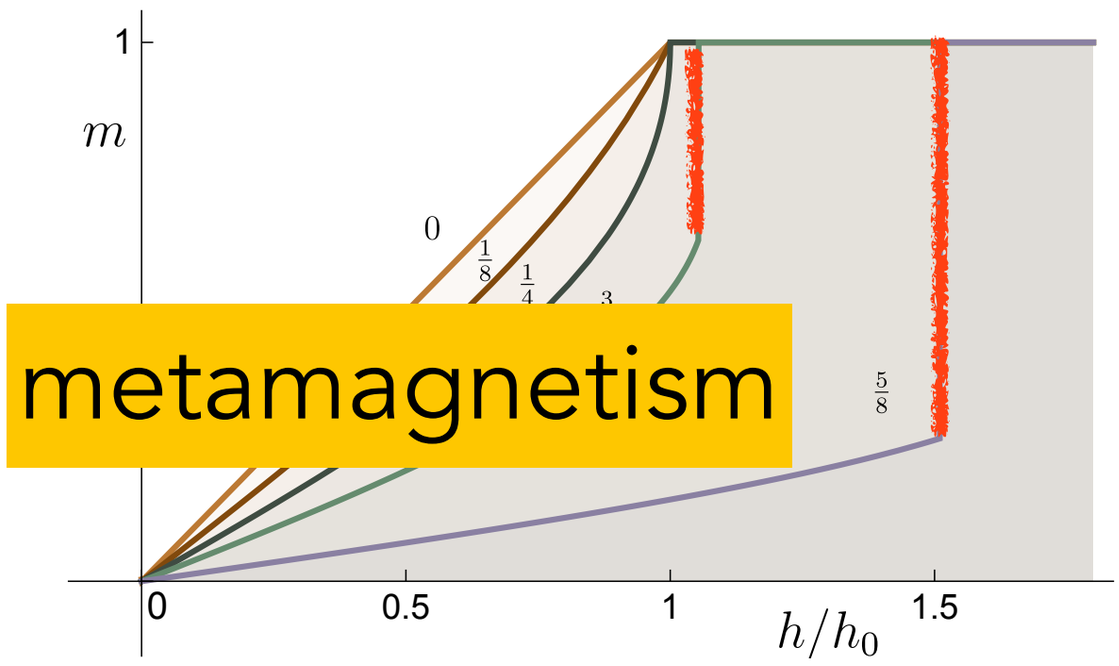
cone state

Obtain

q, Ψ versus h, v, δ

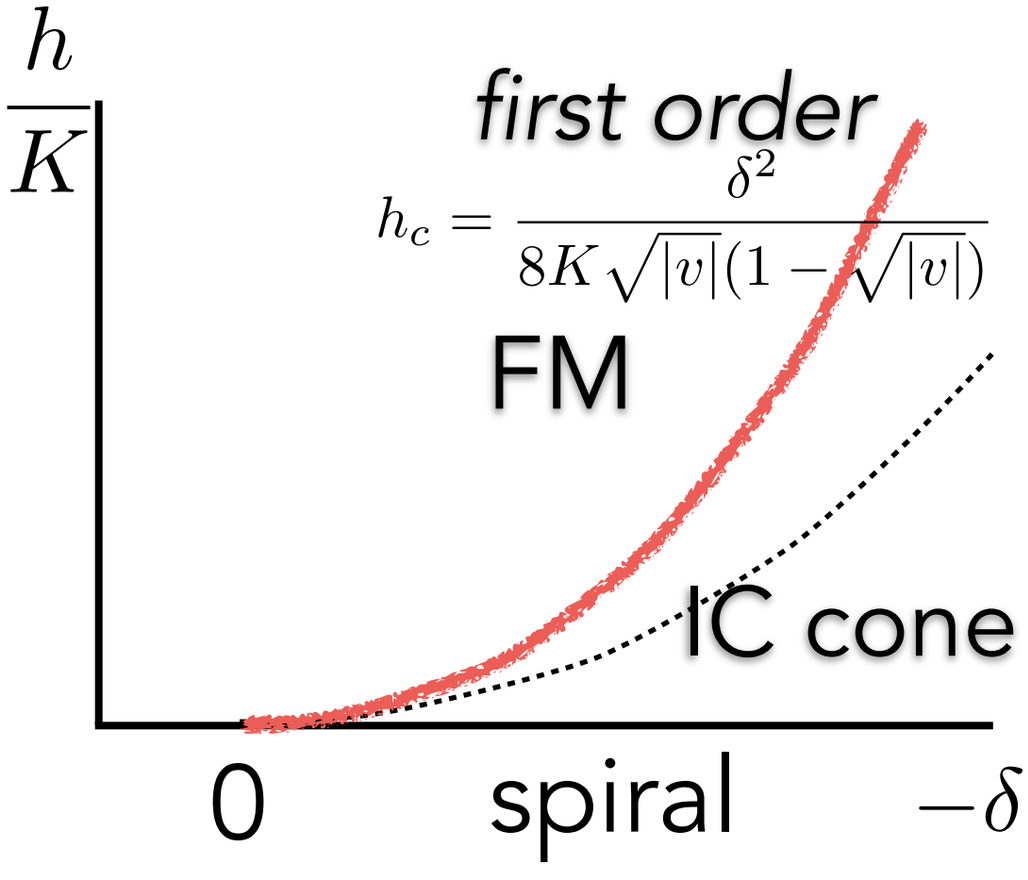


many physical quantities

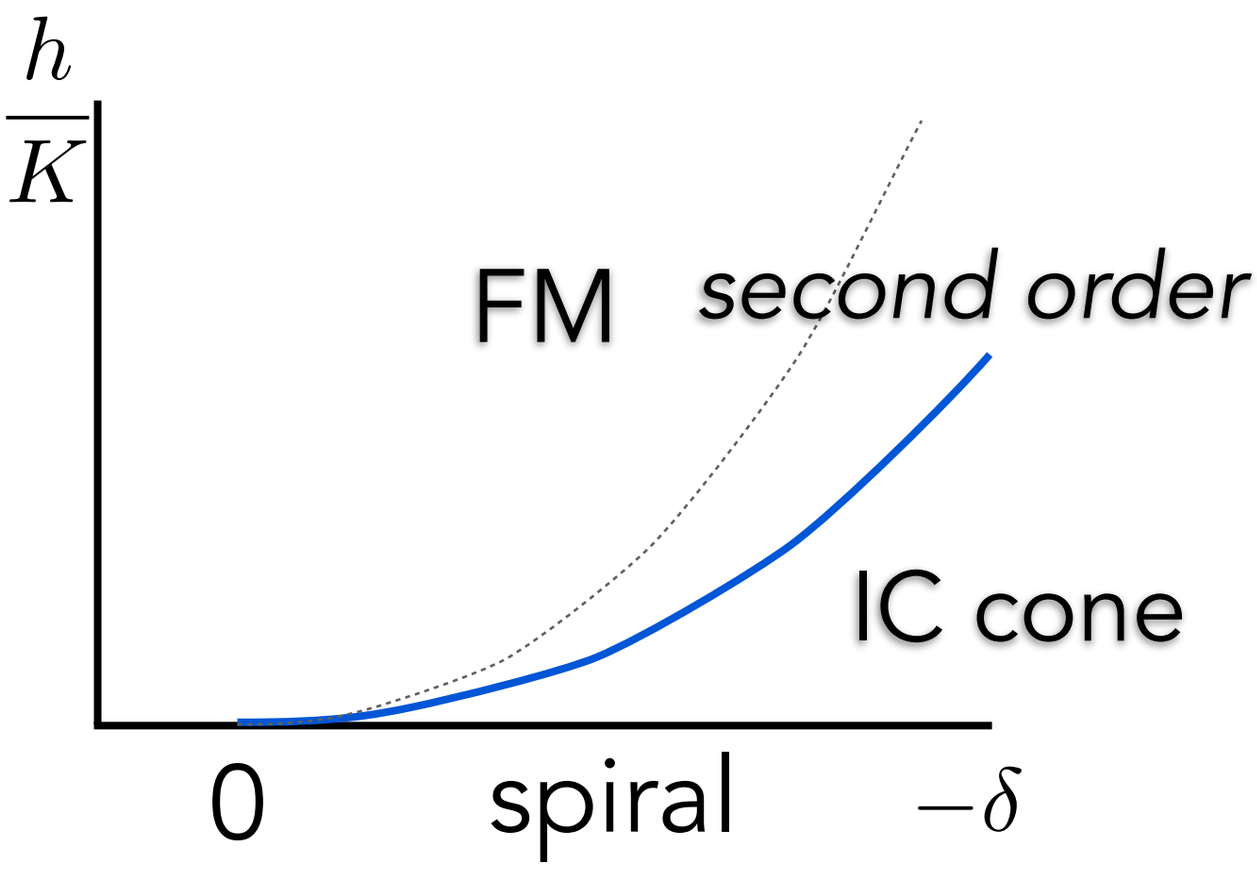


Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$



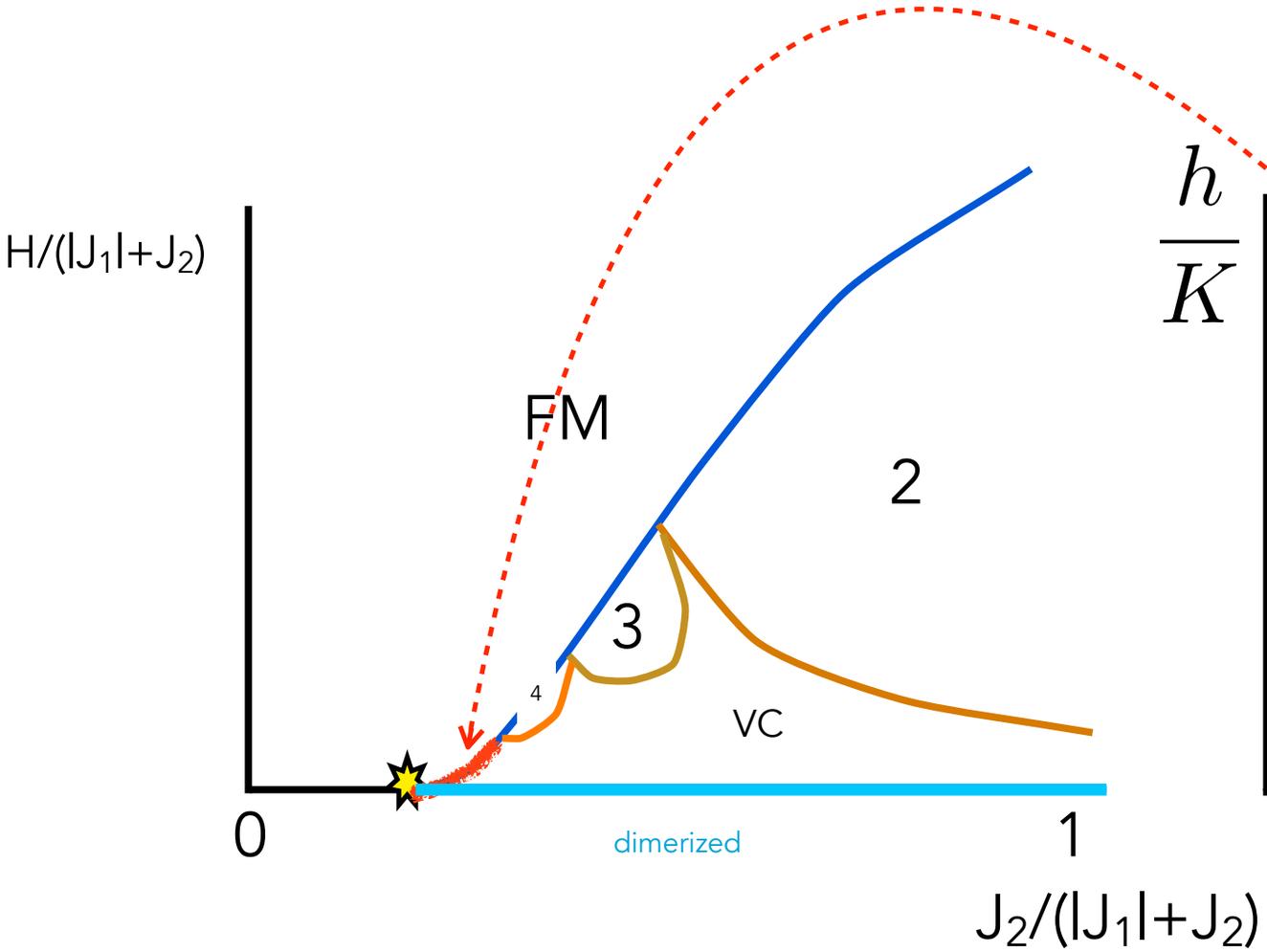
$-1 < v < -1/4$



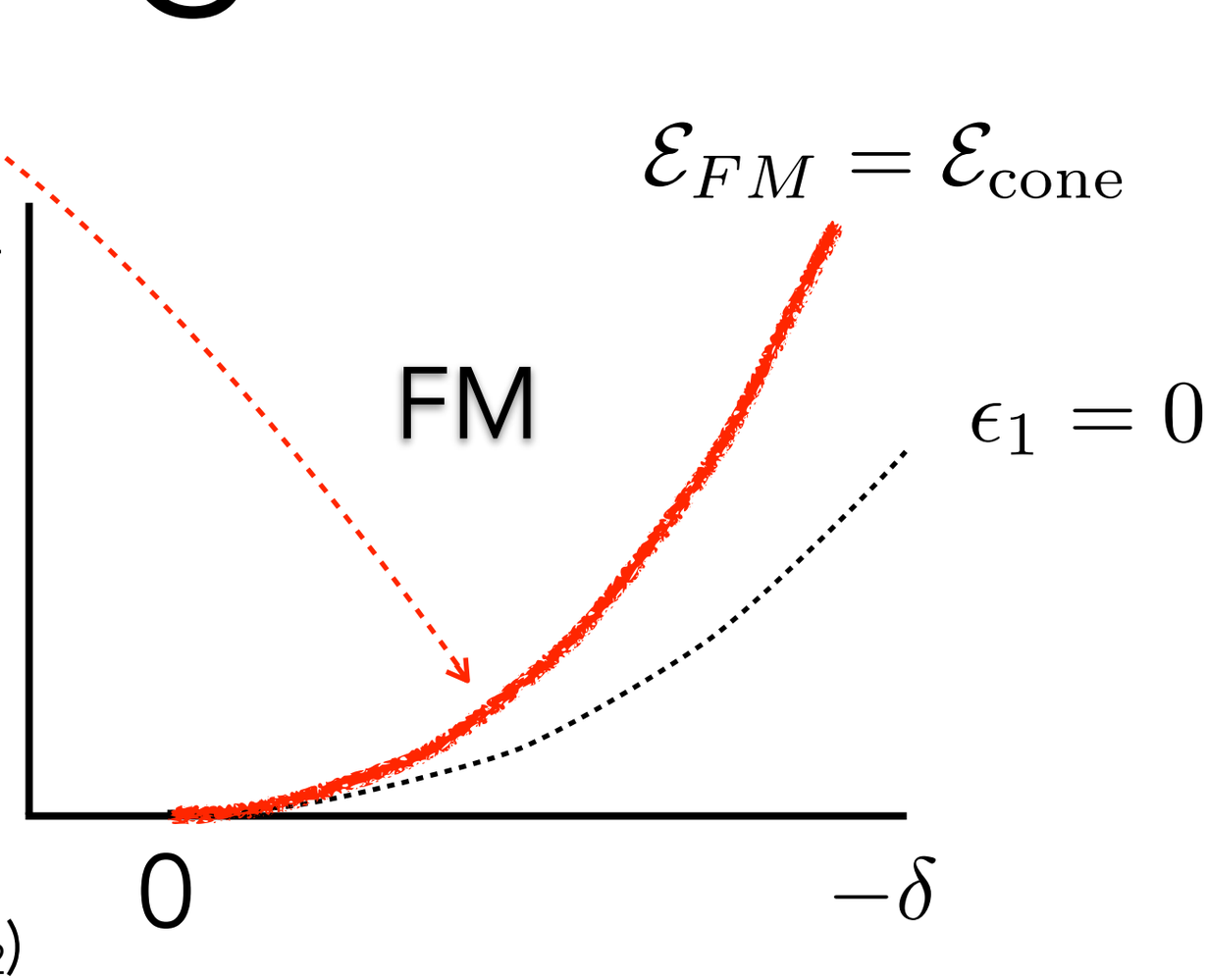
$v > -1/4$

N.B.: at saddle point level there is no scale for **δ**

Missing?



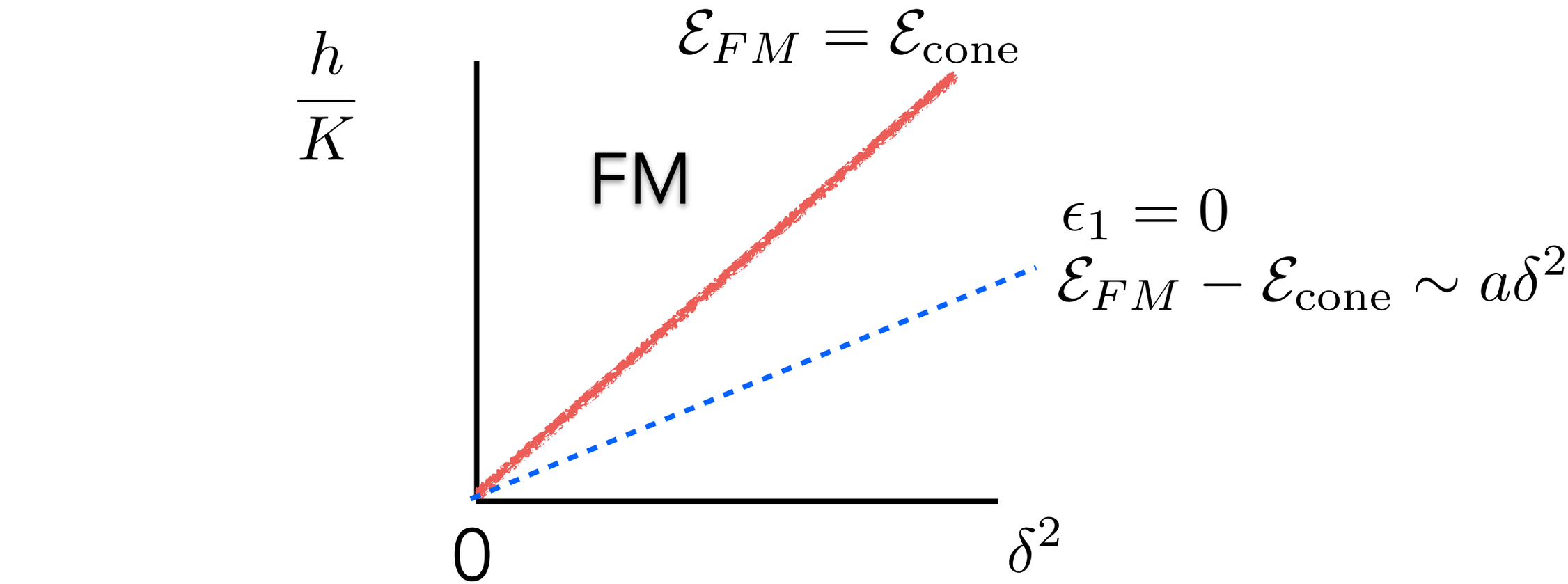
J_1-J_2



saddle point

misses metamagnetic endpoint
and multipolar phases

Metamagnetic endpoint?



Quantum corrections penalize E_{cone} but not E_{FM}

$$\sim \mathcal{E} \times \sqrt{\delta/K}$$

$$\Delta\mathcal{E}_{\text{cone}} = +f(v)\delta^{5/2}$$

Quantum corrections

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

transformation to rotating frame

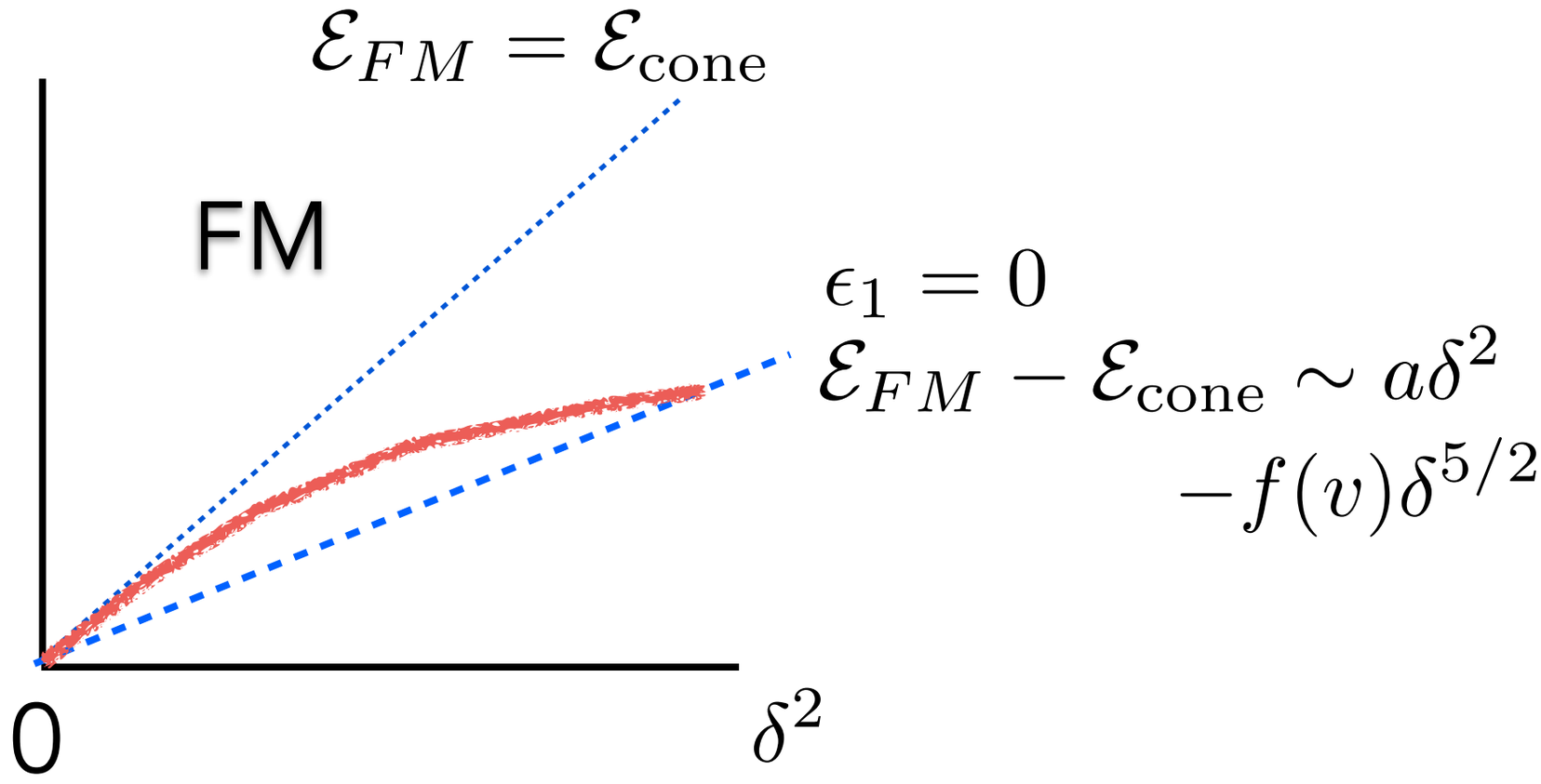
$$\hat{m} = \sqrt{2 - \frac{\bar{\eta}\eta}{s}}[\frac{\bar{\eta} + \eta}{2\sqrt{s}}\hat{e}_1 + i\frac{\bar{\eta} - \eta}{2\sqrt{s}}\hat{e}_2] + (1 - \frac{\bar{\eta}\eta}{s})\hat{e}_3,$$

effective Bogoliubov Hamiltonian

$$S = S_{\rm sp} + \int dx\, d\tau\ \{\overline{\eta}\partial_\tau \eta + H(\overline{\eta}, \eta)\} \quad + O(\eta^3)$$

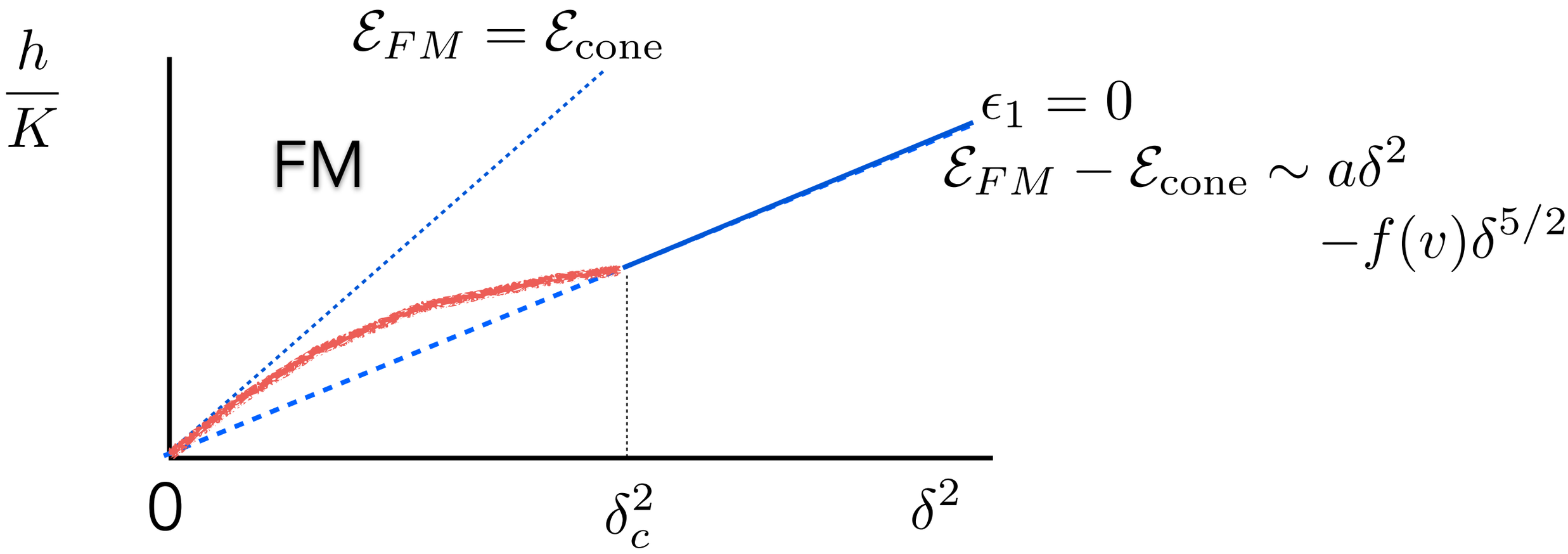
diagonalization gives correction to GS energy

Metamagnetic endpoint?



Corrected first order curve bends slightly downward to intersect second order line

Metamagnetic endpoint?

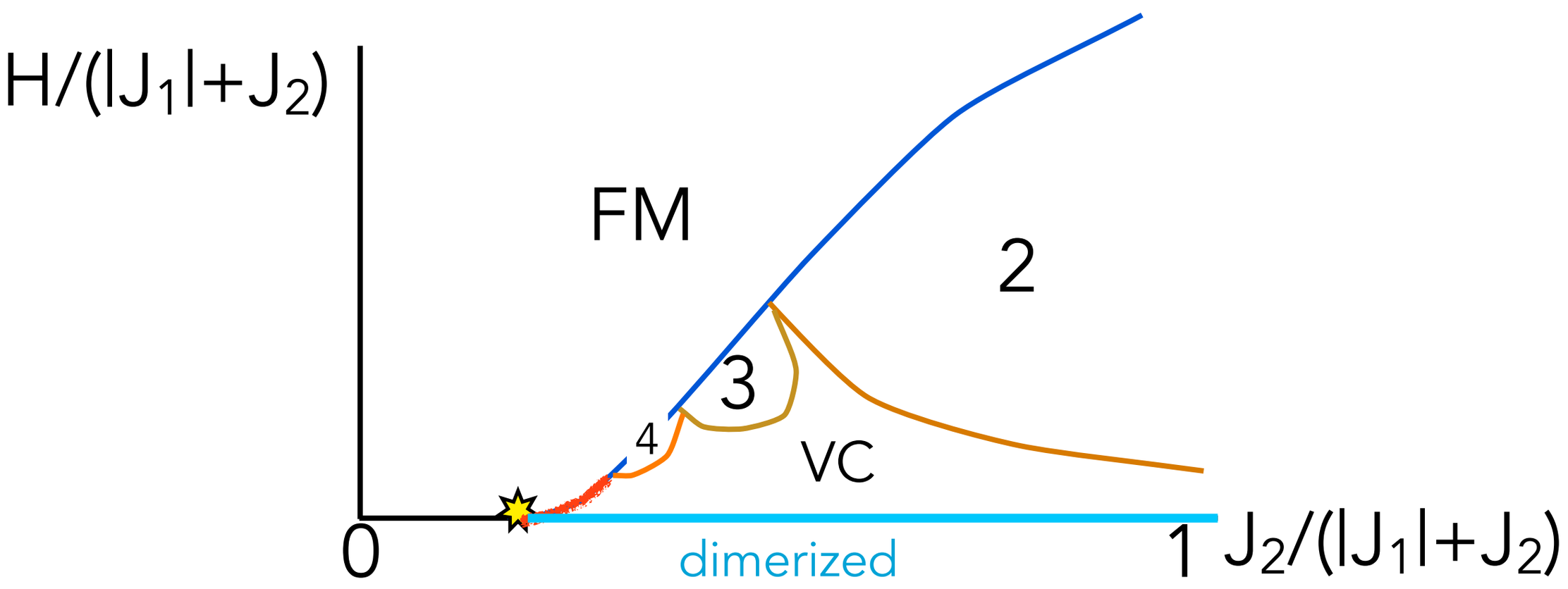


Control? $v = -1/4 - \varepsilon$

$$\mathcal{E}_{FM} - \mathcal{E}_{\rm cone}\big|_{\epsilon_1=0} \sim \varepsilon^3\delta^2 - \varepsilon^2\delta^{5/2}$$

$$\delta_c \sim \varepsilon^2 \ll 1$$

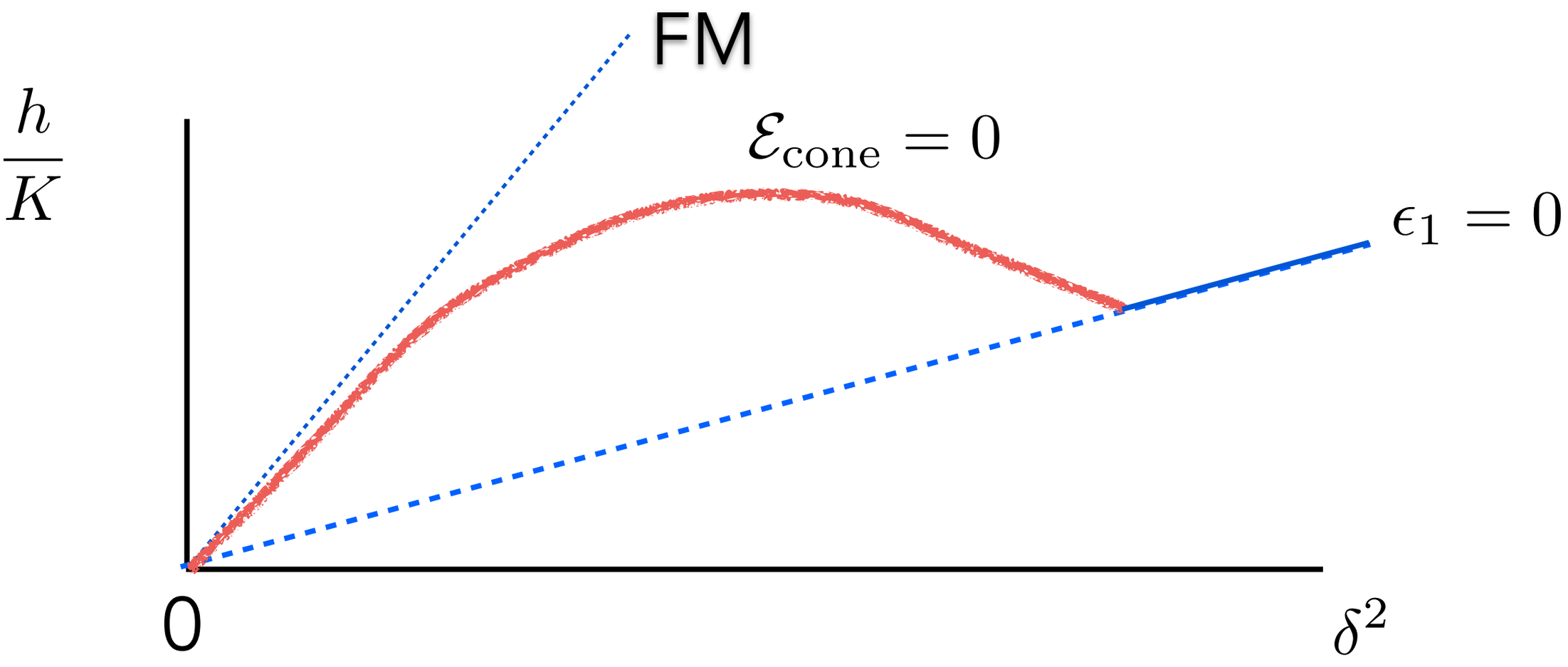
Multipolar phases



Still need to understand multipolar phases!

Instabilities

- Choose $E_{FM}=0$



What about multi-particle instabilities?

Low density limit

$$\hat{m}^x + i\hat{m}^y = \left(2 - \overline{\psi}\psi\right)^{1/2} \psi \qquad\qquad \hat{m}^z = 1 - \overline{\psi}\psi$$

Low energy $\psi \sim \psi_1 e^{iqx} + \psi_2 e^{-iqx}$

$$\mathcal{L} \sim \overline{\psi}_a(\partial_\tau + h - \frac{\delta^2}{2K} - 4\delta\partial_x^2)\psi_a$$

$$+\gamma_1[(\overline{\psi}_1\psi_1)^2 + (\overline{\psi}_2\psi_2)^2] + \gamma_2\overline{\psi}_1\psi_1\overline{\psi}_2\psi_2$$

$$\gamma_1 = \frac{\delta^2}{4K}(1+4v) \qquad\qquad \gamma_2 = \frac{\delta^2}{K}(5+4v)$$

$$\sim -\varepsilon\delta^2 < 0 \qquad\qquad \sim +\delta^2$$

Low density limit

$$H = -4\delta \sum_i \frac{\partial^2}{\partial x_i^2} + 2\gamma_1 \sum_{i<j} \delta(x_i - x_j)$$

$$\gamma_1 \sim -\varepsilon\delta^2 < 0$$

attractive delta-function gas!

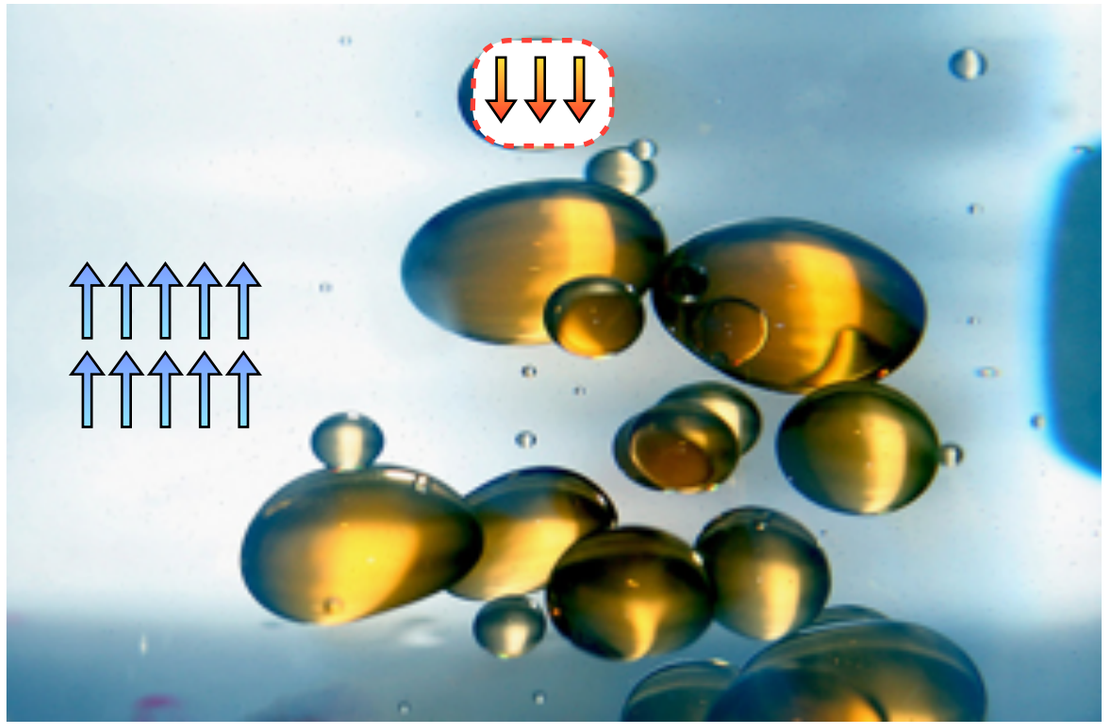
$$\epsilon_n = \epsilon_b \frac{n(n^2-1)}{6} \qquad \epsilon_b = -\frac{\gamma_1^2}{8\delta} = -\frac{\varepsilon^2\delta^3}{8}$$

collapse: bound states have size

$$\ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon\delta}$$

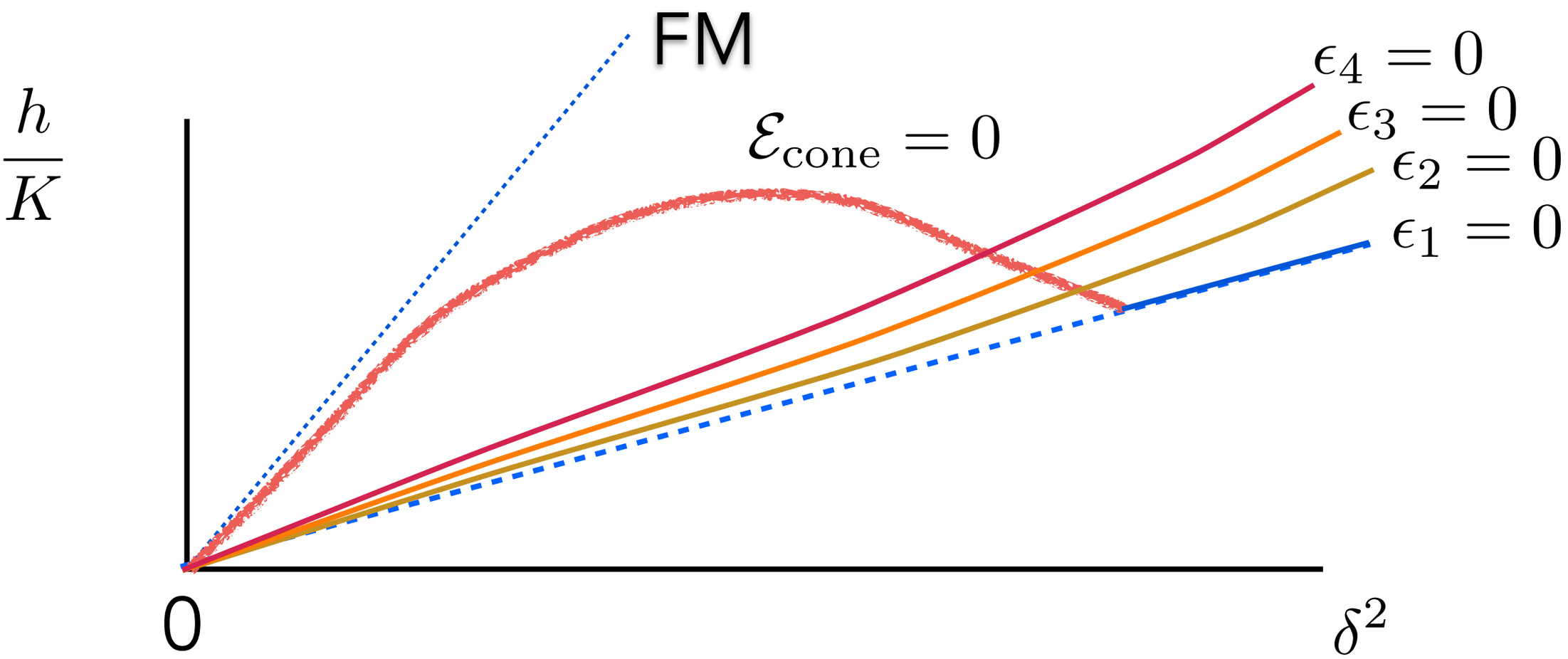
Why bound states?

- Metamagnetism = first order transition
  - Metastable bubbles of one phase inside the other
- Bubble = multi-magnon bound state



Instabilities

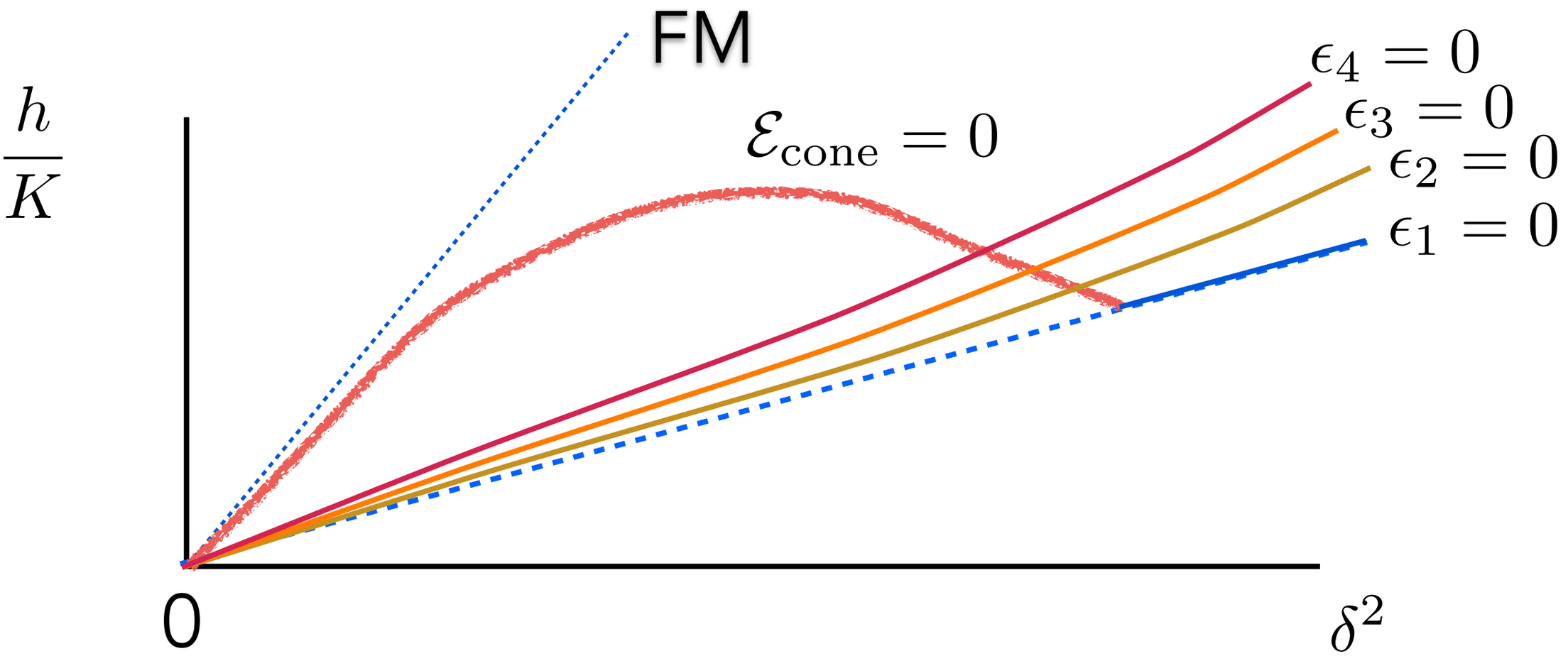
- Choose $E_{FM}=0$



At low density level it appears higher bound state instabilities dominate

Instabilities

- Choose $E_{FM}=0$



But the bound states cannot get arbitrarily deep - low density approximation is violated

A guess

- n magnon bound state

$$\ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon\delta}$$

- Matching to de Broglie wave length

$$\ell_0 \sim \frac{1}{q_0} \sim \frac{1}{\sqrt{\delta}}$$

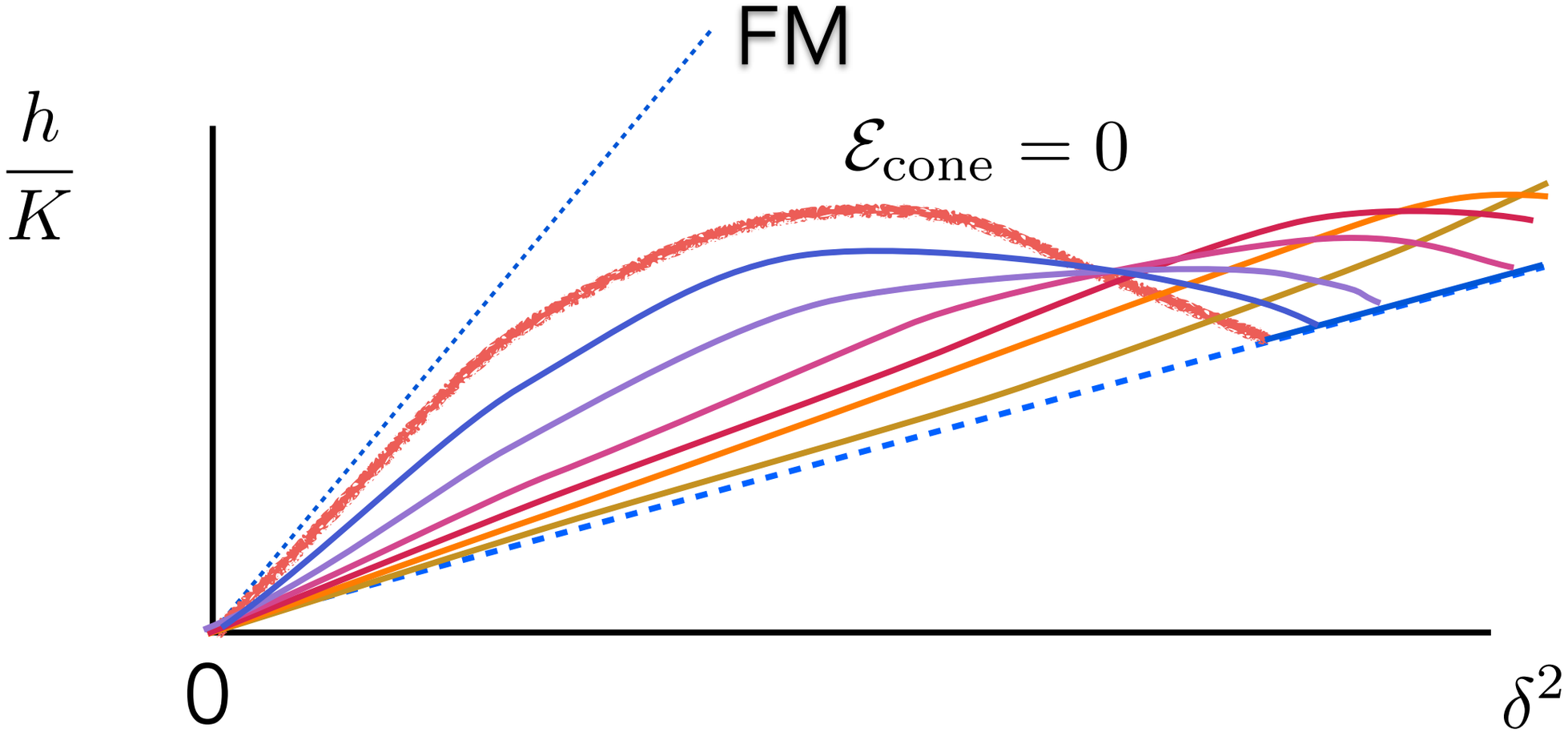
- Suggests maximum bound state

$$n_{\max} \sim \frac{1}{\epsilon\sqrt{\delta}} \sim \frac{1}{\epsilon^2}$$

(at this scale, 3-body interactions enter)

Instabilities

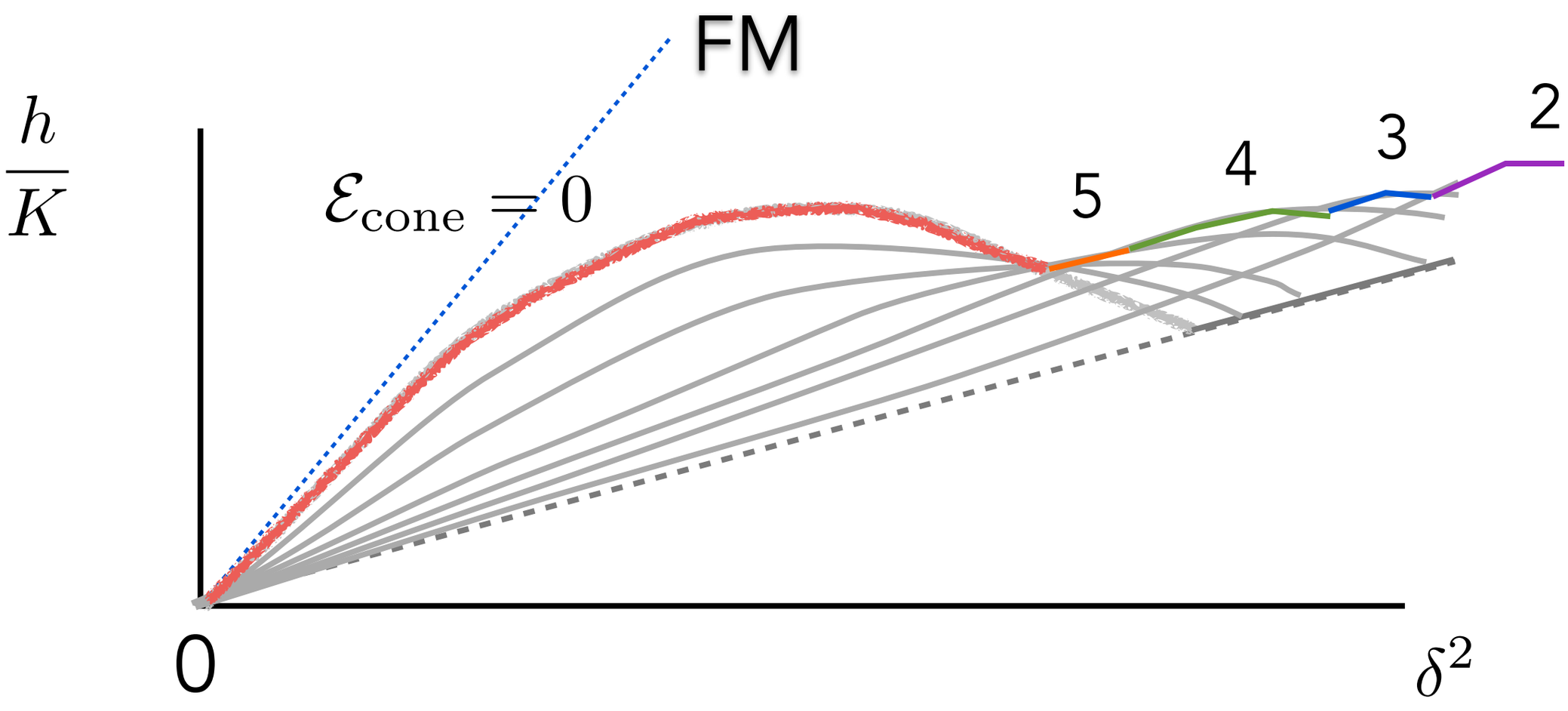
- Choose $E_{FM}=0$



Expect that n-boson bound states bend with increasing n to approach continuum line

Instabilities

- Choose $E_{FM}=0$



Expect that n-boson bound states bend with increasing n to approach continuum line

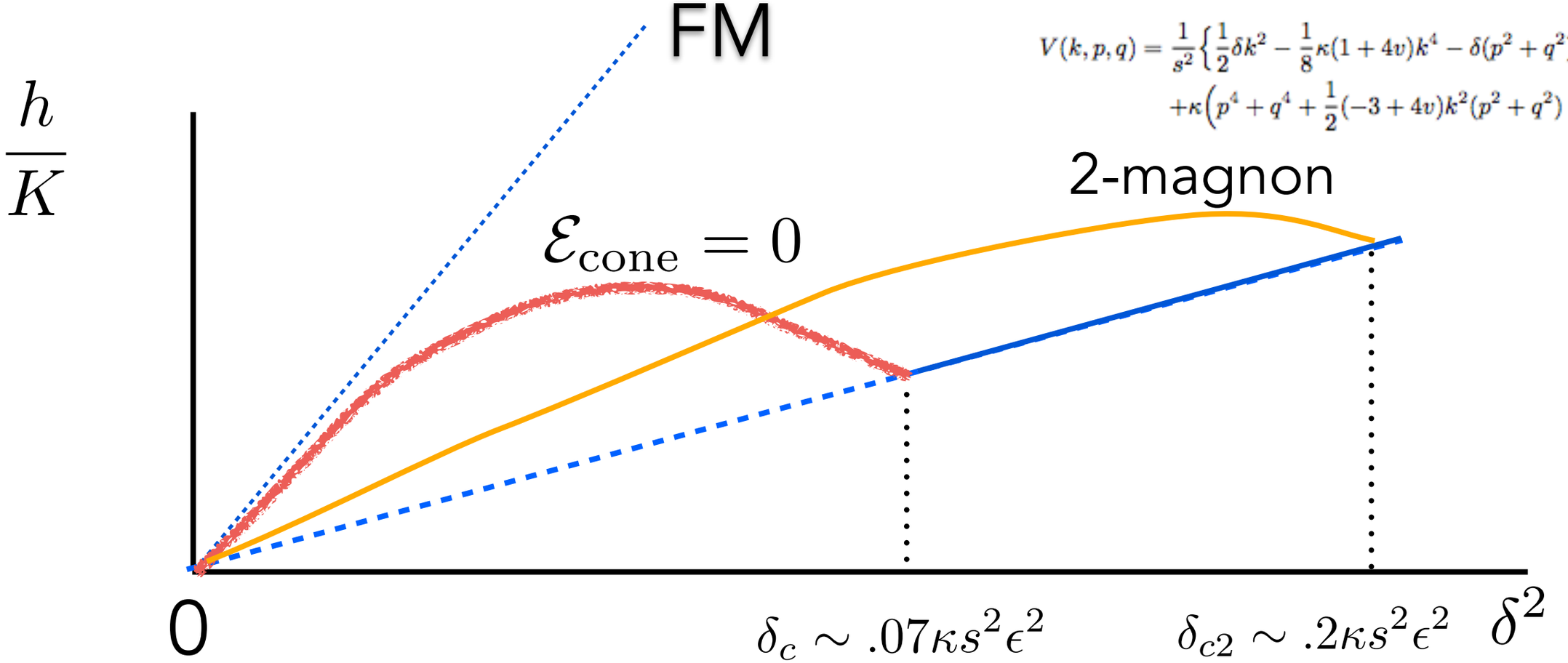
2-magnon check of the proposed scenario

- Compute *exact* 2-magnon energy in QFT

$$H = \sum_k \epsilon_k \bar{\eta}_k \eta_k + \frac{1}{2L} \sum_{kpp'} V(k,p,p') \bar{\eta}_{k/2+p} \bar{\eta}_{k/2-p} \eta_{k/2-p'} \eta_{k/2+p'},$$

$$\epsilon_k = (h + 2\kappa k^4 - 2\delta k^2)/s$$

$$V(k,p,q) = \frac{1}{s^2}\Big\{\frac{1}{2}\delta k^2 - \frac{1}{8}\kappa(1+4v)k^4 - \delta(p^2+q^2) + \\ +\kappa\Big(p^4+q^4+\frac{1}{2}(-3+4v)k^2(p^2+q^2)+4(3-2v)p^2q^2\Big)\Big\}$$



Separation of metamagnetism and multipole formation

d>1

$$S = \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

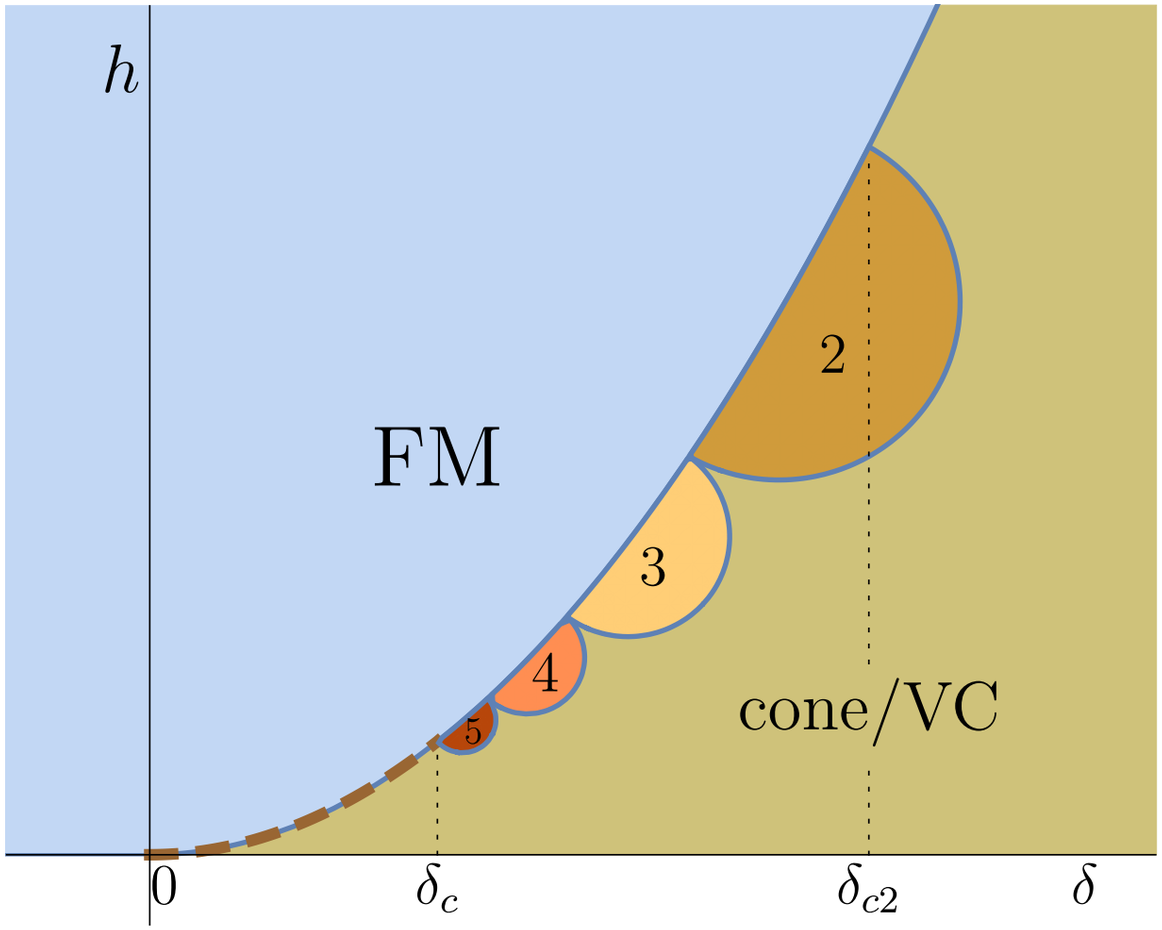
- Rescaling:

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau \qquad y \to \frac{\sqrt{cK}}{\delta} y$$

$$S = \frac{\sqrt{K^d C^{d-1}}}{\delta^{d-1/2}} \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

∴ Similar theory applies in d>1, and very similar conclusions apply

Summary



Lifshitz point is a "parent" of many phases

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

Other frustrated ferromagnets

- In 1+1d, we could figure out (nearly) everything by numerically exact methods (DMRG) - not clear we need QFT
- But in d>1, we have fewer tools but plenty of experiments
  - Hope that QFT is more versatile technique to study those situations
  - Asymptotic solubility property is very general and holds for d>1.

Summary

✓ Nematic physics of frustrated ferromagnets

✓ Inter-chain driven competition between 2d nematic and 2d SDW orders.
Nematic remains to be seen.

✓ Explored Lifshitz point as a "parent" for multipolar states and metamagnetism